(Examples of drawings made by the students in class. Include efforts to draw the block in perspective as well as efforts to draw and label top, front and side views. Also include traced drawings.)

As the drawings are collected, the teacher keeps drawings separate by each half of the room. Drawings from the first half of the room are then shuffled and given to the second half. Drawings from the second half of the class are shuffled and distributed to the first. This is the teacher's way of ensuring that no one receives one's own drawing back again.

Teacher: As you look at the drawing that I pass to you, see if you can match the block in the drawing to a block in the set of Geoblocks. When you think you've found the block, put that block on top of the drawing, then wait until everyone else is through.

When everyone has either found a block to match the drawing or has given up on finding any match at all, the students return the drawings to their originators, one half of the class at a time.

Teacher: Everyone on this half of the room take the drawing that I gave you and the block that you think the drawing describes back to the person whose name is on the drawing. See if the block that you take to the person who did the drawing matches the block from which the drawing was made.

The first half of the class takes the drawings back to the originators. Then the second half of the class pays a similar visit to the first.

Teacher: What made some drawings easy to use and some more difficult?
Student: I couldn't tell how big the block was supposed to be. The drawing was bigger than any of the Geoblocks.
Student: I couldn't tell what size my block was supposed to be, either. It wasn't too big or too small. There just weren't any blocks the same size as the one in the picture.
Student: It needed more sides drawn. There were too many different blocks that looked like they could be the block in the drawing. I couldn't tell for sure which one to choose.
Student: The person that did mine couldn't draw! I couldn't tell what this was supposed to be a drawing of.
Teacher: I can only accept comments that will help a person do a better job of describing the block next time. I cannot accept your saying the person couldn't draw. Please make your comments more constructive in helping the person make a more useful drawing next time.
Student: Okay. It would have helped if the person who made this drawing would have traced around the outline of the block, so all the lines would have been straight and I could tell what the block was supposed to look like and how big the block was supposed to be. The drawing was all bumpy.
Teacher: Thank you. That is more useful to know.
Can anyone tell me something about a drawing that they had that made it easy to find the right block?
Student: When the block was traced.
Student: And when they said how many sides it had.
Student: And when they wrote words like top and bottom and side, so you could tell what you were looking at.
Student: This person traced all the sides and then drew a regular picture of the block. That made it pretty easy, but I don't think that would have helped me draw my block better, because I can't draw a block that well.
Teacher: I think you can learn to draw the block that well, but if you can't yet, then tracing works pretty well for now.

As the students discuss the easy and the confusing parts of the drawings, the teacher keeps the focus of the discussion on what can be done to make each drawing a better drawing next time.

The discussion focuses on learning and not on blaming. It is not necessary to determine if the drawer or the reader of the drawing is at fault for failure to identify the right block. We can all learn how to describe objects visually on paper so that others can identify the object the picture represents.

Teacher: Now draw a picture of a different Geoblock so that someone else in class can use your picture to identify the Geoblock you drew. See if any of the ideas we discussed help you make a picture that more readily identify your block.

From simple to complex...

(illustration 6-6-4)

(Six views of a two-Geoblock structure: top view, front view, right side view, back view, left side view, bottom view. Each view is labeled top. front, right side, etc., as appropriate.)

Teacher: Here are six different views of a Geoblock structure. Show me the blocks you think these drawings represent.

(illustration 6-6-5)
(Two Geoblocks placed next to the drawings of the Geoblocks.)

Teacher: Now draw a picture of two Geoblocks placed side by side or one atop the other so that someone else in class can use your picture to identify the Geoblocks you drew.
Will you always need to draw six views?
Is there a pattern to the kind of structures that might need fewer than six views?
We'll see what you find out.

As students become skillful in drawing recognizable descriptions for two-block shapes, we ask them to describe three.

(illustration 6-6-6)
(Six views of a three-Geoblock structure: top view, front view, right side view, back view, left side view, bottom view. Each view is labeled top. front, right side, etc., as appropriate. Three Geoblocks placed next to the drawings of the Geoblocks.)

Our students describe one block until they can describe one block well. Two blocks together follow one. Describing three or four blocks together may come this year or may wait for next year's teacher to try. We do not rush our students along some predetermined path. The quality of learning is more important than the quantity.

Black-eyed peas...

An inservice instructor was teaching a lesson on place value to a group of teachers. The teachers had been doing "plus-one" activities with beans and cups on place-value trading boards. The inservice instructor usually taught the lesson using red kidney beans. At this particular inservice, however, the instructor was using the black-eyed peas she had purchased on a recent journey to the south.

A conversation overheard:

First inservice participant, quietly to her friend: I can't do this place-value lesson with the children in my class.
Second inservice participant, quietly in response: Why not?
First inservice participant: Where will I get the black-eyed peas? There are no black-eyed peas where I live.

