Chapter 9

Measurement, Estimation and Time

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Before We Begin

Who, what, where, when, why and how...

Our students come to us already knowing about measurement no matter what grade we teach:

Who can make the highest tower with the blocks?
Who is taller, my sister or me?
Who is heavier to pick up?
Who can go without blinking the longest?
Who can hold her breath longer underwater?
Who has the biggest piece?
Who can run the fastest across the field?

What clothes or shoe size do I wear?
What happened to all the snow and ice outside?
What time does the movie start?
Where shall we put the bases for our kickball or our baseball game?
Where do we stand to catch the ball before the ball is kicked or hit?
Where is the buried treasure hidden?
When does my birthday come again?
When will we get to where we are going?
When have I climbed so far up the tree that I would be afraid to jump down to the ground?
Why did you make the water in my bath so hot?
Why do I have to go to bed right now?
Why does water freeze?
How much cereal goes in before overflowing the edges of the bowl?
How many shovels of sand at the beach will fill the bucket?
How close must another child be before the snowball that I throw makes a hit?
How many of my dolls can I fit in bed with me?
How do we mark the hopscotch squares or marble-shooting ring?
How many mud pies have we made?

Measurement is a natural part of our students' lives.

Driving down the street...

Slide into the driver's seat. Put the key in the ignition. Read the gauge. Is there enough gas to get where we are going? Do we have a map inside our head or on a piece of paper of how to get from here to there? What time is it now? What time will it be when we reach our destination?

Start the engine. Give the car some gas. Do all the gauges on the dashboard show us temperatures and pressures as they should?

Put the car in gear. Wait at the road's edge. Watch the cars streaking by. Judge the speeds and the distances of each one. Pick the best spot for merging. Accelerate quickly. Keep out of harm's way.

How quickly or how slowly are the cars around us driving? How closely should we follow? How much space do we wish the car behind would leave? The answers to the spacing questions change with changing speed.

Gather all the data for each change of lanes. Is there enough room for us to make a move? Can we see the other cars in our rear- and side-view mirrors? How fast is everyone around us driving? Is anybody coming up too fast? Judge each lane change with care. Accelerate, decelerate, accelerate again. Move artfully and cautiously from lane to lane.

The left turn we must make awaits us just ahead. How fast are the cars coming in the lanes that we must cross? Is there enough time to turn? Is there a space in the oncoming line of traffic that might give us more time to make our turn? While we are waiting will a car in our own lane hit us from behind? Did we signal our turn soon enough so that the cars behind have the time to stop? Maybe we should not risk this turn across the traffic here. Maybe we should drive up to the signal and make a U-turn instead. If we do the math for this left turn wrong we could end up in a wreck.

Parallel parking on the street. Is the empty parking space we see ahead long enough to fit our car? If we squeeze in, will we be able to squeeze out again? Maybe a bigger space is what we need, or perhaps a smaller car. The diagonal spaces in the parking garage may be better suited to our skills. The sign above the garage entrance says clearance of 8 feet. We are sure our car can drive straight in. Could a Winnebago Camper fit in this garage?

Drive defensively. What does it mean? Defensive means planning in advance for all the crazy things that someone else might do. Will she stop too fast? Will he turn without signaling? Is the car in the next lane drifting too close? Will a child in that group of children on their bicycles swerve into our path? How much room do we have to avoid a child's swerve, without hitting something else? Will that person staggering on the sidewalk stagger into traffic too? Can we slow our car in time if he does? Thinking mathematically all the time is what it means to drive. Correctly estimating measurements is what it takes to stay alive behind the wheel.

Length, width, height, capacity, speed, distance, acceleration, deceleration, time, temperature, angles, map making, map reading, scale drawings, comparisons, conversions, estimation, consumption, applications and more. Think of all the measuring that we do just to get ourselves from here to there.

Measurement is...

Measurement is:

A tool for solving problems.
A way of finding out.
A method that we use to show our students that mathematics is a useful part of everybody's life.
All around us all the time.

Measurement is:

length	width	height	depth
area	volume	circumference	perimeter
weight	capacity	mass	density
speed	distance	acceleration	deceleration
time	clocks	calendars	stop watches
temperature	humidity	sunshine	rainfall
latitude	longitude	azimuth	altitude
shapes	angles	tangents	cosigns
map making	map reading	scale drawings	constructions
comparisons	conversions	consumption	applications

Measurement is something that we use.

Lesson One

Purpose	Learn that measurement is a part of everything we do.
Summary	We create a measuring environment in our room by making measurements a tool for finding out.
Materials	Depends on the environment that exists in our room or that we create. Anything from rice tables and water play, to the questions that we ask the and materials available to support the opportunities that arise, to paper for "Is taller than...".
Topic	Measurement is in the environment we create.
Topic	Measurement is in the questions that we ask.
Topic	Measurement is in "Is taller than...".
Homework	We provide parents with examples of the kinds of measuring questions they can ask at home.

Lessons may have gone before...

What do our students know of measurement before they come to school? Regardless of age and advantages or disadvantages at home, experiences with measuring are a part of every child's life. Words like *big, bigger, biggest; small, smaller, smallest* are used with meaning by children, rich or poor. Pencil marks on the wall can measure a child's growth just as well as any formal measuring tool. Children experience volume and capacity by shoveling sand into a pail or by scooping dirt into a cardboard box. Measurement surrounds all children from their youngest years.

