Chapter 8

Graphing, Probability and Statistics

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Before We Begin

Categories to be taught...

This chapter is about graphing, probability and statistics. The following chapter is about measurement, estimation and time. Three categories to be taught now, three to be taught later. Which three should we teach first? The decisions we make are arbitrary ones. Learning does not stop at or wait for the lines we draw.

We graph data to display mathematical information in a more visually comprehensible manner. The information we graph comes from the measurements we make: Who likes which kind of ice cream? How much have you grown this year? Which month has the most birthdays in it for people in our class? How many days of sunshine or rain have we had since the start of the school year?

Some kinds of measurement are of length, volume, or capacity; some of are of feelings, choices, or opinions. Some kinds of measurements are the results of our experiments, our experiences, or our desire to find out. All kinds of measurements are measurements we can graph.

The graphing chapter comes before the measurement chapter, but we do not keep measuring apart as we are learning how to make and use graphs. The sorting and classification chapter comes before the graphing chapter, but graphing is already a natural part of the sorting and classifying that we do. Graphing does not wait for the graphing chapter to begin.

Graphing, probability, statistics...

(illustration 8-0-1)
(A list of names of ice cream flavors with numbers placed along side the names. A graph of the data the numbers represent.)

The numbers contain the information—how many children prefer each type of ice cream. The pictorial representation of the numbers makes it easier for us and our students to see which flavor of ice cream the children in class prefer. Graphing is the process of gathering the data and arranging it in an orderly way. We record the data or information in pictorial form, so we can make better sense of the numbers.

What we see in one graph we may see in another. If the most common birth month in our class is September, will September be the most common month for the classroom next to ours? If September is the most common month for the students in the class next door, could we use the data from these two classes to help us make predictions for the class of students down the hall? Probability is our tool for seeing patterns in the data we collect.

Statistics is the branch of mathematics dealing with the analysis and interpretation of masses of numerical data. Statistics is interpreting, analyzing and predicting.

Graphing has already begun...

We do not teach our students how to sort and classify. They have been sorting and classifying without our help since they were born. We do not teach what students already know, we focus their attention as we add knowledge to the knowledge they already have.

(illustration 8-0-2)
(Reproduce the illustration of the first sort of the buttons in Lesson One of the Sorting and Classification Chapter. Add a note in the caption that the illustration is from the earlier Sorting and Classification Chapter.)

Sorting is also graphing. In Chapter 5, our students watched us group buttons by our unannounced sorting rule as they guessed what our rule might be. If we had chosen to graph our rule, our graph could look like this:

(illustration 8-0-3)
(The buttons from the previous illustration placed in two parallel columns. Buttons in one column matched side by side with buttons in the other column.)

Regardless of how we recorded our sortings, graphing had already begun. Graphing is a way we use to record information to make the meaning of the numbers clear. We will eventually record the information in pictures or images, but the first graphs that we make are all real.

For a sort, we ask:

What pattern are you using to divide the buttons?
Why did your pattern put this button in that group?
Where will this next button go?
Can you divide the buttons a different way?
How many different ways do you think you can find?

For a graph we ask even more:

How many buttons do you have in each group?
How many buttons do you have altogether?
Which group has more?
Which group has less?
How many more or less in each one?

Our purpose for sorting is to expand thinking and vocabulary. Our purpose for graphing is to gather information on the numbers involved.

We already know...

When *Mathematics Their Way* or *Mathematics... a Way of Thinking* teachers gather for Reunion Conferences, they share samples of students' work from their classrooms. The room set aside for the sharing is filled with hundreds of graphs. If we visit the class of a teacher basing his or her program on manipulative math, we would most likely find graphs hung on most every wall. Graphs are such useful tools for teaching, their use is now common place.

(illustration 8-0-4)
(Collage of photos from a Reunion Conference projects room. The photos of graphing done by teachers should show many different graphing possibilities both in terms of specific graphing ideas and in the variety of ways topics might be graphed.)

The sequence of introducing our students to graphing is already contained in *Mathematics Their Way* and *Mathematics... a Way of Thinking*. We already know how to graph.

Lesson One

Purpose	Learn to use graphing as a tool for finding answers to questions.
Summary	Students learn to turn their curiosity into data to graph. Graphs made now will be used again in Lesson Four.
Materials	Materials depend on the questions students ask.
Topic	Students make graphs in response to questions asked or curiosity expressed that leads to numbers that can be represented pictorially.
Homework	Ideally, our students learn to use graphing as a tool for displaying information at or from home.

Ideas for the next graph...

One problem that confronts us when we teach graphing is thinking of topics for the graphs we have our students make. We need not be concerned. Our goal is making problem solvers of our students. Thinking of a topic is a problem for our problem-solving students to solve.

The reasons why we graph are:

To find out things we want to know.
To link school math to the math outside of school.
To learn how to ask questions for ourselves.

The topics for graphing come from what our students or ourselves might like to know. We ask:

What would you like to find out?
What would you like to graph?

Being curious is a natural part of being human and alive. Being curious is all it takes to know the next topic for a graph.

Kevin and the teacher, of course...

Kevin to his teacher: You call on Brenda more than anyone in class!
Kevin's teacher to Kevin: I do not!

What other answer could a teacher give?

The teacher noticed soon after Kevin's statement that every time she called on anyone, Kevin's head disappeared behind his flip-top desk. After school, the teacher flipped the top of Kevin's desk to see what Kevin had been doing. The teacher felt compelled to know what Kevin had been up to. Is looking in a student's desk something we should not do?

Kevin had learned his graphing lessons well. On a list of all his fellow classmates, Kevin had been placing check marks next to the name of every student the teacher called upon. Kevin's graph showed the student called on the most. The name with the most check marks by it was Brenda's, of course.

From that day forward, the teacher was much more fair when calling on her students. The check marks on the graph were spread more evenly around. The teacher still favored one student, knowing it would show on Kevin's graph. The favored student was no longer Brenda. The teacher had her sense of humor. That favored student now was Kevin, of course.

A tally of the ice cream bars...

A mother at a parent conference related to the teacher a story of an investigation that took place at home. The mother bought packages of ice cream bars for special family treats, but it seems the bars were always gone too soon.

The daughter of the house, a student in the teacher's class, had tracked the disappearing ice cream by placing this note on the refrigerator door.

(illustration 8-1-1)
(The note with no graphing data added to it yet. The note says something like please put an X next to your name every time you take an ice cream bar.)

The graph was dutifully filled in by members of the family.

(illustration 8-1-2)
(The note with the graphing data added. One family member had clearly been eating more than his fair share.)

A holiday...

Teacher: We will have a holiday next Monday because Monday is Veterans Day. Does anyone know what a veteran is?
Student: Someone who fought in the war.
Teacher: A veteran can be someone who fought in the war, but you don't have to have fought in a war to be a veteran. A veteran can be anyone who has served in the military.
What do you think we might like to find out about Veterans Day? What do you think we might be able to graph?
Student: My grandfather was in Vietnam.
Teacher: What do you think we could graph about that?
Student: How many other grandfathers were in Vietnam.
Student: My uncle was in Vietnam.
Student: My father's aunt was there, too.
Teacher: Think what you might like to graph about our uncles and aunts and grandfathers and grandmothers and anyone else.
Students: How many relatives we have who were in Vietnam.
Teacher: Okay. We will make up a questionnaire to send home for your parents to fill out to see how many people were in Vietnam.

The teacher can refine this question now or wait until the first batch of data comes in before asking more specific questions. The graph will have only one column if the only question sent home is "How many relatives were in Vietnam?" If the parents are also asked to label the relatives as grandfathers, grandmothers, aunts, uncles, brothers, sisters, fathers, mothers, cousins, nephews, nieces and so on, a different kind of graph can be formed. If the branches of service are asked for as well, then yet another graph might appear.

One idea is an idea in a stream of ideas. Ideas are all around us in every subject all the time. From a graph of relatives in Vietnam can come a stream of thoughts:

What other wars did relatives serve in?
Were there veterans who never went to war?
What does it mean to say a war is popular?
Could a graph show us what "popular" means?
Was the war a popular war?
Which wars were popular and which were not?
What graphing questions related to our graph might come from other subjects?
Have we studied anything in social studies that could relate to our Vietnam graph?
What other wars have occurred throughout our history?
How does the Revolutionary War relate to anybody's lives?
How many of us had ancestors who were in this country when the Revolutionary War began?
How many of us had relatives in this country when we went to war with ourselves in 1861?
Which sides did those relatives fight on?

What was the war they call the Great War?
Is anyone we know still alive who fought back then?
What country did our parents or our grandparents fight for?
Who fought in the Second World War?
On what side?

