Chapter 1

Understanding

Table of Contents

Mathematics is in 1
Our goal....2
What we have to understand....2
As wise as we need to be 3
One busy teacher says 3
The lessons in this book....3
Assessment....4

Mathematics is in...

Mathematics is in the estimates we make for when we should leave home to pick up a friend at the airport or deliver a child to a Little League game. It is in the calculations we make when we wish to double a recipe. It is in the fitting of suitcases into the trunk of our car as the vacation begins. It is in paying monthly bills, calculating car mileage, making a dress, putting a model together, making and flying a kite. It is in bowling or playing miniature golf or shooting pool or playing cards. It is in calculating the restaurant tip or knowing how long the food in the pantry will last. It is in planting seeds or trimming hedges. It is in the jigsaw puzzle on a rainy Sunday afternoon.

Mathematics is in the bets we make with our lives. Drive or fly? Buckle up? Wear a helmet? Smoke or quit? Drink or abstain? Butter or margarine? Sugar or Nutrasweet? Environment or heredity?

Mathematics is in our lives as teachers. Projected enrollments, projected revenues. Budget shortfalls, district policies. Children, working at stations or seated in rows. Art projects, science projects, musical gatherings, cooking together. Planning and doing and planning again.

Mathematics is in our businesses. Projected income, inventory on hand. Balance sheets, financial statements. Are we growing? Are we shrinking? Designing, creating. Estimating, doing. Shape, size, quantity, value. Patterns from the past, plans for the future.

Mathematics is in our sports and recreation. Shapes of fields, lengths of pools. Speed of the roller coaster, cost to build Space Mountain. Salaries of players, cost of a ticket. Averages, percents, ratios and numbers. Standings and winnings and emotional ties. Camping out. Finding birds to watch. Capturing the sunset in our mind. Fibonacci sequences from the branches of a tree. Patterns of the zebra stripes at the zoo.

Mathematics is in our medicine. Genetic engineering, computerized images. Body temperature, the pressure of our blood. Orthoscopic surgery, laser scalpels. Artificial hearts, ball bearing hip joints. New methods, new techniques. New costs, new wonders. New concerns. When does life begin? When does it end? Math gives us knowledge. Does it give us enough?

Mathematics is in our engineering and our architecture. The structures that surround us: homes, schools, office buildings, concert halls, sports arenas. The machines that transport us: cars, buses, trucks, planes, trains, ships, roads, runways, railways. The devices that connect us: telephones, faxes, television, radio, radar, sonar, fiber optics, satellites, microwaves. The instruments that serve us: calculators, copy machines, computers, dishwashers, garbage disposals, ovens, vacuums, traffic lights, record players, video recorders. Mathematics is metals, plastics, microchips, microwaves, stress analysis, streamlining. If we created it, designed it, invented it, analyzed it, harnessed it, unleashed it, or changed it, we did it with math.

Mathematics is in our cities, states and provinces. Street maps, highway maps, traffic flow. Sewage treatment, garbage collection, water supply. Services provided, services missing. Air-traffic control, noise abatement. Parks for children, parks for adults. Swings, slides, rides, flowers, trees, paths. Grass to run on, grass to play on, grass to sleep on. Community planning, community feeling, community will. Nothing by accident, everything by plan.

Mathematics is in our ecology. It is in the dynamics of an oil spill. The winds, the currents, the gallons of oil, the assessment of damage, the shape of the slick. It is in the pages of an environmental impact study. Modeling, simulating, estimating, guessing, deciding our fate. What happens to the air or the water when we change this or change that? How can we know? Who can we trust?

Mathematics is in our weather. Rainy? Cloudy? Sunny? Overcast? A day at the beach, or a party indoors? A good year for crops or famine for all? Where will the hurricane come ashore or the tornado touch down? When will the drought end or the flood waters recede? Our satellites see, but what do they know?

Mathematics is in our news. Graphs, percents, averages, polls. Trends, forecasts, statistical significance. Opinions formed, opinions swayed. Science news, social news, sports news. Local news, foreign news, political news. Making sense out of sense. Making sense out of nonsense. Mathematics is the information source for an informed citizenry.

Mathematics is in our government. Leading economic indicators, gross national product. Inflation index, unemployment index. Forecasts of prosperity, forecasts of recession. Should we tax more or less? Billions of dollars, trillions of dollars. How much debt is a debt we can afford?

Mathematics is in the sky. Paths of planets. Distances from earth to moon. The effect of dark spots on the sun. Spaceship launches and landings. Numbers of stars, numbers of galaxies. How long since the Big Bang? Is there gravity in space, or is it all just relative? Will we be traveling to Mars?

