Students: Revote! Vote again!

The students in this third-grade class could find that the answer to 32-8=24. Then why did almost all of them vote for twenty-six? Because Zachary called out "twenty-six" before most had done their own thinking, and Zachary was a person they believed.

The teacher accepted twenty-six as the class answer because it is the answer the class set for itself. Two students who thought for themselves could not accept what the class had decided.

How did Elizabeth and Julie feel when their classmates voted to accept an answer the two of them knew was not right? How did Elizabeth and Julie feel once they had persuaded their classmates to vote for the answer the two of them had found? And, how did the other students feel about having accepted twenty-six without thinking, just because they heard it from someone they believed? There is more to the learning of math than learning what numbers to write. When we want our students do to their own thinking, we must let the answer be the answer that they find.

A father and his son...

A conversation overheard between a father and his eight-year-old son at a basketball game:

Son: What's the score?
Father: 15 to 17.
Son: Who's ahead?
Father: We are.

Next basket is scored.

Son: What's the score?
Father: 15 to 19.
Son: Who's ahead?
Father: We are.

Next basket is scored.

Son: What's the score?
Father: 15 to 21.
Son: Who's ahead?
Father: We are.

Next basket is scored.

Son: What's the score?
Father: 17 to 21.
Son: Who's ahead?
Father: We are.

Next basket is scored. And on and on.

There was a point in this conversation when the father told his son to be quiet and watch the game. There was no point when the father asked the child to use the information he already had to find his own answers.

If each basket made from within the three-point line added two points to the score, couldn't the child keep track of the score himself? If the team that the father and his son were cheering for was ahead at 15-17, couldn't the child figure that the team would still be ahead if the score were 15-19? Couldn't the father have asked the child after each basket:

Which team just scored?
How many points did they get?
How many points do they have now?
Which team is ahead?
How can you tell if you are right?

The father was too engrossed in the game to pay much attention to the child. Giving answers to the child or telling the child to be quiet was much easier than teaching the child how to get answers for himself. But what excuse might we offer for doing the thinking for the students that we teach? Are we too engrossed in making sure the answer is correct to let our students find answers for themselves?

More than they need to know...

Teacher: Carlos, please bring three books from the back shelf up to the front of the room. Brenda, please bring four books to the front of the room. Sheila, please bring seven cups to the front of the room.

Write the problem for the number of books that were brought to the front of the room.

Here are the problems that I see written on most boards. Which of these problems tell the numbers for the books?

1	4	3	3
1	5	4	<u>4</u>
<u>1</u>	<u>7</u>	5	
		<u>7</u>	

Where do all the different numbers come from? Students writing 1+1+1 have added Carlos, Brenda and Sheila. Students writing 4+5+7 have added all the books and cups. Students writing 3+4+5+7 have added all the people, books and cups. And students writing 3+4 have just added the books.

When we create problems for our students to solve, we include more than they need to know because we want our students to do more than just add or subtract. We want them to sort out the information they need from the information that they do not.

Teacher: Kyle, Aaron and Brenda were bored, so they asked their mother if they could go to the movies. Their mother said to call the theater to see what time the movie started and how much it cost. Brenda called and found that if they waited for the 5:30 movie, it cost a dollar each, but if they went to the 7:00 showing, the movie cost five dollars a piece.

How much did it cost them to go to the movies?

Some problems have more information than we need, some don't have enough. Some students would answer the problem anyway. Some might say the children spent three dollars on the movie, because their mother would want them to save money. Some might say the children spent fifteen dollars on the movie because they were going to have dinner first and could not get to the movie by 5:30. Some might add one dollar to five dollars, because this looks like an adding problem and 1 and 5 are the numbers they see to add. Some might wonder if the mother went with the children. Some students might think to ask for more information before they give answers to the problem at hand.

Subtraction, too...

Teacher: Russell please bring eight books to the front of the room.

How many books did Russell bring to the front of the room?

Students: Eight.

The teacher writes 8 on the overhead.

Teacher: Denise, please come up and take two of Russell's books.

Now how many books does Russell have altogether?

Students: Six.

The teacher writes:

$$\begin{array}{r} 8 \\ \underline{-2} \\ 6 \end{array}$$

The pattern to the problems is the same. We make up problems to be subtracted using the children in our room, or the materials we ask our children to carry. Our students write the problems and their answers on their chalkboards for each new problem that we give. They turn their boards face down on their desks until everyone has an answer.

We use phrases like, "How many altogether?" or, "How many do we have now?" whether we add or subtract. We want our students to listen for more than the words "plus" or "minus" to know what the numbers ask them to do.

Adding and subtracting...

Teacher: Kyle and Ashley, will you each please bring four books to the front of the room and stack them on my desk.

Everyone, please write the problem on your chalkboard for how many books Kyle and Ashley stacked on my desk.

$$\begin{array}{r} \mathbf{4} \\ \underline{\mathbf{+4}} \\ \mathbf{8} \end{array}$$

Teacher: Now, Ashley, please take two books back to your desk and Kyle please take one book back to your desk.
Everyone, please write on your chalkboard the whole problem that shows how many books are now on my desk.

Knowing the answer is easy. There are five books on the teacher's desk for all to count. Writing the problem is not as easy as counting to five.