What do our students know of measurement from school?

From *Mathematics Their Way* comes:

Free exploring volume by filling jars with water or with rice.
Comparing common objects on scales made from empty cartons of milk.
Extending Pattern Block walls all the way across the room.
Making dough for cookie numbers to learn about, then eat.
Recording different paths from here to there with rubber bands on geoboards.
Counting pendulum swings or water drops as measurements of time.
Using more or less or equal to compare familiar objects in the room.
Using butcher paper to cut the shape of every student in the class.
Measuring everybody's body parts with yarn.
Playing on a teeter-totter to get a real feeling for weight.
Watching lids in water to compare the time it takes each lid to sink.
Competing for who can hold his or her legs straight out the longest while sitting in a chair.
Pouring the contents of jars from one into another to see which volumes overflow and which do not.
Using a pan balance to place objects in order by heaviest to lightest.
Listening with eyes closed to intervals of time, quietly signaling the next interval to arrive.

Calibrating with marking pens to show how far each added cup of rice fills each bottle.
Graphing as way of recording measurements to be compared.
Measuring how far a bean bag can be thrown.
Keeping track of the growth of plants and seeds.
Observing changes in days, weeks and months as the years progress.
Counting out one hundred days of school.
Marking water's slow evaporation from a jar.
Covering areas with squares.
Covering distances with Unifix Cubes.
Creating volumes with wooden cubes.
Measuring displaced water as rocks descend into a plastic tub.
Timing games with metronomes.
Estimating lengths of string to go the length or width or height or all the way around an object.
Estimating how many spoons of rice it would take to fill the jar.
Estimating how swiftly toy cars accelerate down a ramp.
Checking all the estimates with more precise measurements.
Watching patterns grow or shrink.

From *Mathematics... a Way of Thinking* comes:

Measuring with nonstandard units.
Using everything from string to body parts as a measuring device.
Placing selected measurements in order by size.
Inventing techniques to make the measurements more precise than measurements sometimes are.
Redefining and refining the measuring techniques.
Finding ratios of length.
Creating rules for measuring as the need for them arises.
Discovering the need for standardizing measuring units.
Using measuring as a problem-solving tool.
Making maps.
Reading maps.
Learning how to draw to scale.
Knowing the importance of adding descriptive labels to measurements.
Using measurement to keep track of growth and change of all kinds.
Linking measuring to graphing. Measuring is a process of finding out. Graphing is a way of displaying measurements to analyze.
Measuring in metric, with standardized units used throughout the world.

From working in our classrooms comes:

The buildings made while free exploring Power Blocks and Pattern Blocks and cubes.
Opening activities each day that use measurements all year long.
Reasonableness and estimations as ways to know what might be right.
Geometry and geometric constructions of all kinds.
Graphing of measurements before the lessons on measurement begin.
Applications for adding and subtracting that show what kinds of problems numbers can solve.
Multiplication and division taught with areas of squares.
Opportunities without limit. Averages of all kinds.
Algebra and coordinate graphing to see the patterns in measurements.
Science lessons that use measurement as a tool.
Homework designed to bring all kinds of mathematics from school to home and back.
Mathematics from the life of every child, regardless of the unit being taught.

A lesson we do not teach...

We teach our students how to measure, but measurement does not always have to be a lesson that we teach. How do we teach a lesson that we do not teach?

(illustration 9-1-1)
(Rice table or box set up for play with children actively involved.)

Ingredients of a lesson not taught:

A box filled with rice or sand.
Jars and bottles of all sizes.
Small cartons and boxes strong enough to survive children's play for more than just a day.
Small buckets and smaller shovels.

Measuring spoons of all shapes and sizes.
Funnels to make the pouring back and forth go more smoothly.
Pan balances durable enough to take a beating.

Whether they know it or not, children playing in the rice or sand are learning. When children play at home, no one may be focusing their attention on the measuring taking place. When children play at school, we observe their creative exploring and we focus their attention on the measuring.

The questions that we ask direct our students' learning:

Which jar is tallest?
Which jar is shortest?
Can you show me how you know?
What does tallest or shortest mean?
Which jar is the heaviest when filled with rice?
Tell me which of these two jars will hold more rice and which will hold less?
How can you find out?
Fill a bottle to the top with rice. Use a funnel to help you pour all the rice from the filled bottle into a bottle of a different size. Do you think all the rice you have to pour will fit inside?
Fill some other jars with rice. Before you put the rice inside, tell me which jar you think will hold the most? Which jar will hold the least?
How can you prove which holds more and which holds less?
Which jar is bigger?
Which is smaller?
What do you mean when you say big or small?
Does biggest mean the jar holds the most? Or, does biggest mean the jar is taller than the rest?
How much more rice does this jar hold than that?
Are there any jars that hold the same amount of rice?
How many scoops of rice does each jar hold?
Can we graph the scoops that each jar holds to see what we have learned?
Could we use the pan balance to put the jars in order from the lightest to the heaviest?
Would the order be the same if the jars were filled with rice?

As our students ponder the problems that we pose, we ask them to keep written records of their work. Students who cannot yet write may use pictures to show what they have found.

