Chapter 4

Beginning Number

Table of Contents

Questions from Teachers

Before We Begin

Too late for the beginning...

Counting up, counting back, counting on, counting off. Cardinal numbers, ordinal numbers, reading numbers, writing numbers. From the beginning of our students' days in school, we teach our students numbers. But beginning days in school are too late for the beginning. Children learn what numbers are before they enter school. They already know their ones and twos and threes. Is there anything left for us to teach?

A friend tells me: "My son knows his age. Ask Billy how old he is."
I ask: "Billy how old are you now?"
Billy says: "Three."

I am a teacher. I ask more questions than the question I was told to ask. Its what teachers do.

"**And how old is Daddy?**"
"**Three.**"
"**And how old am I?**"
"**Three.**"
"**And how old is Grandpa?**"
"**Three.**"

The start of Billy's learning did not wait for school, but Billy's learning left a thing or two for us to teach.

They do not know what we think they know...

Our students bring the capacity to learn with them to school. They also bring with them all the learning they have already done. And, they do not know what we think they know.

Aaron, age seven, shines a flashlight beam into the unflushed toilet. The beam touches what he calls "poop." He puts the entire flashlight under the faucet to clean it off. His reason: "So the light beam won't be all smelly."

Marcus, age nine, thinks because he has ten toes, his teacher has twelve toes. Why? "Because the teacher is bigger."

Ashley, age six, hears her toy player piano's rendition of *Happy Birthday To You* for the first time. She sings the song as the piano plays the familiar tune and she adds in her own name at the appropriate place. She asks: "How did the piano know my name?"

Sam, age ten, draws a picture.
Teacher: "Sam, why have you put two suns in your picture?"
Sam: "Because there are two suns."
Teacher: "Why do you think there are two?"
Sam: "There is the one that I see up above the front of my house and there is the one that is up above the back of my grandmother's house. I see it there whenever we visit her."

Karyn, age five, sits listening at the tape recorder to a story read by a friend of her teacher. The friend pops in to pay a quick visit to the teacher. Karyn's expression while listening to the tape and also hearing the friend and the teacher talking is one of puzzlement. She finally says to the friend: "But you can't be here [*meaning in the classroom visiting*] because [*pointing to the tape recorder*] you are in here!"

Our students can think and they can reason, but their thinking is not like ours. They are ready to absorb what we choose to present, but what they learn will not always be what we think we have taught.

Da da wee...

A parent pushing his twin sons down the street in their side-by-side strollers hears them say, "Da da wee!" as they point in unison somewhere off in space. "Yes," he says in response, "Nice birdies!" as he tries in vain to guess what "da da wee" might mean. After several more days of pushing strollers, Daddy finally figures out that "Da da wee!" means "Go that way!" His sons have been trying to tell him where they'd like to go. Once he understands, he pushes the strollers in the direction his sons indicate as often as he can, while saying "Yes, we'll go that way", or "No, we can't go that way, because..." to every "da da wee" that he hears.

As a child learns to speak we talk to the child and encourage the child to talk to us. We accept all kinds of approximations of speech as good enough, while helping the child clarify and refine his or her skills. We talk and listen and listen and talk. We allow the child to grow older and marvel at the changes that come with age. We accept "da da wee" until "go that way" comes along.

When teaching speech, we know we do not have to teach everything. We could not if we tried. We do not make lists of all the meaningful words and all the appropriate sentences and all the possible skills. We know it is not necessary to compile a scope and sequence of all the learning to be done. We cannot even think of all there is to know.

To teach beginning number concepts, we set up a number-rich environment that surrounds the child with counting and with meaning. As a child learns about numbers we talk to the child about numbers and encourage the child to talk to us. We accept all kinds of approximations of understanding as good enough, while helping the child clarify and refine his or her skills. We talk and listen and listen and talk. We allow the child to grow older and marvel at the changes that come with age. In number, too, we accept "da da wee" until "go that way" comes along.

When teaching number concepts, we know we do not have to teach everything. We could not if we tried. We do not make lists of all the possible skills. We cannot even think of all there is to know.

No one knows anything...

Parents do not need to assess what their infant might already know. They start at the beginning when the infant does not know anything. Our students are not infants when we first see them. They may be five or seven or nine or twelve years old. Their learning has already begun. How can we assess what our students have learned so far? How can we know what they understand and what they do not?

An initial assessment that we may use in any grade is one that takes no time. When we are done, we know just what to teach. The assessment is that no one knows anything. As fast as we can say "assessment," the assessment is complete.

When we start with the assumption that no one knows anything, we do not need to worry about what was or was not taught the year before. We start at the beginning. As we move through each new lesson, our students show us what they understand. When we find a concept not yet understood, we stop right there and master it right then.

The beginning number lesson for a nine year old may not look the same as for a five year old. Beginning numbers for a five year old may be counting up and back to ten and making designs with toothpicks and Power Blocks. A nine year old might start, instead, with geoboards and shapes. Regardless of how the lesson looks, beginning number is the place where all our students start.

Our commitment is to leave no child behind. If we do not start at the beginning, can we really meet this goal? When we start where all can do it and stop when some cannot, we are showing all our students we will bring each child along. Can thirty children help each other if we do not show them how? When we start at the beginning, we can guarantee the end.

What we want the child to know...

If we were given the responsibility of teaching speech to a child, would it even occur to us that the child might not learn to speak? We do not use a list of skills or a book of what to teach when we help an infant learn. Teaching language is a natural part of us. Teaching beginning number concepts is just as natural.

What our students need to know about numbers in this chapter is not complex. They need to:

Use numbers for counting up and counting back.
Know that numbers are cardinal (1 and 2 and 3) and ordinal (1st and 2nd and 3rd).
Know that numbers mean one-to-one correspondence.
Know that numbers mean ways to make twos and threes and fours and more, all the way up to ten.
Know that numbers can represent quantity or magnitude or position or something else again.
Know what numbers look like—recognizing numerals from zero to nine and more for later on.
Write their numbers well—drawing the numerals from zero to nine and more for later on.
Know that numbers are part of all our lives.

We create the environment that ensures the knowledge will be gained. Mathematics is simple and basic and straightforward. The lessons that we teach keep it so.

