Chapter 3

Patterns and Connections

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Before We Begin

Something we see...

Day follows night. Will it always?
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A pattern is something we see or know that helps us to predict what we might see or know next. Patterns also help us know what may have come before what we see now. Patterns provide a framework for organizing information so that it becomes usable knowledge. Our advancement as a civilization can be measured in the patterns we have seen, described, and acted upon.

We have a science for biology, a science for chemistry, a science for weather and a science for medicine. We have a science for patterns as well. We call it mathematics. Mathematics is the science we use to assist us in discovering, recording, analyzing, describing and predicting from many of the patterns we see. It is also a science for acting upon.

Knowing we can know...

The human mind is a pattern-seeking device. From our earliest years, we search out patterns in the world that surrounds us so that we may bring order to chaos. We seek to discover relationships between seemingly unrelated bits of knowledge. We seek to make connections. We seek to understand. And as we seek, we find patterns.

The experiences children have had before they reach school have already had an impact on their beliefs in their own powers as learners. Some of our students come to us believing they can know. Children who know they can know are children who see patterns and make connections. Children who know they can know are children who learn.

First parent:

Parent and child are grocery shopping together. The experience is treated as a grand adventure. The child is riding in the shopping cart. There is a steady stream of dialog.

"You may pick out one kind of cookie for us to buy today. I will push the cart past all of the cookies while you look at the different kinds. Then I will push the cart back again so you can tell me which one you would like to choose.

"We need cereal today. No, we won't get that one. It has too much sugar in it. We will pick one that will be good for you. You may choose between the Cheerios and the Corn Flakes. Which do you want?

"I am going to pick out some avocados. I want ones that are not quite ripe yet, so they have to be hard. See if this is a hard one.

"What kinds of fruits should we buy for snacks? Shall we buy apples? Grapes? Bananas? Watermelon? Yes, we can buy carrots and celery, too, but they are vegetables, not fruits."

Second parent:

Child rides in the shopping cart. Little if any conversation takes place between parent and child. The parent looks weary. The child has been given a lollipop to suck on so that she will be quiet. The experience is not an adventure. It is a chore in a life filled with endless chores.

First parent:

Parent pulls child's hand away from reaching for the handle of the pot of boiling water on the stove.

"Don't touch. Hot! If you touch the handle, the pot might fall over and the hot water will spill out and burn you!"

Second parent:

Parent slaps the child's reaching hand.

"No!"

No further explanation of the slap or the "No" is given.

First child:

"Why is the sky blue?"

"Where did I come from?"

"How does the picture get inside our TV set?"

"What time is it on the moon?"

Endless whats, wheres, whys, hows and whos fill the child's waking hours. The parent does his or her best to answer questions for which the parent may not even have an answer. Patience is tested. Questions outlast answers, though answers are given endlessly.

Second child:

The questions start out the same, but no one provides the answers. The parent is busy. There are too many other responsibilities and too little time. Too many other children and too few resources. Too many questions and too few answers. The child learns not to ask. The mysteries remain mysteries.

First parent:

Parent sits with child in lap for the reading of a bedtime story. The parent points to the words as he reads. The child knows that the story the parent is telling is coming from the words on the page and not from the parent's imagination. The child has heard this story so many times before that she can provide correcting reminders if the parent deviates from the printed text.

As the story is being read, the parent and the child talk about the illustrations and the action in the story so far.

Second parent:

The child is not read to at all at home. The parent cares about the child's learning but does not know the connection between being read to and learning to read. No one read to the parent when the parent was a child.

All children learn. All people learn. But what do we learn? What effect do our differing backgrounds have on our ability to use our minds to make sense of our world? And what can we teachers do to counteract the negative effects some of these differing backgrounds may have had on the learning lives of our students?

In any interaction, our mind attempts to draw out patterns and make connections. It makes no difference if we are in a grocery store or we are being read to or we are building with a set of blocks in the play room. It makes no difference if we are seeking answers to questions we have asked or we are contemplating answers to questions asked of us. For all of us, the mind is a pattern-seeking device.

The differences in learning ability that we bring with us to school are not a reflection of our mind's desire to seek patterns. The differences are a reflection of the opportunities we have been given to understand what we see. The more opportunities we have to make sense out of things, the more we learn that we have the power to learn.

School has the power to multiply the understandings of the students who know they can know. School has the equal power to provide understanding for the students who have not yet learned to believe that they can learn. School has the power, but school does not always use the power that it has.

Those of us who were good at math in school learned to see the patterns and make the connections. We may have been shown these relationships by a teacher or a parent who understood the underlying structure of mathematics, or we may have come to these realizations on our own. Either way, math was easy for us because math made sense to us. We knew we could know.

Many of us who feel we are terrible at math were not given the opportunity to understand mathematics. We were only given rules and facts to memorize. Our success or failure in school math was based on our ability to remember what we had been told. If we memorized well, we were good students. Poor students were those of us who could not memorize. We did not understand. We did not see the patterns. We did not make the connections. We did not know we could know.

We now have the opportunity to allow our students to see what we may never have seen. Mathematics is simple and basic and straightforward. Patterns make it so.

To think mathematically...

We are used to thinking of mathematics as a curriculum of numbers and operations. Numbers for addition, subtraction, multiplication and division. Numbers for fractions, decimals and percents. But a numbers curriculum is an arithmetic curriculum, not a mathematics curriculum.