(illustration 9-1-2)
(Samples of written records of rice table explorations and discoveries. Include examples of Reading Program writing and of children's work illustrated by drawings without words.)

We ask a question and watch how our students solve the problem. We learn the next question we might ask from watching what our students do. We do not confine the questions that we ask to the questions on our list—we let our students be our guide. We let our students teach us how to teach the lesson that we do not teach.

When our students think they know all there is to know about rice, we can change the rice to water and begin the experiences all over again. Do we need to ask different questions for water than we asked for rice or sand? For the questions that remain the same, do the answers remain the same as well?

Paper cylinder...
We know to introduce our students to rice and water play in their earliest years of school. At what age do our students become too old to learn from the rice or water that containers hold?

(illustration 9-1-3)
(Show the starting piece of paper and the two cylinders formed when it is rolled first the long (or tall) way and then the short (or wide) way. To make it more clear that the same paper size is used to make both cylinders, illustrate the starting piece of paper turned again but still flat before making the second cylinder.)

Which of the two cylinders made from the single piece of paper do we think would have the greater volume or capacity?
Since the same amount of paper is used to make each one, shouldn't the volume be the same?
Are we sure?
How do we prove that our answers are more than just opinions?
Should we try to prove the similarities or the differences between the two by using the formulas for cylinders that we learned in school? How many of us remember the formulas that we learned?

Would it be easier to make the cylinders and fill them each with rice to see? Or, are we too old to learn from rice?

How old are our students when we end their play with rice or water? Learning does not stop at five or six or seven. How old are we? We may think we know the answers to the questions about cylinders, but do we really know? There are discoveries left for us to make regardless of our age.

Opportunities that arise...

How can we create an environment that helps our students learn the measuring skills important in their world? We do not have to know exactly what to teach. In measurement, there is no one thing our students need to know. We start by looking for the opportunities that arise.

There are two kinds of opportunities. Ones that already exists:

How much have I grown?
How long is it until we eat lunch?

And, ones that we create:

Is there a pattern for perimeter and area that you could use to know the biggest area you can enclose within 40 feet of fence?
Draw a map from the door to your desk.

Not every skill we teach has to have immediate application to the child's life. Perimeter is not something a child needs to know to cross the street safely or to calculate the correct change for the purchase of a candy bar. The focus of a lesson on perimeter may be the mathematical skill of learning to search out patterns in events. Students who learn to look for patterns and make connections are learning to transfer their knowledge to new situations. Perimeter may not be something of immediate use in a child's life, but thinking always is.

When we find a problem we feel will add to our students' understanding of the patterns and connections in mathematics, we make that problem a part of what we teach. We ask our students to explore the perimeters of shapes for the patterns they can see and the connections they can make—not because we expect our students to build fences around their yards when they go home that day.

A memorized fact is not useful for knowing the next fact we must learn. Memorizing $3 \times 4 = 12$ helps us know that $3 \times 4 = 12$. It does not tell us the answer to 3×5. But if we understand that 3×4 means we have three groups of something four times, we can reason that 3×5 means we have three groups of something five times. We can reason answers to problems that we might not have seen before.

When we focus on the patterns and connections in the measuring opportunities that arise, we base our teaching on thinking and on reasoning. We do not need to cover every measuring skill.

Wondering...

We do not rely solely on wondering when deciding what measurement to teach, but starting with what children may wonder about is a good place to begin.

I wonder:

How is rainfall measured? By sticking rulers into puddles on the ground?
How is snowfall measured? Do we squash the snow down first?
How is the water content of a snow pack measured? Do we have to melt the snow to know the water that it holds?
How do we measure wind?
How do we measure heights of mountains or distances to the sun and stars? How do we know that we are right? Has anyone paced off the distances to see?
How do we measure ocean depths or tides?
How do we clock the speed of sound or light? How do we use these measurements to know how far the thunderstorm is from where we are standing now?
How fast does a body grows? Do hands grow as fast as feet?
How fast does hair grow? How can we measure hair's speed?
How fast does a stalk of corn grow? Does it grow faster in the day or in the night?
How high up can a kite go? As high as the string we use?
How high are the birds? Do they fly above the clouds?
Is there any limit to the questions we can ask?
Will students ever run out of things to wonder about, regardless of their age?

Which questions should we ask?...

We take the opportunities that arise. We make the opportunities that do not. The opportunities are as limitless as our imaginations and the imaginations of the students in our class.