The holiday for veterans does not have to lead us to study war. The questionnaire sent home asked questions about relatives. The questions could have focused on the relatives themselves and not asked any questions about a war.

Where do all our relatives come from?
How many countries do we represent?
How long have we all been here?
What were the reasons we came?
How many were in the California Gold Rush?
How many in the Oklahoma rush for land?
What kinds of different jobs have members of each generation had?
What other things of interest have our ancestors done?
What can we learn about our past that we can share with one another, even if the things we share do not always lead to something we might graph?

Can we apply the questions that we asked about our relatives to ourselves as well? What experiences do our students have in common with each other?

Where was each student born?
What was the time of day of birth?
Are more babies born at night or in the day?
Are we the oldest or the youngest in our family or somewhere in between?
How many brothers and sisters do we have?
Who has traveled the farthest from their place of birth to be in our room today?
What is the farthest we have traveled from our present home?
What would our students like to be when they grow up?
What favorites do we share? Favorite kinds of food, places to go, TV shows, movies, music groups, colors, or whatever other favorites come to mind.

We do not always have to ask what our students might like to graph. We can learn about our class by listening to our students talk. If there is a time for show and tell, we listen to what our students share. If there is an activity time at the end of every day, we listen to the conversations that arise. We listen to our children as they talk among themselves before the start of school, as they eat lunch, or as they visit with each other at recess. If our students write journals everyday, we can learn the things that interest our students by reading what they write.

Who saw that TV show last night?
Who else eats at that restaurant?
Who else has gone on a trip like that?
Who else has had to go to the doctor or the dentist?
Why did you have to go?
Who likes which football or baseball or hockey or basketball team the best? Do you think where we live now or lived before makes any difference in the teams we like?
What kind of car is your favorite kind?
What kind of car does your family have, or does your family have a car at all?
What kind of cars do the teachers drive?
What are the movies we have seen?
What was the scariest or funniest or the most boring movie you ever saw?
Who celebrates Christmas or Hanukkah or another holiday? Who does not?
How often do we brush our teeth?
What time do we go to bed at night?
What time do we get up?
Who are we anyway?
What do we wonder about?
Are our thoughts the same or different than those of everybody else?
Does what we like change from grade to grade?
Do our older or our younger siblings like the same things that we do?
What do we have in common with children in a cross-town or a cross-state school?

We can graph the children in our class altogether, or each child alone can be the basis of a graph.

How much growth this year?
How much taller or heavier each month?
How many hours of TV each night?
How many minutes of homework?
How many minutes spent each day reading at home?
How many minutes spent at home writing?
How many days at school and how many days missed.
How much progress in our physical fitness tests?

Our students might suggest graphing the number of pages they read at school each day, but a graph like this might become a way to match unfavorably one student's accomplishments against another's. To make their graph look like all the others, the slower readers might read only pages with few words on them or books with the fewest pages. A graph like this might lead our students into making choices in reading for reasons other than the fact that the reading is required.

If a graph can be used to compare one student unfavorably with another, we can suggest another way to graph. Our students can graph the reading each student does, but we can suggest that students graph the number of minutes they spent reading or writing. Our suggestion gives each student a chance to excel. The slowest readers can read for as many minutes as the fastest ones can. The tortoise can keep up with the hare.

If a graph about Veterans Day can lead to more graphs, then what might we graph about the other holidays or special events we share?

Labor Day.
State's Admission Day.
Back-to-school night.
Columbus Day.
Field trip to anywhere.
Halloween.
Thanksgiving.
Christmas.
Hanukkah.
Martin Luther King's Day.
President's Day.
Valentine's Day.
Easter.
P. T. A. potluck.
P. T. A. membership drive.
School play or talent show.
Student fund-raising event.
Mother's Day.
Father's Day.
Independence Day.

Every subject we teach and every period of the day holds questions waiting to be graphed.

Coming to school time.
Attendance.
Lunch count.
Opening activities.
Reading and language arts.
Math.
Science.
Social studies.
Health education.
Physical education.
Recess.
Free choice or activity time.
Going home time.

We are surrounded...

As adults, we are surrounded by graphs everyday. Which of the graphs we see around us can give us ideas for what to graph in class?

Opinion polls on the nightly news.
Popularity polls of all kinds.
Attendance at the baseball games.
Which team is number one. Who says so and why?
Fund-raising charts for the charity drive.
Top-grossing movies for the previous week.
Academy Awards for movies, Emmys for TV, Tony Awards for Broadway plays, prizes of all kinds.
Olympic medals.
Changing world records over time.
U.S.A. Today polls in nearly every section of the paper.
How much wheat is produced in each country? How much is consumed?
Population growth.
Census data of all kinds.
Exit polls on election day.
Measures from a scientific study.
Government spending and government revenue.
Corporate charts of growing income and expenses.
Dow Jones Industrial averages everyday.
The progress of an individual stock.
Ecological issues of all kinds.
Trash disposal, water usage, electricity and gas consumption.
Comparative car mileage from the E. P. A.

Graphs catch our attention quickly and give us information fast. A graph can tell the reader at a glance what would take many words to say. We read the graphs to discover in a flash how much the country spends on this or that, or what our fellow citizens think.

Graphs appear in every morning paper and on the television news shows we see at night. Graphs are used by presidents speaking from the Oval Office or by generals speaking from a battlefront. Corporations boasting of this year's successes show us graphs in pictures designed to make their numbers understandable to all. Advertisers tell us one aspirin sells better than another by showing picture graphs of giant pill boxes. Numbers from the census data tell us much about our country, but we get more meaning from the numbers when we see them graphed. Breaking distances for cars as speeds increase are much more visually impressive when we see the distances in a chart.

The best graphs for our students...

The best graphs for our students to make are ones that occurs spontaneously, because we want our students to graph what they find interesting on a given day. Our students may not yet have all the knowledge about graphing they might need to make a perfect graph. Does an infant wait to speak until every word is understood? Learning does not have to wait until we can get it right. Learning only waits until we want to know. It is more important to seize the moment than to worry about who knows how to make a perfect graph. Students learn the structure of graphing as they graph.

Lesson Two

Purpose	Learn how to display information in a variety of ways.
Summary	Students invent more ways to graph data than they had thought to use before.
Materials	Examples of a variety of graphs, materials in the room, and student creativity and inventiveness.
Topic	Examples of different kinds of graphs are shared as students think of ways to graph they have not used before.
Homework	The search for different kinds of graphs is continued at home.

More than parallel lines...

When we ask our students to sort, we do not tell them how to divide their piles. We trust that they will find a way. We give examples for buttons or keys on the overhead and as we sort children in class. Then we ask our students to think of their own ways.

To help our students answer number questions for a graph of boys and girls, we match the girls and boys one by one in parallel lines. When we graph favorite fruits or kinds of shoes on our graphing canvas squares, we match the data one by one in parallel to help students see how the numbers compare.

(illustration 8-2-1)
(A fruit graph and a shoe graph. The fruit graph and shoe graph are on two separate graphing canvases. Note to include an explanation about the *Math Their Way* graphing canvas in the caption, since there has been no previous reference to the canvas in this book.)

Graphing is a way of displaying information that makes the meaning of the numbers clear. But the graphing canvas or graph paper can lead our students to believe that the parallel lines these graphs produce the only way to graph. There are more ways than parallel lines to display the meanings that numbers have.

When we taught *Beginning Number* with the geoboards (page 058), we asked our students to see how many different shapes they could make that had an area of two. We said, "Think of all the ways you have found to make two. Now, think of new ways you have never thought of before." When we teach our students how to graph, we say, "Think of all the ways you have found to graph so far. Now, think of new ways to graph that you have never thought of before."

The ones about numbers...

(illustration 8-2-2)
(A heaped up pile of fruit. The dialog that follows may have to be changed to match the actual illustration.)

Teacher: What kind of question do you think we might ask about our pile of fruit?
Student: What different kinds of fruit are in the pile?
Student: Which kind of fruit has the most?
Student: Which kind of fruit has the least?
Student: How many pieces do we have altogether?

The questions students ask most likely mirror the questions the teacher has asked the students in the past. Our students are very obliging about giving back what we have given them.