To think mathematically is to think analytically, not just about number, but about everything. When we reason mathematically, we try to make sense out of what we see. We try to connect each new experience to other facts we know. To think mathematically means asking, "What will happen if I do this?" and then finding out. It means asking, "What is going on here? What is the explanation?" and knowing there are explanations we might find. Learning about numbers is part of mathematics. But learning about numbers is not the goal. Thinking and understanding are the goals.

As we help our students and ourselves to learn to look for patterns and make connections, we provide the elements necessary for meaningful learning at school:

We use learning backgrounds our students bring with them to school. If the background is good, we capitalize on it. If the background is incomplete, we add the missing pieces.

We expand vocabulary.

We look for reasons.
We teach only what makes sense.
We make predictable certainty a part of the school day.
We encourage creativity and inventiveness.
We help our students define the problems.
We help our students learn to find solutions for themselves.
We help our students learn to share their knowledge freely with everyone in class.

The first step...

The goal of this chapter is to provide teacher and student alike specific skills in searching for patterns, so that we may begin to formalize our understandings of patterns and their connections. Without knowledge of patterns and the connections they make possible, each new skill taught in mathematics passes into obsolescence as the next skill is taught, and mathematics appears as a series of disconnected and meaningless bits of information useful only for passing tests.

The first step in learning to think mathematically is learning to look for patterns. Searching for patterns forms the core of all our lessons. We search for patterns everywhere, in every subject area. Patterns are in the formulas we discover, the shapes we investigate, the experiments we perform. Searching for patterns is where we begin. It is a search we know will never end.

Lesson One

Purpose	Learn what is meant by "pattern." Learn the A-B system of describing patterns.
Summary	The whole class invents patterns, then smaller groups devise ways to share. Pattern searches extend to the environment.
Materials	Students in the room. Patterns in the world.
Topic	A-A-B.
Topic	A-A-A-B.
Topic	A-B, A-B.
Topic	A-B-C and more.
Topic	Five minutes now and then.
Homework	We send the search for patterns home.

Two goals...

Lesson One has two main goals. First, to introduce our students to the concept of pattern. Second, to provide our students a common framework for describing patterns using an A-B format.

We introduce our students to the concept of pattern by letting them see and hear patterns. When we show our students patterns and ask our students to show us patterns in return, we are setting up an environment that makes learning about patterns as natural as learning to speak. When we were growing up, we learned what cars were by seeing a great many cars. Children learn about pattern by being shown a great many patterns. We could tell a car from a truck long before we could provide a verbal definition of *car* or *truck*. Our students learn what patterns are the same way we learned to tell cars from trucks.

We describe patterns using As and Bs and eventually Cs and Ds so that we can provide connecting links between seemingly unrelated events. What pattern does a penny share with the whole earth? The distance around a penny is *pi* times its diameter. The distance around the earth is *pi* times its diameter, too. The pattern is the same: $C = \pi d$.

Mathematics is the science of patterns. Mathematicians use letters and numbers to describe the patterns that they see.

(illustration 3-1-1)
(Collage of mathematical formulas. A mix of easily recognizable formulas and more advanced mathematical equations. Sample formulas: $a=1/2(bh)$, $e=mc^2$, Pick's theorem, $(a \times a)+2ab+(b \times b)=(a+b)(a+b)$, formula for acceleration, formulas from later chapters, $a=bh$, formula for area of trapezoid, $x+y=10$, etc.

We teach our students to look for patterns. We teach our students how to record their discoveries as mathematicians record discoveries. We teach our students to record with the As and Bs and Cs.

Listen...

Teacher: I want you to listen as I clap and pat a pattern. When you think you can tell the pattern I am clapping and patting, I want you to join in.

The teacher begins clapping and patting an A-A-B pattern—two claps and a pat, two claps and a pat, and so on. Children join in as they feel they can clap and pat along with the teacher and their classmates. The teacher continues the steady A-A-B rhythm until at least a majority of the children have joined in.

Teacher: I did not tell you what pattern I was clapping and patting, but you were able to join in with me because you could hear a pattern that helped you predict what I was going to do next. A pattern is something that we see or hear that can help us know what is coming next. The pattern I was clapping and patting is called an A-A-B pattern. Listen.

The teacher begins the clap-clap-pat, clap-clap-pat rhythm again while saying:

Teacher: A-A-B, A-A-B, A-A-B, A-A-B, A-A-B. When you think you can clap and pat the pattern and say A-A-B along with me, please join in.

The teacher continues clapping, patting and saying, A-A-B to accompany the claps and pats. Children join in as they feel they can clap, pat and say along with the teacher and their classmates. The teacher continues the steady clapping, patting and saying rhythm until many of the children have joined in.

Teacher: We made an A-A-B pattern by clapping and patting. Let's see if we can do it a different way.

The teacher may provide a few more examples or begin asking for suggestions from the children. Enough children may understand the concept of an A-A-B pattern so that no further teacher examples are needed. Not all examples provided by the children may evidence understanding of the A-A-B concept.

Teacher: Rhonda has suggested that we try clapping and then patting our head. Rhonda, please show us what you would like us to do.

(illustration 3-1-2)
(Four pictures. Two claps. Two pats on head.)

Teacher: Rhonda has clapped twice and patted her head twice. Let's see if Rhonda's pattern is an A-A-B pattern. Rhonda, you do the claps and the head pats and we'll say the A-A-Bs.

Rhonda again performs the motions. The teacher and some children say:

Teacher: A-A-B-B, A-A-B-B, A-A-B-B.
That is a very good idea Rhonda. But two claps and two head pats makes an A-A-B-B pattern. We'll save your A-A-B-B pattern for another day. Today I only want us to think of A-A-B patterns. Can anyone think of a way to change Rhonda's idea into an A-A-B pattern?