What would we say in response if we were the second inservice participant? What would we substitute for black-eyed peas to teach place value in our class? What would we substitute for Geoblocks if there were no Geoblocks in our room?

We could ask our students to draw any object in the room no larger than their paper. The rule for the drawings could be:

Shapes only are allowed.
No words except *top*, *side* and *front*.

(illustration 6-6-7)
(Examples of objects from the class drawn top, front, side, with the actual object placed next to it. Possibilities: book, ruler, clock, Power Block, simple Lego structure, crayon box, baseball, hockey puck.)

What is allowed?...

As our students describe one or two or three or more blocks, what resources may they use to create their geometric images? They may trace the blocks. May they draw their block on graph paper with straight lines already printed on? May they assist their tracing with a ruler's steady line? May they measure before they trace then draw the measurement and not trace the block? Is it permissible to use measurement before measurement is taught as a separate lesson in our class?

May they draw their object using just one view, if the single view is a perspective classmates recognize? If a student used one view and only one, could someone tell from the drawing which way the student was facing the block as he or she drew it? What is "perspective geometry" anyway? What does "perspective" mean?

May our students draw an object bigger than their paper, even though an earlier rule might have said the object should not be larger than their paper? What if they find a way to draw a larger object to a scale that fits their paper?

Can our students draw something far larger than their paper and still convey the geometric image of the object drawn? Could they draw their house or apartment or their school from the front and side? Could they imagine and draw what it must look like from the top?

May students work together to make their drawings, or is drawing something they must do alone? Do architects work in isolation or in teams?

What is allowed? What is learned? What is geometry?

Lesson Seven

Purpose	Learn what an angle is and how to measure it.
Summary	Students learn to use angles in giving instructions. They learn to measure angles and use a protractor as a measuring device.
Materials	Students in the class, paper, protractors, straws, sticks, calculators.
Topic	Each new question asked or material explored is like a lesson of its own.
Topic	Students direct each other using paces and turns.
Topic	Students find right angles in the room.
Topic	Angles are measured with straws and sticks, as lists of successively larger angles are made.
Topic	Protractors are explored.
Topic	360°

Turns...

Teacher: Everyone stand up. Show me what you think it means to turn one and only one complete turn all the away around.

Students demonstrate their individual understanding of the teacher's words.

Teacher: Did you all turn the same way?
Student: You didn't say we had to.
Teacher: True. But if I wanted you all to turn the same way, what would be a good way for me to tell you to?
Student: Tell us to turn left or right.
Teacher: Sometimes people say clockwise or counter clockwise to give direction, but we'll use left and right.
Show me what you think it means to turn one half turn to the left.

Fraction questions do not have to wait for fraction lessons to begin. Our students know the meaning of half a piece. Do they know the meaning of half a turn?

Teacher: Now, a half turn to the right.

No matter the grade level of our students, it is useful to assess who knows left from right.

Teacher: Everyone please turn a quarter turn to the left.
Is everybody now facing the same way as everybody else?
How many quarter turns to the left will it take until everybody is facing me again?

Teacher: I have put a treasure in the room that I will now direct Sammy to find. Do not get too excited about the treasure. Its only a piece of paper with an X drawn on it.
Sammy, please stand up.
Turn right one quarter turn.
Take a small step forward.

Turn right another quarter turn.
Take five paces straight ahead.
Now take one more pace.
Turn left a quarter turn.
Take two more paces forward.
Turn right a quarter turn.
The paper with the X on it is right in front of you.
Now everybody pick a partner to give instructions to.

When we have demonstrated how to give directions, our students may direct each other around the room using paces and turns.

The students giving the instructions pick an object somewhere in the room to which they will guide their partners. If the class does not divide evenly into twos, a group of three can have the first two students provide instructions to the third.

Periodically, we stop our class and ask what methods they have found so far that make the instructions easier to follow. It may be that quarter turns have been divided into eighths. It may be that paces have been replaced by measured steps. It may be that students familiar with programs like Logo are giving instructions using numbers like 90 or 180 instead of quarter turn or half. Logo and other kinds of knowledge are a part of students' lives. We encourage our students to use whatever they might know, from wherever their knowledge might come.

As a variation on the theme, we may ask our whole class to provide instructions to a single student. The single student steps outside the room while the class selects the object to be guided to. When the student steps back in the room each member of the class, in turn, gives one instruction to the student. The cumulative instructions are the student's guide. When the class can guide the student well, we may blindfold the student and have our class guide the student around the room again.

As a further variation, we may ask our students to give their instructions in writing. The written instructions prepared for this activity may look much like instructions written for measurement and mapping. (*Measurement, Estimation and Time*, Lesson Two, page 000.) Similarity between lessons is something that we welcome. Mathematics is connections waiting to be made.

What is an angle?...

