Chapter 7

Beginning Addition and Subtraction

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Before We Begin

Stories...

Jeannie says, "I'm terrible at math! Don't give me any problems to do!" But at twenty-five, she owns her own hair salon and has eight stations leased. She runs the shop, pays the rent and knows the market value of every square foot of floor. She knows the number of customers needed to pay for each leased station and the average time each customer sits in a chair. She knows taxes, tips and totals by the day, week, month and year. She knows when to order her supplies, so she never will run short. She knows about marking up and making profits on each sale. She makes projections for the week, month and year and she measures how close her projections come to the reality at hand. When told she uses mathematics all the time, Jeannie says, "I know I do, but I use a calculator. I'm terrible at math!"

Peter says, "Please help me calculate the right amount of cloth to buy to cover my round table and have the cloth drape down to the floor." The helper draws a picture of the table, then draws the cloth upon it. Once sketched out, the amount of cloth needed is readily apparent. Peter says, "But that's too easy. I could have done it that way by myself. What I was trying to remember was the formula for circles, so I could work it out with math."

Daniel says he's, "Good at math." He passes all his tests with 95% or more. But Daniel writes:

$$\begin{array}{r} 33 \\ \underline{-7} \\ 36 \end{array}$$

How could Daniel, who is "good at math," think 33 can grow to 36, when he takes 7 things away? Daniel borrows 1 from the 3 in the tens column and places the 1 next to the 3 in the units column to get 13. He subtracts the 7 from 13 and gets 6. Daniel then forgets he has borrowed 1 from the 3 in the tens column and brings the 3 down to the answer row unchanged. His answer is 36. He remembers the rules almost all the time. His memory fails him no more than 5%, which is why he scores a 95.

Thinking is not required in the math Daniel learns at school. A good memory is all he really needs. Is 95% the standard for who is good or great? Or, should we count as "good" the child knows when an answer makes no sense?

Andrew says to his mother, "My teacher says I should only be doing times and dividing. I already know all about adding and subtracting." Andrew's mother says to Andrew, "Will you please tell me a story for this problem."

$$\begin{array}{r} 8 \\ \underline{-3} \\ 5 \end{array}$$

Andrew says, "There were eight dinosaurs. Three more came along and joined them. Then the three new ones died."

Andrew's teacher says Andrew already knows all about adding and subtracting. Does Andrew know, or does his teacher care, that his words do not tell a story for 8 — 3?

Why does Jeannie think she is terrible at math? She could not manage her hair salon without the math she does so well. Does the calculator she uses mean she is not doing math? Peter wants an answer to a problem, so someone draws a sketch. But Peter wants the formula that he was taught in school. Which has greater value? The answer that we need or the method that we use? Daniel gets his answers right almost all the time. What value is there in an answer if the answer makes no sense? Andrew, too, can get the answers. His teacher taught him how. What does Andrew's teacher think it means to know all there is to know about adding and subtracting?

1,000 words...

Let's imagine that our schools expected every student to know the spelling and pronunciation of 1,000 Latin words by age ten. Let's imagine, too, that this expectation had been passed from generation to generation for at least two hundred years. Our parents and their parents and their parents' parents had faced these 1,000 words. As students, we, too, had our turn to say and spell each one. And, as teachers now, we are expected to teach and test these same 1,000 words.

The 1,000-word curriculum could have started in a time when students read in Latin. Knowing the 1,000 words back then helped the students in a most practical way—it helped them read. Now, even though the reason for knowing the words had faded with the years, the measure of our education remained the learning of these 1,000 words.

It does not matter that the words' value to the child had disappeared with time. It does not matter that the words were just for school. The "school worth" of each child is measured by the child's knowledge of the 1,000 words.

If the spelling of the 1,000 were the measure of school worth and the only skill assessed, would not our lessons focus on the spelling of each word? Even if we had the time to teach the meanings of the words, would the parents of our students put up with lessons about meanings, if no questions about meanings were on the test? Would we say to every parent, "There is more to learning and to life than what is on the test!" And would the parents say right back to us, "Teach my child what I was taught. I want my child to be a school success." And would we hear from inside ourselves, "Just get the students ready for the test."

Would all of our students learn the words with equal ease? We know the students with good memories for spelling would do well. Would our students' memory for spelling improve enough with age to help any of them change their class ranking? Where spelling is concerned, the relative positions for each child in the group remains the same from year to year to year. The child low in first grade would still be low in second, third and fourth.

We know some people, perhaps people just like us, would not learn all the words given them. We know some people, perhaps people just like us, could not see the value in the lessons they were taught. Perhaps there would be so many people just like us who would not learn the words, and would not care that they had not, that it would be permissible for all of them to say without embarrassment, "I am terrible at Latin! I cannot spell a word!"

But, it would not be the Latin at which the people would be terrible. We speak Latin all the time. If we said that we were "terrible at Latin," the *terrible* we say comes from the Latin *terribilis*. We would be using Latin words to say how bad we were. Latin is a part of nearly all we say. Using Latin in our lives gives us little difficulty (*difficultas*). The only difficulty we would have would be with the 1,000 words that we would abhor (*abhorrere*). What we would have learned to hate was not really Latin, it would just be the 1,000 words we had to learn for school.

Could it be?...

Mathematics is simple and basic and straightforward. How do we convince ourselves of the truth of this statement if there are any of us who believe that we are "terrible at math"? How can we believe that something is easy if it was never easy for us? If we believed we were no good at algebra or word problems or geometry or anything we called "math," how can we expect ourselves to make this same subject simple and understandable for the students that we teach?

We know children come to school speaking full sentences. We know, too, that some people leave school at the end of nine, ten, eleven, or more years knowing how to speak, but not knowing how to subtract. Since language is much more difficult to master than subtraction, and language is learned outside of school, could it be that school itself is responsible for the learning in mathematics that has failed to take place?

Could it be that none of us who learned we were terrible at math were ever really terrible at all? Is it possible that all we needed was someone to present math to us in the same natural and meaningful way as was the language we learned at home? Could it be that we were not taught math at all in school, but only the 1,000 problems we had to learn? Yes it could.

As real as language...