Lesson One

Purpose	Learn to count up and back by ones.
Summary	We establish a counting environment.
Materials	Anything available to count.
Topic	Counting up.
Topic	Counting back.
Topic	Counting every time there are things to count.
Homework	We encourage parents to count with their children at home.

Counting and counting and counting...

Teacher: We are going to count how many children are here today. Everyone please stand up. When I touch the top of your head, we will all say your number and you may sit down. See if you can remember the number that we say for you.

When the class is counted, the teacher writes the final number on the overhead or the chalkboard for all to see. It is not important that anyone knows how to read the number. It is important that the children see the counted number written down.

Teacher: I wonder what would happen if we counted everybody all over again, but this time we started on the other side of the room.
Would the number we said for you when I touched you on the head before still be the same?
Would the final number for the children in our class today be the same?
What do you think? Let's do it and see.

Why do you think the number we said for you changed this time?
Did anyone's number stay the same?
Would your number change again if we counted again?
Will it change every time?
Could we ever get your number to come out the same?
Why do you think the number for all the children in our class stays the same no matter which way we start our counting?

Lesson One is not a lesson. Lesson One is a way of life. We want our students to be able to count, so we create the environment that makes counting meaningful. Lesson One is counting and counting and counting, because the way we learn to count is to count.

Creating the environment...

Language is such a natural part of our lives that we give no thought to creating an environment for learning language. We make counting as natural for our students as is language by making counting as natural a part of their environment as is language.

When we talk to an infant, we do not limit our speech to the words the infant knows. We speak as adults. When we count with our students, we do not confine our counting to the numbers our students might already know. We count as adults.

To make counting a part of the environment, we plan our days around counting. Things to count and reasons to count already fill our world, just as reasons for using language are already there. All we have to do is look. All we have to do is count.

What might we count? Our rooms are all filled with countable things.

Books, pencils, crayons, rulers, paper, scissors, and anything else we can touch.
Attendance. Who is here? Who is not? How many altogether?
Lunch count.
Bus count.
Milk and snack count.
Notes to go home, counted by rows, handed out one at a time.
Windows and doors in the hall.
Bricks and squares from walls and floors.
Cars that are parked or cars that pass by our school.
Brothers and sisters and uncles and aunts and people who live in our homes.
How many Pattern Blocks or Power Blocks or Unifix Cubes might we have?
A ticking clock's time. Seconds we hear. Minutes we see. Hours until we return.
Wherever we walk, we step as we walk. For each step that we step, we're careful to count only one.
When we are young, we count until ten before starting all over again. When we are old, we count till we are there and we estimate numbers before we begin.
We count it whatever it is.

Parents know that talking is how we learn language and talking is part of the life of their child. We help parents learn that math is a language as well. We begin by helping each parent make counting a part of the language of home:

Numbers of pieces while setting the table.
Cookies to be shared.
Numbers on the microwave countdown.
Minutes until dinner is ready.
Minutes until show time begins.
Hours to be slept that night.
Number of swings on the swing.
Up and down rides on the see-saw.
Clowns at the circus.
Lions at the zoo.
Numbers of fingers in the whole house.
Numbers of stairs up the stairs.
Numbers of Legos, numbers of Barbies, numbers of toys lying around.
Numbers of pages read before bedtime.
Board games. Card games. Car games. Are there any games where there is nothing to count?
Basketball scoreboards with points added on.
Numbers on football time clocks.
Numbers of balls, numbers of strikes, numbers of outs, numbers of players at bat.
Numbers of hops, numbers of skips, numbers of jumps.
Numbers of everything anyone does.

We do not have to put numbers and counting in the lives of our students. Numbers are already there.

Saying means showing what is said...

Reading means knowing what is read. Just saying the word is not enough.

Counting means knowing what is said. Just saying the number is not enough. The teacher in the example at the start of this lesson touched a child as each new number was said. The touching matched the children counted on a one-to-one basis with each number. Even the children who did not understand every number could learn that each number said went with a child. We count and we touch. We count and we point. We count and we show that what we say matches what we see.

The teacher counted the class with the help of the whole class, or maybe the teacher counted alone. The ones who could count could join right in. The ones who could not, could listen and learn. The class counts together, the class counts with meaning, and the class counts as far as the teacher can go.

Meaning comes gradually for language and for numbers alike. But meaning comes sooner and better when what we have learned is a part of our lives. When we count windows and doors and people and posts, our children can count doors and windows and posts and people wherever they are. The counting we teach can follow our children home. Our students can go to sleep at night counting and wake up counting as well.

Counting up and counting back again...

Teacher: We are going to count how many children are here. As I touch the top of your head, we will all say the number you are and you may stand up. See if you can remember the number we say for you as you stand up.

With the help of the class, the teacher counts the students, one person at a time.

Teacher: How many children are now standing up?
Student: Twenty-eight.
Teacher: Okay. This time we will start with how many there are all together and count back down to none. As I touch the top of your head, we will all say what number you are and you may sit down.

We practice counting up. We practice counting back as well.

(illustration 4-1-1)
(Seventeen Power Block squares, fifteen in one group and two in the other.)

Seventeen squares. Two are removed. A child who can only count forward will count to fifteen to find what remains. The child who can count forward and backward knows how to count what is gone to find that fifteen remain.

We learned the alphabet as a series of letters in a fixed order. We learned the sequence from A to Z, never from Z to A. Now, whenever we alphabetize a list or use the dictionary we must often say the entire alphabet from its beginning just to know if "P" comes before "R." A child who can count up and back can find his or her place in a book with much greater ease when page numbers make sense going both ways. Space launches count down. Microwaves count down. Time clocks and game clocks count down. New Years start with countdowns. Our students can count down as well.

Assessment for counting...

We ask our students to count how many. We lead all the counting we do. When we see that our class can count along with us, we let our whole class count without us at all. When we see that our whole class can count well without us, the counting they do is done in small groups. The small groups grow smaller as counting skills improve. We keep on asking, "How many?" We also ask, "What number do you think will come next?"

At age three, Billy knew about "three." Do we have any doubt that by age four, Billy will know something else about "three?" And at age five, will Billy know even more? Would we have worried or would we have smiled as Billy at three proclaimed that no one in his world was older than he?

We count and we count and we count. We make meaningful counting a part of each day. We give our students the time that they need to grow and to understand. We know where we are going. We know what it is our students will learn. We do not doubt that the learning takes place. We keep what we teach and how we assess as simple and natural as the learning itself. From watching our students, we know what we know. Watching and listening are all the counting assessment we need.