Teacher: What is a rectangle?
Student: Like a box that isn't square.
Student: Something you draw that has four sides.
Student: And the four sides have to be even with each other.
Teacher: What do you mean by "even with each other"?
Student: You know, going the same way.
Student: He means parallel. Four sides, opposite sides equal to each other in length. Also opposite sides parallel to each other.

As students add more detailed explanations of a rectangle, will it occur to anyone of them to add in corners that have angles of 90°? Most definitions given by elementary school students describe the sides and not the angles.

Do students ignore the angles of shapes because they do not know what angles are? Or do they ignore the angles because angles are not something they have talked about in school? We do not have to know the cause, we only have to know the cure.

An angle is the measure of a turn. Children and adults who have used Logo on a computer screen to design geometric shapes know that turns are described in directions and degrees. An angle is also the meeting place for two lines.

What knowledge do our students have of angles from their lives outside of school? What meaning do they give to the word *angle*?

Teacher: What is an angle? Can anyone describe to me in words what they think an angle is?
Student: Two lines that touch.

(illustration 6-7-1)
(Drawing of two wiggly lines coming together at a single point.)

Student: No! Two straight lines that touch.

(illustration 6-7-2)
(Teacher draws one line and then draws another directly on top of it. This will have to be described in the accompanying caption, since a line drawn on top of a line looks a lot like a single line.)

Student: No! Not on-top touching. Meeting touching. They touch at the ends. Two straight lines touching at their ends.

(illustration 6-7-3)
(Straight line with a curved line touching the straight line at both ends.)

Student: No. See, the line on top isn't straight, its curved. Both lines have to be straight.
Teacher: Okay. Two straight lines touching. What about this?

(illustration 6-7-4)
(Two straight lines crossing.)

Student: That's four angles all together. There is a different angle between each time there are two lines. You said to draw only one angle. Two straight lines touching just at their ends.

(illustration 6-7-5)
(Two straight lines touching—an angle.)

How do we know?...

(illustration 6-7-6)
(Two angles. Each drawn with sides the same length. The angle on the left is bigger than the angle on the right.)

Teacher: Which of these two angles is bigger?
Student: The one on the left.
Teacher: How do you know?
Student: Because it is. It opens up farther.

(illustration 6-7-7)
(Illustration from above with the sides of the smaller angle on the right drawn longer.)

Teacher: Now which angle is bigger?
Student: They're still the same size. All you did was make that one have longer sides.

Will any of our students know this at the start? As our students measure angles, they will see that the right angle in the corner of the room is no bigger and no smaller than the right angle in the corner of a box. They will see that when we measure angles, we are measuring how far a line has turned, not how long the line might be.

Teacher: Fold your paper anywhere, like this.

(illustration 6-7-8)
(Piece of paper with a corner folded over at an odd angle.)

Teacher: Look at the edge of your fold. Estimate where the middle of this edge might be. Now fold your paper again so that one half of that edge of your first fold line is exactly on top of the other half of the edge.
Check with your neighbor and have your neighbor check you, so I can see if I have made my instructions clear.

(illustration 6-7-9)
(Second fold of paper. A right angle is formed by the paper fold. Indicate in the caption the corner of the fold that the following dialog is discussing.)

Teacher: I want you to use the paper you have folded as your angle-measuring device. See how many objects or places in the room you can find that are exactly the same angle as the angle you just folded. Make a list of everything you find. You may work with a partner if you wish. If you need a word spelled, you may bring me your spelling notebook.

We start with folded paper and an angle that is easy to identify. We can measure angles endlessly.

Teacher: What angles can you find that are bigger than the angle of your folded paper? Which angles are smaller? How do you know?

What is meant by bigger? What is meant by smaller? The concept of angle develops slowly over time. There is no hurry in the teaching that we do.

Teacher: The angle that we folded is called a "right angle." It is called "right" no matter which way it is turned. The angles that you found that were smaller than a right angle are called "acute angles." The angles that are larger than a right angle are called "obtuse angles."

We introduce vocabulary when the vocabulary has a use.

Teacher: Here is a device we can use for comparing angle sizes.

(illustration 6-7-10)
(Equipment: One straw, two sticks or dowels thin enough to fit inside the straw, some masking tape. Tape the sticks, one each, protruding from each end of the straw. Bend the straw in the middle. This is an angle measuring device.)

Teacher: How can we use this device to help us know which angles might be bigger or smaller than another?
Make a list of at least ten objects in the room. The first object that you put on your list should have at least one small angle. The next object should have an angle bigger than the angle that you measured for object number one. The third object should have an angle bigger than the angle on object number two. Each new object that you add should have an angle bigger than the angle on the object just before it. You may work with a partner in creating your list.

The act of comparing angle sizes helps students focus on the attributes being measured and should be done initially without using any standard units of measure.

By degrees...