When Rhonda's suggestion is changed to meet the requirements of an A-A-B pattern, Rhonda leads the class in doing the motions she has suggested.

It is important...

By telling our students what kind of A-B pattern their suggestion represents, we provide everyone in class with a variety of examples of what we mean by pattern. We also provide illustrations of how we describe patterns in an A-B language.

Not all children may grasp what is meant by an A-A-B pattern at first. It is not important that every child understand immediately. It is important, however, that every child be made to feel secure in offering suggestions. Every suggestion made by a child will represent a pattern of some kind, even if it is not an A-A-B. Children whose suggestions do not turn out to be A-A-B patterns are told what pattern their suggestions do represent and the class is asked to change the suggestion to an A-A-B.

Regardless of a child's level of understanding, we have the power to give each child who contributes a suggestion the feeling that he or she has thought of a pattern to share. If the suggestion was not "right" at least it was not "wrong."

Each student's belief in himself or herself as a learner plays a major role in a student's being able to learn. When we accept as a starting point the patterns each students describes, regardless of whether or not the description matches the pattern we have requested, our students' belief in themselves as learners remains intact.

Understanding comes with time. When we give learning all the time it needs, we make it possible for the children who do not learn today to learn tomorrow. Our students were not expected to speak full sentences at birth. Their parents gave them time to learn. We are as patient as parents with the learning of every child in our room.

Teacher: Can anyone think of a different way?

We can increase each student's opportunity to share by decreasing the size of the group in which the sharing is to be done. We start by having students share their ideas for A-A-B patterns with the whole class. Once we feel enough students understand, we can have each child in a group act out his or her suggestion to others in the group. The group, in turn, may select one or two of these ways to share with the whole class. As the groups work, we wander around the room observing who is making contributions and who has yet to understand the contributions being made.

When is it clear?...

We continue to ask for suggestions of A-A-B patterns from our students until it is clear who does and who does not understand the concept of an A-A-B pattern. But when is it clear? What should we be looking for? And how will we know when we have seen it?

Our eventual goal for our students is that they see patterns that we have not pointed out to them. Our goal for Lesson One is more basic. We want our students to be able to see and hear the patterns we illustrate for them. The most direct way to assess who understands what we mean by an A-A-B pattern is for us to take note of who can create an A-A-B pattern for the class to act out.

As our students volunteer different ways of creating A-A-B patterns, we note who can provide examples. We note, too, the students who can tell others how to change their examples to fit the A-A-B format. The notes we take may be mental or they may be written. They may be as simple as a check mark next to a child's name or as formal as a paragraph written at the end of the day for inclusion in a child's portfolio.

Assessment need not always be recorded. It also need not always be formalized. But assessment needs always to happen. For each child, we ask ourselves:

Can this child create an A-A-B pattern?
Does this child need more time to absorb the concept?
Do I yet have any idea about this child at all?

If we can answer with confidence for a child, we know we know that child. If we are not sure where a child is in his or her understanding, we know we need to focus more attention on that child.

Thirty children, thirty teachers...

We assess our students and find there are children who do not yet understand. The same assessment also shows us that there are many children who have the understanding that we are trying to impart. The children who understand are our resource for bringing understanding to everyone in class.

For every child in a class of thirty children thirty teachers are available to convey the concepts to be taught. The thirty are the teacher and each child's twenty-nine classmates. None of us is as smart as all of us. All of us make sure we leave none of us behind.

When we have identified the children who need more help in creating A-A-B patterns, we form work groups for creating examples by matching children who need help with children who understand. When the groups are formed, we ask:

Teacher: Can any group think of ways to make A-A-B patterns that no group has thought of before?

We do not have to be called "teacher" to teach. Children working together have the opportunity to share with one another their own understandings in their own ways.

Lesson One forever...

Traditionally *Lesson One* might mean:

> Do this page in the workbook on Monday. On Tuesday, turn the page and work on Lesson Two.

In this book, each lesson describes an activity that may be used repeatedly in variations. The repetition continues until the concept of the lesson is understood.

Teachers using the Reading Program know that its basic structure remains unchanged as children progress through successively more difficult levels of activity. The element of repeated variations means that, once the students learn the basic lesson or activity, little if any reteaching is necessary as the variations are introduced. Teacher time is not spent covering the procedures. Teacher time is, instead, focused on helping students master the concepts.

The element of repeated variations is a part of Lesson One as well. As most students demonstrate an understanding of A-A-B patterns, A-A-A-B patterns are introduced as a variation and the lesson is repeated.

Teacher: I want you to listen as I clap and pat a pattern. When you think you can tell the pattern I am clapping and patting, I want you to join in.

(illustration 3-1-3)
(Hands clapping three times, patting once, clapping three times, patting once, etc.)

Teacher: I did not tell you what pattern I was clapping and patting, but you were able to join in with me because you could hear the pattern. The pattern I was clapping and patting is called an A-A-A-B pattern. Listen.

The teacher begins the clap-clap-clap-pat, clap-clap-clap-pat rhythm again while saying:

Teacher: A-A-A-B, A-A-A-B, A-A-A-B, A-A-A-B, A-A-A-B. When you think you can clap and pat the pattern and say "A-A-A-B" along with me, please join in.

The steady clapping, patting and saying rhythm continues until many of the children have joined in.

Teacher: We can make an A-A-A-B pattern by clapping and patting. Let's see if we can do it a different way.
Larry has suggested that we try stomping our feet three times and then turning all the way around. Larry, please show us what you would like us to do.
Let's see if Larry's pattern is an A-A-A-B pattern. Larry, you do the stomps and then turn around and we'll say the A-A-A-Bs.
Can anyone think of a different way?