We teach addition and subtraction in the elementary grades. We teach it every year. But why? Why must we keep teaching addition and subtraction again and again? A child's vocabulary of words keeps on growing. We feel no need to reteach the words the child learned to say the year before.

Child: What was that word you said for when families keep doing the same thing? Like when we go with all your brothers to the beach each summer?
Parent: Tradition.
Child: Yes. I love traditions.

When children learn a new word, they use it with delight. Hearing it and using it is how the word stays learned. Words follow our students everywhere because our students use the words to communicate everywhere they go.

When we teach our students to look for patterns in mathematics, they then amaze us and themselves with the patterns that they see. Patterns follow our students everywhere, because they learn to see patterns everywhere they go. Problems on the workbook page stay on the page. Because there is no requirement for students to use what they have learned once the page is done, the problems have no meaning in their lives. Because the problems have no meaning, what is on the page this year will be on the page next year as well.

The purpose of the lessons that follow is to make adding and subtracting as much a part of our students daily lives as are the words they use or the patterns they see. Opportunities for adding and subtracting are already a common part of our world. When our students learn to use arithmetic as they already use their words, we will not have to spend each new year teaching last year's addition and subtraction once again.

What does it mean to begin?...

Parent: You may have three cookies for dessert. Here is one cookie. How many more cookies may you have?
Child: Two more.

How old does a child have to be to answer the parent's question? Did the child add or subtract to find the answer? Would we ever hear the child say in response, "Mommy, do I add or subtract?" Does the child even know that we give the name *mathematics* to the thinking that is required?

We do not teach our students how to add or subtract. They can add and subtract before they come to school. We only expand upon and refine the skill our students already have. Since our students already know about adding and subtracting, what do we mean when we say we are teaching beginning addition and subtraction? What does it mean to begin?

A child can calculate how many cookies are still owed by starting with one and counting out the two more until three. Or, a child can start with the three and take one away to find that two more are coming. Or, a child can invent some other way. But the most likely ways to figure an answer involve counting up or counting down.

The answers for beginning addition or subtraction problems are found by counting—counting up or counting back. There is no regrouping involved. The Chapter on *Beginning Number* was for learning how to count. The chapter on *Beginning Addition and Subtraction* is for using how to count. We learn that what we do at school is what we do at home, and what we do at home works as well at school. We learn that when we count with care, the answers will appear. And, we learn something our children already knew before they came to school—we are already good at math.

Lesson One

Purpose	Learn to create and check addition problems.
Summary	Students create addition problems that they can check by counting.
Materials	Squares or tiles, chalkboards, calculators, paper.
Topic	Creating addition problems with handsful of squares.
Topic	Creating addition problems with handsful of squares and checking the answers with calculators.
Homework	Once the process is understood, our students create addition problems at home.

A handful of squares...

As the class begins, each child has a chalkboard and a bowl of Power Block S-1 squares or tiles. As the teacher instructs she mirrors the actions her words dictate with squares placed on the overhead.

Teacher: Today, we are going to learn how to make up our own addition problems. Please take one handful of squares and put it on your desk.

Now, count the number of squares in your pile and write the number on your individual chalkboard. When you have the number written, place your board face down on your desk, so I will know you are ready.

The chalkboards allow all of our students equal involvement in our lessons. Fast workers turn their boards down quickly, but no board is held up for view until all students have finished. The face-down boards mean we leave no one behind—everyone can participate, not just those students with the most frantically raised hands.

Teacher: Please hold up your boards.

Now, take another pile of squares and put it on your desk. This pile should also be only one handful big. Please, keep this pile separate from your first pile.

Count the number of squares in your second pile and write your number on your chalkboard beneath the number you already have on your board. When you have the number written, place your board face down on your desk, so I will know you are ready.

Okay hold up your boards.

Some students will have written their first number so large that very little room is left for a second number on their boards. Some students will automatically erase their first number before writing their second. Some students will write the numbers side by side. Some students will write the new number above, and not below, the first number. Some students will not understand what is to be done at all. And some students will actually write the second number below the first, just as the teacher had hoped all students would do.

Teacher: This is our first time for making up addition problems with the squares, so we must be patient with ourselves. Look how I have written my example on the overhead. I wrote the number for my first pile at the top of my board. I wrote the number for the second pile beneath the first number. Beneath means it goes below it, like this. I also left room for more numbers in case I wanted to take another pile.

I will look around the room and tell you whose boards I think have the piles recorded the way I want you to learn how to record them. Brenda's does... Carlos's does... Denise's does... Julie's does... Aaron's does... Kyle's does...

Please look at my example on the overhead or look at the boards of the people whose names I called and see if you can change the way you recorded your two piles to be more like those. The number for your first handful at the top, the number for your second handful beneath it, and room left beneath for more numbers.

The chalkboards allow us to see how well we are communicating to our students. When our students hold up their boards, they show us how well they understand what we have asked them to do. We know from what we see when we need to find another way to communicate what it is we wish to say.

Teacher: We could take another handful of squares, but for now, two handsful will be enough. Please slide your two handfuls together, like this and count how many you have altogether.

I want you to record on your chalkboards how many squares you have altogether. The way we show that the number you are writing is the total number is to draw a line under the numbers for the handfuls, like this. Then we write the total for all the squares and put it beneath the line, like this.

(illustration 7-1-1)
(The teacher's overhead example with the answer written beneath the line. Show all stages of the problem.)

Teacher: Count the number of squares you have when you slide your two handfuls together and write the answer on your individual chalkboards. When you have the number written, place your board face down on your desk, so I will know you are ready.

The teacher waits until all students have written a number and turned their boards face down.

Teacher: Okay hold up your boards.

We are teaching a process of creating and recording addition problems with squares. The emphasis is on the procedure and not yet on the correctness of each answer. Correctness comes from careful counting. Careful counting comes from practice. And practice comes from the problems we ask our students to create.

Our students continue to create and record addition problems on their individual chalkboards until we can see from the boards that the process is understood. When our students understand the process, they are ready to begin creating problems on their own.