Teacher: What kinds of standard measurements do we use when we measure how long things are?
Students: Inches... Feet... Meters... Miles... Yards...
Teacher: What kinds of standard measurements do we use when we measure weight or mass?
Students: Ounces... Pounds... Grams... Kilograms... Tons...
Teacher: What is the standard unit we use when we measure angles?

If our students already know the answer is degrees, there is one less thing we have to teach this year.

Teacher: We measure angles with degrees. The degrees for angles are not the same degrees we use for taking temperatures. When we measure distances, we use a ruler or tape measure. When we measure degrees for angles we use a protractor.

(illustration 6-7-11)
(A protractor. The following dialogue assumes that the protractor has an arrow at the midpoint of the straight edge for positioning. The assumption is also made that there are double rows of numbers on the protractor.)

Teacher: I've drawn an angle on the overhead. Where do you think the straight edge of the protractor might go on the angle?
Student: On one of the sides of the angle.
Teacher: Why?
Student: Because it's straight, too.
Teacher: Where do you think this arrow in the middle of the straight edge of the protractor is meant to point?
Student: In the middle of the side of the angle.
Teacher: Like this?

(illustration 6-7-12)
(Angle with the straight edge of the protractor on one of its sides. The center-point arrow on the protractor pointing to the midpoint of the angle's side.)

Student: Yes.
Teacher: Then what size would the angle be?

Student: Either 0 or 180. Probably 180, because it's too big to be 0.
Teacher: Then let's measure a different angle.

(illustration 6-7-13)
(Angle larger than the angle in the illustration above with the straight edge of the protractor on one of its sides. The center-point arrow on the protractor pointing to the midpoint of the angle's side.)

Teacher: Is this angle the same size as the previous angle?
Student: No it's bigger.
Teacher: What size does the protractor say it is?
Student: Zero or 180. So, that can't be the right way to measure it.
Teacher: Does the angle change in size if we make the sides longer?
Student: The angle's still the same.
Teacher: So, if we make the sides longer or shorter, and the angle is still the same, what part of the angle has not changed?

If our students do not know, we tell them that what remains unchanged is the turn the lines have made.

Student: The amount of turn the lines have made?
Teacher: We can measure the amount of turn by putting the protractor's arrow at the place where the lines meet. The meeting point for the lines is called the vertex. Let's see what happens if we put the arrow at the vertex.
Now, where do you think the straight side of the protractor goes?
Student: On the bottom line of the angle.

(illustration 6-7-14)
(Protractor correctly placed on the angle.)

Teacher: How many degrees are in this angle?
Student: Do we use the top number or the bottom number?
Teacher: Look at the side of the angle where we put the straight edge of the protractor. What numbers are there?
Student: Zero and 180.
Teacher: Which number do you think represents a more reasonable place to start?
Student: Zero?
Teacher: That makes sense to me. We usually start counting with the smallest numbers first.
Student: Then what are the other numbers there for?
Teacher: Let's measure this angle first and then we'll see. What is the size of this angle?
Student: It looks like it's pretty close to 30 degrees.
Teacher: Then let's see what happens if we put the protractor on the other side of the angle.

(illustration 6-7-15)
(Protractor correctly placed on the opposite side of the same angle.)

Teacher: What size is the angle now? Remember, we start with zero.
Student: About 30 degrees again.
Teacher: Which numbers did you use this time?
Student: The other ones.

Students who have had experience measuring angles informally have little difficulty comprehending that smaller or larger angles and smaller or larger numbers on their protractors go hand in hand.

Teacher: Now I want you to try measuring angles that you make for yourselves. Remember, the arrow points to the vertex. What's the vertex?
Students: The place where the lines meet.
Teacher: And the straight edge of the protractor goes along one of the edges of the angle. Does it make any difference which edge you use?
Students: No.
Teacher: Why not?
Student: Because the angle is the same size either way.

Angular explorations...

Once our students can measure angles, we give them problems to explore. Calculators are allowed.

Use your protractor to find the number of degrees for all the different angles in your Pattern Block or Power Block set. If it makes it easier for you and your protractor, you may trace each block on paper and measure the angles of the tracing and not the block itself.
How many different angles in each set?
Which angle is the most common in the set?
Which angle occurs the fewest times?
What is the sum of all the angles for each piece?
Does every piece have a different sum?
Is there a pattern we can find for the sum of all the angles in some pieces that might tell us the sums of angles for other pieces in the set?
Is there a pattern we can find for the number of sides of a polygon and the total for degrees?
What is the sum of all the angles around a tessellating point?

(illustration 6-7-16)
(Show two or three examples of what is meant by a tessellating point.)

Which shapes fit together to form a straight line?
What is the sum of the angles of these shapes that went in to making the straight line?

(illustration 6-7-17)
(Show what is meant by shapes that fit together to form a straight line.)

We can tear the corners off this one triangle and fit them in a line.

(illustration 6-7-18)
(Show a carefully cut paper triangle intact. Then show the same triangle with the three corners torn off. Then show the three corners fitted together so that their bottom edge is a straight line.)