Lesson Two

Purpose	Learn to look for patterns in the counting numbers.
Summary	We post numbers for students to search for patterns.
Materials	0-100 column of numbers.
Topic	Search the numbers from 0 to 100 for patterns.
Topic	Learn to say the names of numbers.
Topic	Look at the 00-99 matrix once again.
Homework	We send numbers home so the search for patterns may be shared.

Numbers are different than letters...

The English alphabet has twenty-six letters. Twenty-six letters are all we need to write any English word. No matter how simple. No matter how complex. Our number system has ten digits in it, from zero through nine. Ten digits are all we need to write any number. No matter how simple. No matter how complex.

We do not expect our students to understand the order of the letters in our alphabet. We teach the order by rote. We do not expect our students to understand the order of the digits from zero to nine. This order, too, is learned by rote. But numbers are different than letters. Zero through nine are digits to be memorized. For numbers beyond nine, there are patterns to be seen.

Numbers in columns...

(illustration 4-2-1)
(The ideal illustration is of a 0-100 number strip (not a number line, but a number column). It may be difficult to show the whole strip in the illustration. Aim for the ideal, but if necessary, split the strip 0 through 49 and 50 through 100. The number column has 0, 1, 2, and not 00, 01, 02. The strip has identifiable columns labeled "tens" and "units" for reference.)

Teacher: Look at this column of numbers and tell me what patterns you can see.

The students tell what, if anything, they see. The teacher listens and clarifies what is said.

Student: Some numbers are the same.
Teacher: This column is called the "units" column and this column is called the "tens" column. Which column do you mean?
Student: That one.
Teacher: Show me. Come up and point to it.
Student: (Points.)
Teacher: This is the tens column. What numbers are the same?
Student: The ones. And there are twos that are the same, too.
Teacher: How many ones are there here?

The teacher loops the numbers to be counted.

(illustration 4-2-2)
(The ones looped.)

Class: (No answer or random guesses or maybe even a knowledgeable "ten.")
Teacher: Let's count the ones as I point to each one.

The teacher counts to ten with the class.

Teacher: Ten ones are inside this loop. How many twos do you think there might be?

(illustration 4-2-3)
(The twos looped.)

Teacher: Let's count them and see.

If someone comments that there are also many threes or fours or fives in the tens column, the threes, then fours, then fives are looped and counted.

If no one sees any patterns, the teacher helps the process along.

Teacher: This column is called the "units" column. Look at the numbers in this column. What number is at the top?
Student: Zero.
Teacher: What number comes next?
Student: One.
Teacher: What number comes next?
Student: Five.
Teacher: A five and a two look a lot alike. I sometimes get them mixed up. But, this number down here is the five. What is this number that comes right after the one?
Student: Two.
Teacher: And what number comes after the two?
Student: Three.
Teacher: And what number comes after the three?

The teacher continues asking for all the numbers in the units column from zero through nine. After the number nine, the teacher asks:

Teacher: And what number comes after the nine?
Student: Zero.
Teacher: We said zero before. Zero was the number at the top. Are the numbers starting over again? Is there a pattern we can find that will tell us what numbers we will see next.

(illustration 4-2-4)
(0 through 9 looped.)

Teacher: How many numbers are inside this loop? Let's count them and see.

The teacher counts to ten with the class.

Teacher: Does this pattern of numbers happen again?

(illustration 4-2-5)
(Loop, 0 through 9, 0 through 9, 0 through 9.)

In Lesson One, we counted and counted and counted. We said the numbers as we matched the numbers to the people or objects we saw. We counted for comprehension. But comprehension is more than knowing which words to say in what order. Comprehension for numbers also means knowing the structure of the numbers we use.

Names for the numbers...

We look at the numbers for patterns. We talk about all that we see. As we first look, we do not have to know all the right names. When counting what's inside the loop, we do not have to know which of the numbers may be 70 and which may be 17—knowing 0 to 9 is enough. What we learn will soon be connected to what we already know.

Teacher (pointing to the 0-100 column of numbers): Count with me as I point to each number.

The teacher points to each number in the column and says each in turn along with the class.

Teacher: Zero, one, two, three, four, five, six, seven, eight, nine.
These next numbers are the hardest ones for me to get right.
Ten, eleven, twelve, thirteen, fourteen, fifteen, sixteen, seventeen, eighteen, nineteen.
See if you can hear a pattern for how we say these next numbers.
Twenty, twenty-one, twenty-two, twenty-three, twenty-four...

The teacher stops after twenty-nine and gives the students the opportunity to describe any connection they might have heard between the spoken numbers from twenty through twenty-nine and the numbers written in the column. The whole class counts together. The whole class discusses what they see and hear together, as well. What one or two may discover is knowledge shared with everyone in class. It is not important for the students initially to be able to identify patterns or make connections. It is important for the students to know that there may be patterns in the numbers that they might hear.

Teacher: See if you can hear a pattern for how we say these next numbers as well.
Thirty, thirty-one, thirty-two, Thirty-three...

Thirty, forty, fifty, sixty, seventy, eighty, ninety. There is more than one pattern children can hear.

Seventeen and seventy...

Kevin, in the fifth grade, fills in the spaces on his worksheet. The worksheet has pictures of sticks bundled in groups of ten. For one problem, there are seven bundles of sticks with no sticks left over. The page has a space for Kevin to write in the number of sticks he sees. Kevin writes 17. If there had been seven bundles with one stick left over, we might think Kevin had reversed the numbers. But there is no stick left over at all.

Teacher: Kevin, how many bundles are there in this picture?
Kevin: Seven.
Teacher: And how many sticks did we say were in each bundle?
Kevin: Ten.
Teacher: Can you count the bundles by tens?
Kevin: Yes. Ten, twenty, thirteen, fourteen, fifteen, sixteen, seventeen.

How many years had Kevin gone without anyone's hearing the confusion he had in his mind? Our students do not know what we think they know. As we teach, we ensure that our students see and hear the same numbers we are showing and saying. If our students cannot tell a 13 from a 30 when we say it, then what they learn will not always be what we think we have taught.

The "ty" (tee or dee) at the end of the word *seventy* is clearly not the "teen" at the end of the word *seventeen*. Children who can count to 9 can make sense out of counting to 99. Seeing the numbers and listening for patterns helps children know that there is a sequence that they can learn.