Numbers record mathematics events. Four books added to four books can be recorded as:

$$\mathbf{4 + 4}$$

or

$$\begin{array}{r} \mathbf{4} \\ \underline{\mathbf{+4}} \end{array}$$

Three books taken from eight books can be recorded as:

$$\mathbf{8 - 3}$$

or

$$\begin{array}{r} \mathbf{8} \\ \underline{\mathbf{-3}} \end{array}$$

But the 8 came from 4 + 4—the books Kyle and Ashley brought up—and the 3 came from 2 + 1—the books Kyle and Ashley took back. A student who writes only 8 - 3 has left out part of the story. How is the whole story to be recorded? Here are four of the ways:

$$\mathbf{(4 + 4) - (2 + 1) = 5}$$

$$\begin{array}{rrr} \mathbf{4} & \mathbf{2} & \mathbf{8} \\ \underline{\mathbf{+4}} & \underline{\mathbf{+1}} & \underline{\mathbf{-3}} \\ \mathbf{8} & \mathbf{3} & \mathbf{5} \end{array}$$

$$\mathbf{(4 - 2) + (4 - 1) = 5}$$

$$\begin{array}{rrr} \mathbf{4} & \mathbf{4} & \mathbf{2} \\ \underline{\mathbf{-2}} & \underline{\mathbf{-1}} & \underline{\mathbf{+3}} \\ \mathbf{2} & \mathbf{3} & \mathbf{5} \end{array}$$

We can make the problems we create as easy or as complex as we choose. The deciding factor for the complexity is our students themselves. As they understand one level, we move on to the next. At each new level, we teach the skills our students need to record the events they now see.

Lesson Seven

Purpose	Learn to create and solve story problems.
Summary	Students create their own stories to go along with numbers. First the teacher provides numbers, then numbers are taken from student lives.
Materials	Writing paper, drawing paper.
Topic	Students create addition stories to share.
Topic	Stories are shared before the next addition creations are produced.
Topic	Students create subtraction stories to share.

Topic	Stories are shared before the next subtraction creations are produced.
Topic	Addition and subtraction are mixed.
Homework	Look at home for number problems to bring to school.

Writing stories, drawing pictures...

$$\begin{array}{r} 4 \\ \underline{+5} \end{array}$$

Teacher: What is the answer to this problem?
Student: Nine.
Teacher: Who would like to tell me a story to go with these numbers? Larry.
Larry: There were four boys playing, then five more boys came over to play. Then there were nine boys playing.
Teacher: What are they playing?
Larry: Baseball.
Teacher: Let's draw a picture to see if Larry's story matches the numbers.

(illustration 7-7-1)
(Stick figure drawing of four boys in one group and five in another. The figures are playing baseball. Bases are drawn and one of the figures has a stick bat.)

Teacher: Does Larry's story match the numbers?
Students: Yes.
Teacher: How can you tell?
Student: Because there are four boys and there are five boys and altogether there are nine boys. That's what the numbers said.
Teacher: Another story. Stephanie.
Stephanie: I had four dolls. Debbie had five dolls. So, altogether, we had nine dolls.
Teacher: Let's draw a picture to see if Debbie's story matches the numbers.

(illustration 7-7-2)
(Stick figure drawing of one girl with four dolls and one girl with five dolls.)

Teacher: The girls that I have drawn do not look anything like Stephanie and Debbie. They are just supposed to represent the girls in Stephanie's story.
Does that story match the numbers?
Student: No, because there are eleven people in the drawing. That's not four plus five.
Stephanie: You're only supposed to count the dolls. You don't count the people. There are only four plus five dolls. The two girls are extra.
Teacher: Both stories match the numbers. Can anyone make up a really different story to go with the numbers? Aaron.
Aaron: Can I make one up for subtraction instead?
Teacher: Okay. Tell us the numbers for your story first.
Aaron: Nine take away four. There were nine punk rockers. Four of them got drunk and died. How many were left?
Teacher: Thank you Aaron. That was a very different story. How can we draw a picture to show if Aaron's subtraction story matches his numbers?
Aaron: Can I draw it?
Teacher: Sure:

(illustration 7-7-3)
(Drawing of nine stick figures with spiked hair. All with guitars. Four of them on the ground dead.)

Teacher: Does Aaron's story match the numbers nine minus four?
Students: Yes.

$$\begin{array}{r} 2 \\ +\underline{1} \end{array} \qquad \begin{array}{r} 5 \\ +\underline{7} \end{array} \qquad \begin{array}{r} 4 \\ +\underline{6} \end{array} \qquad \begin{array}{r} 3 \\ +\underline{8} \end{array}$$

Teacher: I want each of you to write a different story for each of the problems I have written on the overhead. If you do not know how to spell a word, you may bring me your spelling notebook and I will write the word for you. If you want to draw pictures to go along with your stories, you may. See how different you can make your story from anyone else's.

The students write and draw. The teacher adds words to the spelling notebooks as needed.

Brenda's story...

Brenda: Teacher, may I write a multiplication story?
Teacher: Yes, but be sure to write the numbers for your story, so we can tell if your story matches your numbers.

Brenda writes:

$$\begin{array}{r} 3 \\ \underline{\times 4} \\ 12 \end{array}$$

I had three doll houses and four doll houses. Three times four is twelve. So, I had twelve doll houses altogether.

Teacher: Do you think you could draw a picture to show that three doll houses times four doll houses is twelve doll houses?

(illustration 7-7-4)
(Drawing of three houses and four houses.)

Teacher: How many houses do you have in your drawing?
Brenda: Seven.
Teacher: Does your drawing show that three times four is twelve?
Brenda: No.
Teacher: What is three plus four?
Brenda: Seven.
Teacher: How many houses in your drawing?
Brenda: Seven.
Teacher: Why don't we make your story an addition story instead.
Brenda: Okay.

Brenda wrote of a concept she did not understand. What students write helps us assess what they understand and what they do not.

At the end of the writing time, the stories are collected. The best examples of creative story writing are selected for reading to the whole class just before the lesson is repeated on another day. If we reward inventiveness, our students will allow their inventiveness to shine through.

Two-way street...

On one day, we have our students write addition stories. On another day, the stories are about subtraction. On still another occasion, we have our students write about addition and subtraction. When our students are exploring multiplication or division, we have our students make up stories about these operations as well.