We look for A-A-B patterns, A-A-A-B patterns, A-B-A-B-A-B patterns and A-B-C patterns. With each new lesson's variation, does the lesson really change?

Lesson One everywhere...

We teach our students to look and listen for patterns whenever we have a moment to spare. We clap and pat patterns for them to hear as we wait outside the cafeteria before lunch. We softly snap and clap as we sit waiting for an assembly to begin. We scout out in advance the paths of pattern walks. When we later guide our students on the journey, we know in advance the patterns we will see. When our students learn to look, we will learn that our students can find more for us to see in our walks than we ever knew was there.

We teach an A-B search that transcends the boundaries of all curricula we present. In every area of curricula, we see what we can find. We find what we can see. Lesson One is everywhere we look.

For homework, our students are asked to find the patterns in their lives: knife-fork-spoon, knife-fork-spoon, knife-fork-spoon; house-garage, house-garage, house-garage; fence wood-empty space, fence wood-empty space, fence wood-empty space.

Math is simple. Math is basic. Math is straightforward. Math is everywhere.

Lesson Two

Purpose	Learn to relate A-B patterns to materials.
Summary	Students create and share A-B patterns. Students also learn to check their neighbors to ensure that everybody understands.
Materials	Pattern Blocks, Power Blocks and any other material we might select.
Topic	Pattern Blocks and A-A-B.
Topic	Power Blocks and A-A-B.
Topic	Pattern Blocks and A-A-A-B.
Topic	Power Blocks and A-A-A-B.
Topic	Other materials. and A-B patterns.
Homework	We encourage our students to make and share A-B patterns with materials available at home.

Deja vu, deja vu...

In Lesson One, we introduced our students to the concept of A-B patterns. The materials we used were the students themselves. In Lesson Two, we use the concept of A-B patterns to alter how our students look at math manipulatives.

Teacher: Today we will be using Pattern Blocks to make A-A-B patterns. I am going to make an example of an A-A-B pattern. Please see if you can copy the pattern I am making and extend it out as far as you can.

(illustration 3-2-1)
(Red-red-green pattern on the overhead. Use overhead Pattern Blocks.)

Teacher: My pattern is: Red-red-green, red-red-green, red-red-green.

(illustration 3-2-2)
(Children copying the teacher's pattern and extending it out. Show some children doing it right and some doing it wrong. Add a caption acknowledging the presence of both right and wrong copies in the illustration.)

The teacher describes the pattern as red-red-green, trapezoid-trapezoid-triangle, flat-flat-pointed, or even as A-A-B. Any verbal description that reflects what is being made is acceptable.

Teacher: Please have your neighbor check you and you check your neighbor to see if my instructions are clear. Please check to see if we all agree on how to copy and extend the pattern I am making.

Checking your neighbor...

Our students learn to check each other's work as soon as there is any work that can be checked. We tell our students that we are asking them to check one another's work because we know our words are sometimes confusing. We tell them, too, that if a fellow student is not doing what we have asked, it is our fault and not the fault of the student.

Our goal is to ensure that all our students understand. We do not ask our students to check one another so that they may turn each other in. We ask them to check one another so that they may help each other learn. If we have not been clear, we ask our students to help us to explain.

Teacher: I will be walking around the room to see how well I have explained what I want you to do. I know that what I say is not always easy to understand. If you do not understand, I will not be upset with you. It is my responsibility to make my meaning clear. But if you <u>do</u> understand and you are <u>not</u> helping others to understand, I <u>will</u> be upset with you.

As students, we were taught not to help one another. We were taught that helping each other would somehow keep the person receiving the help from doing his or her own learning. The only one authorized to respond to our wonderings and our confusions was our teacher. Did learning not to help really help us learn?

Outside of school, we use all the resources available to us to find the answers to our questions. If someone stops to ask us for directions, we give the best directions we know how. Would we say to

ourselves: "If we answer, they will not learn." If we are lost, we stop and ask for guidance. We ask people who do not know until we find someone who does. Would we say to ourselves: "If we ask, we will not learn." Outside of school, we answer what we know whenever we are asked.

Helping is required in our classroom. Learning is something we all do together. We do not reward the first one finished for leaving everyone else behind. We praise the person who helps everyone else get through. Our students learn from us that they are to take responsibility for everyone in class. We care that they care for one another. We start by having neighbor checking neighbor. We end by having an entire class that learns.

A different way...

Teacher: Let's see if we can do it a different way. Can anyone think of another way to make an A-A-B pattern with the Pattern Blocks?
Student: Orange-orange-blue.
Teacher: Okay. Aaron has suggested orange-orange-blue. Let's make orange-orange-blue.

(illustration 3-2-3)
(Children making orange-orange-blue, orange-orange-blue, etc.)

Teacher: Let's see if this is an A-A-B pattern.

The teacher and the students says A-A-B while pointing to the orange and blue shapes.

Teacher: What other ways could we describe this pattern?
Student: Square-square-diamond.
Student: Flat-flat-pointed.
Student: Okay-okay-squashed.

As was true for Lesson One, not all children may grasp what is meant by an A-A-B pattern, but more will now than did before. We listen to suggestions that do and do not represent A-A-B patterns. If the pattern is not A-A-B, we tell our students what pattern their suggestions do represent and we ask the class to change the suggestion into A-A-B.

Surrounding the child with the concept...