We look at the column of numbers from 0 through 100 and describe the patterns we see. We listen to the numbers from 20 to 99 and describe the patterns we hear. For the numbers between 10 and 19, the patterns we see are not the patterns we hear.

It would make sense to teach words like *tenty*, *tenty-one*, *tenty-two* and *tenty-three* for the numerals 10, 11, 12 and 13. But what we must teach is the language we speak, not a language that might make more sense. We teach words like *eleven* and *twelve* and *thirteen*, knowing these are the numbers that are the hardest for our students to get right. There are always patterns in numbers. There are not always patterns in words.

Lesson Three

Purpose	Learn the difference between 1, 2, 3 and 1st, 2nd, 3rd.
Summary	We use numbers in language to convey meaning.
Materials	The language we use.
Topic	Language use is not lesson bound. The use of cardinal and ordinal numbers is done consciously everyday.
Homework	We send parents a list of words to emphasize at home.

Cardinal then ordinal...

Cardinal means:

1, 2, 3.
Counting how many.
Weighing how much.
Measuring how high.
Budgeting money.
Telling time.

Ordinal means:

1st, 2nd, 3rd.
First prize.
Second seat from the left.
Third place.
Fourth Street.
Fifth grade.

We use cardinal numbers for counting or for answering questions like, "How many are there?" We use ordinal numbers to indicate order or place. We do not need to teach the definitions. Our language is not that precise. We learned the word *car* by seeing cars and hearing the word car. We learn the words *one*, *two*, *three* and *first*, *second*, *third* by seeing and hearing examples of what the words mean.

We teach our students language by using language. We teach our students about ordinal numbers by using ordinal numbers as we speak:

I would like the first person in each row to hand the take-home notes to the others in your row.
You are all in one long line, but we need to form two lines for going to the assembly today. Every second person please take one step forward.
Why do you think this grade is called third grade? (Or first or fourth or whatever the number is.)
When we divide into work groups for our art activity, I want the first person in your row to pass out the paper, the second person to pass out the paint, the third person to pass out the water and sponges, and the fourth person to pass out the paper towels. The fifth person will be in charge of passing out the clean-up materials at the end of the period.

Lesson Three is not a lesson. Lesson Three, like Lesson One, is a part of life. We want our students to understand the use of ordinal numbers, so we create the environment that makes ordinal numbers a natural part of all we do. Lesson One is counting, because the way we learn to count is to count. Lesson Three is using words, because the way we learn to know what words mean is to hear them used.

Lesson Four

Purpose	Learn the fiveness of five.
Summary	We surround our students with the concept of numbers from three to as far as we decide to go.
Materials	Power Block squares or tiles, toothpicks, wooden cubes, Pattern Blocks.
Topic	3 with squares.
Topic	3 with tooth picks.
Topic	3 with wooden cubes.
Topic	3 with Pattern Blocks.
Topic	4 with squares.
Topic	4 with tooth picks.
Topic	4 with wooden cubes.
Topic	4 with Pattern Blocks.
Topic	5 and more with each material, in turn.
Homework	We send the number stations home to be done with any material present in the house.

Doing numbers...

(illustration 4-4-1)
(Three Power Block S-1 squares. Boxes or tubs of Power Block squares visible.)

Teacher: Today we will be doing numbers. The number we will be doing is three. Please check your neighbor and have your neighbor check you to see if you each have three squares. Then, please tell me how many squares you have.
Students: Three.
Teacher: Watch as I make a design with my S-1 squares on the overhead.

(illustration 4-4-2)
(Square design on the overhead.)

Teacher: Do you think I can make a different design?
Students: Yes.
Teacher: Show me with your three squares what other design you think I might be able to make.

(illustration 4-4-3)
(Several designs, some touching, some not.)

The teacher walks around the room looking at the students' designs.

Teacher: You sure can think of a lot of different designs that I might make with my three squares.
I am going to add a rule for the designs we make. My rule is that each of the squares in the designs we make must touch another square. No square can be put off all by itself.

The teacher illustrates this rule by making a series of three square designs on the overhead. For each new design, the teacher asks the students if the design fits the rule. Since the rule the students are trying to understand is the teacher's rule, the teacher provides responses of "correct" or "incorrect" as the students say "yes" or "no" to the designs.

For each design in turn, the teacher also asks the students to explain why the design fits or does not fit the rule. If no one can provide an explanation, the teacher explains whether or not the rule fits the design. Once the students understand the rule, the teacher gives each work group a set of squares and the students begin making designs for three on their own.

Teacher: I would like to see how many different ways you can think of to make square designs for three squares. So, each time you make a new design, leave it made. Make your next design using a new set of three squares.

See if you can make up a way that is different from the ones made by anyone else in your group.

Our meaning clear...

We provide our students with manipulative materials in mathematics so that each child has the opportunity to experience for herself or himself the concept we are teaching. We also give our students manipulative materials to use so we may see each child's thinking acted out.

We say, "Make designs for three." As our students work, we can see in the materials how well our students understood our words. We walk around our room watching to see who understands the assignment and who needs further explanation. If only one or two students do not understand, we provide additional clarification to these few individually. If many do not, we clarify our instructions for the class—phrasing the assignment in a different way.

If we cannot think of a different way to phrase our explanation, we learn new ways by working with a single child who does not understand, until our meaning is clear to just that child. As we work with this one child, we learn to make the lesson clear to every other child in our room. We may not always make our meaning clear the first time, but we have more times than one to help our students understand.

Our students check instead...

Teacher: I am really impressed with all the different ways you have found to make designs for three squares. Look around your own group and see the different ways you all have created. You are making so many designs that there are just too many for me to check. I need you all to help me check each other's designs so I can see if I have made the rules clear to everyone.

Please check your neighbor and have your neighbor check you to see if all of your designs have three squares in them. Please also check your neighbor and have your neighbor check you to see if each of your three squares is touching another square in your design.

We ask our students to check each other's work because we cannot move around the room fast enough to check it all ourselves. Mathematics is simple and basic and straightforward. Students checking students helps us keep it so.

What doing numbers does...

Counting numbers or saying numbers is not the same as understanding numbers, just as reciting the alphabet is not the same as reading words. Billy could say, "Three." He could hold up three fingers. He could give three as an answer to a question that asked for a number. And, if he were asked, he could even count "One, two, three". But, what did Billy understand? How was he to learn that three was more than just a word to say?