We connect concepts to symbols. There must be a concept before we can connect. We know from *Beginning Number* that concept-connecting-symbolic is not a one-way street. Concept-connecting-symbolic means we use symbols to record events we have already seen. Symbolic-connecting-concept means we start with the symbols and construct what they represent. We show our students numbers and ask for the words that describe what the numbers mean. Concept-connecting-symbolic, symbolic-connecting-concept—two-way street.

The math that is already there...

We ask our students to invent stories to expand their range of creative expression, but creativity is not our only goal.

Teacher: Today I want you to tell me some mathematics stories from your own lives. I will give you a couple of examples from my own life to show you what I mean.
When I was your age, my family always went to my grandparents' house for Christmas. My grandmother and my grandfather and my great aunt lived there. My aunt and uncle and my three cousins would go there for Christmas, too. In my family, there was my father and mother and my three brothers.
How many people were at my grandparents' house for Christmas?

That is a mathematics story from my life. I want you to tell me math stories from your own lives. You do not have to be able to figure out the answer to the story you tell. Figuring out the answers is something we can all do together later on.

In my story, it might be easy to count all my relatives to find out how many people were at my grandparents' house, but we each gave each other at least one present. What if I had asked you how many presents there were under the tree? That is a harder answer to figure out. Or, what if I asked you how much money was spent on all the presents? That would be even harder to know.

So, think of the stories you want to write. You do not have to worry about knowing the answers yet. You may use your spelling notebooks if you need help with words.

Even though some answers may already be within our students ability to calculate, the answers to the problems our students report are not yet important. There were fourteen relatives at my grandparents' house for Christmas. We can count to find the answer. Fourteen people exchanged presents with fourteen people, but no one gave presents to him or herself. Did the parents give more than one present to each of their children? How many presents might there have been? What might the gifts have cost altogether? Some answers take more time to find.

When we ask our students to write about the math they find in their own lives, we are helping them learn to focus their attention on the math that is already there. We are helping our students focus our own attention as well.

If our students need help in seeing the math in their lives, we can use what we know from when we were children to think where they might look. We can remind our students what their lives include:

Birthday parties, pizza parties, journeys to the ice cream store.
Playing baseball, basketball, football, stickball, hockey, soccer, volleyball, gymnastics and any other sport around. In school, at home and on all the local Little League teams.
Dodgeball, four square, hopscotch, jump rope, jacks or Pick-up Sticks, every kind of tag or a game of hide-and-seek.
Monopoly, Trivial Pursuit, Go to the Head of the Class, Clue, Parcheesi, Yahtzee. Games in boxes that families play.
Ice skating, dancing, swimming and the water slides.
Part-time jobs baby-sitting or pulling weeds from the neighbor's lawn.
Allowances, spending, saving, shopping at the grocery store or mall.
Arcade games out in the world or Nintendo games at home.
Playing with friends, visiting relatives, sitting home alone.
Bed times, meal times, favorite times, worst times, show-and-tell times, great adventure times.
Sad times, glad times, bored times, overjoyed times, sitting still times.
Picnics, camping, hiking, visiting the park. Swings and slides and Jungle Gyms.
Flying kites, making models, playing dolls, playing war.
Flying in a plane, riding in a bus or train, subway journeys, traveling in a car.
Vacations to Disney World or Disneyland or Opreland.
Adventures in the snow or water or sand.
Circuses, fairs, carnivals, pageants and parades.
Aunts, uncles, fathers, mothers, cousins, siblings, relatives of all kinds.
Doctors, nurses, dentists, broken bones, scrapes and fevers, numbers of cavities to be filled.
Weather, wet or dry. Lightning storms, hail storms, hurricanes and tornadoes.
Pets, big and little, lost and found.
Favorite movies or TV shows.
Reading books in libraries or at home, reading comics or reading the sides of cereal boxes.
Moving from one city to another or from one house to the next.
Visiting friends in hospitals or in jail.
Time and weight and measure. Growing every week and month and year.
How long to travel from there to here.
What else can we or they think of?

Rich or poor or somewhere in between, there is math in all of our lives. We ask our students to write and draw. We encourage creativity. We encourage seeing the math that is all around.

Our students write. Our students draw. We solve some problems now. We solve some later on.

Lesson Eight

Purpose	Learn to think about the reasonableness of answers.

Summary	Students learn to ask: "Does the answer I have found make sense?"
Materials	The materials used depend on the questions asked. Calculators.
Topic	How did you get your answer and how do you know it is reasonable?
Topic	We pose problems to help our students understand "reasonable".
Homework	Where might a use for "reasonableness" be found at home?

Reasonableness...

The skill of estimation often appears on lists of skills to teach. We show our class a jar of beans and ask each student to give a number for the number of beans inside. Then we count the beans to find the number really there. The estimates we receive are guesses at the number in the jar. We do not ask each child to give a reason for a guess. We know the guesses will improve as each child gains experience.

There is a place for estimation in mathematics—we estimate beans in jars and measurements of length and time and weight. An estimate is a refined guess. Estimates are a part of arithmetic, as well. When we add 452 to 537 on our calculator and the answer 242,724 appears, we know the answer is not right. The estimate we make of the answer to expect is closer to 1,000.

When we are teaching beginning addition and subtraction, do we accept estimates for the answers that our students find? Will we accept 20 as close enough for 9 + 9? Estimation is for making guesses and guesses do not give us an answer for 9 + 9 that we would accept. For arithmetic, we teach reasonableness instead. Reasonableness means knowing when an answer makes sense and giving reasons why. No guessing is allowed.

Monica and Andrew...