Is this the only triangle whose three corners fit together in a line?
What is the sum of all the angles in this triangle?
Are there any triangles with a different sum?
What do we learn when we measure the angles made by hinged mirrors?
What shape appears when the image of a straight line is reflected in hinged mirrors placed 90° apart?

(illustration 6-7-19)
(Show a top down view of two mirrors at 90° angles to one another, with a line drawn between them as if forming the cross bar of the letter A. Do not show the reflected image, just the setup.)

What angles can we set the mirrors to, to have the images of the line be regular polygons?
Is there a connection between the size of the angles and the number of sides in the polygons?
Can you plan the angle for the mirror and predict the number of sides that you will get? Will dividing 360° by the number of sides on the polygon give you a clue?
What other patterns are there to see?
What else can you discover?
What rules can you invent?

360°...

Teacher: How many degrees on your protractor?
Students: One hundred and eighty degrees.
Teacher: Your protractor measures half a circle or half a turn. How many degrees do you think are in a full circle or a full turn? You may use your calculators to find out.
Students: Three hundred and sixty degrees.
Teacher: Yes, there are three hundred, sixty degrees in a circle. Why?
Students: ?

Why are there 360° in a circle? Why not 400°? Why not 25°? Were the first circles seen by humans long ago measured in degrees? Does the full moon that we see on a summer night have 360 written on its face? Were there any circles at the start of history that had numbers written in? Who decided how many degrees were in a circle and why?

Teacher: How many different numbers can you divide evenly into 360? You may use your calculator to help you find out. You may work with a partner if you wish. Every time you find a way, call it out and I'll write it on the board.

1
2
3
4
5
6
8
9
10
12
15
18
20
24
30
36
40
45
60
72
90
120
180
360

Teacher: Numbers that divide into other numbers without leaving any remainder are called factors of that number. So far, we have found 24 factors for 360. Are there any patterns in the factors we have found that might tell us if we've found them all? Can anyone see a pattern in the factors if I pair the biggest factor with the littlest, the next biggest with the next littlest and so on, like this?

1	360
2	180
3	120
4	90
5	72
6	60
8	45
9	40
10	36
12	30
15	24
18	20

Teacher: If you would like a hint, use your calculator to multiply each pair of factors and tell me what you get.
Students: 360 (again and again).
Teacher: How many different factors can you find for 359? Finding factors means finding how many numbers divide evenly into 359? Every time you find a way, call it out and I'll write it on the board.

1
359

Teacher: So far, we have found 2 factors. I thought there would be more. Its pretty easy to see that 1 times 359 is 359.
How many different factors can you find for 358?

1
2
179
358

Teacher: Let's pair these factors up the same way we did for 360.

1	358
2	179

Teacher: It looks like 360 can be divided evenly by many numbers, but not every number can. Of all the numbers between 1 and 360, I wonder if 360 has the most factors. Can we find any number between 1 and 360 with more than 24 factors? That's a lot of numbers to find out about. If we start with the little numbers first, maybe we can find some number patterns that make the search easier.

Work with partners or groups. See how many different numbers you can find out about today. Any patterns that you find that make your work a little easier, we'll share with everybody else in class.

How long would it take our students to find the numbers that divide evenly into all the numbers between 1 and 360? It might take one child days and days. Thirty children in a room can do it thirty times as fast. Thirty children with parents at home to help can do it even faster.

What might be learned through all this effort besides how to use a calculator to divide and multiply?

People working together to find many answers is a more efficient way to get many answers than working by oneself.
When we find patterns in the answers, the answers are easier to find.
Numbers that are big do not always have more factors than numbers that are small.

We also learn that there is a reason that a circle has 360°. People were looking for patterns in numbers long before we were born. 360° has more factors than any smaller number has. More factors means more ways a circle of 360° can be divided equally without using fractions to describe the equal measurements. Perhaps there was more thought put into math by the people who made math up than we thought.

A question for another day might be: If 360° has more factors than any smaller number does, what is the next larger number that has more factors than 360°?

Lesson Eight

Purpose	Learn to use a compass, protractor and straightedge to explore geometric properties.
Summary	We ask questions and pose challenges for our students that guide their explorations with compass, protractor and straightedge.
Materials	Compass, protractor, straightedge, paper.
Topic	See what you can make.
Topic	Make two or more circles the same size.
Topic	Use a compass to compare the lengths of lines.
Topic	Make two angles that are the same size.
Topic	Copy an angle.
Topic	Divide a line in half.

Constructions...

(illustration 6-8-1)
(Compass for use in drawing circles.)

When we hand out compasses to our students we know to teach them safety rules, just as we teach them safety rules for scissors or pencils or any other object that has the potential for injury. Once the safety rules are understood, the compass is another manipulative for our students to explore.