How do parents convey to a child all the meanings of the word *car*? Do they teach *car* by saying "car" to their child again and again when no car is present for their child to see? Do they put the notion of what a car is inside their child's head by writing out car and having their child memorize the word? We know there are no special lessons that parents use to teach their children the meaning of a word. If cars are in the environment of the child and people around the child talk about what the child sees, the child will learn the meaning of the word.

What doing numbers does is to put numbers in the environment of the students in our room. We surround our students with numbers and with other students who talk about the numbers seen. Doing numbers is how our students learn that three is more than just a word to say.

As our students make their designs for three, they count to three again and again. They count each time they design. They count when they check what their neighbor's work. They count and they count and they count.

They see the quantity three in a thousand designs. They see it in what they make. They see it in what is made around them. They see it in squares and toothpicks and cubes and blocks and everything else in their lives. They count and they look and they see for themselves what a three might be like.

Three is not two. Two is smaller. Two is a part of all of the threes they have made. Three is not four. Four is much bigger. Three is a part of all of the fours they will make. Three can be made in a hundred new ways, and each of the hundreds of ways is just three. Three can be tall or wide or off at an angle. Three can be steps or look like small houses or parts of a fan or prints from a bird walking by. All of the ways that a three can be seen have one thing in common—each of them is a three.

Three for all...

Our initial assessment is that no one knows anything. So, *three* is the number we start with for all our students. For some students, three is easy. For others, three is hard. We start with three for all our students because there is more to the learning than the numbers we teach.

Three for all means that all our students learn the structure of the lesson for doing numbers at a level they can do. Three for all means that all of our students, regardless of their level of ability, feel a part of the lesson. No one is separated out for how much faster he or she might go. No one is separated out for what he or she can or cannot do.

When all of our students do *three* together, help is available for those who do not understand from those who do. Three is easy. Not all numbers are.

When we do threes, all our students together do threes. When we do fours, all our students together do fours. Would we say "car" to some students and not to all others? Or, would *car* be a word we would speak to all in our class? Would we show just a handful of students the cars passing by? Or, would we give all our students the same chance to learn? Leaving a child behind as the others move forward teaches that child—and all other children—that we do not believe he or she can learn. Everyone learns when everyone helps. There is more to the learning than learning the numbers we teach.

Squares then toothpicks then...

We do the number three with Power Block squares. We follow the squares with toothpicks, then wooden cubes, then Pattern Blocks. Four materials. Four different chances for us to make our meaning clear.

Once our students know how to make and check designs for three using squares, they have already learned the structure for making three with each new material we introduce. The structure is the same, but the learning is different.

Teacher: Today we will be doing numbers again. The number we will be doing is three. Please check your neighbor and have your neighbor check you to see if you each have three toothpicks. Then, please tell me how many toothpicks you have.
Students: Three.
Teacher: Watch as I make a design with my toothpicks.

(illustration 4-4-4)
(Toothpick design on the overhead.)

Teacher: Do you think I can make a different design?
Students: Yes.
Teacher: Show me with your toothpicks what other design you think I might be able to make.

(illustration 4-4-5)
(Several designs, some touching, some not.)

The teacher walks around the room looking at the students' designs.

Teacher: You sure can think of a lot of different designs that I might make with my three toothpicks.
I am going to add a rule for the designs we make. This is the same rule I made up for the squares. My rule is that each of the toothpicks must touch another toothpick. No toothpick can be put off all by itself.

The teacher illustrates this rule by making a series of three toothpick designs on the overhead. For each new design made the teacher asks the students if the design fits the rule. The teacher provides responses of "correct" or "incorrect" as the students say "yes" or "no" to the designs.

Teacher: I would like to see how many different ways you can think of to make toothpick designs for three toothpicks. So, each time you make a new design, leave it made. Make your next design using a new set of three toothpicks.
See if you can make up a way that is different from anyone else's in your group.

The teacher walks around observing who understands the assignment and who needs further help.

Teacher: I am impressed with all the different ways you have found to make designs for three toothpicks. You are making so many designs that there are just too many for me to check. I need you to help me check each other's designs so I can see if I made the rules clear.
Please check your neighbor and have your neighbor check you to see if all of your designs have three toothpicks in them. Please also check your neighbor and have your neighbor check you to see if you remembered the rule about each of your three toothpicks having to touch another toothpick in your design.

The toothpick lesson is the square lesson. The designs are different, but the counting and seeing are the same. The thinking and the learning are the same, as well. Children who understood the square lesson will understand the toothpick lesson. The toothpicks give children who did not previously understand a second chance to comprehend.

Wooden cubes...

Teacher: Today we will be doing numbers again. Today's number is three. Please check your neighbor and have your neighbor check you to see if you each have three wooden cubes...
Watch as I make a shape.

(illustration 4-4-6)
(Wooden cube shape on the overhead.)

Teacher: Show me with your wooden cubes what other shape you think I might be able to make.

(illustration 4-4-7)
(Several shapes, some touching, some not.)

What rule do you think I am going to add for the shapes we make?
Students: Each of the cubes must touch another cube.
Teacher: Let's see how many different ways you can think of to make cube shapes for three cubes. Each time you make a new shape leave it made.
I need you all to help me check each other's shapes. Please check your neighbor and have your neighbor check you to see if what I said to do is clear.

The wooden cube lesson is the toothpick lesson. The toothpick lesson is the square lesson. Because the structure of the lesson remains the same for each new material, children who understood for an earlier material continue to understand now. Understanding, once gained, does not go away.

Each new material allows us to bring understanding to more students. Children who understood the square lesson understand the wooden cubes lesson. Children who did not understand squares and who then made sense of toothpicks will also understand cubes. Each child who understands becomes a teacher for all the children who do not yet.

Surrounding literally means surrounding. When our students work with squares they are literally surrounded with ways to make three. When they use toothpicks, our room is once again filled with shapes for three. The cubes, too, fill our room with hundreds of threes. We surround the child with the concept. Three is everywhere. What doing numbers does is put numbers in the environment of the students in our room.

Pattern Blocks...