The students in the third-grade class had been given calculators and were asked to free explore the kinds of problems to which they might find answers. Monica created a problem for herself that everyone in class was asked to solve.

$$\begin{array}{r} \mathbf{932} \\ \underline{\mathbf{x361}} \end{array}$$

Teacher: Monica, have you come up with an answer for your problem yet?
Monica: Yes. Twenty-seven.
Teacher: Tell us how you found your answer.
Monica: I times'd one times two.
Teacher: That's two.
Monica: Then I times'd six times three.
Teacher: That's eighteen.
Monica: Then I times'd three times nine.
Teacher: That's twenty-seven.
Monica: So, the answer is twenty-seven.
Andrew: Twenty-seven is too small.
Teacher: What do you think I should do to get an answer?
Andrew: Multiply the twenty-seven again.
Teacher: By what?
Andrew: By twenty-seven.
Teacher: That's seven hundred twenty-nine.
Andrew: No, its still too small. Times it by twenty-seven again.
Teacher: That's nineteen thousand six hundred eighty-three.
Andrew: Its still too small. Do it again.
Teacher: That's five hundred thirty-one thousand, four hundred forty-one.
Andrew: Yes, that's it. That's big enough. That's the right answer.
Class: Yes, that's right.

Neither Monica nor Andrew know yet how to find answers to large problems on their calculators. Monica also does not know that a big number times a big number should have an answer that is much bigger. Andrew knows at least that 27 is too small and so is 729 and so is 19,683. 531,441 is about as big as the answer ought to be. Andrew's answer is a better estimate. Is either answer reasonable?

What we mean by reasonableness...

(illustration 7-8-1)
(A jar of beans.)

Teacher: How many beans do you think are in this jar? Please tell me why you think the number you give me is reasonable.

The students give the teacher all their numbers and all their reasons for the numbers. When the discussion is finished and the beans have served their purpose, the teacher pours the beans from the jar without counting them. The lesson is over. The actual number of beans is never to be known.

How would our students react if we did this to them? Would they yell and scream at us if we poured the beans out without counting? Would our students fill the jar back up on their own and do the counting for themselves? We always place such emphasis on finding the right answer, how can our students accept as "reasonable" something that is wrong? We need to help our students learn the words to describe why some answers make more sense than others. We need to help our students understand what we mean by reasonable.

0	**17**	**19**	**170**	**1,900**	**17,000**

Teacher: Which of these six answers is the most reasonable answer to nine plus nine? And, why do you think so?
Student: Nine plus nine is eighteen. You don't have eighteen written up there.
Teacher: I know. But of the six answers I have written, please tell me which is the most reasonable and why? Reasonable does not have to mean exactly right.
Teacher: Is zero a reasonable answer?
Students: No!
Teacher: Why?
Student: Because you can't have nothing for the answer when you add nine and nine together.
Teacher: Is seventeen thousand a reasonable answer?
Students: No!
Teacher: Why?
Student: Because its way too big for nine plus nine.
Teacher: Is seventeen reasonable?
Student: No, because the answer is eighteen.

It is easy to understand why the extremes are not reasonable at all. But for students for whom the only answer is the answer they think is right, no number that is not eighteen is reasonable. So, we select problems for which no *right* answer is known.

Teacher: How many children do you think are in the class next door? Is there no one? Are there a hundred? Or do you think it is a number in between?

Now we have a question we can ask for which the answer is not known. For each number the students give, we ask them why they think their number might be reasonable. We will not go next door to count.

Teacher: How many children do you think are in our school? Zero or a million? Or do you think it is a number in between? What number might it be? Why?

Problems we might pose...

How do we teach reasonableness? Reasonableness is not on a list of skills. It is two questions we can ask of all the answers that we find:

Does the answer make sense?
What reasons can I give?

What problems might we pose to help our students understand the meaning of the word reasonableness?

How many children are there in the class next door?
How many children in our school will ride the bus or pay for lunch today?
How many teeth are in the mouths of all the children in our room? How many teeth in the class next door or in the school?
How many cars pass our school or our homes each day?
How many brothers and sisters not yet in school do we have at home? How many would there be for all the classes at our grade? How many first-born or only children are in our school?

How many desks might there be in our school? How many books in each desk?
How could you find how many words there might be in a book if you do not count each word?
How many books in the library at our school? Or, in the library downtown?
How much TV do you watch in a week? How many shows? How many hours? What do you think the number might be for others in our school?
What is a reasonable score for the sports game on TV tonight?
How much of what kinds of food do you think will be served in the cafeteria today? How many cartons of milk? How much food will be thrown away?
How many children would be in a line to reach from wall to wall in our class? How many to stretch the length of the hall?
How many pairs of pants in a store? How many boxes or cans on the shelf?
How many people eat at the local McDonald's in one day? How many hamburgers are sold?

We can count people or things in our own room to gather information we might use. We can choose problems that go beyond our own room so that the answer cannot be found by counting. If we count to find the answer, we teach our students that the right answer is all we are interested in. We do not wish to teach that reasonableness means right; reasonable may not be exactly right at all. To teach the meaning of reasonableness, we do not count to check. We talk only about what we think the number might be.

We can tell when our students know what reasonableness means by asking them a question:

0 17 19 170 1,900 17,000

Teacher: Which of these six answers is the most reasonable answer to nine plus nine? And why do you think so?
Student: Either seventeen or nineteen, because they are the closest to eighteen. The other numbers are way too big or too small.

Using reasonableness...

Brenda looks at 33 - 7 and writes the answer 36.

Teacher: Brenda, look at your answer and tell me if it is reasonable. Tell me why you think so.

Roxann looks at 33 - 7 and writes the answer 26.

Teacher: Roxann, look at your answer and tell me if it is reasonable. Tell me why you think so.

Brenda's 36 is not exactly right. Reasonableness does not have to mean exactly right. But 36 is larger than 33. Should an answer for subtraction be larger or smaller than the number at the start? What is reasonable to expect?

We teach our students to look at every answer they find and ask themselves if the answer makes sense. We ask the question all the time. We ask Brenda, whose answer is not right and we ask Roxann, whose answer is. If Brenda is the only one we ask, we teach our students that when we ask if their answer is reasonable it means they have the answer wrong.

Thinking shared...