Teacher: I have given you each a piece of paper so that you may record the addition problems that you create. I want you to use the squares to make up addition problems for the next twenty minutes.
Student: Can we take more than two handfuls?
Teacher: Yes, if you want to. Remember, though, when you record, you put the numbers for each of the handfuls above the line and the number for how many squares there were altogether beneath the line.
Student: Do we have to use the squares to make up the problems?
Teacher: You must make up your problems with the squares, but if you want to try to predict the answer before you slide the squares together, you may. If you predict first, I still want you to check your answer with the squares, so you can prove if your prediction was right. When I come around to your desk, I will ask you to prove some of your answers with your squares.

Working for time...

When our students understand the process of creating addition problems with their squares, we give them twenty or thirty minutes to make up their own problems to solve. We give them numbers of minutes and not numbers of problems or numbers of pages.

When students are handed a page of problems to do, some students can complete the page even before the teacher has finished handing out the page to everyone else. Other students take so long to work the problems that some teachers ask them to stay in at recess to finish. Is it not true that the ones who are last to finish today will be the last to finish tomorrow? How can we feel we are good at math if good at school math is only a measure of speed? What does the number of problems we do on a page have to do with how well we understand?

Giving our students a number of minutes to create problems means that when the minutes are up, everyone is finished. No one is ever left behind. Everyone can feel successful. No one need feel overwhelmed. When recess comes, we can all go out and play. No one has learned that he or she was no good at math today.

When students work for time and not for the number of problems, the students who wish to may make up much larger problems to solve. Students who wish to may try three handfuls or more. Students who wish to may keep their problems small. Students who wish to may count more slowly and give the thought they need to each answer that they find.

Credibility...

Some students add better than others. Some students surprise us with how little they know. If we used squares for only the students who need help with adding, then use of the squares would mean we had separated some students out.

Our goal is that all our students learn. We know that how our students feel about themselves as learners has as much to do with their learning as the lessons that we teach. We insure the materials we use have credibility in the eyes of all the students in our class. When even one student needs squares, we give squares to every student in class. We leave no student behind. We separate no student out.

Checking everyone…

Our students make up their own problems with squares. If our students count carefully, the answer they write is already correct. There is no need for us to check again what our students have already checked. When students understand adding, what purpose is served by marking papers after school? We can find better uses for our time.

As we walk around checking our students, we look to see if the problems are written clearly. Can we tell where one problem ends and the next one begins? Is the student counting carefully? Is the counting of the squares what we would call, "one to one"? If we see a student whose answers are not what we want them to be, we work with that student to improve the counting technique. Adding means counting very carefully. Careful counting is what we assess as we walk around our room. The answers in math are not the teacher's. The answers are within the materials we use.

Problems out the door…

Sometimes we have more students than time, and our time to assess runs short. If we do not have time to observe each student, we assess with a problem out the door.

After the students have cleaned up and are ready to go home at the end of the day, we give each student a small blank piece of paper.

Teacher: I am going to give you a problem to do so I can tell how well everyone understood our math lesson today. I need to know how well you understood, so I can know how clear my instructions were and what I might need to change for tomorrow.

Please take out your squares. I am going to give you a problem to do. I want you to work the problem and write it on the piece of paper I put on your desk. When you have finished the problem, please hand your piece of paper to me as you go out the door.

So that I can know how well everybody understood my lesson today, I need each person to find his or her own answer to this problem. So, you must be absolutely quiet as you find the answer. If you talk, you will have to wait until everyone else has left and I will give you a new problem to do.

First, count out a group of eight squares and put them on your desk. Do you remember how to record what you have done? Don't tell me what you think you should do. No talking is allowed. Just do what you think you should do.

Next, count out a group of six squares and put them on your desk next to the first group. Do what you think you should do to record the squares in the second group.

Now slide the groups together and count how many you have altogether. Think how our lesson today showed you to record the total.

Please hand me your paper as you go out the door.

As our students hand us their answers, we make one of three comments to each student in turn: correct, try again, or close enough.

Correct means the answer is the right answer to the problem. *Try again* means the answer is not the right one for the problem given, but we feel the student has the understanding to get the correct answer. In our view, the student may have worked too hurriedly or without enough care, and a second effort should produce the desired result. *Close enough* means we feel the student's answer indicates the student does not understand at all. A student who does not yet understand can still leave school for the day with a positive feeling to take home. The *close enough* student will be the focus of our attention the next day in class.

We use problems out the door to help us know whom we should be helping and whom we can ask to help the others in the room. We use the problems to give us a glimpse of every student's work on those days when we did not have the time to see everyone for ourselves.

The problems we give can be anything from counting the handfuls of squares to finding areas of shapes on a geoboard. Mathematics is simple and basic and straightforward. Our assessments are as simple as the mathematics that we teach.

Calculators…

Calculators free us from the drudgery of the computational process, just as washing machines free us from the drudgery of doing laundry by hand. We do not have to pound our dirty clothes on the rocks by the creek. We do not have to balance our checkbooks or figure our taxes with paper and pencil. When the math becomes more complex. The spreadsheet on a computer is a calculator supreme.

The calculator cannot give us answers to questions we have not learned to ask. If we ask our students to use a calculator to find out how many months they have been alive, how many students would even know where to begin? Do they know the number of months in a year? Do they know that the months are multiplied by age? Do they know that their age alone will not give them an answer? Do they know to add on the months since their last birthday? And, what if the question asks weeks or days of life and not months? The calculator can do the math, but it cannot do the thinking involved.

We begin using calculators with our students as soon as our students can find their own answers to the problems we pose. When our students can count by ones, we can ask our students to use calculators to look at the patterns that appear as they add one after one after one. As our students are learning to count by twos or fives or tens, they can add twos or fives or tens on their calculator and predict the numbers their calculators will flash next on the screen. As our students count back from a number, they can learn to use their calculator to count back with them as well.

When our students learn to slide two handfuls together and count the number of squares they have, they can use their calculators to check the counting they do. When our students record the answer for squares, they can write the answer their calculator finds alongside it.

(illustration 7-1-2)
(Show a page of problems recorded by a student with the calculator answer written next to or near the square answer.)

Questions we ask when calculators are in use:

Have you found any problems where the answer you counted for squares is not the same as your calculator's answer?
Why do you think the two answers do not agree?
Which of the two answers do you believe? Why?
How could you tell if a calculator's answer were right if no squares were around to verify the calculator's work?