Teacher: The number we will be doing today is three.
Please look in your tub of Pattern Blocks and pick out three blocks. The rule for Pattern Blocks is that you must use two different colors.
Student: Can we use only one color if we want?
Teacher: No. The rule is that you must use two colors.
Please check your neighbor and have your neighbor check you to see if you each have three Pattern Blocks. Please also check to see that you each have two different colors. You do not have to have the same two colors as your neighbor.
Watch as I make some designs on the overhead.

(illustration 4-4-8)
(Pattern Blocks design on the overhead. More than one design is made because there are so many possible combinations of the two different blocks the students select. Use overhead Pattern Blocks.)

Teacher: What different designs do you think I can make? Show me with your Pattern Blocks.
What rule do you think I am going to add for the designs we make?

Students: Each of the Pattern Blocks must touch another Pattern Block.
Teacher: Let's see how many different ways you can think of to make Pattern Blocks designs for three Pattern Blocks.

(illustration 4-4-9)
(Many designs for three Pattern Blocks. Show walls and stacks, so that the dimension of height is included.)

Teacher: I need you all to help me check each other's designs. I need you to check three different things. First, check to see if each design uses only three blocks. Second, check to see if each design uses only two colors. Third, check to see if all three of the blocks are touching.
What three things did I want you to check? First? Second? Third?

How long do we linger?...

Our initial assessment is that no one knows anything, so we start with three. But once we begin, we can see who is learning and who needs more help. How long do we linger at three? How do we know when it's time to do four? How long do we give our children to learn *car*? And how do we know when it is time to teach *truck*?

We are surrounding our students with the concept of number. We start with three because three is so easy. But learning three does not end when we start four. There are threes in each four that each child makes. If a child needs help with the fours, we teach the child to count out the threes and add just one more. Designs for some children can be three that are touching and one that is not. Fives can be made with threes and with twos. We are the teachers. We make our rules according to what our students need.

Our students are learners. All will learn three when we give them the time. They are still learning threes when we start making fours. They are still learning fours when we start making fives. When we move on together, our students are all learning that we believe they can learn.

How far do we go? Threes are in fours, fours are in fives. Sixes, eights and tens are made up of threes and fours and fives. We do the basic three-four-five. Then we might do six next or we might jump to eight or nine. We let our students be our guide to how long we linger and how far we go.

We look for the learning that we ourselves do...

We assess our students by judging how well they understand the concept of making designs for three. When they as a class can make hundreds of threes, we ask them to make hundreds of fours. After hundreds of fours come hundreds of fives.

We assess ourselves by seeing how well we communicate our meanings to our students. At the same time, we look for the learning that we ourselves do.

All children can learn, and we can learn from all children. We seek out the students who do not understand and we work with them in groups of ones or twos or threes. We experiment with ways of expressing what it is we wish these students to do so that they, too, can understand. The materials we use make it possible for us to see how our students think, even when we may sometimes believe they are not thinking at all.

The challenge of our profession is finding ways to help each of our students learn. The joy of our profession is in making learning possible for all.

Lesson Five

Purpose	Record the number concepts learned in Lesson Four.
Summary	Students learn a different way to record each material. The recordings have a use in Lesson Six.
Materials	Power Block squares or tiles, toothpicks, wooden cubes, Pattern Blocks, a different kind of recording paper for each material, paste or glue. Blackline masters for recording cube shapes.
Topic	The lesson focus is on recording.
Topic	Recording 3 with squares.
Topic	Recording 3 with tooth picks.
Topic	Recording 3 with Pattern Blocks.

Topic	Recording 3 with wooden cubes (upper grades).
Topic	Recording 4 with squares.
Topic	Recording 4 with tooth picks.
Topic	Recording 4 with Pattern Blocks.
Topic	Recording 4 with wooden cubes (upper grades).
Topic	Recording 5 and more with each material, in turn.
Homework	Homework for Lesson Five waits for Lesson Six.

Recordings...

Our earth orbits the sun whether or not we choose to measure the time the orbit takes. Objects we toss in the air fall back to the ground whether or not we can explain their fall. There are countable trees in the forest whether or not we choose to count them. Rainfall can be predicted, whether or not we know how to predict it. What we measure or count or sort or classify with mathematics exists whether or not we know how to measure or count or sort or classify.

Mathematics is the language we invent to help us better organize what we see in the world around us, so that we may discover patterns, make comparisons, know how many or how much, or make sense out of seeming nonsense.

We can see the mathematics in the world all around us. Height of the mountain. Force of the wind. Paths of the planets. Numbers of people. But there is too much to see to remember it all. We record it to store it, to use it for later. Once recorded, we can see discoveries to be made.

Which mountain is tallest in all of our world? We cannot move mountains side by side, but we can move the recordings of the measures we make. Which wind is strongest and when should we hide? We know from our records when we need to find shelter. Where is the sun and where is the moon? We know from our records when the next eclipse will be here. How big is our city? When will it be too big for its water supply? Do we know from our usage records when the water will run out?

We record in symbols: numbers, letters, drawings, diagrams, graphs, charts, maps, blue prints, equations, formulas. Shorthand methods of displaying our data. Words that we write are not the things they describe—they are only the symbols we use to record the idea.

We have them do numbers all over again...

Our students do numbers, then we have them do numbers all over again. Each design in Lesson Four—designs for squares, designs for toothpicks, designs for whatever we use—lasts only as long as the lesson itself. Made for today, gone by tomorrow. No lasting connection from one to the next. In Lesson Five we make threes and we record the threes we make.

We record to teach recording. We also record to send our students back through the same lessons again. Students for whom four was not easy or for whom five required much thought, now go back to the beginning and learn three again. Students for whom all numbers were easy go back as well. We have much more to learn about number than we learned the first time. Recording can show us patterns and connections in what we thought were designs.

Squares...

Teacher: Today when we do numbers, I want you to record the designs you make so that we may save them. The number we will be doing today is three. We will be making designs for three with the Power Block squares. For each design you make today, please record it using the squares of paper I have given you. Please put your name on each of the recordings you make.

(illustration 4-5-1)
(Examples of designs with Power Block squares and recordings of the designs, with squares glued to paper.)

Later on, we will sort the designs, classify them and describe them in writing using numbers and words. Later on, we will see that the numbers for squares, toothpicks, cubes and blocks have patterns in common—patterns we might not have seen on an earlier day.

Toothpicks...