Measuring circumference with strips of yarn and comparing circumferences to the areas inside: Can we plot the points for circumference and diameter?
Measuring long distances with string: How much kite string would it take to stretch from our classroom to the cafeteria? How much to stretch from our classroom to the bus stop? Which distance would use more string?
Measuring with trundle wheels: How do rolling wheels measure distances on the ground? What other rolling wheels tell us how far we have gone? Can rolling wheels tell us how fast as well?
Playing house in kindergarten: Do children assign the roles of father, mother, baby sister and baby brother by size? What measuring do students do to know the biggest and the smallest child? What other measurements will we hear our students make as we listen to their play?
Setting up a classroom store: What measuring do we need to make the store? What measuring do we need once the store is made?
Sharing letters with pen pals from anywhere: Can we construct a life-size drawing of the pen pal from the descriptions and the measurements received?
Growing plants and raising animals. How much bigger is the vine this morning than it was just yesterday? How shall we measure bigness of a vine? Is our classroom hamster growing as fast as is our plant? What are the ways we measure growth? Does growth mean taller, wider, heavier, all three at once, or something else?
Miscellaneous facts from the books we read: A humming bird eats twice its weight in food each day. A shrew may eat a hundred times its weight. What is the weight of all the food we eat each day? What is that weight compared to us? How much more would we have to eat to consume twice or a hundred times our weight? What would our grocery bill look like?
Cooking at home, at school, or in the restaurant, and reading recipes: Can we cook without measurements? How many measurements do we have to make?
Keeping track of temperatures inside the room and out: What does 72° feel like? Can we step outside and feel what the temperature is and then check our feelings with a measuring device? Can we train ourselves to feel what the thermometer tells us we should feel? Or, will we most likely get it wrong?
Measuring the impossible: How tall is the flag pole at our school? How can we measure its height without climbing up to see? How can we measure building height without sitting on the roof? How can we measure velocity of wind or the speed of passing cars? Do we always know the ways to make these measurements ourselves, or do we need a little help? In school, as in life, we do not always know the answers to all the questions that we ask.

For every question that we ask, we also ask:

What pattern did you see?
Can you write a description of what you have found?
Can you write the proof you used?
Can you write the discoveries you have made?

Writing is a way we have of sharing what we find. Writing is a part of everything we do in math.

Is taller than...

Teacher: Look at the wastepaper basket by my desk. Name something that is a little bit taller than the basket.
Student: Your desk.
Teacher: Name something a little bit taller than my desk.
Student: You.
Teacher: Name something a little bit taller than me.
Student: The door.
Teacher: Name something a little bit taller than the door.
Student: The ceiling.
Teacher: Name something a little bit taller than the ceiling.
Student: There isn't anything taller than the ceiling.
Teacher: There are lots of things taller than the ceiling.
Student: Not in our class, there aren't.
Teacher: Did I say the things you named had to be inside our classroom?
Student: The tree outside our window.
Teacher: Name something taller than the tree.
Student: The flag pole.

Teacher: I am going to give each team of two students a lined piece of paper. What I want you to do is make a list of "taller than" things, so that each item on your paper is taller than the item on the line before. Fill your paper from the top to the bottom. No fair skipping lines. You may use your spelling notebook if you need help with a word.

If students cannot yet write, they may work together as a class taking turns adding taller and taller items to a spoken list. The teacher may write out what is said. Reading Program students can work in teams to stamp out written lists.

Teacher: How do you know your list is right?

Is taller than may be followed by:

Is shorter than
Is narrower than
Is lighter than
Is shorter than
Is slower than
Is earlier than
Is younger than

Is wider than
Is heavier than
Is longer than
Is faster than
Is later than
Is older than

When we wish to discuss the meaning of some words we can ask our students to make lists:

Is bigger than
Is smaller than

We may ask our students to make their lists for measurement, but there are other lists our students can contemplate:

Is bluer than
Is harder than
Is nicer than
Is more fun than

Is brighter than
Is easier than
Is tastier than

Is there any limit to the lists that we can make?

Lesson Two

Purpose	Learn to make and read maps.
Summary	Students make maps and use and critique maps made by others to refine their own map making techniques.
Materials	Geoboards, geoboard recording paper blacklines, wooden cubes, graph paper, paper of all kinds.
Topic	Record geoboard shapes on paper.
Topic	Build and describe with cubes on graph paper.
Topic	Repeatedly making and critiquing maps to get from here to there.
Topic	Drawing and critiquing maps of class.
Homework	Students make maps of home and of routes to get from here to there.

Hand in hand...

When we help an infant learn to speak, we do not start with lessons, we start with talking. When a child who can speak says, "I goed", we correct with gentleness. "I went." Lesson One is the talking. Lesson Two is the refining of the talking skills. Lesson One is about creating an environment for measuring. Measuring continues all the time. Lesson Two is about refining measuring skills. Lesson One and Lesson Two go hand in hand.

Mapping...

How were we taught to use maps when we were in school? We filled in names on dittoed pages or answered questions from hanging maps on the wall. Our mapping lessons were not a part of mathematics—they were part of geography or history. We only read or used the maps that someone else had made.

There are maps for:

- City roads
- Countries in the world
- Buried treasure
- Tourist sites
- Explorers' paths
- Solar system
- Population density
- Rainfall quantities
- Walking trail guides
- Subway lines
- Instructions for the journey from here to there
- Counties in the state
- Fault lines in the earth
- Blueprints for a house
- Theater seating plans
- Ancient drawings of our world
- Stars up in the sky
- Currents in the sea
- Land topology
- Fire exit paths
- Routes that planes or buses take

In the graphing chapter we said:

> We graph data to display mathematical information in a more visually comprehensible manner. The information we graph comes from the measurements we make: length, volume, capacity; feelings, choices, opinions; experiments, experiences, or our wanting to find out. All kinds of measurements are measurements that we can graph.
>
> The numbers contain the information. The pictorial representation of the numbers makes it easier to see what the numbers represent. We record the data or information in pictorial form, so we can make better sense of the numbers involved.