When we give our students a problem to solve, we encourage our students to work in pairs or groups to find an answer. Sharing their thinking multiplies the learning that takes place.

Pairs or groups of students working together:

Show every student the ways that other students think.
Create more possible ways to solve the problem than could occur to any one child.
Allow the possibilities to blend with one another to create new ways to solve.
Increase the use of language as each student justifies his or her own thinking to the other members of the group.
Make reasonableness something students discuss with one another.
Multiply the teaching power in the room.

Children need to write about their thinking, but first they need to think about their writing.

Our students work in groups to solve a problem, but each student writes his or her own report. Not all people working in a group have the same understanding as every other member of the group. We assess the writing or the drawing to gauge the student's understanding.

For every problem we ask:

How did you get your answer and how do you know it is right or reasonable?

Students may use calculators to help them, just as we do in our own lives. Working out the answer on a calculator is no proof that the answer is right. Reasonableness means the answer makes sense. Reasonableness is not a lesson in a chapter, it is a part of math and a part of life.

Lesson Nine

Purpose	Learn to connect school math to life.
Summary	We look for problems that exist around us for our students to solve.
Materials	Problems from ours and our students' lives.
Topic	Twenty problems or just one.
Homework	What problems can our students find at home?

Mr. Smith...

Teacher: I am going to give you a piece of paper with a Mr. Smith problem written on it. Please read the problem and work out its answer with your math group. When your group has figured out an answer, each of you please write out how you solved it and why you think your answer may be right. When all of the groups are finished, we will vote on a class answer and discuss how you came to your conclusion.

Mr. Smith leaves his home at seven o'clock in the morning to walk to the store nine miles away. After one hour, Mr. Smith has walked three miles. What time will it be when he gets to the store?

The teacher in this class has given her students an excellent problem, in an excellent lesson. The problem involves time and miles and reasoning. It gives no written clue to the arithmetic to be used.

To solve this problem, the students must:

Read the problem.
Think about the arithmetic they will need to use.
Work together to find a solution.
Use more than one arithmetic operation to calculate their answer.
Justify the answer they find.
Write about how they found their answer and their answer's reasonableness.
Be prepared to vote for the class answer and defend to others the answer their group has found.

We can easily change the problem to add more challenge for a later grade:

Mr. Smith leaves his home at seven o'clock in the morning to walk to the store nine miles away. After <u>fifty</u> <u>minutes</u>, Mr. Smith has walked three miles. What time will it be when he gets to the store?

The Mr. Smith problem is an excellent problem, but Mr. Smith is no one that our students know. Do we ever make up a Mr. Smith problem to help our students learn the words they speak?

For a vocabulary lesson would we say:

Mr. Smith has favorite foods. What might the names be of the foods he likes to eat?

For a language lesson would we ask our students questions like:

Mr. Smith watches TV when he gets home from work. What are his favorite shows?
Could he tell us what happened on one of his favorite shows last night?

For language, we talk to our students about what they like to eat and what they like to do when they get home from school. We do not mention Mr. Smith.

Mathematics is as naturally connected to the lives of our students as is the language that they speak. The problems that our students contemplate in mathematics should come from their own lives and not from the life of Mr. Smith.

Walking a mile...

How long does it take our students to walk a mile? How long does it take each one of us? How can we find out? Can we mark off a quarter-mile path and walk it four times or mark off an eighth of a mile and walk it even more? Can we time the walk for ourselves and for the students in our class? How much faster might the mile be traveled on a bike? Do we know how far it is from our school to the store? Or to downtown? Or to the nearest mall? Would the odometer in our car give us a clue?

Teacher: How long did it take us to walk a mile this morning?
Students: Mostly about twenty minutes.
Teacher: Okay, lets assume it takes twenty minutes to walk a mile. Yesterday, I drove between school and the record store on Main Street to see how many miles away it is. The store is five and a half miles from here. If you left right after school to walk to the record store, what time would it be when you got there? What time would it be when you got back? Should you allow time for going inside and shopping? Would you be back before it was time for your dinner?

The structure of the problem for a child walking to the record store is the same as that for Mr. Smith's journey to the store. The students think about the arithmetic they will need to use. They work together to find a solution to the problem. Each writes about and justifies the answer that the group finds. We expect reasonableness. Class answers are voted on and defended to others who do not agree with the majority. The structure is the same, but the connections students make are not. Once the process is learned, our students can think of distance and speed and time of arrival whenever they go from one place to another. Mathematics is patterns and connections, but with Mr. Smith there are no connections to be made.

Conversation between two parents...

A conversation between two parents waiting for a P.T.A. meeting to begin.

Ms. Brown: Did Aaron get the "Mr. Smith Problem" for homework last night, too? Kyle did and I had real trouble helping him with it. I hated word problems when I was in school. I could never do them.
Ms. Jones: Let me see it.

> Mr. Smith leaves his home at seven o'clock in the morning to walk to the store nine miles away. After fifty minutes, Mr. Smith has walked three miles. What time will it be when he gets to the store?

Ms. Jones: What time did you leave home to come to this meeting?
Ms. Brown: At about three o'clock.
Ms. Jones: You left at three to be here at three-thirty. How did you know when to leave?
Ms. Brown: Because it takes me about fifteen minutes to get here and I allowed extra time for parking and walking to the room.
Ms. Jones: How much time did you allow for parking and walking?
Ms. Brown: About ten minutes.
Ms. Jones: Then, you expected to be here about five minutes early.
Ms. Brown: I added in five minutes, to allow for traffic or parking farther away if there were too many parents here already or whatever else might come up.
Ms. Jones: Here's a word problem for you to work out. Mrs. Brown leaves home at 3:00 to travel to the school. She drives for fifteen minutes. It takes her ten minutes to park her car and walk to the meeting. She arrives at the meeting five minutes early. What time does the meeting begin?
Ms. Brown: That's not a word problem. That's the kind of thing that I do all the time.