Teacher: The questions you have asked are good questions. We will see if we can find the answers to them in a little bit. Who can think of a question to ask that none of us has thought of to ask before? Who can think of a really unusual question?
Student: Which fruit tastes the best?
Student: Which one has the most seeds?
Student: Which ones grow on trees?
Student: What color were they before they turned ripe?
Student: Which one can you throw the farthest?
Student: Which ones make into pies?
Student: Which ones can you eat the most of without getting sick?
Teacher: Let's see if we can answer some of your questions. We'll start by answering the ones you asked about numbers. How can we display the fruit so that we can see the answers to which type of fruit has the most pieces and which type has the least?
Student: We could put all the fruit into piles.
Teacher: The fruit is already in a pile.
Student: Piles of fruit, with each kind of fruit in its own pile, all grouped together the same.
Teacher: Okay, let's try it.

(illustration 8-2-3)
(The fruit in piles, sorted by kind. Adjust the dialog to match the fruit in the actual illustration.)

Teacher: Which fruit has the most pieces?
Students: Apples.
Teacher: How many apples?
Students: Seven.
Teacher: How many more apples are there than bananas?
Students: There are seven apples and four bananas.
Teacher: That is true, but how many more apples are there than bananas?

The students ability to answer this question depends on their experience with the meaning of the question. We might teach our students what we mean by "How many more?" by placing the apples and bananas in parallel lines, but parallel lines are not the only way we have to compare numbers.

Teacher: How many bananas do we have?
Students: Four.

Teacher: Okay, I need four volunteers to come up and form a circle. Ashley, Roxann, Kyle and Aaron, please come up.
Are there enough bananas so that each of my four volunteers can take one apiece?
Students: Yes.
Teacher: Each of you please take one banana. Are there enough apples so that my volunteers can each take an apple?
Students: Yes.
Teacher: Please take one apple apiece.
Is each person holding one banana and one apple?
Students: Yes.
Teacher: Are there any bananas left over?
Students: No.
Teacher: Are there any apples left over?
Students: Yes.
Teacher: How many are left over?
Students: Three.
Teacher: Then three is how many more apples there are than bananas. Which has the fewest?
Students: Oranges.
Teacher: How many oranges?
Students: Three.
Teacher: How many fewer apples than oranges?

The questions are easy for the teacher to ask. How many more? How many fewer? How many altogether? Are the questions as easy for the students to answer?

Teacher: Putting the fruit in piles is one way we can tell which kind of fruit has the most pieces. Who can think of another way?
Student: Put them all in a line.
Teacher: Show me what you mean.

(illustration 8-2-4)
(The fruit all in one long line. No order to the line.)

Teacher: Which fruit has the most?
Student: Apples.

The students already know there are more apples. But knowing the answer is not the same as being able to prove the answer. Once our students understand the concept of how many more or how many fewer we can ask them to prove their answers. We can also ask them to find ways to prove more than or less than that no one has thought of before.

Teacher: Show me how you can tell there are more apples by looking at your line.
Student: We just count the apples.
Teacher: Yes, but we already know there are more apples because we already counted them. When we put the fruit in piles, we could see there were more pieces in the apple pile than in the other piles without having to count. How can we tell just by looking at the line that there are more apples?
Second student: Put them together in the line.
Teacher: Show me what you mean.

(illustration 8-2-5)
(All the fruit in a single line. All the apples, then all the bananas, then all the oranges, etc.)

Teacher: Does putting the fruit all in a line make it easier to see how many pieces of fruit there are of each kind?
Students: Yes.
Teacher: Who can think of another way?

(illustration 8-2-6)
(Two or three more ways of displaying the fruit, including the classic bar graph format.)

If the class has already used a graphing canvas, some students may suggest using the canvas to organize the fruit. If parallel lines are suggested, this way, too, may be used. We use whatever way our students suggest. The object of the lesson is not to exclude familiar ways. The object is to expand the thinking involved.

The more we ask our students to look for different ways to graph, the more ways they learn to show what numbers represent. We ask our students and ourselves to think of new ways we and they have never thought of before.

We can help our students' expand their thinking by the questions that we ask. We can also help our students see a variety of ways to chart data by bringing in examples of different kinds of graphs we find in newspapers and magazines. *USA Today* uses imaginative kinds of graphs nearly everyday. What examples of creativity might our students find to share?

Lesson Three

Purpose	Learn how to ask questions for a graph.
Summary	We assemble unseen graphs to guide students in learning how to ask what it is they want to know.
Materials	Shield for the graphs, cut-off milk carton boxes to create the hidden graphs.
Topic	A graph is assembled behind a shield as students ask questions about data that remains unseen.

Asking questions...

(illustration 8-3-1)
(Illustration of the materials from Math...a Way of Thinking Lesson 15-8. The materials are the cut-off milk cartons with students names or pictures on the bottoms and the big cardboard box to be used as a shield. The captions include descriptions of the materials and brief directions for how to make them. Indicate in the caption that the cut-off milk cartons are called graphing cubes in the text that follows.)

Teacher: Today I am going to build a graph behind this shield. Once I finish building, you can ask me questions to see how much you can find out about the graph I have made. Your questions will be your only way of finding out about the graph.
What shall my graph be about?
Student: About who likes pizza best.
Teacher: Do you mean which person in class likes pizza better than anyone else in class?
Student: No. I mean who likes pizza better than anything else.
Teacher: Do you mean anything else, including summer vacation or going to Disney World?
Student: No. I mean anything else, like any other kind of food to eat.
Teacher: Okay the question I will graph is "Who likes pizza better than any other kind of food?"
One person at a time, please bring me your graphing cubes and whisper to me your favorite kind of food.

(illustration 8-3-2)
(Show the teacher listening to a child whispering his or her favorite kind of food while handing over a graphing cube. The graph is partially made behind the shield. There are two groups of graphing cubes behind the shield, each representing one of two choices. The choices are: 1)- pizza; 2)- any other kind of food. The non-pizza choices are not separated out from one another by kind of food mentioned. The two different groups of short stacks are clustered so that no one stack is visible above the shield.)

Teacher: I have finished my graph. You may now ask me any questions you wish to go along with the graph that I have made.
Student: Did pizza win?
Teacher: I'll write the questions you ask on the overhead. I'll wait to answer all the questions until after you have finished asking everything you want to know.

The teacher writes out the questions regardless of whether or not the students in class can read. In Reading Program classrooms, the teacher can stamp out the questions so that everyone in class can read what has been written.

Teacher: Another question?
Student: Which food came in second?
Teacher: Another question?
Students: (Silence)
Teacher: I am only going to answer the questions you ask. Once I start answering your questions, I will not accept any new questions, so you have to think of anything you want to know now. Are you sure you don't have any other questions?
Student: Which food came in third?

Teacher: I will not be able to answer that question with my graph, because all I graphed was who likes pizza best and who likes any other kind of food best. All the other foods are in one group. Another question?
Students: (Silence)
Teacher: Okay, if there are no more questions, please read me the questions you asked one at a time and I'll use my graph to tell you the answers.

Older children will have no collective difficulty reading what the teacher has written. Younger students in Reading Program classrooms will be able to manage the collective reading very early in the year. In other classrooms of very young children, if the students collectively are not yet comfortable with reading, the teacher reads, and those who can, read along.

Students: Did pizza win?
Teacher: No.
Student: Then which food won?
Teacher: Remember, I said that once I started answering the questions you asked me about my graph, I would not accept any new questions. Which food won is not a question you already asked, so it will have to remain unanswered. Please read me the next question.
Student: Which food came in second?
Teacher: From the graph I made, pizza came in second. All the other foods came in first.

The teacher then demolishes the graph behind the shield.

Students: Wait! We want to see the graph!
Teacher: You asked all the questions about my graph that you wanted to and I answered all the questions you asked, so you already know everything about the graph you wanted to know.
Student: But how many people voted for pizza?
Teacher: If you wanted to know that, then you should have asked the question.
Student: That's not fair. I thought you were going to show us the graph at the end.
Teacher: Remember, I said when I was finish building my graph behind this shield, you would be able to ask me questions to find out about the graph I had made. Your questions would be your only way of finding out about the graph.
Student: But we didn't know you weren't going to let us see the graph after.
Student: Which food was the winner?
Teacher: My graph was about pizza. It was not about any other food, so my graph could not answer which food was the most popular.
Student: Let's do it again. Make another graph and let us ask questions again.
Teacher: What do you think my graph should be about this time?
Student: Favorite kinds of food. Who likes which kind of food the best.
Teacher: Okay the question I will graph is "Who likes to eat which kind of food the best?" One person at a time, bring me your graphing cubes and whisper to me your favorite kind of food.

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Teacher: I have finished my graph. You may now ask me any questions you wish about the graph that I have made.
Student: Which food was the winner?

As the students ask, the teacher writes the questions on the overhead or the chalkboard.