In the measurement chapter we say of mapping:

> We map data to display mathematical information in a more visually comprehensible manner. The information we map comes from the measurements we make; areas, distances, routes; snowfall, crop growth, the path of a hurricane; results of observations, experiences, or our wanting to find out. All kinds of measurements are measurements we can map.
>
> The measurements contain the information. The pictorial representation of the measurements makes it easier to see what the measurements represent. We record the data or information in pictorial form, so we can make better sense of the measurements involved.

Mapping gives meaning to the measuring we teach. Mathematics is more than numbers. Mathematics is patterns and connections. Mapping connects math to life.

Making and recording...

When our students made shapes on their geoboards and recorded the shapes on paper, their experiences with making maps had already begun.

(illustration 9-2-1)
(Geoboard with a simple shape on it and a piece of geoboard recording paper next to it. The geoboard recording paper is the same size as the geoboard. The shape on the geoboard is to match the shape described in the following dialog.)

Teacher: I want you to help me record the shape I have made on my geoboard. Please tell me how many nails over and how many nails up this corner of my shape is.
Student: One over and one up.

If the students cannot collectively describe the location of the point, the teacher teaches them to count the nails across and then up to find the position.

Teacher: I will mark that nail on my recording paper.

(illustration 9-2-2)
(A small circle drawn around the dot that is one dot over and one dot up from the bottom left hand corner of the recording paper.)

Teacher: How long is this side? How many nails up from the start does it go before it turns?
Student: Another nail up.

There are several ways to describe the next nail's position: one nail up; two nails from the bottom row; two nails up the side, then one nail over. It does not make any difference how students describe the position so long as they describe it accurately.

Teacher: I'll draw this side on my recording paper. It goes from the dot that is one up and one over to the dot that is two up and one over.

(illustration 9-2-3)
(The first side drawn on the recording paper.)

Teacher: Look at the shape. Can you describe at which nail the next side ends?
Student: Two over and one, two, three, four up.

(illustration 9-2-4)
(The next side drawn in.)

Teacher: Now which side do I draw?

The teacher continues asking for help in drawing the shape until it is recorded completely.

(illustration 9-2-5)
(The geoboard shape on the board and copied completely on to the recording paper.)

Our students record their geoboard shapes on paper so they may:

Learn to record accurately the shapes they make. Students who experience perceptual difficulties benefit the most from learning to look from point to point in the shapes that they record.
Have written records of the shapes they make to which they add their proofs when finding areas.
Practice in an informal way locating points on a grid using the coordinates of how many over and how many up. This skill is used again in the coordinate graphing that is a part of algebra.
Learn beginning elements of mapping: copying shapes onto paper and drawing shapes to scale.

Once our students demonstrate that they understand the recording process on paper the same size as their geoboards, we substitute paper that has the dots in smaller spaces.

(illustration 9-2-6)
(Three sizes of geoboard recording paper. First size, one to one. Second size, Four boards to a page. Third size twelve boards to a page.)

Recording on graph paper...

When our students record geoboard shapes on geoboard recording paper they focus their attention on the nails. They record the points that mark the nails where sides begin and end.

Once our students are comfortable recording shapes on various sizes of geoboard recording paper, we add graph paper to the material they may use to capture shapes.

(illustration 9-2-7)
(Several geoboard designs. Show both the geoboards and the recordings on graph paper. Use different size graph paper for each geoboard design recorded to make clear that the squares on the graph paper used are meant to be variable.)

Graph paper was not made with the geoboard in mind. It is not as easy for students to describe where to draw the lines. But "not as easy" does not mean not possible.

Teacher: Please help me record the shape I have made on my geoboard. How can I tell where to record this side on my graph paper?
Student: First, draw a geoboard shape on your paper.
Teacher: What do you mean?
Student: Draw a line that goes around five squares across and five squares up.
Teacher: Show me what you mean.

(illustration 9-2-8)
(Section of graph paper with a five by five square drawn on it.)

Teacher: Now what?
Student: Count two squares over and two squares up and draw the side.

Teacher: Where do I draw it?
Student: On the square two over and two up.
Teacher: But where? There are four sides on the square. Which side do I use?
Student: The side on the left.

It may not occur to students to make a drawing of the geoboard on the graph paper before giving us instructions. Our students may not initially be able to describe where to draw the side. We ask the questions. If our students need help in providing answers, we provide the help they need.

Once our students understand the use of graph paper for recording geoboard shapes, we stop using geoboard recording paper and use only graph paper to record all the shapes our students make.

(illustration 9-2-9)
(A geoboard shape copied onto geoboard recording paper and onto graph paper.)

Graph paper has other mapping uses as well.

Behind the shield again...

In the chapter on Sorting and Classification (page 084), our students learned to build behind the shield. Skills learned are skills used.

(illustration 9-2-10)
(Show the graph paper, the cubes and the shield.)

Teacher: Today you will be building behind your shields again. This time you will be using wooden cubes and sheets of graph paper. Please find two or three partners to build with. Have one member of your group build a shape on top of his or her graph paper, describing to the rest of the group how to make the same shape. When it is time to pick up all the shields for your group, everyone should have exactly the same shape on his or her paper.
I'll give you an example of what I mean.
Put your shields up.
Start at the bottom left-hand corner of your paper.
Student: Which way should we turn our paper?
Teacher: I forgot to add the rule that you cannot ask any questions. Since I did not remember to tell you which way to turn your paper, you'll just have to turn it any way you want to.
Start at the bottom left-hand corner. Count two spaces over and three spaces up. Put a cube in that space.