Ms. Brown had to solve a more complex math problem than Mr. Smith just to get to the meeting on time. She solves these kinds of problems all the time. But Ms. Brown does not count what she does as math. Mathematics is something she learned in school. Mathematics is something that Ms. Brown feels is something she cannot do.

Twenty problems or just one...

Twenty problems on a workbook page or twenty problems from the life of Mr. Smith will provide our students practice with whatever skill we have taught. When the twenty problems have been solved, we give our students twenty more. How much practice will our students need to remember what we teach from one year to the next?

Just one problem in our class can give our students math to think about everywhere they go. One problem can become twenty problems or a hundred problems more. Measuring how fast our students walk can lead to questions like:

> Once you know how far away any place is, can you calculate how long it will take you to walk there?
> How far away is too far to walk?
> Which is more important in knowing how far away is too far to walk:
>> The time it takes?
>> The miles on your feet?
>> Having to walk back?
>
> Can you figure out the distance to places you go by keeping track of time?
> We know the distance to the record store. What other distances can you find out about?
> Shall we make our homework finding distances from home to everywhere you go this week?
> How does your car know how far it has traveled?
> Can you measure distance the same way your car does?
> What are the biking times or skating times to different places?
> Can we make a chart of distances and of travel times?
> Should our chart of distances allow for the different times it takes us all to walk?
> Is the time for biking or roller skating or driving the same for everyone?
> How shall we record the range of speeds for all these ways of travel?
> What does thirty miles an hour mean?
> If you know what time your family starts out in the car and what time your family gets where it is going, can you determine how many miles the journey might have been?
> What else might you need to know to figure out the distance your family traveled?
> What is an average speed?
> Is riding around in the family car the only way we can find the distances we travel?
> How far is it across our country?
> How much time would it take to go from coast to coast on foot or in a car?
> How long does it take to get from here to anywhere in an airplane? At what speed?
> What time will it be when we get to where we are going?
> What time will it be when we return?
> Does it take the same time to go from east to west in a plane as it does from west to east?
> What is the jet stream and what difference does the jet stream make when flying?
> How fast does the space shuttle travel?
> How many miles does the shuttle travel in one orbit?
> How long would it take the shuttle to reach the moon?
> How long would it take the shuttle to reach the local record store?
> How fast does light travel?
> What is the speed of sound?

Questions lead to questions. There is no limit to what we and our students may ask. There is no limit to the problems we can create.

Easy to say, easy to do?...

It is easy to say that Mr. Smith problems can be replaced by problems from the lives of our students, but is it easy to do? We hope to draw problems from our students, but hopes are not always fulfilled. If we tell our students how many relatives were at our house for Christmas, what they tell us in return is how many relatives were at their house for Christmas or Hanukkah or the Fourth of July. If we praise a student for writing about dinosaurs or punk rockers, then dinosaurs and punk rockers may appear in stories from everyone in class. How do we help our students to see more mathematics than the mathematics that we say we see?

How? Creativity for a child is a way of life. Just watch a child at play. A stick becomes a sword, or cane, or pen, or boat. A scrap of cloth becomes a cape, or a parachute, or a doll's dress, or a flag.

Teacher: Aaron wrote about punk rockers. What I liked about Aaron's story was that he was being creative. Aaron thought of something to write about that no one else in class thought to say. Aaron found a different way. What different way can you think of today?
I told you about the relatives who came to my grandparents' house for Christmas. What example can you think of for numbers that is different from the one I gave? How really different can you think to be?

We ask our students:

What different ways can you think of?
How different can you be?

We ask ourselves:

What different ways can we think of?
How different can we be?

There is nothing unique in having ideas. We have them all the time. We can be as creative as our students are.

A question waiting to be asked…

Mathematics is patterns and connections and problems in a stream. Every problem is connected to another. Every idea we have sparks more.

In Lesson Seven, we asked our students to tell us stories of the math in their own lives. Below are three of the thoughts from Lesson Seven along with some questions we might ask:

Birthday parties, pizza parties, journeys to the ice cream store.

How many people at the party?
How many boys? How many girls?
Were there more children than there were adults?
How many more?
How many paper plates and napkins and cups?
How many knives, forks and spoons?
What do you think it costs to put on a party?
What was the price of all the food?
What was the cost of the plates? Can we use the plates again?
What was the average cost of the party for each child there?
How many pieces of candy in the bag? How many will each child receive? What do we do if the candy does not divide evenly?
How many children to share the cake or the cookies? Will there be enough? How can we be sure?
How much ice cream should we order? How many scoops in a half-gallon?
How much is an ice cream cone?
If each scoop sells for a dollar, how much profit does the ice cream store make on its cones?
How many cups of juice will pour from the bottles that we have?
What kinds of pizza should we order? How many slices in each size?
If it is delivered, when will it come?
What does the pizza delivery person make as a salary for riding around in his or her car? How many pizzas could he or she buy with the pay?
How many kernels of popcorn popped from the bag of popcorn? How many kernels did not?
How much did the popcorn expand when popped?
If we need money to put on a party, how much could we raise at a bake sale?
What does it cost to make a cupcake or cookie? What could we sell each one for?
What games could we play at a party? What kind of entertainment would we want?
Is there any math in the games that we play?
If we take our party to the bowling alley, or to the local arcade, or to the beach, or to the water slide, or to the ice skating rink, or to the local park, what kinds of questions can we find?
The plate for the cake is a circular plate. What else at the party has the same shape? What other shapes can we see again and again?
Do we see patterns for numbers or shapes that we think we have seen before?

Playing or watching baseball, basketball, football, stickball, hockey, soccer, volleyball, gymnastics and any other sport. In school, at home and on all the local teams.