Teacher: This is a compass. See what you can make. If you wish to, you may use your straightedges and protractors as well.

(illustration 6-8-2)
(Student designs with compasses.)

Questions we might ask to expand the range of geometric constructions our students create:

Can you make two or more circles the same size?
How do you know what size the circles are?
Can you use your compass to measure the length of a line?
Can you use your compass to tell if another line is the same length?
Can you use your straightedge and your compass to draw two angles that are the same size?

(illustration 6-8-3)
(Show how to use a compass and a straightedge to copy an angle. An example can be found on page 392 of *Geometry: An Investigative Approach*.)

Can you use a straightedge and a compass to copy a triangle?
Can you use your compass and your straightedge to divide a line in half?

(illustration 6-8-4)
(Show how to use a compass to bisect a line. An example can be found on page 394 of *Geometry: An Investigative Approach*.)

Can you use your compass and your straightedge to divide an angle in half?

(illustration 6-8-5)
(Show how to use a compass to bisect an angle. An example can be found on page 396 of *Geometry: An Investigative Approach*.)

Can you use your compass and your straightedge to construct a perpendicular line?

(illustration 6-8-6)
(Show how to use a compass and straightedge to draw a perpendicular line. An example can be found on page 397 of *Geometry: An Investigative Approach*.)

Can you use your compass and your straightedge to construct a parallel line?

(illustration 6-8-7)
(Show how to use a compass and a straightedge to make two parallel lines. An example can be found on pages 399-400 of *Geometry: An Investigative Approach*.)

What else can you construct?

What is it that we really want our students to know from our lessons on geometry? Do they really need to know how to measure angles or draw circles before they leave our class? What is the feeling we wish our students to have about geometry from the lessons in our room? Do we expect them to memorize predetermined rules, or do we want them to gain the understanding that comes from exploring lines and shapes? Can we free ourselves to follow the direction of the child, not the direction of the book? If we ask, "What can you create?" can we accept the creations and not wish for something else?

We teach geometry to construct images of mathematical relationships in our students' minds. Mathematics is more than numbers on a page.

Lesson Nine

Purpose	Learn to be aware of the geometry in our lives.
Summary	We ask our students to look more closely at what they already see.
Materials	Paper for writing lists.
Topic	Make a list of rectangular shapes. What other shapes can we list?
Topic	List the tessellations that you can find.
Topic	What shall we look for today? Why are the things that we see the shape that they are?
Topic	What angles, shapes, lines in a bicycle?
Homework	This is a lesson on looking and wondering. Looking and wondering questions can be sent home everyday.

Awareness means...

Look at the clouds. Why do the clouds form billowy curves and not crystal-like squares? Look at the lightning. Why does the lightning form jagged, streaked lines and not curves? Look at the designs the wind blows on the hot desert sand. Why is this the pattern of wind on the sand and not any other? What shape is an apple, an orange, or a pear? Why have these tree-growing fruits taken unique shapes and not the same shapes as each other? Imagine the mass of a mountain or the depth of a canyon nearby. How did the mountain rise up without falling over? How did the canyon get so deep without filling in? Look at the structure of trees. How do the trees know to grow only as tall as their own species and not any taller than that?

We know how to teach about numbers and shapes. But how do we and our students learn to see the mathematics in clouds?

Awareness means learning to look.

Teacher: Today, I want you and your partner to make a list of all the objects you can find that have rectangular shapes. You may use your spelling notebook if you need assistance in spelling a word.

What other kinds of awareness questions might we ask for shapes?

Are there more rectangular shapes in our room than any other kind?
If there are more rectangular shapes, is this just because we are in school?
What might be the most common kind of shape inside of school or out?
How many different kinds of shapes might we find just looking out the window?

Teacher: What tessellations can you find? Think of where you might look at school, or at home, or on the way to and from anywhere.

Where are there tessellations?

In the fabrics used for covering furniture and making clothes.
In wrapping paper and wallpaper.
In tiles and bricks that make up walls and floors.
In fences and roofs and sidewalks.
In leaves, flowers, animal skins and fish scales.
In aerial view photos of cities and farms.

Teacher: What objects or designs or angles or lines can you make a list of today? Can you find an example of a shape or an angle or a line that no one else in our class has thought of before?

We can make lists for:

triangles	squares	rectangles	quadrilaterals
trapezoids	parallelograms	rotations	reflected images
tessellations	symmetry	flips	slides
right angles	acute angles	obtuse angles	straight angles
vertical lines	horizontal lines	parallel lines	perpendicular lines
diameters	arcs	perimeters	circumferences
semicircles	circles	ellipses	curved lines
cubes	spheres	pyramids	boxes of all kinds

Teacher: Today, I want you and your partner to look at just a single object and make a list of all the kinds of angles, shapes and lines that you can find. The object for today's observations is a bicycle. You may use your spelling notebook if you need assistance in writing words.