Mathematics is in the lives of our children. Counting, comparing, sorting, classifying, sharing, drawing, throwing, kicking, running, hopping, skipping, jumping, keeping score, cooking, eating, playing, working, building, spending, saving, thinking, reasoning, planning. Measuring space, distance, volume, area, growth, minutes, hours, days, years, ages, the speed of sound, the speed of light. Naming shapes, using words. Triangles, circles, squares. More than, less than, about the same. If, then, or, else, both, neither, either, and the same. Mathematics is our children's tool for knowing and for knowing they can know.

Mathematics is in all that we see, all that we do, and all that we are. Art and science. Space and earth. Play and work. Numbers or shapes. Solo or in concert. Inconceivably large or infinitesimally small. Past, present, future. It is how we make sense out of all that there is.

Our goal...

Our goal for ourselves is to have the mathematics that we teach be the mathematics in our students' lives. Our goal for our students is that they understand and use the mathematics that we teach.

When we were students in elementary school, we studied arithmetic and called what we learned mathematics. Since our own education focused only on arithmetic, it can be difficult for us to grasp the full implication of what it means to study mathematics. Arithmetic is computation. Mathematics is a way of thinking. When we were in school, mathematics was what we memorized to pass the test. In our lives, mathematics is what we use to broaden our understanding of the world in which we live.

We ask our students to accept the unknown as an invitation to explore, discover and invent. So, too, we ask ourselves to risk not knowing the answers to every question asked, as we accept the challenge of creating a learning environment in our classrooms as exciting and as challenging as life itself.

The view of mathematics we received in school was of rules and formulas taught as absolutes. We were taught mathematics as certainties and clear right answers, even though real mathematics is fraught with uncertainties and approximations. Whereas school math focuses on numbers, real math focuses on the search for patterns that help us bring order to the chaos in our lives.

The mathematics that we teach in school focuses not on formulas and calculations, but on the search for patterns and their interconnections. We teach the skills of thinking, reasoning and problem solving as the most basic skills of all. We teach these skills not just to produce answers to problems about numbers, but to produce answers to problems in life. Through mathematics, we give all of our students the learning power that comes from knowing they can know.

What we have to understand...

What do we have to understand before we understand enough to teach? Do we have to understand the workings of a child's mind? For days a child may come to school not grasping the point we are trying to make. Then, one day, the same child comes to school and shows us that she understands. Must we know exactly what goes on inside the child's mind that changes confusion into learning before we understand enough to teach?

Learning happens naturally. Would the learning happen whether we were there or not? Would a child learn to speak if no one was around to speak words to the child? Would the child learn to walk if no

one was there to extend a helping hand? Learning happens naturally, but learning does not happen without help.

We have to understand enough to help.

Learning comes from the environment of the child. If English is the language spoken, English is the language learned. If reading is a part of the child's every day experience, the child learns to read. If the mother and the father share their love for cooking or for camping out, their children will be cooks or campers. The environment that surrounds the child determines the learning taking place.

We have to understand enough to set up an environment that permits the learning to take place.

How much mathematics do we have to understand? We may have felt that we were terrible at math when we were children learning math in school, but mathematics is a natural part of all our lives. We already understand enough mathematics to create the environment that our students need. We already understand enough to help a child.

As wise as we need to be...

Textbooks and workbooks are reassuring. They tell us what lesson we should teach tomorrow and the day after that and the day after that as well. We never have to wonder what comes next. Our wondering has been done for us. All we have to do is turn the page.

We may not think that we are as wise as textbooks appear to be. We may not think that we always know every skill in math the textbooks imply that we should teach. We may not think we always know when it is time to move from teaching one concept to teaching what comes next. We may not think we are as wise as textbooks seem to be, but we are as wise as we need to be.

No book can know our students for us. No book can tell us what is right to teach on the second Tuesday of month three. Only we can know our students. Only we can decide what to teach tomorrow once we know the learning that has taken place today. We know the answer for what to teach tomorrow is not waiting at the turning of a page. We know there is more to teaching than being reassured.

One busy teacher says...

One busy teacher says:

> I need to use the lessons from the textbooks and the workbooks for my students. I have no time to plan so many different lessons for each day. There are so many different subjects that I teach.

Would a busy mother say:

> I cannot teach my baby to talk today. I do not have the time to make a list of words for her to learn. There are too many diapers to change. There is too much shopping to be done. There are too many different meals to prepare. My house needs cleaning endlessly. If you could write the list of words out for me, I might have time to teach an hour's talking from nine o'clock to ten o'clock today.

The busy mother teaches talking all day long and gives no thought to the teaching taking place. The busy mother knows that learning is not a lesson to prepare. Learning does not have to come from books. Learning does not need a script. Learning comes from living life. Teaching is a frame of mind.