How many children on your team?
What are the positions they play?
Are there numbers on the uniforms? Is there a pattern to what the numbers mean?
How many play at once? How many sitting on the bench?
How many strikes? How many balls? How many points for a touchdown or a goal or the different kinds of baskets that can be made?
How do you score a point? How many points can be scored?
How long is the game? How many minutes on the clock?
What kinds of things does the scoreboard keep track of? Are all game scoreboards the same?

How many periods are used in a game and what are the periods called?
How many time-outs are allowed? How long is each one to be?
Why do the last two minutes of a basketball or a football game often take far longer than two minutes to play?
If you play in a league, how do you know which team is winning and how far that team is ahead?
How can a team have a half-game lead when no teams play a half game?
What do uniforms cost?
What is the price of a ball?
If the game is on TV, how many time-outs for advertisements?
How much does each advertisement cost? How much is earned for the game?
How many sports use something round to hit or to throw? How many sports do not?
What is a sport and what is just a game?
If the basketball team has scored 20 points, how many baskets might that be? Is there another way to score the 20?
Which sports have lots of ways to make points? Which have very few?
Which sports are watched the most in our country? Which are watched the most in the world?
What is a batting average or a passing percent? Can you calculate your own?
What does it mean when they say, "Percentage complete"?
How many shots made? How many shots missed? How many shots altogether? How many more or less made than missed?
How many children play Little League baseball in our country? How many play sports in high school? How many play sports in college? How many play on the major league teams?
How long is the average career of a professional player? What is the average salary?
What is the price of a ticket to a game? How many people paid for yesterday's game? How much did all the people pay to see the game?
What kind of information about teams and the players is found in the sports section of the paper each day? What do the numbers mean?
How are numbers used to score a figure skating or gymnastics event?
How are computers used to analyze the motions of runners and throwers to help them run or throw better? What patterns in motion do the computers help each athlete see?
How do calories and carbohydrates help athletes know what they should eat?
Does what we eat have anything to do with how well we run or swim or kick or throw?
What does math have to do with the calories we consume?
Can a person understand sports without knowing math?

Pets, big and little, lost and found.

What kinds of pets do we have?
How many pets of each kind?
Which kinds of pets are most common?
How many pets for our class? How many pets for our school?
How many pets in the block where you live?
How many pets in our city? How can we tell if our answer is a reasonable one?
How many pets have been lost? How many pets have been found?
What kinds of foods do our pets eat?
How much do they eat everyday?
What is the cost of the food that they eat?
What other costs are involved?
How much time do you spend with your pet?
What does your pet do when you are not there?
Is your pet as smart as you are?
Can your pet tell time?
How long will your pet live?
Will your pet outlive you and me?
What does it mean when they say, "dog years" or "cat years"?
What does a pet cost to buy?
How many puppies or kittens or baby fish or whatever would a female pet be likely to have?
How much has your pet grown since it was a baby? How much more will it grow?
Does growth mean taller or heavier? Do we have to choose between the two?
Does your pet grow faster or slower than you?
How fast can your pet walk or run? Who is faster, your pet or you?
How long would it take your dog or your cat to walk to the record shop?
How can you measure the speed of a fish or a turtle? A dog or a cat?
What is the fastest or strongest pet you could have?
What is a pet? Can a pet be a bug in a jar?
How many kinds of dogs or cats are there?

What makes a dog not a cat?
Does your pet's heart beat faster or slower than yours? How do you take the pulse of a fish?
What else can we find about the pets we have?

Everywhere we go where money is spent or time passes or scores are kept or food is consumed or people have a good or bad time, math can be used to measure the experience. The questions are all around us. All we have to do is ask.

With a little practice...

Mathematics is simple and basic and straightforward. With a little practice, we can see how easy it is to draw problems for our students to solve from the math that is part of our lives.

Teacher: I am fifty-two. How old was I when you were born?

We have our students work in groups to find their answers. Will all their answers be the same? Students who can write record their answers in writing. They can also record methods they used to find the answers. The class discusses what their groups have found and all the ways they used.

With a little practice, we can see what new questions might follow from the first:

How old will you be when I am one hundred?
How old were your parents when you were born?
Do you know anyone who is that age right now?
Who else in class has parents the same age as yours?
How old were your older brothers and sisters when you were born?
How old were you when your younger brothers and sisters were born?
How old will you be when your youngest brother or sister is in this grade?
What is the difference in years from the oldest to the youngest of the children in your family? Is that the same difference as is was for the brothers and sisters of your parents or grandparents?
How many years altogether have all of us in class been alive?
How many months or weeks or days have you been alive?
How old will all your relatives be when you are graduated from high school?
How many years older or younger are your parents than I?
Am I old enough to have taught your parents in school?
How old were your grandparents when your parents were born?
What is a family tree?
What other questions do you think we could ask about age?

With a little practice, there is no end to the questions we can think of to ask.

Our assessment for ourselves...

Our assessment for our students is how well they put to use the mathematics that we teach. Our assessment for ourselves is how good we can become at creating problems for our students to solve that do not involve Mr. Smith.

Ms. Brown uses mathematics all the time, but she does not call it math. She shops and cooks and pays the bills. She knows the time of day and knows how long before her children will be home from school. She knows the distances from here to there and the times it takes to drive depending on the traffic. She knows clothes sizes and calorie counts and which jar is the better buy. She knows the ages, heights and weights for every child in her home and how much each child has changed since the year began. She puts the money in the bank and takes it out again. She can do about a million things that she does well, and all require math. Even though she uses mathematics every day, no one ever taught her to think of the math she does as math.

Our assessment of ourselves is how good we become at seeing the math in everything we do. We do the math that we have always done, and we learn to see the math in what we do.

Summary

Opportunities that arise...

As we teach about adding and subtracting, we do not forget that the activities we teach are not as important as the mathematics itself. Lessons are useful, but lessons are not meant to replace the opportunities that arise for adding and subtracting numbers naturally.