Teacher: The number we will be doing today is three. We will be making designs for three with the toothpicks. For each design you make, please record it on the paper I have given you. Please record only one design on each piece of paper. Please put your name on each of the recordings you make.

(illustration 4-5-2)
(Examples of designs with toothpicks and recordings of the designs on paper.)

For squares, the students pasted squares onto paper. The squares were the right size. No drawing skill was required. For toothpicks, the students must use their perceptual skills to put a line on the page where they think the line should be. The lesson with toothpicks gives us an early opportunity to see which students have difficulty making their pencil or crayon record on paper what they want it to.

Toothpicks are a lesson in drawing. We show a student whose line is too long or short how to get it right. We show a student whose line goes in an opposite way how to turn it around. We show students for whom drawing is a challenge how to draw what they think they cannot draw.

We save our students records to use in later lessons. The focus of this lesson is the recording. Knowing how to copy onto paper the lines that we see is a skill of great use. There is more to be learned in *Beginning Number* than ways to count three.

Pattern Blocks...

Teacher: The number we will be doing today is three. We will be making designs for three with the Pattern Blocks. For each design you make, please record it using the Pattern Block shapes I have given you. Please record only one design on each piece of paper. Please put your name on each of the recordings you make.

(illustration 4-5-3)
(Examples of designs with Pattern Blocks and recordings of the designs.)

Problems occur with recording three-dimensional designs when the shapes we glue down are in two dimensions. Our students may deal with the limits imposed by the recording materials in any way that they devise. We do what we can with the materials we have while we look for ways to do more.

Wooden cubes...

Teacher: The number we will be doing today is three. We will be making shapes for three with the wooden cubes. Recording cubes is pretty hard to do. We'll try it and see if we can do it.

Our students may not always be ready for everything we decide to teach. Shapes made with wooden cubes are in three dimensions. Capturing three dimensions on a two dimensional surface is not easy. How many of our students could we expect to master drawing in perspective when we ourselves may have trouble drawing what we see?

(illustration 4-5-4)
(A set of three wooden cubes with a perspective drawing sitting next to the set.)

We might decide in advance that recording cube shapes on paper is just too difficult for our students. But the best way to assess whether something is too difficult is to let our students try it. We make clear to our students in advance that the task we are presenting is difficult, and we feel free to stop the lesson if no one understands. We learn what is hard and what is easy by what our students show us and not by what we might have thought.

Teacher: Recording shapes for wooden cubes is like drawing blueprints for a building. Usually people don't learn to draw blueprints until they are much older. When you work together in teams, though, you can help each other overcome the difficulties.
Watch as I make a cube shape and record it on the overhead.

(illustration 4-5-5)
(A cube shape at the top of the blackline master recording sheet.)

Teacher: Copy this shape at your table. Check your neighbor and have your neighbor check you, so that we can see that we are all starting with the same shape.
Now, stand above your shape and look down at the top of it. Draw what you see looking straight down on your recording sheet.

(illustration 4-5-6)
(Top view of the shape drawn on the recording sheet.)

Teacher: Check your neighbor and have your neighbor check you, so that we can see if we all agree what the top of the shape looks like.
Now, look at the front of your shape. Draw what you see on your recording sheet.

(illustration 4-5-7)
(Front view of the shape drawn on the recording sheet.)

The process is repeated for the right and left sides and the back.

The bottom is saved for last. Students speculate what the bottom might look like. If possible, the teacher carries the shape around the room on a piece of glass or clear plastic, so students may see the bottom for themselves. Or, one student in each group may hold the shape up and other students report on what the bottom looks like. Teacher and students then record the bottom on their recording sheets.

The class does two or three shapes, following the teacher's lead each step of the way—checking each other's work constantly, to see if the instructions are understood. When students can record 3-D shapes presented by their teacher, they begin making up their own shapes and recording them on paper.

Lessons are contained within lessons. In learning to draw cubes, our students are using mapping skills and spatial relationship skills. Mathematics is connections. Learning is not compartmentalized.

Lesson Six

Purpose	Record in words and numbers while learning to envision what the words and numbers mean.
Summary	Students imagine then write words and numbers to describe numeric designs.
Materials	Recordings from Lesson Five, paper for writing words, separate paper for writing numbers. Number lines. Number line templates. Spelling Notebook introduced.
Topic	Writing words for recordings of 3 squares.
Topic	Writing numbers for recordings of 3 squares.
Topic	Creating square designs for numbers.
Topic	Writing words for recordings of 3 toothpicks.
Topic	Writing numbers for recordings of 3 toothpicks.
Topic	Creating toothpick designs for numbers.
Topic	Writing words for recordings of 3 Pattern Blocks.
Topic	Writing numbers for recordings of 3 Pattern Blocks.
Topic	Creating Pattern Block designs for numbers.
Topic	Repeating the three step writing and creating cycle for the numbers 4 and 5 and maybe 6.
Homework	We send the word and number books home to be read by our students to any audience that can be found.

Universal symbols...

When we see the written words *Kyle* or *candy* or *catastrophe*, what we imagine for these words may not be the image these words were meant to record. We need not know all the Kyles or taste all the candies or experience all possible catastrophes before we learn the universal symbols for all the Kyles and candies and catastrophes that exist. In Lesson Six, our students add universal symbols to their recordings of the designs.

Teacher: Today we will be looking at the recordings you made for your square designs for three and we will record them in a different way Here is a recording of a square design for three. What does this look like to you?
Student: Stairs.

(illustration 4-6-1)

(Three squares in a stair step pattern. Two squares in the bottom row. One square in the top row to the right. First picture, "stairs" written underneath. Second picture, "stairs" stamped underneath. (Reading Program stamps). Caption says: "The teacher writes or stamps the word stairs beneath the design.")

Students using the Reading Program collectively have no trouble reading the stamped word *stairs*. Even with no Reading Program present, the teacher writes the descriptive word beneath the recorded design, regardless of reading ability or the grade level of the class.

We do not limit the words we speak to the words we think our children already understand. We do not limit the words we write to the words we think our students can already read.

Teacher: What else does this design look like to you?
Student: A big building next to a little building.
Teacher: What else?
Student: A backwards letter L.
Teacher: What else?

We ask for more than one description to let our students see that there is more than one way to describe a design. For now, though, we write down only the first description we are given.