We envelop infants with words and meanings. We surround them with the concept of speech. As a result, infants learn to speak. We teach our students the concept of pattern in the same natural way. We envelop them with pattern. We establish an environment that gives them time to learn. With time, learning always happens. It always will. We surround the child with the concept of pattern. We surround the child with the concept of mathematics. We surround the child with the concept that everyone will learn.

(illustration 3-2-4)
(Collage of a variety of ways children create A-A-B patterns with Pattern Blocks)

Teacher: I am really impressed with all the different ways that you have thought of to make A-A-B patterns. Stop what you are doing for now and walk around to everyone else's desks so that you can see all the A-A-Bs you have collectively invented.

The students walk around the room looking at the patterns their classmates have constructed. The walking around is part of the process of surrounding each child with the concept. It is also part of the process of expanding each child's awareness of the many different possibilities. None of us is as smart as all of us. A walk around the room makes this plain to see.

Teacher: What would happen if you stood the pieces up instead of laying them down?

(illustration 3-2-5)
(Wall of Pattern Blocks in A-A-B pattern)

Teacher: If you can do it with two colors, can you do it with just one?

(illustration 3-2-6)
(Pattern Blocks, all red, some standing, some lying down.)

Teacher: Can you do it a way none of us has thought of yet?

(illustration 3-2-7)
(A over A next to B)

Teacher: How many ways can you do it?

The children create and then wander among the creations of others in their class. They add to what they thought was possible from the new possibilities that they see. We encourage them to go beyond our limits to set and then surpass new limits of their own. We have much to teach one another. We have much to learn from one another as well.

Power patterns...

Teacher: Today we will be using Power Blocks to make A-A-B patterns. I am going to make an example of an A-A-B pattern. Please see if you can copy the pattern I am making and extend it out as far as you can.

(illustration 3-2-8)
(Small triangle-small triangle-small square pattern on the overhead)

Teacher: Small triangle-small triangle-small square, small triangle-small triangle-small square, small triangle-small triangle-small square.

(illustration 3-2-9)
(Children copying the teacher's pattern and extending it out. Have some children doing it right and some doing it wrong. Add a caption noting the existence of right and wrong copies.)

Teacher: Please have your neighbor check you and you check your neighbor to see if my instructions are clear.
Let's see if we can do it a different way. Can anyone think of another way to make an A-A-B pattern with the Power Blocks?
Student: Little square-little square-big square.
Teacher: Sam has suggested little square-little square-big square. Which squares do you mean?
Student: Two S-1s and an S-4.

The letters and numbers on each Power Block are useful identifying marks if the students can read them. A child who cannot might hold up the blocks and say:

Student: These three.
Teacher: Sam has suggested little square-little square-big square. Let's make Sam's pattern.

(illustration 3-2-10)
(Children making little square-little square-big square.)

Teacher: Let's see if this is an A-A-B pattern.

The teacher and the students say A-A-B while pointing to the square shapes.

Teacher: What other ways could we describe this pattern?
Student: Window-window-door.
Student: Cookie-cookie-cake.
Student: Baby-baby-mother.
Teacher: Can anyone think of a different way to make an A-A-B pattern with your Power Blocks? Work with the person sitting next to you to see what new way you can create.

(illustration 3-2-11)
(Collage of a variety of ways children create A-A-B patterns with Power Blocks.)

Teacher: Stop what you are doing for now and walk around to everyone else's desks so that you can see all the A-A-Bs you have invented.

The students walk around the room looking at the patterns their classmates have constructed.

Teacher: Can you do it in a way none of us has thought of yet? How many new and different ways can you invent?

A-A-A-B today...

Teacher: Today we will use Pattern Blocks to make A-A-A-B patterns. I am going to make an A-A-A-B pattern. Please copy the pattern I am making and extend it out as far as you can.

(illustration 3-2-12)
(Orange-orange-orange-blue pattern on the overhead. Children correctly copying and extending the teacher's pattern.)

The pattern for the lesson is the same, regardless of the material used. The Teacher says:

I am going to make an example of a pattern. Please copy and extend the pattern that I make.
Have your neighbor check you and you check your neighbor to see if my instructions are clear.
See if you can make my A-B pattern in a different way.
(So and so) has suggested (such and such a way). Let's see if (such and such a way) is the pattern that we want.
What ways can we describe the pattern made?
Stop what you are doing now and walk around to everyone else's desks so that you can see all the ways that you have invented.

As we did in Lesson One, we look for A-A-B patterns, and A-A-A-B patterns, and A-B-A-B-A-B patterns, and A-B-C patterns, and any other kind of pattern we may choose.

Teacher: Today we will use Power Blocks to make A-A-A-B patterns. I am going to make an A-A-A-B pattern. Please copy the pattern I am making and extend it out as far as you can.

The pattern for the lesson is the same, regardless of the material used.

Lesson Two, like Lesson One, is not the kind of lesson that is over in a day. We start by using Pattern Blocks to make A-B patterns. How much does the lesson change when we use Power Blocks? How different is the lesson for Geoblocks or junk box shells or toothpicks or squares? Patterns are everywhere. Lesson Two is in every material that we use.

Lesson Three

Purpose	Learn how to record patterns.
Summary	Students record patterns and use the recordings of others to reproduce and extend the patterns.
Materials	Pattern Blocks and Pattern Block shapes or stencils; Power Blocks.
Topic	Pattern Block A-B patterns recorded.
Topic	Pattern Block A-B patterns copied and extended.
Topic	Power Block A-B patterns recorded.
Topic	Power Block A-B patterns copied and extended.
Homework	What materials for recording do our students have at home? What materials might we send?