We do not wait until the time is right or until the proper lessons have been taught. If our students want to know how much lunch money is collected, we use each student's calculator to add the dollars and the cents. Even if they cannot do the place value and even if they cannot read the answer found, we show our students that adding and subtracting are what we use to find the answers that we need. Our focus is on the opportunities that arise and on those that we create.

Aaron, age eight, goes to the refrigerator to get a bottle of root beer to share with his friend Kyle. As Aaron begins to pull out a bottle, he says to Kyle, "There are five bottles of root beer in the refrigerator. How many will be left when I take this one out?"

Five bottles of root beer are in the refrigerator whether Aaron chooses to count them or not. Four bottles will be left when one is withdrawn, whether Kyle can answer the question or not. Thinking mathematically means seeing the mathematics that is waiting to be seen.

Questions from Teachers

1. How do we assess?

How do we not? As soon as our students can hold a pencil or a crayon we have them write about what they are doing and what they are thinking and how they solved the problem. Students using the Reading Program start stamping out their stories before they can even hold a pencil well. Writing words is not the only way to write. Students show us how they solved the problem by drawing pictures of what they did. We add selected written records from our students to the portfolios we keep. What our students write or draw is part of our assessment.

As our students work, we move about the room looking, wondering, asking questions, paying close attention to what we see and hear. We take notes on what we see. When students cannot write for us, we write for them and add it to our records. What we see and hear is part of our assessment.

(illustration 7-Q-1)
(The Math Their Way assessment of the teacher hiding two blocks of five in his or her hand and asking the child to tell how many are missing from the five.)

The teacher assesses one child. The child is shown five blocks. Two blocks are then hidden in the teacher's hand. The teacher shows the three remaining blocks and asks the child, "How many am I hiding in my hand?"

The assessment is a good one if we only have one child to teach. When we have more than one student to teach, we do not leave our other students unobserved. We use a more efficient way to discover what is known and what is not.

(illustration 7-Q-2)
(The students in class working in teams of twos with an occasional group of three. The students take turns hiding blocks from one another while asking each other the same assessment questions the teacher would have asked them one at a time. The teacher is in the picture walking around the classroom with a clipboard observing and taking notes.)

To assess a class, we have each student start with five blocks, or whatever other number the teacher may decide, and then hide some of the blocks from view. The second student figures out how many blocks are hidden by looking at the blocks that remain in view. The students take turns hiding blocks as the teacher walks around observing how each student plays.

When the assessment is something that the students share with one another, the teacher has more time to observe who understands and who does not. The students themselves have the opportunity to learn from watching one another play this game of blocks.

We do not seek perfect answers. Evidence of understanding does not mean getting every problem right. We do not expect children to be perfect in every aspect of life at ages five, seven, nine, or ten. We have nothing of importance to gain by handing papers back wrong answers circled in bright red. If we choose to circle answers, we circle what is right. We focus attention on what can be done and not on what cannot.

What we seek is evidence of understanding. What we give is time to comprehend. Learning takes time. When there is time, learning takes place. We do not rush the process. We provide the learning experiences and collect notes on the progress made.

2. In Lesson One, all students worked for the same amount of time and not for the number of problems to be solved. But they were making up their own problems with squares. When we make up the problems, what is a good number of problems to give? How long should our students work?

We give one problem and then we give the time for solutions to be found. The thinking is the most important part. We want every student to be involved, so we do not give a second problem until all our students have had an opportunity to solve the first.

The groups of students working on the problem will not all finish at the same time. We tell our students in advance how to use their free time. The conditions that we place on what is chosen are the conditions that match our present needs. If we have been making puzzles for the tangram box, *Mathematics... a Way of Thinking*, (page 278), then making tangram puzzles might be the choice we give. If we have been looking for areas of two on our geoboards (page 058), then geoboards might be the choice our students have.

We make it clear to all our students that early finish is no guarantee of better work. All in class will have the time to find a solution. Quality is what matters. There is no prize for speed.

When all have solved the problem and the class has discussed their answers, we give a second problem. We give the time to solve this problem, too. There is an inverse correlation between the number of problems on a paper and the amount of thinking involved. The more problems, the less time to think.

3. Within each lesson there appears to be no grade-level separation. The advanced problems are mixed in with the easier ones. In Lesson One, for example, taking handfuls of squares to make up addition problems is included right along with finding series of consecutive whole numbers. How do we know which problems to do for which grade levels?

Regardless of the grade level we teach, we start at the beginning. Starting at the beginning tells us what our students know. The assessments that we make as we watch our students work tell us what our students can and cannot do. Our students tell us when and if we should move on. We know we will leave no child behind.

As teachers, we are always in control of the level of difficulty of the problems that we give. We move forward as we think our students are ready for more challenge. We move back if we find we have given too much. As teachers, we know what is right for the students in our class.

4. I use something I call "Mad Minute" to time test my students on the basic facts. I find it very useful for drilling my students on quick recall. The students are given a minute to do the problems on each of several increasingly difficult pages. One page per minute, one minute per day. Once they can do one page at a mastery level, they pass on to the next sheet of problems for the next Mad Minute drill. My students love it and they learn so much from this test of speed. What harm is there in this?

We say that our students love the tests like "Mad Minute," but we know the only ones who love the tests of speed are the students who do well. What is the feeling for the other students in our class? Can we pretend the students who do not fare as well find this failure something they should love?

The argument given in justifying the need for speedy recall, is that there will be situations in our students' future where answers to arithmetic problems will be required with great speed. Our students will not always have their calculators by their sides. We must drill for speed to ready our children for these unnamed future trials.

What are these events of such great importance? Grocery shopping? Charging with a credit card? Writing checks for paying monthly bills? Finding an office in which to work where no calculators are allowed? Are these the events for which we are willing to sacrifice the confidence of our students? Are there really any situations our students will face at any age in life where no time is given to work out the answer? School is the only place where arithmetic is a test of speed.

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