Students may or may not notice that there is more than one way to interpret this instruction.

(illustration 9-2-11)
(At least two different interpretations of two over and three up. In the caption for the illustration state that while the teacher meant count two over and then, when counting three up, count from the bottom square, some students consider that since they already counted the bottom square, the three-up starts one-up from the bottom.)

The teacher could clear up the confusion by having the students label the edges of the graph paper with numbers and/or letters similar to those in the children's game of Battleships. But confusion is often a part of learning. When it is the students' turn to build and describe, they may decide to invent a system of coordinates for themselves. If not, then in time we will assist them in learning better ways to pinpoint position on their paper.

Teacher: Put the next cube in the space immediately to the left.

To the left of what? In the space on the left side of the paper? On the left side of the cube? How much easier it would be just to say, put the cube in space (3,3). But the students cannot ask questions and the teacher has not yet added coordinates to the instructions given.

The teacher continues giving instructions to the class until the class understands the instruction game they are to play. The students then take turns building behind their shields and describing to their group mates the constructions that they make.

After the students have had the opportunity to give and follow instructions, the teacher leads a discussion of what makes some instructions harder to follow and what makes some easier. At this point, if the thought has not occurred to any student, the teacher suggests the use of coordinates to describe positions on the paper.

Our students record shapes on geoboard paper by counting nails over, then up. They record shapes on graph paper of various sizes by counting the sides over, then up. When we were students long ago, the coordinates we learned in algebra went over, then up, as well. Mathematics is connected—from the earliest free explorations to the college calculus class.

Map of the class...

Teacher: Draw a map of the class.

Will our students ask:

How big shall it be?
Can we work together in teams?
What do you mean by a map?
What should we include?

Does it make any difference what answers we give? Is learning only possible on one size of paper? Do we accept only one definition of "map"? If we ask our students to draw us a car, how many cars do we expect to see?

Each of the maps our students make will be as different as the students themselves. Our questions apply to all the maps, even though the maps will not all be the same:

Tell us about your map.
What do all the things on your map mean?
Did you use measuring while making your map?
What have you added to your map that you have not seen on any other map in our class?
What did you leave out that someone else has put in?
Could someone new to our class look at your map and know what it is about?

Maps involve scaling—nothing on maps is drawn in real size. Maps involve measuring, too.

Teacher: Draw a map of the classroom so that someone else could use your map to find your desk. The one rule for your map is that you cannot write anybody's name on it.
Student: Can we use paces to tell people how far to walk?
Teacher: The one rule for your map is that you cannot write anybody's name on it. That is the only rule. Anything else is possible. Anything else is permissible.
Student: Can we work together?
Teacher: The one rule for your map is that you cannot write anybody's name on it. That is the only rule. Anything else is possible. Anything else is permissible. If you want to work with someone else, you may, but you will each have to have your own map to your own desk.
Student: Can we...
Teacher: What will my answer to your question be?
Student: The one rule for our maps is we cannot write anybody's name on it. That's the only rule. Anything else is okay for us to do.

Because everyone in class already knows where everybody else sits, a student's name is an automatic indicator of location, no matter how vague the map itself might be.

The teacher gives the assignment. The students make the maps. Once all the maps are made, the teacher collects them, shuffles them and hands them out again. In the collection process, the teacher makes mental note of who was working with whom, so that when the maps are redistributed, no one receives a map that he or she may recognize.

A convenient shuffling technique is to give the maps collected from the children on one side of the room to the children on the other.

Teacher: Use the map I have handed you to find the desk the map directs you to. When you find the desk indicated by the map, sit at that seat. If you cannot find the desk, just come up and stand by me until everyone else has found a seat.

As the class follows the teacher's instructions, some students will be fortunate enough to find the seat their map directs them to. It is likely that there could be as many students wandering around looking puzzled as there are students sure they have found the right seat. It is also likely that there may be two students who arrive at the same desk, pointing at each other's maps trying to decide who is the one really meant to sit here. The most bewildered students will end up keeping the teacher company in the front of the room.

Teacher: What made some maps easy to read and some maps more difficult to follow?
Student: The person that made the map did not put where their desk was. All they drew was desks. It didn't say what desk to go to.
Student: It had paces on it, but it didn't tell where I was supposed to begin.
Student: Mine had paces on it too, but I don't think my paces were the same size.
Student: I couldn't tell what all the drawings were supposed to be.
Student: It had little tiny feet drawn on it to tell me where to walk, but when I walked that way I couldn't find any desk. I ended up at the wall in back.
Student: There was too much erasing on it. It was too messy to see where to go.
Teacher: Please tell me something about a map that was easy to follow.
Student: Mine had all the desks in class and one desk had a circle around it.

For each comment a student makes, the teacher asks:

Teacher: What might have made the map easier to follow?

As the students discuss the easy and the confusing parts of the maps, the teacher keeps the focus of the discussion on what can be done to make each map a better map next time. The discussion focuses on learning and not on blaming. It is not necessary to determine if the map maker or the map reader is at fault for maps that are difficult to follow. None of us is as smart as all of us. Working together, we can all learn how to make maps that all of us can read.