Our students will see that the calculator's answer is not guaranteed to be right. With squares, careful counting brings correct answers. Calculators provide a means for our students to check their answers, but a calculator for checking cannot replace the thinking that they do.

To make calculators a natural part of the mathematical lives of our students, we make them available in the desk of each child in our room.

Lesson Two

Purpose	Learn to apply the skills of addition.
Summary	We give our students problem-solving questions and number patterns to explore.
Materials	Squares, Unifix Cubes, calculators, lined paper.
Topic	Problems in the middle of a stream.
Topic	Start with, go bys.
Topic	Start with, go bys, both.
Topic	Consecutive whole numbers.
Topic	Odd and even numbers.
Homework	What is understood may be continued at home.

In the middle of a stream...

When we were taught arithmetic in school, we were given problems for which to find answers.

14	13	195	1,241
+7	+24	+256	+3,985

Each of these problems was given to us as if it existed in isolation, unconnected to the numbers all around. But each set of numbers represents a problem in the middle of a stream of problems that we could have been asked to solve. 14 + 7, for example, is a part of these streams:

20	19	18	17	16	15	***14***	13	12	11	10	9
+1	+2	+3	+4	+5	+6	***+7***	+8	+9	+10	+11	+12

Or,

2	4	6	8	10	12	***14***	16	18	20	22	24
+1	+2	+3	+4	+5	+6	***+7***	+8	+9	+10	+11	+12

Can we see the patterns in these streams of problems? Can we tell what problems come next? Is there a pattern to the answers that we find? Are there other streams that contain the problem 14 + 7? Can our students see the patterns that we see?

The handfuls of squares in Lesson One helped our students refine and expand their ability to add. Lesson Two puts the problems from the handfuls into the stream.

Once our students understand the basic technique of creating and checking their own addition problems, we give them questions to explore that make use of what they know.

2	**4**	**6**	**8**	**10**	**12**	**14**
+1	**+2**	**+3**	**+4**	**+5**	**+6**	**+7**

Teacher: Is there a pattern to the problems I have written on the overhead?
First student: The bottom number goes up by one each time.
Second student: The top number goes up by two.
Teacher: Can you tell me the next problem I will write?
Students: Sixteen plus eight.
Teacher: If there is a pattern to the problems, is there a pattern to the answers, too?

Anything we do with numbers in a regular way produces a pattern to be seen. Any problem we start with can become a problem in a stream by increasing the top number by one, two, three or more. Or by making the top number very large and having it decrease incrementally every time we add. The bottom number can decrease as the top number goes up, or the bottom number can go up, or even stay the same. It does not matter what we do, if we do it in a regular way. There are patterns everywhere.

We start by making up the problems for our students. But what we do, they can do as well. We invent the rules for making problems grow or shrink. Our students are inventors, too. When our students understand the process of inventing, we have them fill their pages with the streams that they create.

As our students work, they may use their squares or their ability to count to find the answers. They use their calculators and their neighbors to check the answers that they find. We want our students to practice adding. Technique, once developed, is used in mathematical ways. We give our students questions to explore that make use of the mathematics that they know. The streams of problems we and they create offer more challenge than the problems on the workbook page.

Start with, go by...

Teacher: I am going to take a small handful of squares. I want my handful to be less than ten.

(illustration 7-2-1)
(A handful of four Power Block squares on the overhead. The handful is put in a single row.)

Teacher: I put my handful in a row. My handful has four in it. Please take four squares for your handful, as well.
What we are going to do today, I call, "Start with, go by." Whatever number we start with, we also go by. What number did I start with?
Students: Four.
Teacher: Okay, then I will go by four as well. Take four more squares and add them to the four we already have.

(Illustration 7-2-2)
(Four more squares added to the squares on the overhead. Each group of squares is in a side by side row of four.)

Teacher: I have placed the squares in rows. Since I will be going by fours, putting the squares in rows makes it easier for me to see that I have added four more. How many do we have altogether?
Students: Eight.
Teacher: Add four more. How many do we have now?
Students: Twelve.

Teacher: And how many are we going to add next?
Students: Four.
Teacher: When we do Start with, go by, I want to be able to look for patterns in the numbers we find, so I am going to record how many we have each time we go by four. How many do we have now?
Students: Sixteen.

4
8
12
16

Teacher: What number will I add next?
Students: Four.
Teacher: And after that?
Students: Four.
Teacher: Okay. I want you to do the Start with, go by for four at your own table. Keep adding four and writing down how many squares you have altogether until you get all the way to the bottom of your paper. No fair skipping lines! You may work with a partner if you wish.

(illustration 7-2-3)
(A student's desk with the squares in rows of four and lined recording paper showing 'plus four' all the way down the page.)

Teacher: Look at the numbers on the right and tell me if you can see any pattern there.

"Number on the right" is the units column.

Student: It goes: 4, 8, 2, 6, 0; 4, 8, 2, 6, 0; 4, 8, 2, 6, 0. It just keeps doing those numbers.
Teacher: Check your Start with, go by numbers with your calculators. See if you get the same answers and the same pattern.

Not all students will see a 4, 8, 2, 6, 0; 4, 8, 2, 6, 0 pattern. Some may have abandoned counting the squares and have added abstractly. These students may have ended up with no pattern at all in their haste to reach the bottom of the page. Even so, a majority of students will have a pattern to share with their classmates.

Teacher: I will record the pattern you found so we can remember it in case we find it again as we do more Start with, go bys. I will write the set of numbers that you told me kept repeating and repeating.

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
			4						
			8						
			2						
			6						
			0						

Teacher: Let's see what kinds of patterns you can find for other Start with, go bys. You can use any number between one and ten. Record your numbers and look at the number on the right for any patterns you might see.
Once you have gone all the way to the bottom of your recording paper, you can use your calculator to check the numbers you have found.
I will give you about fifteen or twenty minutes to do the Start with, go bys and then I will ask you to tell me what patterns you might have found for your numbers.
You may work in teams if you wish, and if you think you see a pattern, you can ask others if they think they see the same pattern.

Basic questions, basic statements...