Teacher: Another question?
Student: Which food came in second?
Teacher: Another question?
Student: Which food came in third?
Teacher: Another question?
Students: (Silence)
Teacher: I am only going to answer the questions you ask. Once I start answering the questions you have asked, I will not accept any new questions, so you have to think of anything you want to know now. Are you sure you don't have any other questions?
Student: How many different foods got votes?
Teacher: Another question?
Students: (Silence)
Teacher: Okay, if there are no more questions, please read me the questions you asked one at a time and I'll use my graph to tell you the answers.
Student: Which food was the winner?
Teacher: Ice cream.

Student: Which food came in second?
Teacher: Hamburgers.
Student: Which food came in third?
Teacher: Pizza.
Student: How many different foods got votes?
Teacher: Seven.

The teacher demolishes the graph behind the shield.

Student: Wait! What were the seven foods?
Student: How close was pizza to ice cream and hamburgers?
Student: How much did ice cream win by?
Teacher: Sorry. The time for asking questions is over. The graph is already gone.

We could remove the shield and let our students see our hidden graph, but we choose not to. When the questions that our students ask are their only means of finding out, students of all ages can learn the art of asking what they really want to know.

We repeat the process of assembling graphs behind a shield and then disassembling them unseen as often as we feel we should to make our message clear. Asking questions is a way of finding out. Better questions find out more.

Lesson Four

Purpose	Learn to ask questions for the graphs that students make and see.
Summary	Students learn to add written questions to their graphs. The lesson on asking questions is also a lesson on learning to speak math and learning to ask math questions.
Materials	Graphs from Lesson One, cut-off milk carton boxes.
Topic	Students add questions to graphs already made.
Homework	What math questions can our students bring from home?

Writing questions for each graph…

The first graphs we make disappear when the children who are in the graph sit down, or the favorite snacks we have set out on the graphing canvas are consumed, or the buttons go back into the box. We use pictures or symbols to represent people or foods or favorite shows to make graphs that we can save. And, nearly as soon as we record in pictures, we record the questions that we ask.

Teacher: Today we'll make a graph for the months in which people were born. Please bring me your graphing cubes one at a time so I can make the graph.

(illustration 8-4-1)
(A fully completed graphing cube graph for birth months. The graph is labeled with the twelve months of the year.)

Teacher: You have asked questions for graphs you could not see. Now I want you to think about questions that you can ask for a graph that you can see. Look at the graph we have made and tell me what questions we can answer with this graph.

We teach our students to ask questions about their graphs so they may learn to ask questions about all the graphs they see. The graphs we see in our own lives come with statements already written alongside telling us what we are to learn from the numbers. We could simply teach our students to write statements about their graphs and learn to read all the other statements on all the other graphs that they will see. We prefer instead to teach our students to ask questions of their graphs, so they will learn that questioning is a part of math. Questioning is a part of life.

Older students who can write, write questions to accompany every graph they make. Spelling notebooks make it possible for all to write, even when all cannot always read. Younger students in Reading Program classrooms stamp out the questions they would ask. Students may work in teams for writing, so that students who cannot write or stamp alone are never left behind. If few students are comfortable enough with writing, the teacher may write or stamp the questions for each graph.

Writing does not wait until everyone is an expert with a pen. Reading does not wait until all the skills are taught. Writing and reading are a part of everything we do at school. We do not wait to teach a child to talk until the child knows the meaning of each word. We do not wait until the child can pronounce each sound with clarity. Learning how to talk begins as soon as we are born. There is no need to wait to ask a child to write or read in school until some magic point in time. We need only accept whatever writing is produced as good enough for now. Spelling notebooks or Reading Program stamps help the child along.

Once our students have learned to ask questions for their graphs, we have them add questions to all of the graphs that they have made from Lesson One or will make as the year goes on.

Graphing helps our students form a link between the math they learn for an hour everyday at school and the math that surrounds them everywhere they go. Writing questions for the graphs means thinking is required from everyone who sees the information. We do not give answers. The answers are in the information we display.

Who can think of another way to graph?
Who can think of another question we might ask that can be answered from the graph?

Speak math...

When we teach our students to look for the questions their graphs can answer, we are teaching more than which questions go with which graphs. We are also teaching our students and ourselves to see the questions to be asked, not just for graphing, but for math.

A conversation between a parent in the grocery store and the parent's young child riding in the shopping cart:

Parent: What kind of fruit shall we get for snacks at home?
Child: Apples and bananas.
Parent: (Loading apples in the shopping cart.) What color are these apples?
Child: Red.
Parent: (Loading bananas in the shopping cart.) What color are these bananas?
Child: Yellow.
Parent: What kind of vegetables shall we get for a snack?
Child: Oranges.
Parent: An orange is a fruit. We already have apples and bananas as our fruit snack. A vegetable is like carrots or peppers.
Child: Carrots and peppers.
Parent: (Loading carrots in the shopping cart.) What color is this bunch of carrots?
Child: Orange.
Parent: Any other color?
Child: And green tops.
Parent: (Loading peppers in the shopping cart.) What color are these peppers?
Child: Green and red.
Parent: Okay, now let's get some breakfast cereal. What kind would you like.

Child points to a box of cereal that the parent will not allow as a choice.

Parent: No that one has way too much sugar in it. Pick between the Cheerios and the Rice Krispies and the Corn Flakes.
Child: That one.
Parent: What is its name?
Child: Cheerios.
Parent: Do you see this word on the box? It says Cheerios.
Let's get some milk, too.

Parent places a half-gallon carton of milk in the child's lap.

Parent: Can you find the word that says milk on this carton?

Not all parents ask as many questions. Not all children know as many answers. But we are all natural teachers and we are all natural learners. We all learn language from the language that is spoken to us.

We speak language, but do we speak math? A conversation between the parent and the child with the parent speaking math:

Parent: What kind of fruit shall we get for snacks at home?
Child: Apples and bananas.

Parent: (Loading apples in the shopping cart.) How many apples did I put in the cart?
Child: Three.
Parent: (Loading bananas in the shopping cart.) How many bananas did I put in the cart?
Child: Three.
Parent: How many apples and bananas do we have altogether?
Child: (No response)
Parent: Let's count them together. One... two... three... four... five... six. How many do we have?
Child: Six.
Parent: What kind of vegetables shall we get for a snack? A vegetable is like carrots or peppers.
Child: Carrots and peppers.
Parent: (Loading carrots in the shopping cart.) Which is longer, the carrots or the banana?
Child: The carrots.
Parent: (Loading peppers in the shopping cart.) Which is bigger, the peppers or the apples?
Child: The peppers.
Parent: How do you know?
Child: Because.
Parent: Can you show me which is bigger?
Child: (Holds a pepper next to an apple.) See!
Parent: Okay, now let's get some breakfast cereal. Pick between the Cheerios and the Rice Krispies and the Corn Flakes.
Child: That one.
Parent: What is its name?
Child: Cheerios.
Parent: How many boxes did we get?
Child: One.
Parent: Let's get some milk, too.

Parent places a half-gallon carton of milk in the child's lap.

Parent: What do you think the heaviest thing we have in the cart is now, besides you?
Child: (After lifting everything in the cart) The milk!

How do we learn to speak math? The math that we do everyday, we do inside our heads. The parent in the first example talked about colors and categories and choices and words, as the parent did all the counting or weighing or pricing in his or her head.

Apples are a fruit that is red. Do we think to ask aloud how many apples there are? Carrots are orange vegetables in a bunch. Do we think to ask aloud how many there are in each bunch? Do we ask if each of the bunches we see is the same? One roll of towels is $2.49, the next brand is $2.59. Do we think to ask aloud how much money we might save? Do we think to ask about the shapes or the sizes or compare the weights of each thing?

The teacher telling:

Teacher: Please line up for lunch.

The teacher asking questions, speaking math:

Teacher: How do we usually line up for lunch?
Students: By buying and bringing.
Teacher: Okay. Yesterday, we had each line put itself in alphabetical order by first names. Today, line up by height. Shortest to tallest.

• • •

Teacher: Which line is longer?
Students: Buying.
Teacher: How much longer is the buying line than the bringing line?
Students: Four people longer.
Teacher: How much shorter is the bringing line than the buying line?
Students: Four people.
Teacher: If everyone in the buying line had paid the full price, how much would the buying line have spent on lunch today?

Whether the teacher asks a price question like this and whether the class can find an answer depend on the teacher, the class and the calculators available in the room. The particular questions asked are not as important as the asking. We are surrounded by mathematics all the time. The questions we ask help us see the mathematics that is already there.

Teacher: What other questions can we think of to ask about our lines?

We use graphs to teach us to ask questions. The questions we ask extend beyond graphs.