Endless lesson...

The instructions for the lesson are:

Make a map to where you sit.
Someone else will use the map you make to find your desk.
You will use someone else's map to find where it leads you.

Everyone discusses what makes some maps easier to read than others. If an opaque projector is available, the teacher uses it to project selected maps on a screen so that everyone can see examples of how other students chose to indicate locations in the room.

The cycle is repeated again and again to test suggested new techniques. Make, use, critique. Make, use, critique. Make, use, critique. Simple cycle. Endless lesson.

For each new cycle we can change the rules we make up or the questions we ask to add new challenges. If one student has already drawn a map with all desks in identifiable positions and marked her desk with accuracy, other students can use this plan as inspiration the next time maps are made. To encourage new inspirations, we add the rule that no desks may be drawn. The student who first drew all the desks in rows will have to find another way to locate herself in class.

Students who become experts in making maps to their desks may then make maps for destinations outside of class. We can ask our students to:

Make a treasure map from the door of our classroom to a secret place in our school. Make your map so that someone else can use it to find your secret place and bring back a special marker you have hidden there.
Make a map from our school to your home, so that someone else could use your map to walk or drive to where you live. Use the map you make to guide yourself home today. If you walk with a friend, see if he or she can follow what you draw.
Make a map of our entire school showing where all the classrooms are.
Work together to make a cooperative map of the neighborhood around our school. You can each make the part of the map that shows where your house is. How can you make it so that when you put the separate pieces of the map together, it will still be readable?
(Note: The *Mathematics... a Way of Thinking* lessons on cooperative map making [14-14 and 14-15] should be taught in advance of asking students to make a cooperative map.)
Make a map that tells someone how to get from A to B to C. (A and B and C are any points within the classroom or around the school or school yard.)
What other maps can you make?

Mapping leads to measurement...

Once we have provided the tools and taught the techniques, we lead mapping into measurement.

What kinds of questions do we ask that lead the way?

Today I want you and a partner to make a map of the room using Unifix Cubes (or dowels, popsicle sticks, or other available materials) to measure all the sizes and the distances for your map. How do you think you might use the cubes to help you draw a map? Do you have to count all the cubes by ones, or can you count by sticks of ten instead?

Use Unifix Cubes (or some other standard unit of measure) as you draw a map that shows how to find your desk. Are cubes better for measuring than paces are? Why do you think so?

We used graph paper to help describe the cube constructions that we built behind the shields. Can we use graph paper to help us find a better way to draw a map? If each square on the graph paper stood for fifty or a hundred cubes, how many cubes across is our classroom? How would we record measurements made with Unifix Cubes on a graph paper map? Can we use graph paper to help us draw maps outside our class? Would we still use Unifix Cubes as our measuring tool outside our room?

What kinds of information are on store-bought maps that you do not have on the maps that you have made? What did the map makers do to make their maps easier to read? How do the map makers get all the streets in a whole city to fit on a single piece of paper?

How is a ruler used? What do all the markings on the ruler mean? Could we use a ruler to help us make a map? What is a tape measure? How is a tape measure used?

(illustration 9-2-12)
(Trundle wheel.)

This is called a trundle wheel. It rolls along to measure distances. Each time the wheel goes around once, it makes a clicking sound. With a trundle wheel, we can measure the clicks from here to anywhere. How can we record a trundle wheel's measurements on our map? Can we use graph paper and have every square represent a click? What if there are too many clicks to fit on a piece of paper? Do we use a bigger piece of paper? Or can the squares on our paper stand for more than just one click?

(illustration 9-2-13)
(Picture of a compass)

This is called a compass. Has anyone ever seen a compass before? What does it mean to travel north? Is north the same for everyone, or does north depend on where you are? Is north like right and left? When we are facing each other, right for me is left for you. Is north for me north or south for you? Is there a north, south, east and west on any of the maps we have available in our room? Can we use a compass to help us with directions on the maps we make?

Have you ever seen a weather map in a newspaper or on the TV news at night? What do all the temperatures mean? Can we make a temperature map for our school? Is every place at school the exact same temperature as every other place?

Unifix Cubes, rulers, tape measures, trundle wheels, compasses, thermometers. We introduce the measuring tools we feel our students need. Our choices are governed by what we want our students to know and what we feel they are ready for. We use the maps our students make and the rules that we create to refine our students' measuring skills. We follow no preestablished sequence to know which measuring device to introduce on the second Monday of the month, or which skill to present on the third Tuesday of the year. We teach measuring to our students as naturally as parents teach language to a child. We decide the next step to take from watching what our students do. We add the tools as the need for tools arises.

Mapping is the blueprint from which a building is made. It is in the toy cities our students make with Geoblocks. It is how we find the constellations in the starry sky at night. It is how the kindergarten child finds her way to the office with the milk count for the day and then finds her kindergarten class again. It is how we know what weather to expect. It is how we describe to one another how to get from here to there. Creating an environment for measuring is our goal. Our purpose in teaching mapping is to present mathematics that our students use. What is used is learned.

Lesson Three

Purpose	Learn to make and use balances.
Summary	Students learn to make and use different kinds of balances for weighing. Each balance is explored for longer than a day.