Mathematics is patterns recorded...

Mathematics is patterns. Mathematicians record the patterns that they find. Our students are mathematicians. As our student mathematicians begin to understand what we mean by pattern, we provide them ways to record the patterns that they find.

Recorded work is used to:

Teach our students from the very beginning that mathematics and recording go hand in hand.
Form the basis of student-made task cards for future assignments. Students learn to use what other students write as a beginning point for extending patterns.
Provide a way for our students to capture and share a permanent record of their creativity and inventiveness. Recorded work can be added to each student's portfolio. Written records are a means of sharing with parents the abilities we are assessing in our class.
Save for sharing beyond the present year. Written work forms a link between siblings passing through our class in past and future years.

Teacher: I am really impressed with all the great ways you have found to make A-A-B and A-A-A-B patterns with your Pattern Blocks. But when I try to tell my teacher friends after school of all the creative ways you have found to make patterns, I have trouble remembering all the different ways you have shown me. Today, I will show you a way to record your patterns so that we will have a system that will help us keep track of at least some of the patterns you have been inventing.

Recording is something that takes place at every level in school. We match the methods and materials of recording to the levels of our students. Young children might glue Pattern Block shapes to tag board. Older children might use stencils to draw and then color the shapes on to strips of paper. Regardless of the method used, the message is the same. If we make it and we like it, we record it and we share it.

Teacher: Once you have recorded your patterns, we'll see if you can use a recording that someone else in class has made to copy and extend the pattern their recording represents.

(illustration 3-3-1)
(Show three progressions, complete with captions.
The first progression is of making Pattern Block walls using Pattern Block cutouts and photo copied pictures of the children as their "names." Include in this progression comments from the sequence in Math Their Way, page 38 forward and the Newsletter, page 9.13 forward. Carry this and the following progressions through making and then using the recordings as task cards. The second progression is of older children using a template made from a coffee can plastic lid to draw and then color in their design. The third progression shows children tracing around shapes. Regardless of level, show the child's name and the A-B designations written on each finished recording.)

The student or the teacher writes the first and last name of the child who creates the pattern on the pattern card. This is done even if the student cannot yet write or read the written name. Our students create. Their work is recorded. Their names as creators are transcribed for all to see.

As each child makes a pattern card to record a favorite pattern, the child also writes on the card the A-B letters that indicate the pattern made. This writing is required even from a child who cannot write. Our students learn to say "A-A-B." They learn to write what they are saying. The use of letters to record mathematics is a natural part of our lessons from the start.

Power written down...

Teacher: Today, we are going to record your Power Block patterns so that we can keep track of some of the patterns you have been inventing.

(illustration 3-3-2)
(Show several progressions, complete with captions for the illustrations. Use both shapes and colors in the patterns. Two sets mixed together permit the same size shape to be used to represent A-A-B by alternating colors. The captions for the illustrations can indicate the various methods. Illustrations should include a variety of levels of work form kindergarten to upper grade. Older children can use templates or just trace around the shapes. For younger children, shapes can be cut out as there were shapes cut out for the pattern blocks. To do this, however, the teacher limits the number of different Power Blocks with which the children may create patterns. Show the child's full name on each finished recording, regardless of grade level. Also, show the A-B designations written on each task card, regardless of level.)

We assess...

As our students learn about pattern, we assess the following levels of understanding for each child:

Can the student reproduce and extend an A-B pattern that someone else has made? Does the student have a basic understanding of what we mean by A-B?
Can the student create ways of showing a pattern that go beyond copying the patterns shown? Can the student find ways that no other student has found?
Can the student see A-B patterns in the classroom or at the school that no one else has found?
Can the student see A-B patterns in his or her environment at home? Is what is learned at school a part of the student's understanding of the world outside of school as well?

As we assess our students, we assess ourselves as well:

Do we see A-B patterns at school that no child had pointed out to us?

Do we see patterns in our environment outside of school? Is what we teach in school a part of our awareness of the world as well?
Do we share what we see with our students? Do we show them that we, too, are constantly searching and constantly discovering?

Sometimes assessment is simple and straightforward. We can watch our students to see who can copy and extend a pattern we have made. Sometimes assessment is more complex. We may not know for weeks or months if a child has learned to see that patterns are a part of life. Sometimes we can assess by walking around the room. Sometimes the only way to assess is to look and listen for days and days and days. There is no rush. We have all year to hear. We have all year to see.

Moving on...

We set up an environment in our classrooms that surrounds our students with concepts. For patterns, the environment is established when our students can extend the A-B sequences we give to them or they give to each other. When we reach this point we are ready to move on.

Moving on does not mean our students stop learning about pattern. It does not mean we stop looking for what there is to see. Moving on means we make pattern a part of all the lessons in all the chapters that are to come.

Lesson Four

Purpose	Learn to look for patterns in numbers.
Summary	Students look at number charts for patterns and describe the patterns they see.
Materials	Blacklines for assorted number charts and the number charts themselves.
Topic	00-99 matrix.
Topic	1 -100 number strip.
Topic	10 X 10 multiplication matrix.
Topic	25 X 25 multiplication matrix.
Topic	Pascal's triangle.
Topic	12 month calendar, with all months visible.
Homework	We send number charts home for viewing, too.

The never ending lesson...

(illustration 3-4-1)
(Show a 0-99 matrix with 00, 01, 02, etc. in the top row, as opposed to 1, 2, 3. Matrix has "top right, top left, bottom right, bottom left" on it for reference as well as the appropriate Reading Program sound stamps. Note that the reference words are added even if the children cannot read the words, because that's how children learn to read words.)