School learning is no more complicated for a child than the learning that the child does at home. A child's learning how to write her name, or how to add two twos, or how to spell *cat*, is no harder than learning how to talk, or how to dress, or how to eat with spoon or fork.

A mother can teach a child to talk without a daily lesson plan. As teachers, we help our students learn as naturally and as well.

The lessons in this book...

The lessons in this book follow two parallel definitions of the word:

Lesson means a period of instruction. The period of instruction is the time it takes the lesson to be learned.
Lesson means something learned. The lesson is not over until the learning has occurred.

The lessons in this book are taught until the lesson has been understood by everyone, without exception. Teaching children is more important than covering every skill on a list of skills. It is better to teach a little clearly than to teach a lot confused.

The following passage is taken from the *Baratta-Lorton Reading Program Teacher's Manual*, page 368 (Center for Innovation in Education, 1985):

> How quickly should my students proceed through this program? What is the calendar for the year?
>
> These questions are always to be asked about any school program. But for this program, no timeline is provided. The activities that this manual has described are meant to be a flow of events, the pacing of which changes with every group of children and each different teacher. If the activities described in this manual can be said to represent a range of events from A to Z, then what is important is to proceed from A to B well and then to proceed equally well from B to C. There is no rush to get to Z. Let Z take care of itself. Z is not the goal. Learning is the goal.
>
> If we are to err in the rate at which we introduce activities to our students, let us err in favor of allowing children too much time to learn rather than too little. The more time we give our students, the more time our students have to sort out the concepts in their own way. When we rush the child's learning, we are actually undermining that child's access to his or her own thinking.
>
> The most common observation made by people who have witnessed this program in action in the classroom is that the children quite visibly enjoy what they are doing. Part of this enjoyment exists because there is no fear of failure associated with the learning that is taking place because the learning is allowed to take place at each child's natural rate, with no pressure exerted on the child to move ahead too soon. If we are patient, learning [mathematics] can be as natural as learning to talk. Learning takes time. When there is time, it takes place.

Assessment...

When we teach a child to talk, every word the child speaks is an assessment of what the child knows. Rarely is there any need to focus on assessment as a separate event. The assessments that parents make are indistinguishable from their interactions with their child. If the child experiences any difficulties in learning to talk, many difficulties can be analyzed by listening to the child speak.

When we teach a child in school, everything we ask the child to do is an assessment of what the child knows. Rarely is there any need to focus on assessment as a separate event. The assessments that we make are indistinguishable from the lessons that we teach. If the child experiences any difficulties in learning, many difficulties can be analyzed by working with the child.

When we plan our lessons, we plan to teach the mathematics that people who use mathematics use. People who are big and people who are little, too. We teach the math that children use. We care that what we teach is understood. We care that what we teach is used. We care that our students can communicate what they know.

When we assess ourselves, we judge how useful the skills are that we teach. We judge how good we are at enabling everyone to understand. We judge how effectively our students have learned to communicate and share. We give ourselves a passing grade when we can say: Everyone has learned. No one has been left behind.

The *Curriculum and Evaluation Standards for School Mathematics* (National Council of Teachers of Mathematics, 1989) advocates the application of four standards to the content of a mathematics curriculum:

Mathematics as problem solving.
Mathematics as reasoning.
Mathematics as communication.
Mathematics as connections. (Connections means making linkages within mathematics and between mathematics and the real world.)

Our assessments of our students' mathematical abilities are based on these standards. We assess our students' ability to:

Solve problems.
Reason.
Communicate.
Make connections between what they have learned before and what they are learning now or are about to learn.

Our assessment of our own mathematical abilities is based on standards for ourselves. We assess our ability to:

See that mathematics is more than isolated problems on a page. Can we see that mathematics is a part of all our lives?
See that problems that we pose leads to other problems to be solved. Can we make connections in mathematics?
Let our thoughts become provoking. Can we see the possibilities?
Accept that mathematics does not have to mean just one right answer. Can we accept more points of view than one?
See the mathematics that is all around us all the time. Can we see the patterns everywhere?
As we do with language, integrate mathematics with all the other subjects in the day. Can we make connections outside of school as well?

Our students learn to solve problems, reason, communicate and make connections. We learn to create our own curriculum for math. For every lesson that we teach, we:

Decide the goal. What do we want our students to know?
Decide how to reach the goal. What activities use the skill we want known?
Identify what is important to assess. The assessment is included in the lesson that we teach.
Present the activities. Do our lessons use students' natural abilities to learn?
Measure to see if the goal has been reached. Do our students know what we want them to know?
Keep on teaching until the goal is reached. We leave no one behind.

Teaching and measuring are one and the same. As we teach, we measure. As we measure, we teach. If an activity assesses well what we want learned, the activity is the lesson taught and not a separate measure we devise. The means is also the measure of the means.