The teacher telling:

Teacher: Time to clean up.

The teacher asking questions, speaking math:

Teacher: What time is it now?
Students: Twenty minutes after three.
Teacher: What time do we go home?
Students: Three-thirty.
Teacher: How many minutes do we have to clean up between now and time to go home?
Students: Ten.
Teacher: Is ten minutes enough time to clean up?
Students: Yes.
Teacher: How do you know?
Students: Because we can clean up by then.
Teacher: Will it take you the full ten minutes to clean up, or will there be any time left over?
Students: There will be time left over.
Teacher: Let's see how much.

The students all clean up and return to their seats.

Teacher: What time is it now?
Students: Twenty-six minutes after three.
Teacher: How many minutes did it take you to clean up?
Students: Six minutes.
Student: No. It only took five minutes. You took nearly a minute asking us all of those questions about how long it would take before you let us get started!
Students: Five minutes.

How old do our students have to be before we ask them questions about time? How old do our students have to be before we ask them to find the number of minutes between this time and that? How old does an infant have to be before we start speaking to him or to her?

When we teach our students to ask questions about graphs, they learn that graphs can provide answers. But, there is more to the asking than the questions. The asking teaches our students and ourselves to see events around us as questions we might ask and not as answers someone else has found. The asking teaches our students and ourselves to look for and verbalize the math that is there. Our lessons for graphing are also our lessons for learning how to speak math.

Lesson Five

Purpose	Learn a beginning framework for connecting probability to graphs.
Summary	Students toss cardboard squares, graph the outcomes and predict what future outcomes might occur.
Materials	Cardboard squares and graph, lined, or plain paper.
Topic	One square toss and graph.
Topic	Two square toss and graph.
Topic	Three square toss and graph.
Topic	Four square toss and graph.
Topic	Five square toss and graph.

Probability...

Patterns are everywhere we look. The patterns for the streams of numbers in arithmetic help us see relationships we never knew were there. Graphs record numbers, too. Is every graph we make a problem in a stream? Can we make connections between the different graphs, as we know to make connections for the numbers in arithmetic?

What do graphs of shoes, fruit, relatives in the war, months of birth, and days of sun in March or May have to do with one another? What do these graphs we make at school have to do with the lives of our students outside of class?

This chapter is about statistics, graphing and probability. Graphing is gathering and recording. Statistics means interpreting, analyzing and predicting. We have gathered and recorded. How do we analyze and predict? What does predicting mean for all these unrelated graphs? Probability is our tool for making sense out of the patterns in the data we collect. Probability is what makes each graph connect.

If we found that September was the most common birth month in our room and in the room next door, we might predict that September would be the most common month of birth for the classroom down the hall. Could we guarantee the correctness of our prediction?

If we were right, what would we predict for classrooms number four and five? If we were wrong, what would we predict instead? If the most common month in our class were September and if the most common month in the room next door were a different month, what would we then predict for classrooms down the hall?

If we learned that we could predict the most common month for birth, could we predict favorite kinds of fruit? What do graphs of fruit have to do with months of birth? What do graphs of fruit and months of birth have to do with relatives in war? What do the graphs we make at school have to do with anything at home? How can these separate graphs all be problems in a stream? Probability helps us understand why we can predict for some events but not for all.

We teach our students how to look for the patterns that might exist between and within the graphs that they create. First, however, we establish a basic frame of reference for understanding why we might predict from the data from some graphs and not from others.

Tossing cardboard squares...

Teacher: Whenever I am asked to call heads or tails for a toss of a coin, I always choose heads because I think heads will win more times than tails. But I know that some people call tails because they think tails will win more often. Today, I am going to have you see if I am right to call heads every time. Instead of coins to toss, I have given you square pieces of cardboard. I don't know if we will get the same results if we use cardboard, but coins make more noise than I can tolerate when they land on desktops in school.

Please mark one side of your square to stand for heads and the other side to stand for tails. Once you have your cardboard square marked, toss your square and keep track of what side comes up. You may use any way to keep track that lets you know which side comes up the more often. We will toss the squares for about five minutes before I ask you to tell me which side you had as your winner.

...

Teacher: I am going to make a graph of your results on the overhead. How many of you had heads as the winner? Please raise your hands.
Student: I had a tie.
Teacher: If you had a tie, then hold your hand up for heads and hold it up again for tails.
How many had tails as the winner? Please raise your hands.

(illustration 8-5-1)
(Graph of heads and tails winners.)

The teacher's graph might have either heads or tails as winning more often. It might even be that the teacher's graph has heads and tails as winning equally. Regardless of how the graph looks, the teacher's questions are the same.

Teacher: Which wins more often, heads or tails? Why?

The explanations will be as logical as the teacher's own starting assumption that heads is more likely to win than tails. The class should test any suggestion or hypothesis offered by the students that can be tested by further tosses of the squares.

If testing proves an explanation is "right," then the explanation is accepted, regardless of how faulty we may believe it to be. We need not interject our own assumptions about why one side of the squares may come up more often than another. Odds are that better explanations will be coming from our students themselves as time goes by.

Two-square tossing...

Teacher: I wonder what would happen if we tossed two squares at once. Would heads or tails win? Please find a partner to share with, so you will have two squares to toss. What are the ways that your two squares can come up?
Student: Two heads or two tails.
Student: Or, it can come up a head and a tail.
Teacher: Okay, keep track of the ways that come up so we can see if two heads beats two tails or if the head and tail wins.

It is unlikely that students who have not done this activity before will see that there are really four possibilities: Heads-heads, tails-tails, heads-tails and tails-heads.

Me heads	you heads
Me heads	you tails
Me tails	you heads
Me tails	you tails

Does it make any difference who gets heads and who gets tails? Does it make any difference if there are four ways instead of three? The answer to these questions are in the squares themselves.

Teacher: I am going to make a graph on the overhead of your results. How many of you had heads-heads as the winner? Please raise your hands. If you had a tie for winner, then hold your hand up for each column that was tied. But, don't hold up your hand for a tie if the tie was only for coming in second.
How many had heads-tails as the winner? Please raise your hands.
How many had tails-tails as the winner? Please raise your hands.

(illustration 8-5-2)
(Graph of heads-heads, heads-tails and tails-tails winners.)

Even if a few individual students have heads-heads or tails-tails as their winner, the majority of students will find heads-tails in the lead.

Teacher: Why?

The explanations students offer are tested by further tosses of the squares.

It makes a difference that there are four ways for the squares to land and not just three. If our students do not see that heads and tails have two different ways to happen, and each way might be graphed separately, we do not tell them what we know. The odds are that they will learn it for themselves.

Three-square tossing...

Teacher: I wonder what would happen if we tossed three squares at once. Would heads and tails still win?
You may either find two partners to share with, so you will have three squares to toss, or pairs of you may make an extra square for your group to toss.
I am not sure of all the different ways that three squares can come up. You can keep track of the different ways and tell me when I make my graph on the overhead.

There are three squares, so there are eight possibilities. How many of the eight will our students see?

If you and I and a friend named Sue are partners sharing our three squares, we can each get heads and we can each get tails. There are eight possible ways for our heads and tails to match up:

Me heads	you heads	Sue heads
Me heads	you heads	Sue tails
Me heads	you tails	Sue heads
Me heads	you tails	Sue tails
Me tails	you heads	Sue heads
Me tails	you heads	Sue tails
Me tails	you tails	Sue heads

Me tails you tails Sue tails

Can we see a pattern in the heads and tails that might help us know if we have found all the ways for our heads and tails to match up?

Teacher: I am ready to make my graph on the overhead. Please tell me the ways you found for the heads and tails to come up.

Our students are not likely to see the eight ways that we know.

Student: All heads or all tails.
Student: Two heads and one tail.
Student: Two tails and one head.
Teacher: Any other ways?
Students: No.
Teacher: How many of you had heads-heads-heads as the winner? Please raise your hands. The same rule goes for if you had a tie for winner. Hold your hand up for each column that was tied, but only if it tied as a winner.
How many had heads-heads-tails as the winner? Please raise your hands.
How many had heads-tails-tails as the winner? Please raise your hands.
How many had tails-tails-tails as the winner? Please raise your hands.

(illustration 8-5-3)
(Graph of heads-heads-heads, heads-heads-tails, heads-tails-tails and tails-tails-tails winners.)

Of the eight ways we know the squares may come up, one is all heads and one is all tails. There are three ways each for heads-heads-tails and tails-tails-heads. Our students are likely see only four possibilities and not eight. Of the four that they see, only two win more often than not.

Teacher: Why?

Four-square tossing...