The questions that we ask are any that occur to us as we watch our students work. If we cannot think of questions on our own, we use the basic questions from *Free Exploration* (page 009) as our guide:

What can you find out? What do you see as you write your Start with, go by numbers in a column down your page? Do your calculator answers always agree with the numbers for your squares?
What would happen if...? What will happen if you start with a different number? Will the pattern be the same?
If you can do it with... can you do it with ...? If you can do it with fours, can you do it with threes and fives and sevens?
How many ways can you...? Do we have to stop at ten?
Do you see a pattern? Can you predict what numbers will come next?
Have you seen the pattern before? Is what we see for fifteen what we saw for five?
Use the patterns you see to help you know what will happen next. If two is like twelve, will twelve be like twenty-two?
Find the one (or ones) that do not work. Are there any Start with, go bys for which there is no pattern?
Prove it. How do you know that you are right? Show me.

The patterns our students see now, they will see again in multiplication. When we begin teaching multiplication, we will ask, "Have you seen this pattern before?" Will our students know the patterns are the same? Patterns allow connections to be made.

Start with, go by, both...

Teacher: Here are two columns of numbers from your Start with, go bys.

2	**3**
4	**6**
6	**9**
8	**12**
10	**15**
12	**18**
14	**21**
16	**24**
18	**27**
20	**30**
22	**33**
24	**36**
26	**39**
28	**42**
30	**45**

Teacher: Which numbers appear in both of the columns?
Students: 6, 12, 18, 24 and 30.

2	3
4	**6**
6	9
8	**12**
10	15
12	**18**
14	21
16	**24**
18	27
20	**30**
22	33
24	36
26	39
28	42
30	45

Teacher: Make a column of the both numbers and see what patterns you can find.

6
12
18
24
30

Teacher: What other different Start with, go by columns can you find that have numbers that appear in both?

The kinds of questions we might ask:

What can you find out? There are patterns for the Start with, go bys. Are there patterns for the "both" numbers, as well?
What would happen if...? What would happen if you made the Start with, go by columns longer? Will the column for the "both" numbers grow longer as well?
If you can do it with... can you do it with ...? If you can do Start with, go bys with twos and threes, can you use twos and fours or twos and fives or threes and sixes?
How many ways can you...? How many ways can you match the columns you made with Start with, go bys to make new columns of "boths"?
Do you see a pattern? Does what you see tell you what you can expect?
Have you seen the pattern before? Is what we see for "boths" what we saw for Start with, go bys, too?
Use the patterns you see to help you know what will happen next. Can you predict what the first "both" number will be before you even match the Start with, go by columns? Can you predict what numbers will come next in the column for the "boths"?
Find the one (or ones) that do not work. Are there any Start with, go by columns for which there is no "both"?
Prove it. How do you know your predictions are correct?

In Start with, go by, we teach our students to record the successive answers in a column so that they can examine the answers for patterns. In Start with, go by, both, we teach our students to record their data in a different way, so that they might see a different kind of pattern.

Start	Start	Both	Both

The first "Both" is for the start number, the second "Both" is for the go by number.

Teacher: I am going to record some information from the Start with, go by, boths that you have been doing, so that we can see if we can find any new patterns. I will begin with the twos and the threes columns that we did first. These two places are where I write the Start with, go by numbers for the two columns.
What was the number that the columns had as their first "both" number?
Students: Six.
Teacher: And what were the "both" numbers going by?

For the "boths", the pattern is something the students can discover: that the starting number and the go-by number are the same.

Students: Six.

Start	Start	Both	Both
2	3	6	6

Teacher: Okay, now tell me what you found for the other "both" columns you created.

Start	Start	Both	Both
2	3	6	6
2	5	10	10
3	4	12	12
4	8	8	8
3	5	15	15
3	6	6	6

Teacher: What patterns can you see that might help you know what the "both" numbers would be if you knew the two start-with numbers?

It is not at all likely that our students will be able to come up with an adequate explanation of any pattern they might see. Older students will have had some experience with multiplication. Younger students will not have had as much. Even if students know that two times three is six, multiplying the starting numbers will not always find the "both" numbers. Four times eight is thirty-two. But eight, not thirty-two, is the "both" number on the chart. Multiplication alone cannot predict the "both" numbers they have found.

We record the data our students give us. We examine it for patterns. We listen to the explanations for how to predict a "both" number from two starting numbers. We test all the theories that we hear. We encourage all the thinking that is involved. We save this chart because we will see these patterns again in *Fractions*, Lesson Six. When we were students, we learned that lowest common multiples were important things to know. Did we know that lowest common multiples were a pattern to be seen?

Ways to make eight...

(illustration 7-2-4)
(Eight S-1 squares on the overhead.)

Questions we might ask:

How many ways can you...? How many different ways can you find to divide eight into two groups? Write down all the different ways you find.
If you can do it with... can you do it with ...? If you can do it with eight, can you do it with six or seven or nine?
What would happen if...? What would happen if we said that three plus five is the same as five plus three? How many ways would there be? What would happen if we say that three plus five is different from five plus three? How many ways would there be? Should we say that eight plus zero is a way to divide the squares in two? Is zero a group that we should count?
Do you see a pattern? Are there patterns in the numbers for two groups that help you know if you have found every way? How can we record the data we find to make the patterns easier to see?
Have you seen the pattern before? Are the patterns we see in groups of squares anything like those we saw in Unifix Cubes? (*Beginning Number*, Lesson Seven)
Use the patterns you see to help you know what will happen next. Are there patterns in how many ways you can find to divide six and then seven and then eight, that will help you know how many ways you might find to divide nine or five?
Find the one (or ones) that do not work. Is there any number you can divide into two groups that does not fit your pattern?
What can you find out? Are patterns something we see only for groupings of two piles? What if we place no limit on the number of groups we may use to find ways that add up to eight?
Prove it. Do you believe the predictions you make? Why should we believe them, too?

The numbers we give our students to add do not have to be large. The excitement of discovering patterns exists in the large and in the small.

The patterns we see in ways to make eight or nine or ten will later become equations in algebra that our students will plot on coordinate graphs. (*Algebra*, Lesson Five.) Patterns connect.

Consecutive numbers...

Teacher: Numbers that are next to each other are called consecutive numbers. One, two and three are consecutive numbers. Five, six, seven and eight are consecutive numbers. One, three and five are not. One, two and four are not. Consecutive numbers are like staircases of cubes that go up one step at a time.