Patterns are everywhere. We teach our students to look for patterns all the time. We use the 0-99 matrix to teach our students to look at numbers and see what they can see. Look today, say what you can see. Look again in a week or a month and say what more you see. Look again in another year. Aren't you impressed with how much more you know? Numbers record patterns. The more we know of numbers, the more of their wonder we can see.

Students at any grade level can look at the 0-99 matrix and say what they can see. As long as looking at the matrix is something the entire class does together, no child in school is too young to begin. What readiness do we measure for this activity? The readiness to learn. As long as children and adults are invited to think, no student or teacher is too young or too old to make new discoveries.

Teacher: Look at this 0-99 matrix and tell me what patterns you see.
Student: All the numbers in that place are the same.
Teacher: Which place? Come and point it out to me so I can show everyone else what you mean.

(illustration 3-4-2)
(Matrix with line drawn around all the fours in a column.)

Student: All the numbers go up by one.
Teacher: Which numbers? Tell me what you mean?

(illustration 3-4-3)
(Matrix with an example of the numbers going up by one. This illustration is NOT the same as the one showing that the students know the numbers are the counting numbers in order. This illustration is referring to the fact that all the numbers on the right hand side of a column go up by one.)

Student: The numbers are all the numbers for when we count to one hundred, except one hundred.
Teacher: What do you mean?
Student: Look. One, two, three, four, five, six.
Teacher: Oh! The counting numbers. What other patterns do you see?
Student: The numbers from the top left to the bottom right are all the same.
Teacher: What do you mean by "all the same?"
Student: In the first space, they are zero-zero. In the next space they are one-one. In the next space they are two-two. They keep on going that way all the way down.

(illustration 3-4-4)
(Matrix with line drawn down the diagonal as described in the dialog.)

Teacher: That line is called a diagonal. Are there any other patterns on a diagonal?
Student: The numbers going the other way are one less each time.
Teacher: What do you mean by "the other way?" And what do you mean by "one less each time?"

(illustration 3-4-5)
(Matrix with many lines drawn down the top right, bottom left diagonals.)

Student: When you add the numbers in the nines column, they all add up to a number one bigger than the number before it. Zero and nine is nine. One and nine is ten. Two and nine is eleven and it keeps on going that way.

(illustration 3-4-6)
(Matrix with the column pointed out and the addition answers written next to the column.)

Teacher: Does that happen for any other column? What about the rows?
Student: If you add the numbers in the diagonal that goes from the top right to the bottom left, all the numbers add up to nine. Zero and nine is nine. One and eight is nine. Two and seven is nine.

(illustration 3-4-7)
(Matrix showing that the numbers for the diagonal in the dialog add to nine.)

Teacher: What happens when you add the numbers in the lines that are parallel to that diagonal?

(illustration 3-4-8)
(Matrix showing the lines parallel to the diagonal from the teacher's question.)

Student: When you go from left to right in steps of two down, the numbers are eleven bigger each time, no matter where you start.
Teacher: What do you mean by "steps of two?"

(illustration 3-4-9)
(Matrix showing the over-one, down-one pattern, with an example of each number being eleven larger.)

Teacher: What happens if you take steps of three?

A student describes what he or she sees. The teacher helps the student clarify the description for the class as a whole. The next student describes what she or he sees, and so the process goes. No matter what the students find, the teacher's question is always, "What else can you see?"

The examples above represent a range of the kinds of discoveries children of various ages might make while looking at the 0-99 matrix for patterns. Our students will see much more than has been shown here. All we have to do is let our students look as we let ourselves look along with them.

Learning never stops. Do we already know all the words we are ever to know? Have we already had all the adventures we are ever to have? Do we have to be prodded to keep life's excitement alive? We use the 0-99 matrix to give our students the opportunity to look at numbers for the patterns they reveal. Once we put the matrix on the wall, we never take it down.

Other numbers...

(illustration 3-4-10)
(Recording strip of numbers from 1 to 100, all in a column. Multiplication matrix for 10 X 10. Multiplication matrix for 25 X 25. Pascal's triangle. A 12-month calendar, with all months visible.)

We ask our students to look at the numbers in the 0-99 chart for patterns they can see. We scan other charts of numbers as well. Any set of numbers that we use in class is a set of numbers to be searched. Multiplication matrices in any base. Columns of numbers from place-value lessons. Fractions recorded from Unifix Cubes. Calendars. Exchange rate or conversion tables. Pascal's triangle of numbers for probability. Any numbers we can find. Mathematics is patterns. We search for patterns in all the mathematics that we see and do.

Lesson Five

Purpose	Learn to extend pattern searches beyond the period set aside for math. To connect mathematics to art.
Summary	Students create pattern designs for themselves.
Materials	Art supplies of various kinds. Drawing paper, graph paper, crayons, colored pencils and so on.
Topic	Pattern Block walls.
Topic	Pattern Block mosaic designs.
Topic	String designs.
Topic	Graph paper patterns.
Topic	Other patterned art that we might choose to use.

Without all our plans...

We show our students how to look for the A-B patterns all around them. We post matrices on the wall to search their numbers for predictability. We plan all of our lessons to produce learning. But learning is possible without all our plans. We use the time set aside for art in our class to let our students create and invent what they by themselves feel or see.