Teacher: Let's try tossing four squares at once. You may each make two squares for yourself. Then find a partner, so together you have four squares to toss.
Keep track of the different ways you find so you can tell me the columns I will need for my graph on the overhead.

If you and I and our friend named Sue are joined by her friend named Sam, here are the ways for our heads and tails to match up:

Me heads	you heads	Sue heads	Sam heads
Me heads	you heads	Sue heads	Sam tails
Me heads	you heads	Sue tails	Sam heads
Me heads	you heads	Sue tails	Sam tails
Me heads	you tails	Sue heads	Sam heads
Me heads	you tails	Sue heads	Sam tails
Me heads	you tails	Sue tails	Sam heads
Me heads	you tails	Sue tails	Sam tails
Me tails	you heads	Sue heads	Sam heads
Me tails	you heads	Sue heads	Sam tails
Me tails	you heads	Sue tails	Sam heads
Me tails	you heads	Sue tails	Sam tails
Me tails	you tails	Sue heads	Sam heads
Me tails	you tails	Sue heads	Sam tails
Me tails	you tails	Sue tails	Sam heads
Me tails	you tails	Sue tails	Sam tails

Can we see a pattern in the heads and tails that might help us know if we have found all the ways for our heads and tails to match up?

Is there a pattern for the ways the squares come up? We see that one square has two possibilities. Two squares has four possibilities. Three squares has eight. Four squares has sixteen. The powers-of-two pattern for Power Blocks is the pattern for the cardboard squares, as well.

Squares **Possibilities**

1	**2**
2	**4**
3	**8**
4	**16**

Teacher: Please tell me the ways you found for the heads and tails to come up so that I may make my graph on the overhead.
Student: All heads or all tails.
Student: Three heads and one tail.
Student: Three tails and one head.
Student: Two heads and two tails.
Teacher: Any other ways?
Students: No.

Our students see five ways. We see sixteen.

Teacher: How many of you had heads-heads-heads-heads as the winner? Please raise your hands.
How many had heads-heads-heads-tails as the winner?
Heads-heads-tails-tails?
Heads-tails-tails-tails?
Tails-tails-tails-tails?

(illustration 8-5-4)
(Graph of heads-heads-heads-heads, heads-heads-heads-tails, heads-heads-tails-tails, heads-tails-tails-tails and tails-tails-tails-tails winners.)

Of the sixteen ways we know the squares may come up, only one is all heads and only one is all tails. There are four ways each for heads-heads-heads-tails and tails-tails-tails-heads. There are six ways that heads-heads-tails-tails may come up. Still, our students are only likely to see the five possibilities and not sixteen. Of the five that they see, all heads or all tails is likely never to win.

Teacher: Why?

Five-square tossing...

Even though the students may not yet know the reasons for what they see, they can still anticipate what might happen next.

Teacher: What do you think might happen if we tossed five squares at once? I know that one of the possibilities for five squares would be five heads and another would be five tails. Do you think either all heads or all tails would come up the winners?
Students: No!
Teacher: Before we start tossing the squares, please figure out for me what you think all the possibilities for five squares might be, then tell me so I can add the ways to my graph.

(illustration 8-5-5)
(Graph labeled: h-h-h-h-h, h-h-h-h-t, h-h-h-t-t, h-h-t-t-t, h-t-t-t-t, t-t-t-t-t.)

Teacher: Which ways do you predict will be the winners?

The patterns we see help us anticipate what we might see next, even if we do not always know the reasons why. Students who cannot answer, "Why?" for four or three or two or one can still make predictions for five.

Would we as teachers be able to predict? The pattern we could see for one and two and three and four was 2, 4, 8, 16. The numbers are doubling each time. Is the number of ways for five 32?

Teacher: Now, let's toss the squares and see what we find out.

Lesson Five asks, "Why?" without giving answers. Lesson Six gives our students an explanation of the why.

Lesson Six

Purpose	Learn a connection between ways possible and ways that actually occur.

Summary	Students roll dice, graph results, and learn to predict likely outcomes in advance.
Materials	Dice and graph, lined, or plain paper.
Topic	One die toss and graph.
Topic	Two dice, chart the ways, toss and graph.
Topic	Three dice, chart the ways, toss and graph.
Topic	Four dice, chart the ways, toss and graph.
Topic	One die, chart the ways.

A roll of the dice...

Teacher: Which number do you think will come up the most often if you roll one die?
Students: (Opinions as to the number that will come up most.)
Teacher: Let's find out. Each of you please roll one die and keep track of the numbers that come up. Let's roll for about five minutes. Please keep track in a way that will let you know which number comes up the most. What are the possible numbers?
Students: Zero, one, two, three, four, five.

Note: The dice in this class are numbered from zero to five and not one to six. These dice are also used to create problems for addition, subtraction, multiplication and division in place value lessons later on, hence zero to five and not one to six. Regular dice work just as well.

Teacher: Which number did you have as your winner? If you had a tie for winning, you can tell me the numbers that tied. How many had zero win? One? Two? Three? Four? Five?

(illustration 8-6-1)
(Graph of the winners. Make the graph match the questions that follows.)

Teacher: Why did five win?

Whatever number wins, the teacher's question is still "Why?" The teacher listens for any explanation that will provide an excuse for rolling the dice for another five minutes. When the dice are rolled again, the teacher makes a new graph of winners and asks, "Was the hypothesis borne out? Or, is there another explanation we can test?"

Since no number is any more likely to win than another, the winning numbers each time are apt to be as random as the numbers on the rolling dice themselves. Whatever the result, the teacher's question remains predictable:

Teacher: Why?

Two-dice combinations...

Teacher: Today we are going to keep track of the numbers you get when you roll two dice at the same time. I want you to work with a partner, so mark your die so you can tell it from your partner's. Begin by seeing what different totals you can get from your two dice, then roll and keep track of how many times you get each total.

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Teacher: Let's stop for awhile. I know we are only keeping track of the totals you get when you add the dice together, but I am curious about something else, too. I would like you to help me make a chart of all the ways two different dice can come up.

The class will make charts for two- and three- and four-dice combinations and, eventually, even for one die. Since the charts will not all be completed in the same day or maybe even in the same week, they are recorded on paper to be put up on the wall and saved.

Teacher: First of all, tell me what different totals you have found so far.
Students: Zero, one, two, three, four, five, six, seven, eight, nine and ten.

0 1 2 3 4 5 6 7 8 9 10

Teacher: How many ways do you think there are to get zero? What do you and your partner have to have on your dice to get zero for a total?

Students: We each have to have zero.
Teacher: I'll write that way above the zero on my chart.

0+0										
0	1	2	3	4	5	6	7	8	9	10

Teacher: How many ways to get one?
Students: Zero and one.
Teacher: Zero and one or one and zero?
Student: Its the same thing.
Teacher: But for my chart I want to keep track of what each partner can get. If I were rolling with you, I could get one and you could get zero or you could get one and I could get zero. I count that as two different ways.
Student: Then there are two different ways for zero, too.
Teacher: If I were rolling with you, I could get zero and you could get zero. Is that different than if you get zero and I get zero?

Whether the students can understand the logic behind this question depends upon the experience they have had in thinking through the meaning of words. In any event, 0 + 0 is the same as 0 + 0 and 1 + 0 is not the same as 0 + 1.

	0+1									
0+0	1+0									
0	1	2	3	4	5	6	7	8	9	10

Teacher: How many different ways to make two?
Students: Zero and two, two and zero and one plus one.

Students who volunteer that the one-plus-one combination counts twice do not yet fully grasp the concept of why zero plus zero is only one way to add to zero and not two. What may be clear to one child or adult is not always clear to another. The reason the students have marked their dice is to assist them in identifying the combinations they find. If not everyone understands just yet, enough will understand to help the teacher make the chart.

		0+2								
	0+1	2+0								
0+0	1+0	1+1								
0	1	2	3	4	5	6	7	8	9	10

Teacher: How many different ways to make three?

			0+3							
		0+2	3+0							
	0+1	2+0	1+2							
0+0	1+0	1+1	2+1							
0	1	2	3	4	5	6	7	8	9	10

Teacher: I think I am beginning to see a pattern for the ways to make each number.

number	ways
0	1
1	2
2	3
3	4

Teacher: How many ways do you predict you will be able to make four?
Students: Five!
Teacher: See if you and your partner can find all five ways.

				0+4						
			0+3	4+0						
		0+2	3+0	1+3						
	0+1	2+0	1+2	3+1						
0+0	1+0	1+1	2+1	2+2						
0	1	2	3	4	5	6	7	8	9	10

Teacher: How many ways do you predict you will be able to make five?
Students: Six!
Teacher: See if you and your partner can find all six ways.