(illustration 7-2-5)
(Illustrate the number sequences in the above dialog with Unifix cubes, some in staircases, some not. Label the cube stacks as consecutive or not, to indicate what the "staircase" reference refers to.)

Teacher: What consecutive numbers add up to three?
Student: One and two.
Teacher: Can you show me with your cubes?

(illustration 7-2-6)
(One and two side-by-side in cubes.)

Teacher: What consecutive numbers add up to four?
Student: Two and two.
Teacher: Show me.

(illustration 7-2-7)
(Two and two side-by-side with cubes.)

Teacher: Two and two add up to four, but two and two do not make a staircase, so two and two are not consecutive numbers.
Student: One and three.
Teacher: Show me.

(illustration 7-2-8)
(One and three side-by-side with cubes.)

Teacher: One and three add up to four. Are one and three consecutive numbers?
First student: No. They don't make a one-step staircase.
Second student: Then there aren't any ways to get four.
Teacher: If no one can find any consecutive numbers that add up to four, then let's try five.
Student: One, two, three.
Teacher: Show me.

(illustration 7-2-9)
(One, two and three side-by-side with cubes.)

Second student: That's six!
Teacher: Then we have already found the numbers that add up to six. How about five?
Student: Two and three.
Teacher: Show me.

(illustration 7-2-10)
(Two and three side-by-side with cubes.)

Teacher: Do two and three make a one-step staircase?
Students: Yes.
Teacher: Do they add up to five?
Students: Yes.
Teacher: Okay. Now, working on your own or with a partner, see how many numbers you can find between one and thirty-two that consecutive numbers add up to. Also, see if there are any patterns you can find that might help you predict some of the sequences in advance.

After the students look for consecutive numbers between 1 and 32, they might also look for them between 1 and 64, or 128. The answers may take more than a day to find.

We encourage our students to share their findings with one another. If a student cannot find a way for the number twenty, he or she may ask all the students if a way has been found. A student who finds a pattern that gives answers may share the pattern with all others in class. Learning is not a race. The excitement of discovery is meant to be shared.

Students will find that they cannot make 1 with consecutive numbers—a stack of one is not consecutive. They cannot make 2 or 4 or 8, but we do not tell them in advance what they cannot do. Will they find a way to make 16? Why did we tell them to find all the consecutive numbers to 32 or 64 or 128? What number comes next in this pattern of numbers? Are these numbers doubling? How does the 1, 2, 4, 8, 16, 32 pattern relate to Power Blocks?

Do we always have to know the answers to the questions that we ask our students? Wondering is a part of mathematics. If we always know the answers before we ask the questions, what wondering are we allowing ourselves to do?

Odd and even...

As our students search for consecutive numbers between 1 and 32, one pattern they might find is that any odd number greater than 1 is the sum of just two stacks of cubes. But this discovery assumes they know the difference between numbers that are odd and numbers that are even.

Even numbers are numbers whose stack divides equally in two. Odd numbers always have one cube more or less in their two stacks.

(illustration 7-2-11)
(Stacks of cubes demonstrating even and odd numbers, with the numbers written beneath the stacks.)

Questions we might ask our students about even and odd numbers:

Which numbers are odd?
Which numbers are even?
Is there a pattern to the odd and even numbers that there are?
When you add two even numbers, is the answer an even number or an odd? Does the same thing happen all the time? Why?
When you add two odd numbers, is the answer an even number or an odd?
Does the same thing happen all the time? Why?
What happens when you add an even number and an odd?
Does the same thing happen all the time? Why?

Lesson Three

Purpose	Learn to create and check subtraction problems.
Summary	Students create subtraction problems that they can check by counting.
Materials	Squares or tiles, chalkboards, calculators, paper.
Topic	Creating subtraction problems with handsful of squares.
Topic	Creating subtraction problems with handsful of squares and checking the answers with calculators.
Homework	Once the process is understood, our students create subtraction problems at home.

A bigger handful of squares…

Teacher: Today, we are going to learn how to make up our own subtraction problems. Please take one big handful of squares and put it on your desk. Use only one hand to take the handful.
Now, count the number of squares in your pile and write the number on your individual chalkboard. When you have the number written, place your board face down on your desk, so I will know that you are ready.
Please hold up your boards.
Now, take some of the squares away from your pile. Please keep the pile of what you take away separate from your first pile.
Count the number of squares in the pile you took away. Write this number beneath the number you already have on your chalkboard. When you have the number written, place your board face down on your desk, so I will know you are ready.
Okay, hold up your boards.

Some students may place a minus sign on their problem because they remember being taught that a minus symbol should go with subtraction. Since all of the problems for this lesson are subtraction, we do not need the signs to give us direction. Students may use the signs if they wish, but we do not require signs until there is a need for their use.

Teacher: Draw a line under the number of squares you took away. Now please count how many squares you have left in your original handful. Record this number on your chalkboard under the line. The line shows that the number you are writing is for the squares you have left.
When you have the number written, place your board face down on your desk, so I will know you are ready.
Okay, hold up your boards.

(illustration 7-3-1)
(A chalkboard with the a full problem written in it. S-1 squares placed next to the board.)

We are teaching a process of creating and recording subtraction problems with squares. As was true for addition, the emphasis is on the procedure and not yet on the correctness of each answer.

We have our students continue to create and record subtraction problems on their individual chalkboards until we can see from the boards that the process is understood. When our students understand the process, they are ready to begin creating problems on their own.

Teacher: I have given you each a piece of paper so that you may record the subtraction problems that you create. I want you to use your squares to make up problems for twenty minutes.

A student who takes fifteen squares and then takes five away can count out the answer by counting to ten, but ten is also five down from fifteen.

Teacher: As I have been walking around the room checking your work, I have noticed that some of you have found another way to figure out the answers to your subtraction problems.

(illustration 7-3-2)
(Fifteen squares on the overhead, divided into a group of ten and a group of five.)

Teacher: The total that I started with was fifteen. I took five away. The way I showed you to get your answers before was to count how many were left. But I have seen some of you use your counting backwards skill to find your answers in a different way. Watch what happens if I count the ones we took away.
Fifteen, fourteen, thirteen, twelve, eleven. What is the number of squares in the other pile?
Student: Ten.
Teacher: Then there are ten in this pile.