What other real-world items can our students examine for lines, shapes and angles?

wagons	roller skates	cars	busses
houses	fences	buildings	bridges
floors	walls	doors	windows
books	pencils	writing paper	rulers
baseballs	footballs	hockey pucks	playground apparatus
playing fields	tennis courts	jumping ropes	hopscotch paths
ice rinks	ski jumps	diving boards	swimming pools
sidewalks	roads	runways	rivers
escalators	elevators	stairways	fire escapes
dresses	pants	shirts	shoes
trees	flowers	fruits	vegetables
dogs	cats	horses	cows
mountains	valleys	fields	streams
zoos	museums	circuses	planetariums
shelves	racks	fabric	fibers

water spray

Wondering means...

Wondering means wondering why.

Which lines and shapes are human made?
Which lines and shapes are nature made?
Why did humans or nature choose these shapes?
Would other shapes have worked as well?
Why does the creek-bed mud dry and crack the way it does?
Why is every snowflake different from its neighbor, yet most end up as hexagons?
Why do bees make hexagons and not circles in their honeycombs?
Why are oranges round and not the shape of hexagons?
What are the designs made by frost on the windowpane? Why these designs and not others?
Why does the oak's acorn seed wear a hat?
Why do big sheep horns and sea horse tails and breaking ocean waves all curl the same way?
Why are rainbows curved and not triangular?
How many shapes can we find in a pond? How many reasons can we find for all the shapes?
Are there any shapes that nature makes by accident? Or does every shape in nature have a reason for its being?
Are there any shapes that humans make that are made by accident? Or does every shape that humans make have a reason?
Why are no fish shaped like an elephant?

Asking why is what every child does outside of school. Wondering is a part of every child's life. We ask our students to look for shapes to make them more aware of the mathematics in their lives. We ask our students to wonder why things are the way they are to make wondering a part of school.

Lesson Ten

Purpose	The purpose is a teacher purpose. Our assignment is to find the opportunities.
Summary	We make ourselves aware of the opportunities for geometric experiences that exist. We use the opportunities that we find.
Materials	Our own awareness.
Topic	Opportunities that we find.
Homework	What opportunities can our students find at home?

The house across the street...

The house across the street from school was built as the students watched. What did they see?

Surveyors measuring angles and elevations with their sextant-like devices on long poles.
The leveling of the land. What does "level" mean?
Map-like sketches of the future home, drawn to scale on blue paper.
String and chalk rectangles on the ground, marking the foundation lines.
Wooden molds built to parallel the string. Rectangles turned three dimensional.
Trucks pouring water, rock and sand. Cement shapes and slabs spread flat and smooth to dry.
Long, thin oakwood rectangles nailed together to form the house's frame. How many rectangles does it take to make a house? Are there any other shapes involved?
Pipes and wires strung from room to room inside the future walls. Circles, lines and angles in 3-D.
Lumber triangles hoisted in high rows to form the outline of a roof.
Tessellated piles of wood and stacks of brick to form the outside walls.
Sheet metal tunnels to carry heat to every room from the bottom to the top. Curves at every turn.
Insulation between the inside and the outside walls. Insulation surrounding all the metal tunnels, too. Shapes surrounding shapes.
Sheet rock for every inside wall. Piles of giant rectangles waiting to be used.
Shingles laid one atop another, each nailed securely in its place. Little rectangles nestled together to protect the house from wind and rain and snow.
Appliances pulled and tugged into their new home. How did the builders know the needed space?
Rugs tacked down. Rolls of carpet cut to match the shape of every floor.
Paint spread everywhere. How many gallons to cover all the ins and outs?
Lights turned on and numbers on the door.
Furniture moved in. Geometric rearrangings everywhere.
All the building complete. New neighbors for the school.
Mathematics and geometry are real tools that real people use.

Lesson Ten is a lesson in using opportunities. A house built right across the street is an opportunity to help our students see the value of geometry and all the rest of math. We look for opportunities that let our students see how others use math. We look for opportunities that let our students use geometry and mathematics for themselves. We know that geometry and mathematics are more than numbers on a page. We use the opportunities that arise to pass this knowledge on.

Using opportunities...

Science:

Our science lessons (Chapter 15) are filled with opportunities to use math:

When our students are using geometry in science, we connect their activity with the word *geometry*. We want our students to know that geometry is something they use and something they understand.

Drawing:

What geometry is in a child's picture of a house?

(illustration 6-10-1)
(Two drawings of a house. First drawing: Kindergarten child's two dimensional free-hand rendition of a house, flatly drawn without perspective. Second Drawing: Sixth grade child's ruler drawing of a house, angular view in perspective.)

Writing is a way we have of sharing words. Drawing is a way we have of sharing the images in our minds. Drawing is an application of geometry.