					0+5					
				0+4	**5+0**					
			0+3	**4+0**	**1+4**					
		0+2	**3+0**	**1+3**	**4+1**					
	0+1	**2+0**	**1+2**	**3+1**	**2+3**					
0+0	**1+0**	**1+1**	**2+1**	**2+2**	**3+2**					
0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**

Teacher: How many ways do you predict you will be able to make six?
Students: Seven!
Teacher: See if you and your partner can find all seven ways. Then tell me he ways you find.
Students: Zero plus six, six plus zero, one plus five, five plus one.
Teacher: I think you are beginning to predict from looking at the patterns on the chart and not from using your dice. I hear zero plus six as a way, but you have to be able to show me the ways with your dice. Can anyone show me zero plus six?
Students: No.
Teacher: Why?
Students: There are no sixes on our dice!
Teacher: Then how many ways can you make six with your dice?
Students: Five.

					0+5					
				0+4	**5+0**	**1+5**				
			0+3	**4+0**	**1+4**	**5+1**				
		0+2	**3+0**	**1+3**	**4+1**	**2+4**				
	0+1	**2+0**	**1+2**	**3+1**	**2+3**	**4+2**				
0+0	**1+0**	**1+1**	**2+1**	**2+2**	**3+2**	**3+3**				
0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**

Teacher: That's interesting. Let's see what our pattern looks like now.

number	**ways**
0	**1**
1	**2**
2	**3**
3	**4**
4	**5**
5	**6**
6	**5**

Teacher: What number do you think we'll get next for ways to make seven?
Student: Six again.
Student: Five again.
Student: Four.
Teacher: Let's see how many ways you can find. Remember, the ways you find have to be ways you can make with your dice.

					0+5					
				0+4	**5+0**	**1+5**				
			0+3	**4+0**	**1+4**	**5+1**	**2+5**			
		0+2	**3+0**	**1+3**	**4+1**	**2+4**	**5+2**			
	0+1	**2+0**	**1+2**	**3+1**	**2+3**	**4+2**	**3+4**			
0+0	**1+0**	**1+1**	**2+1**	**2+2**	**3+2**	**3+3**	**4+3**			
0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**

Teacher: Now look at our pattern.

number	**ways**

0	**1**
1	**2**
2	**3**
3	**4**
4	**5**
5	**6**
6	**5**
7	**4**

Teacher: How many ways do you predict we will find for eight?
Students: Three.

					0+5					
				0+4	**5+0**	**1+5**				
			0+3	**4+0**	**1+4**	**5+1**	**2+5**			
		0+2	**3+0**	**1+3**	**4+1**	**2+4**	**5+2**	**3+5**		
	0+1	**2+0**	**1+2**	**3+1**	**2+3**	**4+2**	**3+4**	**5+3**	**4+5**	
0+0	**1+0**	**1+1**	**2+1**	**2+2**	**3+2**	**3+3**	**4+3**	**4+4**	**5+4**	**5+5**
0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**

number	**ways**
0	**1**
1	**2**
2	**3**
3	**4**
4	**5**
5	**6**
6	**5**
7	**4**
8	**3**
9	**2**
10	**1**

Teacher: Thank you for helping me make a chart of all the ways two different dice can come up. Now, remember when we first started, we were keeping track of the numbers you got when you and your partner rolled two dice at the same time. You can go back to rolling your dice and seeing which totals come up the most.
Student: Do you want us to keep track of all the different ways the numbers come up?
Teacher: No. just keep track of the totals. I want to know which total comes up the most.

The students roll their dice for about five minutes before the teacher asks for their results.

Teacher: Which total did you have as your winner? How many had zero as the winner?...

(illustration 8-6-2)
(The numbers 0 through 10 with an X placed above each number found to be a winner. The dialog that follows should match the actual graph.)

Teacher: How come almost all of the winners seem to be four, five, or six? How come zero and ten did not have any winners at all?

"How come?" is just another way of saying "Why?" The question remains constant. The ability to answer what is asked changes over time. Zero and ten do not win because there are too few ways for those numbers to come up with the dice the students roll. Four, five and six have a better chance to win because there are more ways they can come up.

Although we, as teachers, can see why graphs for two dice rolled together give us the winners that they do, there is no guarantee that our students will yet make a connection between our chart and the winners for their dice.

A winning number for rolling one die has more to do with chance than anything we might predict. No one number is any more likely to come up than another. Numbers rolled with two dice have a greater or a lesser chance of winning depending on the number of ways they have of coming up.

We may see that one-die winners happen just by chance and two-dice winners happen more predictably. But do we see yet what the rolls of dice have in common with graphs for shoes, fruit, relatives in the war, months of birth or days of sun in March or May?

Three-dice combinations...

Teacher: Today we are going to keep track of the numbers you get when you roll three dice at the same time. What different totals will you be able to get when you use three dice? Talk with your partner to see what you both think all the totals might be.

The lesson is on rolling dice, but learning is not meant to be compartmentalized. Finding all the different totals may lead our students into a discussion of how they know they have found all the ways there are to find. Individual teams of students can offer their opinions or share the patterns they see.

Students: All the numbers from zero to fifteen.
Teacher: Okay. Today, before you start rolling the three dice to see which number total wins, let's see if we can make a chart for all the different ways the numbers can come up, just as we did for the ways to make the numbers with two dice. Remember, though, we can only use ways to make the numbers that we can actually make with the dice.
You can work with your partner to find the ways.
You and your partner will have to mark your three dice differently, so you can keep track of the ways each separate die can come up.
What are the ways to make zero?
Students: Zero plus zero plus zero.
Teacher: Are there any other ways?
Student: No, because if you add any other numbers, you won't get zero any more.
Teacher: How many ways are there to make one?

As we guide our students through the ways to make three-dice totals, we see that there are more ways to make numbers with three dice than there are with two.

(Illustration 8-6-3)
(The chart for the three-dice combinations. Include the full list of ways as in the two-dice chart.)

number	ways
0	1
1	3
2	6
3	10
4	etc.

When the chart is finished, our students roll their sets of three dice and keep track of the winners. Is it likely that totals of zero or fifteen will end up ahead?

(illustration 8-6-4)
(Graph of the winners. The numbers 0 through 15 with the students' winning numbers recorded above. The dialog that follows should match the eventual illustration.)

Teacher: How come almost all of the winners seem to be six, seven, eight and nine? How come the littlest numbers and the biggest ones did not have any winners at all?

It is becoming easier for our students to see the connection between the chart of ways to make the numbers and the winners for their dice. The assessment that the connection has been made is in each student's ability to predict.

Four dice and predicting...

Teacher: Today we are going to try four dice. Talk with your partner to see what you both think all the totals might be.
Students: All the numbers from zero to twenty.

By now, our students will likely understand there are many ways to make the totals in the middle and few ways to make the totals on the ends. We no longer need to record every possibility to make our point.

Teacher: I still want to make a chart of all the ways the numbers can come up, but the chart we made for threes was getting pretty big. So, this time let's not write down zero plus zero plus zero plus one in four different ways. It really can happen four different ways, but let's just

write it down once. We will only write ways if the numbers are different, not if the numbers are just in a different order.
How many ways to make zero?
Students: One.
Teacher: It seems that no matter how many dice we roll, there is still only one way to make zero. How many ways to make one?

(Illustration 8-6-5)
(Chart for four-dice listing the number combinations that are unique and not simply in a different order.)

number	ways
0	1
1	1
2	2
3	3
4	5
5	7
6	9
7	11
8	etc.

Teacher: Before you begin rolling your sets of four dice, please tell me if you think you can predict which numbers might win. Do you think zero or twenty will win?
Students: No!
Teacher: Why not?
Which numbers do you think will win?

Our students will be able to predict that zero and twenty have little chance. Our students will also be able to predict that the numbers in the middle of the chart will come up winners more often than not.

Mathematics is patterns and connections. Students who look for patterns as a part of their mathematics curriculum can, collectively, see the connection between the combinations of numbers on the chart and the likelihood that that number will be a winner on their graph.

Patterns made with Pattern Blocks or Power Blocks or cubes are very tidy. We can see the numbers or the shapes they represent. Number patterns, too, are organized. Not all the patterns in our lives are quite so orderly, but patterns are there nonetheless. Seeing patterns does not mean finding numbers in a sequence. Seeing patterns means being able to predict.