We demonstrate to our students the art of counting backwards. But we do not tell them which way to use. Either way or any way our students use is fine.

Checking...

When our students know how to take a handful of squares, take some away and count how many remain, they can use their calculators to check the subtraction problems they create. Our students record their answer and write the answer that their calculator finds alongside it.

The questions we ask for subtraction are the same as we asked for addition:

Have you found any problems where the answer you counted for squares is not the same as your calculator answer?
Why do you think the two answers do not agree?
Which of the two answers do you believe? Why?
How could you tell if a calculator answer were right if no squares were around to compare?

Lesson Four

Purpose	Learn to apply skills of subtraction.
Summary	We give our students problem-solving questions and number patterns to explore.
Materials	Squares, calculators, lined paper.
Topic	Starting with 100.
Topic	Problems in a stream, a negative flow.
Homework	What is understood may be continued at home.

Starting with 100...

Teacher: Count out ten squares and place them in a row.
Now, make ten rows altogether, each with ten squares in them. We already have one row of ten. How many more rows will we need to make to have ten rows?

(illustration 7-4-1)
(Ten rows of ten Power Block S-1 squares.)

Teacher: How many squares do we each have altogether?

Counting out one hundred squares may not be easy for all. But counting out ten sets of ten is much more manageable. Finding out how many altogether may be something the class does quickly or it may be something that takes more time. We can give hints, but the answer is our students' to find.

Students: One hundred.
Teacher: When we did Start with, go bys for addition, we began with a handful and kept adding and adding and adding. Now, we are going to start with one hundred and go the opposite way.
Let's start with going by twos.

(illustration 7-4-2)

(A ten by ten square at the overhead. Next to it, a column of numbers for taking away twos from 100. The numbers written are 100, 98, 96, 94, 92, 90, 88, 86.)

Teacher: You can see what I am doing at the overhead. I want you to do the same thing on your own. Start with one hundred on your own table and go by twos, keep taking two away at a time, and write down how many squares you have left until you get all the way to the bottom of your paper. No fair skipping lines. You may work with a partner if you wish.
Please look at the numbers in the column as you record to see if you can find any patterns.

Students may count up to find the answers, but students who count down will have an easier time.

Teacher: As I walk around the room watching you work, I can see that some of you have found interesting ways to count how many you have left without having to count every square each time. Would anyone like to share with us ways to count that you have invented?
Student: First, I took two from the top row. Then I counted by tens to see how many tens I had left. Then I counted how many were left in the top row.

The teacher acts out the student's words at the overhead, so that all may understand the method that the student used. If the student's words are not clear to the teacher, the student acts out the words at the overhead.

Teacher: Another way?
Student: I knew I started with a hundred and I knew I took two away. So, I counted two down from a hundred. Two down from a hundred is ninety-eight. Then I counted two down from ninety-eight. Then I counted two down from ninety-six. And I just kept counting down.
Teacher: Another way?

Some students will need to recount the entire remaining pile every time they take two away. Allowing these students to hear how their classmates have done the counting gives them the opportunity to rethink their strategy. If they still need to count every square, we accept their need.

Teacher: Look at the numbers on the right. What patterns do you find?
Student: It goes: 0, 8, 6, 4, 2; 0, 8, 6, 4, 2; 0, 8, 6, 4, 2;. It just keeps doing those numbers.
Teacher: I'll write the set of numbers that you told me kept repeating and repeating.

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
	0								
	8								
	6								
	4								
	2								

Teacher: See if you get the same answers with your calculator as you did with squares.
What kinds of patterns can you find for other go-by numbers when you start with one hundred? You can go by any number between one and ten.
Use your calculator to check the numbers you have found.
You may work in teams if you wish, and if you think you see a pattern, you can ask other teams if they see the same pattern, too.

When the class has finished its exploring, the teacher asks the class to share the patterns it has found.

A negative flow…

2	**4**	**6**	**8**	**10**	**12**	**14**
-1	**-2**	**-3**	**-4**	**-5**	**-6**	**-7**

Teacher: Is there a pattern to the problems I have written on the overhead?
Student: The bottom number goes up by one each time and the top number goes up by two.
Teacher: Can you tell me the next problem I will write?
Students: Sixteen take away eight.
Teacher: Since there is a pattern to the problems, is there a pattern to the answers, too?

We give our students problems in a stream of problems to be solved. The problems can be for any computation—addition, subtraction, multiplication, or division. We could even use fractions. What other problems in a stream can we create? What streams can our students invent for themselves?

Lesson Five

Purpose	Learn the families of addition facts above ten.
Summary	Students practice number facts, through flash cards and number games. There are no tests of speed.
Materials	Blackline for flash cards; games like Blackjack, dominoes and shaker dice.
Topic	Flash cards.
Topic	Blackjack.
Topic	Dominoes.
Topic	Shaker dice.
Homework	Flash cards are sent home. Games to play at home are recommended, too.

Number facts...

In *Beginning Number*, Lesson Seven, we introduced our students to the number facts from one to ten. Number facts for eleven through twenty are just as useful for our students to know.

9	**9**	**9**	**9**	**9**	**9**	**9**	**9**	**9**
+1	**+2**	**+3**	**+4**	**+5**	**+6**	**+7**	**+8**	**+9**

Teacher: Is there a pattern to these problems? Is there a pattern to the answers, too?

Facts can be discovered. They are not hard to find. They are a part of all the math we do.

Teacher: How old are you?
Student: Nine.
Teacher: How old will you be in one more year?
Student: Ten.
Teacher: In two more years?
Student: Eleven.
Teacher: In three more years?

A sight vocabulary of basic words like *is* or *the* or *at* is not a substitute for knowing how to read new words. But a sight vocabulary of basic words helps reading go more smoothly for a child. In math, we arm our students with answers to the most basic problems, so that each student will have a sight vocabulary of numbers with which to work.

We know it does our students no harm to know the answer to nine plus nine without having to count it out each time. But to do "no harm" means that we never equate the learning of the facts with tests of speed. We teach a sight vocabulary of facts, but we only teach what we give our students time enough to learn.

The families of facts we want our students to learn come from problems in the stream. We place these problems from the stream on cards.