We know that the background our students bring with them for geometry is building. Building means experiencing forms and shapes. As our students build, we can ask them to record in pictures what they have made. Their pictures may be drawings or tracings or paper pieces pasted down. Concepts and images go hand in hand.

Situations that arise:

Computer applications like Logo extend our students' understanding of geometry deliberately. Computer games like the classic Tetris extend our students' understanding of geometry incidentally. Both styles of learning—deliberate and incidental—work together to help students understand and internalize what they come to realize.

The P. T. A. is planning to raise money again this year by selling school T-shirts and sweatshirts. Last year, the shirts had the school name on it and nothing more. Might we suggest to the P. T. A. that a creative use of the talent in our school would be a contest to pick a student-generated geometric design to put upon the shirts?

What other uses of student-made designs might we find? Holiday wrapping paper. Posters to advertise events at school. Designs added to the clothes our students wear. Binder covers. Hall decorations. Illustrations on posters made to advertise books in the school's library.

Are the angles on the playing fields at school correctly measured out? Are all the corners in the baseball diamond 90°? How many other playing fields in sports use right angle at their corners? Which playing areas use angles other than the right? What would happen in an ice hockey match if the wall behind the goal had right angles and not curves? Are races run around square tracks? What would basketball be like if the hoop were triangular and not circular? What would soccer be like if the goal were curved and not rectangular?

Could the parking lot for our school hold more cars if the parking lines were drawn a different way? Are the tables in the cafeteria usefully arranged for seating the most children comfortably? Is there a better way the tables might be faced? How is geometry used to allocate space?

How did Columbus find his way across the sea? Did he use geometry? Why did Columbus think the world was round, not flat? When I look outside, the world looks flat to me.

Why does it make a difference what time of day or night NASA launches a shuttle into space? What is the path around the earth that a satellite follows? Does the satellite's path copy the path that the moon travels around the earth or that the earth follows around the sun?

How do the scientists know where the epicenter of an earthquake is? How do they know if the epicenter is in the middle of the ocean when no scientists live nearby to see the ocean shake? How can they tell how far down below the ground the epicenter is, when none of them go down inside the ground to look?

The puzzle we set out on a table to be pieced together during free time is an application of geometry. What geometry and other kinds of learning do we use when we play miniature golf, or shoot pool, or play air hockey at the local arcade?

Creating opportunities...

We make use of the opportunities that arise. We also create opportunities.

(illustration 6-10-2)
(A simple geometric figure drawn on geoboard recording paper.)

Teacher: Work with a partner to calculate the area of this shape. Be prepared to explain your assumptions and procedures to the rest of the class.

How will the students know who is right and who is wrong? Will they use their geoboards to help them find a way?

Other questions we might ask:

Make up two more figures with different shapes that have the same area as the first figure. Explain how you know your two drawings have the same area as the first.

What is the highest number of right angles a polygon can have? Are there any polygons in which you cannot include a right angle? Give an explanation for the answers you find.

What is the largest area you can put inside a shape made with a string two feet long? The ends of the string must meet so that the area has no openings. Explain why you think your area is the largest you could find.

Can you draw a shape with straight sides and a perimeter of 20 units that will fit inside a shape with a perimeter of 15? Show how or explain why not.

Can you draw a shape with straight sides and an area of 20 units that will fit inside a shape with an area of 15? Show how or explain why not.

Choose two of these shapes to compare. Describe how they are alike and how they are different.

(illustration 6-10-3)
(Drawings of five different quadrilaterals labeled A, B, C, D, E.)

Can you find a pattern that will help you find the number of new squares you need to add to one square in order to make the next-sized square?

(illustration 6-10-4)
(Consecutive squares made with Power Block squares.)

This is a growth pattern. If you find a growth pattern, describe it so that someone else can see it, too. What other shapes have growth patterns that you can find? Describe the patterns that you see.

Which patterns of six connected squares that you cut from your graph paper will fold into a box or cube? Which will not? Predict before you fold.

(illustration 6-10-5)
(A collection of six connected square shapes cut from graph paper.)

Our lessons in geometry help our students recognize and appreciate the geometry that exists in our world. We make awareness and wonder a part of school when we connect the investigating, experimenting and exploring that is a natural part of every child's life.

Questions from Teachers

1. How do we assess geometry?

How do we assess the awareness and the wonder that we teach? If we have increased our own awareness and wonder of the geometry that fills our lives, then we know we have made awareness and wonder a part of every student's life as well.

Our assessments are also drawn from our observations of the work our students do in class. We know they have learned if they can:

Construct images of mathematical relationships.
Use spatial sense and visual imagery as problem-solving skills.
Find lines of symmetry everywhere.
Know the building that they do is part of math.
Build and then describe.
Associate the beauty of nature and the beauty of design with the beauty of math.
Know the properties that different shapes have in common and that different shapes do not.
Know what an angle is.
Make constructions with a compass, a straightedge and a protractor.