(illustration 3-5-1)
(This lesson is contained primarily in the illustrations. The illustration includes examples of children's art. Add captions wherever necessary to indicate the basic materials and question used to generate the art. Pieces of paper to glue, templates to guide pencils, blocks for tracing created designs, etc. Add the caption: "We give our students the means to record what they imagine." Kinds of art to show—Pattern Block mosaic designs, Power Block designs, string designs, graph paper patterns, number matrix pattern coloring, mirror art from geometry (symmetry of all kinds), tessellations and Escher designs, potato art or sponge art (wrapping paper designs.)

A pattern is something we use to predict what will come next. Designs are not necessarily patterns. We do not need to focus on definitions of designs or patterns. Not every pattern has to be an A-A-B. For the art that our students create, we need not debate the right name to call what they do. Our students do not yet need to know how many angels dance on the head of a pin. Math is not confined to what we are told we must know. There is always more to learning than the learning that we see.

Questions from Teachers

1. **Most of my students can see the A-A-B and A-A-A-B patterns, but I'm not so sure about two or three children in particular. Do I wait until I am sure these children see, or do I go on to the next lessons? I hate to hold everyone else back for these few.**

We will leave no one behind. But that does not mean that every child is expected to know everything everyday. We look for patterns in math, we look in reading, we look in science and social studies and in P. E. We look on walks and bus rides and going to and from the cafeteria or the auditorium. We look on the way to and from school. We look around the house and around the neighborhood. We look for patterns all the time, everywhere, all year long.

Our lessons surround our students with the concept of pattern. An awareness of pattern permeates the environment of our room. When this awareness exists at a class level, our purpose is accomplished. The class is ready to move on. When our class understands, our class leaves no one behind.

When a new child comes into our room in the middle of the year, we do not start the whole class over from the beginning. We know the new child learns from our class what our classroom is about. The new child is soon absorbed by our class as if he or she had been there all along. A child who is in our room all year long learns from our class just as well.

When all but one or two or three students understand, we move on to the next area to be taught. The one or two or three move on, as well. We leave no one behind, but not everyone has to know everything on the day we move ahead.

2. Some of my students still want to play with the materials when I am trying to get them to create patterns. What do I do if some of my students are still at the free exploration level when I want to begin work with patterns?

A student who wants to free explore when we have another lesson in mind is no different than a child wanting to stay outside and play when recess is over. Would we let a child continue to play outside until he or she got it out of his or her system? Or, would we help the child to learn that different kinds of behavior are appropriate at different times of the school day?

School learning is not home learning. At home, much of the learning that takes place is non directed, incidental learning. The child may learn about roller skating, riding a wagon, climbing a tree, building with blocks, or how to dress a doll and comb its hair, but there are no requirements that the child learn these things. There is no list of skills to be covered.

School is not home. School learning is directed learning. We, and not the child, are in charge of establishing the curriculum for achieving our learning goals. Free exploration is not something children do until they get it out of their systems. We hope the desire to explore freely remains with our students throughout their lives. However, we must balance our need to keep alive within each child the desire to explore freely with our need to provide the specific learning experiences that enable us to meet our learning goals. We create this balance by:

Providing the initial opportunity for free exploration with the introduction of each new material.
Allowing time for continued free exploration at the end of some periods in which a material is in use.
Providing an activity time at the end of each school day, so that students may select for themselves the materials with which they wish to continue free exploration. (See *Organization and Management* (page 426) for a discussion of activity time.)
Expecting and requiring each child to work diligently at the lesson we have elected to present.

3. The dialogs in this chapter show the teacher using words like diagonal, parallel, symmetry, create, predict, extend, matrix and so on. At what grade level would you recommend beginning to use words like this with students?

At any grade level where children speak and learn.

We make no effort to control the language we speak to infants when they are first learning to understand speech. We speak in full sentences even when they cannot understand a word. Should we select our words more carefully when talking to infants? If we spoke only the words we thought they could comprehend, would they learn any faster? Or, do they learn faster when we speak as if they will soon understand every word?

Our students hear all that we say and make sense of what they can. When they do not know, they may ask so that we may explain. We say "parallel" and draw two straight lines. We say "matrix" and a chart appears. We ask for symmetry and provide images that are symmetrical. Learning language is natural, but we can only learn the language we experience. When our students hear it or see it or say it or sense it, we can be sure they will learn it as well.

4. Is there a particular sequence of A-B patterns that is recommended?

Yes and no.

The particular sequence that we use is the sequence we discover from working with our students. Creating our own sequences takes a knowledge of pattern and of mathematics that we may not feel we possess, but we know far more than we may think we know.

We begin with a pattern. We watch to see if our students can predict. If there are very few who can, then we start again with something less complex. We find an A-B series that our students can begin to understand. We continue at this level until two-thirds or more of our class shows us they are ready for more complexity.

If we start a group of kindergarten children with an A-B-B-C-A-B-B-D-A-B-B-E-A-B-B-F pattern, the children will show us that they are not ready to predict. This starting point is not appropriate for them. We know enough to reduce the complexity of the patterns we present until they can predict. We know that we can learn from anything that does not work—what not to do again. We also know that we can learn from anything that does work—what to do instead. We know that we can learn from what we do wrong—what to do right.

5. What if I don't see the patterns my students see?

We ask our students, "Can you see a pattern?" when we don't even know if there is a pattern to be seen. If we only ask when we already see, can we ever learn anything new? If we only ask when we already know, what will our students learn? Might not they learn that the answer is always "yes," because the teacher only asks when the teacher already knows?

None of us is as smart as all of us. We do not have to know everything in advance. What our students see they share with us. If we do not understand, we ask them to explain and re-explain until we do. We do the same for them. We are resources for our students. They are resources for us, as well.