(illustration 7-5-1)
(Sets of flash cards made up around "problems in the stream". This means the flash cards are grouped by patterns.)

We use the cards any way we can invent. We ask our students to invent more ways.

- Students working singly or with partners can read through their stack of cards. Counting is allowed. Answers guessed correctly go in pile number one. Answers not quite right go in pile number two. Pile number two is then reviewed. Can any cards from pile two now be placed in pile one? There are no grades. There is nothing to be timed. There is only time to learn.
- Students who wish to compete with one another may face off with a set of cards. They may see who can say the most right answers. Competition is voluntary. Counting is allowed.
- Students may take the cards home. They may use the cards for solo practice or for teaching younger siblings what they know.

Number games...

The flash cards we make are not the only way to practice numbers. The card game we call Blackjack is a game that uses all the facts. But because Blackjack can also be used for placing bets, schools have to be careful about using this game. Probability is a part of mathematics and we study probability by studying odds. Understanding Blackjack has much to do with odds. Where Blackjack is allowed, it should be played.

Dominoes can be used for betting, but children playing dominoes in school usually do not bet. When children first play dominoes, they simply match the numbered dots to form trains. But dominoes played correctly uses facts as well. Players add the totals of the dots at the end of all the lines and gain points for multiples of five. We can make up our own rules—we can give a point for the largest total anyone recorded after all the dominoes have been used up, or we can give a point for any total over ten.

Five dice in a leather shaker rattle on the barroom counter top as men and women bet rounds of drinks on the rolls. We do not want to ready our students for the dice scene in a local bar. But any game with dice to be added teaches facts. Our students can use the dice shakers to play any adding game our minds create. They can roll the dice playing against each other and have the highest or the lowest total win. They can add only the dice that do not turn up one. They can add only the dice with the same dots. The rules are ours and our students' to create.

Any games that families play that includes adding are games for putting number facts in each child's mind. We know to tell our students' parents how important it is to read to their children every chance they have. Do we know how important it is to tell our parents the kinds of number games to play?

Our lessons need not be elaborate. The games we play need not be complex. We want our students to know the facts, so we give them the facts to learn and the time to learn them. Learning lasts when we use what we have learned.

Lesson Six

Purpose	Learn to solve word problems that the teacher creates.
Summary	Students learn to draw the necessary numbers from the stories that we tell.
Materials	Students and materials available in class, chalkboards.
Topic	Word problems for adding.
Topic	Word problems for adding, extraneous information added in.
Topic	Word problems for subtracting.
Topic	Word problems for subtracting, extraneous information added in.
Topic	Adding and subtracting mixed.

People problems...

Teacher: Melina and Sarah, please come up to the front of the room and stand on my right.
How many people are standing on my right?
Students: Three.

The teacher writes 3 on the overhead.

Teacher: Zachary and Jacob, please come up to the front of the room and stand on my left.
How many people are standing on my left?
Students: Two.

The teacher writes 2 beneath the 3 on the overhead.

Teacher: How many children are there standing at the front of the room?
Students: Five.
Teacher: How do you know your answer is correct?
Students: Because we can see them, we can count them.

The teacher adds the answer to the problem written on the overhead.

Teacher: You can see how I recorded the people problem I made up. This time, I want you to record the problem I make up on your individual chalkboards.
Everybody at this first table stand up.
Everybody at this second table stand up.
I do not want you to tell me the answer to my question out loud. I want you to write the problem and the answer on your chalkboard.
How many people are standing up? When you have written the problem and the answer, please turn your chalkboard face down on your desk.
Okay, everybody hold up your chalkboard so I may see what you have written.

The answer I see most often on your boards is five. So the class answer to this problem is five. If you do not have five on your board, please have someone who has the class answer on their board show you how they got five.

We make up problems using the children in our room as the objects to be added. Our students write the problems and their answers on their chalkboards and turn their boards face down on their desks until everyone in class has an answer to each new problem that we do. The boards are held up for each new problem and the teacher announces the class answer.

The class answer...

We use the individual chalkboards to give every student a chance to provide an answer to the questions that we ask. When students must raise hands to answer a question and only one or two students are called upon to speak, some students learn not to try. When all students wait to respond until all have an answer, all students are given equal opportunity to participate.

The answer each student writes on the chalkboard becomes each student's vote for the class answer. We observe all the answers and take the most common one we see as the one that we accept, even if it is not the answer we expect.

Let the answer be...

A third-grade class is playing a subtraction game. The rule for this game is that each side must take a number from 1 to 9 from the previous answer. The winning side is the side that makes the other side reach zero as the answer with its last move. The game starts with a number the teacher writes on the overhead.

For the game in this example, the teacher wrote 51 on the overhead. The first moves were 6 and 9 and 4.

$$\begin{array}{r} \mathbf{51} \\ \underline{\mathbf{-6}} \\ \mathbf{45} \\ \underline{\mathbf{-9}} \\ \mathbf{36} \\ \underline{\mathbf{-4}} \\ \mathbf{32} \end{array}$$

Teacher: What shall we take away next?
Zachary: Eight.
Teacher: Zachary, what is eight from thirty-two?
Zachary: Twenty-six.

$$\begin{array}{r} \mathbf{32} \\ \underline{\mathbf{-8}} \\ \mathbf{26} \end{array}$$

Elizabeth: No! The answer is twenty-four.
Teacher: Zachary says it is twenty-six. Zachary, can you prove your answer?
Zachary: I did it on my calculator.
Teacher: Class, how many people think the answer is twenty-six?

A majority of students raise their hands in support of Zachary. They accept his statement that he has proved it on his calculator.

Teacher: The class answer is twenty-six.
Julie: No. Elizabeth is right. The answer is twenty-four.
Teacher: Which people who voted for the class answer would like to prove to Elizabeth and Julie that the answer is twenty-six?

Students cluster around Elizabeth and Julie's desks to prove to them why thirty-two take away eight is twenty-six. Julie and Elizabeth defend the twenty-four they know to be right.

Zachary: I agree with Elizabeth. It's twenty-four.
Kyle: I agree, too. I vote for twenty-four.
Teacher: The class answer is twenty-six until more people think it is twenty-four than twenty-six.