The charts and the predictions our students learn to make teach them to see new kinds of patterns in events. The patterns for dice give them a frame of reference for understanding why they can predict from the data for some graphs but not for others. What our students come to see from the two- or three- or four-dice charts and graphs was not apparent to them as they rolled their single die or tossed their cardboard squares.

Predicting for one...
Teacher: We have made charts for two dice, three dice and four dice, but we don't have a chart for when we rolled only one die. What were the numbers we could get when we rolled one die?
Students: Zero, one, two, three, four and five.

0 1 2 3 4 5

Teacher: Let's make a chart for all the different ways the numbers can come up, just as we did for the ways to make the numbers with two, three and four dice. You can work with your partner to find the ways.
What are the ways to make zero?
Student: You can only make it one way. Its just zero.
Teacher: How about one?
Student: They each just make their own number. They haven't got anything to be added to.
Teacher: When we did the chart for four dice, I asked you to predict which numbers might win. You did a very good job of telling me which numbers would come up the most and which numbers might not come up winners at all. Can you use your one-die chart to predict what number might win when we roll only one die?
Student: They're all the same. There won't be a winner.

Teacher: But when we rolled one die, there were winners every time.
Student: Yes, but the winner was never the same. It kept changing. When we rolled for four dice, the winner didn't change.

Some students may be able to articulate why it is not possible to predict with certainty a winner for a one-die graph. Whether the reasons can be articulated or not, the charts on the wall show the pattern to be seen. The more ways there are for an event to occur, the more likely it is for that event to occur.

Dice and cardboard squares...

If our students can verbally express why they cannot predict for a one-die graph and why they know with certainty that no zeros or twenties will be winner on a four-dice graph, we might ask them to look again at the cardboard squares they tossed in Lesson Five. Why does a one-square toss divide itself between heads and tails as winners? Why, as more and more squares were added, did all heads or all tails gradually disappear from the record of winning ways? What is it in the cardboard squares that makes them act the way they do? What do the cardboard squares have to do with dice?

Lesson Seven

Purpose	Learn to apply the probability from Lessons Five and Six to graphs made or to be made.
Summary	Students review graphs in newspapers and old graphs they have made and engage in one-die/four-dice wondering.
Materials	Past and future graphs.
Topic	The focus of the lesson is on building a frame of reference for viewing past and future data more analytically.
Homework	Look for graphs at home to think about and analyze.

Which of the graphs...

Teacher: It would seem that we could predict which numbers would win when we rolled three or four dice at once because we could see from our charts that some numbers have more ways of happening than others. It was harder to predict for rolling just one die because all of the numbers had the same number of ways they could be made. Since all one-die numbers had an equal chance, we could not really tell which number might come up most often.
We could even predict which combinations of heads and tails would be winners when we tossed three or four squares at once. It was harder to know whether heads or tails would be the winner when we tossed only one square.
I wonder which of the graphs we have already made we could predict from, and which graphs would not look the same way again if we asked a different group of people the same question.
Let's look at the graph we did for months of birth.

(illustration 8-7-1)
(Months of birth graph. Make sure the dialog that follows matches the actual graph in the illustration.)

Teacher: Is there any month that comes up more than any other month?
Students: Yes. September has the most.
Teacher: If we asked another class what month everybody was born in, I wonder if a different month would come up most, or if September would. Could it be that September is really the month that most people were born in, or is September just the most common month for the people in our class?
Is the graph for month of birth like rolling one die or is it like rolling three or four?

Using the frame of reference...

We graph data to display mathematical information in a more visually comprehensible form. The numbers contain the information. The pictorial representation of the numbers for the months of birth makes it easier for us and our students to see which month is the most common birth month for the children in our class. Graphing is the process of gathering the data and arranging it in an orderly way. Probability is our tool for making sense out of the patterns in the data we collect.

Lesson Seven asks our students to look at all the graphs they have made or will make and to think:

Is this graph like the one-die graph, or is it more like the graphs for three or four dice?
How can we find out?

Lesson Seven asks us to look at all the graphs we see each day in the newspaper or on TV or anywhere else that graphs appear, and think:

How can opinion polls be so wrong sometimes? Do the pollsters sometimes show us a one-die graph while we may think they are showing us a graph for four?
What do graphs show us about such things as safety in an airplane or safety in a moving car? Are these one-die graphs or four?
How can the Tobacco Institute look at the same graph as the Cancer Institute and give us different opinion on the harm of smoking cigarettes?
What information is there in all the graphs we see that tells us what kinds of graphs they are?
What questions could we ask the graph makers that would tell us which kind of graph it might be?

As we reexamine all the graphs in our classroom, we can wonder:

Will the graphs we see look the same if we make the graph for another group of students?
Will the graphs look the same if we gather data from our parents or from friends outside of school?
Can we predict from one graph to the next, or is the graph's information only true for this time?

The study of probability is the study of likelihoods. How does the manager of the grocery store know how many checkers to have for the check-out lines, depending on the time of day? Is the manager always right? The district knows how many kindergarten children came to school this year and last. Can the district know how many kindergartners will come next year? When is it helpful to know when the pattern of the data is like a four-dice graph? When is it helpful to know the pattern of the data is more like a one-die graph?

Summary

Connections...

We use graphing to connect mathematics to other activities in school:

Language arts: Keep track of which book is read the most by students in class or write stories to accompany every graph.
Science: Record growth of plants or plot heights of bouncing balls.
Social studies: Show whose ancestors came from where and how long ago they came, or chart population growth.
Health education: Keep track of the foods we eat or the number of hours we sleep each night.
Physical education: Chart individual progress for fitness events or record standings for a team.
Music: Graph favorite kinds of music, singing groups, or the kinds of instruments students play.
Art: Use creative imagery to record information using pictures in any form that makes the meaning clear. Mathematics can be as inventively expressed as we allow our imaginations to conceive.
Show and tell: Keep track of the range of topics students share, or use what is shared as inspiration for a graph.
Recess: Chart activities students choose or the range of snacks they bring from home.
Lunch: Chart the worst school lunches or the frequency and diversity of foods served.
Taking roll: Keep track of how many students are absent each day or week, the reasons they are not here, or why they are late.
Raising money for the P. T. A.: Make a graph showing which class has the most parent members or how much money each class has raised.
Cupcake sales for the fifth-grade class: Decide how much money we wish to raise and calculate how close each cupcake brings us to our goal.
Answering questions about our favorite things: Chart who watched the special on TV last night or what we think the best vacation might be.

Among the most valuable graphs our students create are the ones that occur spontaneously. A child may bring a hockey puck from home for show and tell and talk about the hockey game she saw last night. Every child in class may now want to share all the games they have ever seen. What ways can the class think of to share what they have done? How can we tell from the sharing which games have been seen the most and which are rare?

Graphing may have a chapter of its own, but graphing does not wait for its chapter to begin. We take the opportunities that arise to teach the mathematics that is there. Mathematics is patterns and connections. We make connections whenever opportunities arise.

Questions from Teachers

1. The probability in this chapter involves tossing cardboard squares and rolling dice and comparing the charts and graphs our students make. The only conclusion was that some things are more likely to happen than others. What about odds and ratios and all the rest of probability?

The probability in this chapter helps our students search for broader meanings in the graphs they make. The dice and cardboard squares provide a framework for understanding the significance of the data represented by each graph. Ratios are covered in the fractions chapter later on.

The lessons in this book are not meant to be all encompassing or to impose a limit to what we teach the students in our room. We read books and magazines, attend conferences and inservice workshops, talk to fellow teachers, and have insights of our own. We use what we learn. We use what we know.

For everything we do in mathematics or in life, there is always more that we could have done. We need not teach everything a child will ever need to know in just one year, However, if we feel something is important for our students to know, we teach it even if it has not been included as a lesson in the book.

2. As we teach our students to graph, what is our assessment to be?

Making a graph is a mechanical process. The student gathers data and displays it pictorially. Making graphs is also an exercise in solving problems. How is the information to be gathered? What kinds of questions should be included in a survey? Is the survey to be conducted by interview or will a written form be used? How is the survey to be distributed and to whom? How might the information from the survey be graphed? What interpretation shall we give to the numbers that we find? What other kinds of questions come from our graph? Is the information we have found useful to us?

We assess our students by observing how well they can handle the mechanical aspects of a graph. Can they gather data and display it pictorially? We also assess our students by observing the development of their problem-solving skills. Can they develop ways to gather information? Do they think about the meaning of the numbers they have found? And, most importantly, do our students use graphing as a tool for answering questions for themselves?

Kevin used his graph to prove his "Brenda" theory. The ice cream bar graph on the refrigerator door kept track of disappearing bars. When our students use their graphing skill to find out things they want to know, then we have achieved our goal.