Teacher: You have told me that the design can look like many different things. But I have only written down the first way that you said. When you begin looking at the recordings you made for your own designs, you will see many different things each design might be. But I want you to write down only one of the many ways you see.
Please do not write the words on the paper with your design on it. I will give you a different piece of paper for writing your description.

Students using the Reading Program are familiar with the format of words on one page, picture on the next. The books that our students now make from their words and designs share the same format.

(illustration 4-6-2)
(Two design books, not yet stapled together, showing word, design, word, design, word, design. First book written, second book stamped.)

In the Reading Program, the word-picture-word-picture format allows each child to experience the power of his or her own reading. When the word to be read is on a page all by itself, without any hint from an accompanying picture, the focus is on reading the word, not on guessing the word from the accompanying picture. The word read gives the child a sense of anticipation of the picture he or she expects on the following page. Unless the book has been read before, the exact picture cannot be known. When the child turns the page for the first time, he or she sees evidence of the power of this reading and the delight that reading can bring.

In the books that our students make for their designs, the experience is the same. The words are read before the page is turned. Readers imagine for themselves the designs the words might represent before seeing the actual design.

Writing words...

From the earliest grades, students using the Reading Program learn to listen for the sounds in words and stamp out what they hear. Once they can read and write with sounds, they write the letters for the sounds instead.

Reading Program or not, we make writing words for what is seen a requirement for all. We do not wait for writing to be taught. We provide our students the means of writing every word, even words they cannot read. We accept invented spellings if our students have the knowledge to invent. We can also use spelling notebooks, with a letter on each page.

(illustration 4-6-3)
(Spelling Notebook.)

A spelling notebook is any notebook into which words may be written by the teacher for student use. One way to create notebooks is to photocopy sets of 26 lined pages, each headed by one of the 26 letters of the alphabet. Each child in the class receives his or her own book. The pages may be blank or they may include lists of pre-selected words, like "and", "the", "up", "down", and whatever other words are common to that grade.

Anytime a student needs to know how to spell a word, he or she opens his or her notebook to the page headed by the word's starting letter and brings to book to the teacher for the word to be written in. Not all children will be able to find the right page, but as long as students are encouraged to turn to each other for help, the searching process is a manageable one. None of us is as smart as all of us, and spelling notebooks are better used when we take advantage of the collective knowledge in our room.

We can also reduce our own writing burden by asking who in class knows how to spell a particular word and having that person (once tested orally) write the word in the questioning child's notebook.

The notebook may be used in any subject area where writing is desired. As each student's word list grows, the need for writing additional words is reduced. Since students must open their books to the correct page before asking the teacher to write a word, and since it is usually the case that children learn to read the words written in their own books, the more words written, the less words to write.

The books are more effective if the words they contain are not used as the basis for individual spelling tests. If students know the words are to be the subject of tests, they will be reluctant to ask for words to be added to their books.

We make it possible for all to write, even those who do not yet read. We do not think of why they cannot, we think of ways that they can. We accept whatever is written. Letters in any shape are good enough for now. Writing is a part of mathematics. Writing is part of all we do.

Writing numbers...

Teacher: I am impressed with all the different words you have written to describe your designs. I am going to show you a different way to record the same designs. Look at this design again.

(illustration 4-6-4)
(Repeat the first stair-step design illustration in this lesson.)

Teacher: Before, you told me this design looked like stairs and buildings and letters and all kinds of things. Now, please tell me what numbers you could use to describe it.
Student: One and two.
Teacher: Where is the one and where is the two? Please show me.

(illustration 4-6-5)
(The stairs broken apart, with one square to the left and two squares to the right.)

Teacher: We record that in numbers like this.

1 + 2

Teacher: Mathematicians put the plus sign between the numbers to show the numbers add together to make the number we were doing. What number were we doing when we recorded this design?
Students: Three.
Teacher: And, what is one plus two?
Students: Three.
Teacher: Can anyone see a different way to describe this design with numbers?
Student: One and two.
Teacher: How is that different? Please show me.

(illustration 4-6-6)
(The stairs broken apart, with one square on the top and two squares on the bottom.)

Teacher: Okay. We write that one plus two like this.

1
+2

Teacher: When mathematicians write the number down like this and not sideways, they put the plus beside the bottom number. We still read this as "one plus two." What is one plus two?
Students: Three.
Teacher: Can anyone see a different way to describe this design with numbers?
Student: Two and one.
Teacher: Please show me.

(illustration 4-6-7)
(The stairs broken apart, with the two squares on the diagonal pulled up and away from the one square that is in the lower right corner.)

Teacher: We could write that like this.

2
+1

Teacher: But mathematicians choose between either writing it sideways or up and down like this.

1 **or** **2 + 1**
+2

Teacher: Kris, since you pointed out the two plus one in this design, you may decide whether we shall record it up and down or sideways.

Student: Sideways.
Teacher: What is two plus one?
Students: Three.
Teacher: Can anyone see a different way?
Student: One and one and one.
Teacher: Please show me.

(illustration 4-6-8)
(The stairs broken apart, with all three squares pulled away from each other.)

Teacher: Shall we write that up and down or sideways?
Student: Sideways.

1 + 1 + 1

Teacher: What is one plus one plus one?
Students: Three.
Teacher: Can anyone see a different way?

We ask our students for ways to describe a design using numbers. If our students do not understand our meaning, we describe the design in numbers for them and show them what our numbers mean.

Once our students understand how to record their designs in numbers, they record the numbers they see for their designs. Each design has a page of its own. Words have their page. Numbers, too, have a page.

(illustration 4-6-9)
(A recorded design with an accompanying word page and a separate number page.)

We provide our students the means of writing every number. We put the numbers zero through nine on each student's desk. Each of the numerals has dots for counting, in case the child forgets the numeral's name.

(illustration 4-6-10)
(Number line on child's desk. The appropriate number of dots below each number. Also show the number line templates.)

Writing numbers with all the right strokes in all the right directions is something our students will learn. We do not think of why they cannot, we think of ways they can. We do not wait to record designs using numbers until the numbers can be drawn without a flaw. We accept whatever is written. Numbers in any shape are good enough for now. Numbers are a part of mathematics. Numbers are part of all we do.

(illustration 4-6-11)
(Recorded designs for Power Block squares, toothpicks and Pattern Blocks. Each design has a descriptive word written by it. The numbers 2+2+1 are also written by each design.)