Materials	Materials for making three different kinds of balances. Pegboard strips, washer and centimeter cube weights, fulcrums, balance stands, paper bowls and string, Power Blocks, plastic bag weights.
Topic	Board and fulcrum balances.
Topic	Bowl balances.
Topic	Double-arm balances.
Topic	Student-made balances.
Topic	Weighing skills developed are used.
Homework	Balance explorations can be carried on at home.

Three kinds of balances...

Balances create an environment for learning about weight.

(illustration 9-3-1)
(Three kinds of balances. Board and fulcrum. Bowl balance. Double arm balance. Labels identifying each kind. All three balances made from the same basic 21 hole by 3 hole pegboard strip. Show centimeter cube block weights and "washer" weights.)

The three kinds of balances are made from the same 21-hole-by-3-hole pegboard strips. The fulcrum is wood molding cut in three-inch sections, with sand glued to the top to improve the balancing stability. The balance stand is a dowel screwed into a piece of wood. The dowel has a hook protruding from it to serve as the hanging point for the pegboard strip. The paper bowls for the bowl balance are fastened with string to their paper clip hooks. The double-arm balance weights are hung from paper clips.

After we present the materials to explore, we ask our students what they can discover on their own. To the questions that we know to ask, we add questions that occur to us as we wander around the room, observing the discoveries that our students make.

Students are encouraged to work together in teams. Teams are encouraged to share their discoveries with anyone around. None of us is as smart as all of us. We learn more from working together than we learn by working apart.

Both the pacing and direction of the balancing activities are governed by student interest and discoveries, and not by a schedule of what must be covered when. Each day's activities are followed with a discussion of what each team found out.

Board and fulcrum...

Equipment:

Pegboard strip
Fulcrum
Standard weights: Metal washers of uniform size, store-bought weights, or weights made out of centimeter cubes

Questions to ask over several days:

Can you balance the pegboard on the fulcrum?
How do you know it is balanced?
Can you put some weights on each end of the board and have it balance?
When it is balanced, slide the weights somewhere else on the board and keep it balanced.
Where are all the places you can put weights on your board and still make it balance?
Can you make your board balance if you put one weight on one side and two on the other?
How about one on one side and three on the other?
How many different ways can you make it balance if you don't use the same number of weights on both sides?
Is there a pattern to where you put the weights as you add more weights to one side while keeping only one weight on the other?
Can you make it balance with one weight on one side and no weights on the other?
What are the most weights you can put on one side without having any weights on the other and still make the board balance?
What else can you find out about how your board and fulcrum balance works?
What does all that you have learned have to do with how the teeter-totter on the playground works?

Bowl balances...

Equipment:

Pegboard strips
Balance stands
Paper bowls
String
Paper clips
Bits of clay to act as equalizing weights for boards not perfectly cut
Objects in the room to weigh
Standard weights
Small, clear plastic bags of uniform size, each filled with a cup of a different product (see illustration page 000)
Power Blocks

(illustration 9-3-2)
(A simple drawing of a bowl balance.)

Teacher: I have drawn a bowl balance on the overhead so that you may see what one looks like. Working with a partner if you wish, please construct a bowl balance from the materials available in the front of the room.

We do not provide an actual bowl balance as a model for our students. A drawing is all we show. The first assignment for our students is to discover for themselves the necessary elements in making the bowls balance on the scale. A drawing does not indicate where to tie the string to the bowl or how many pieces of string to tie. A drawing does not show how the bowl is to be hooked to the pegboard strip or how is the strip to be hooked to the balance stand.

Teacher: Can you make your bowls balance while they are empty? What does "balanced" mean?

Some students may feel the balance is balanced when the bowls are level. Some may feel that balanced means when the board is parallel to their desk. Some may feel that everything is balances when the balance is not moving up or down or wiggling around. Coming to an agreement about what it means to be in balance is part of the learning that our students do.

Questions to ask over several days, once the bowl balances are made:

Does it make any difference where the bowls are hooked on your strip of pegboard?
Put some objects in both bowls. What can you do to make the objects in the bowls balance?
How do you know when they are balanced?
If two sets of objects are balanced on your balance, would they be balanced on someone else's balance, too?
Does it make any difference where the objects are placed in the bowl?
What can you do to make the bowl on one side go down?
What can you do to make the bowl on the same side go up?
Which is heavier, down or up?
Put an object in one bowl. Add standard weights to the other bowl until both bowls are in balance. Does the number of standard weights it takes tell you how much your object weighs?
Weigh objects using standard weights. Keep track of the weights used for each object weighed.
How can you tell if one object is heavier than another without placing both objects in your balance at the same time?

(illustration 9-3-3)
(Small, clear plastic bags of uniform size, each filled with a cup of a different product. Sample fillings: Breakfast cereal, pebbles, sea shells, sand, rice, salt, peas, sawdust, bits of chalk, bits of old crayons, marshmallows, cotton balls, flower, popcorn, bits of cut up cloth, tooth picks, etc. The bags may be numbered or labeled for reference. Include in the caption a list of the materials that might be used.)

Weigh each bag and put the bags in order from lightest to heaviest.
Put the bags in order without using standard weights to help you.
Put the bags in order using standard weights to help you.
Did you get the same order for the bags both ways?
What could we fill a plastic bag with that would make it heavier than any of the other bags?
What could we put in a bag that would make it the lightest bag?
Make a standard weight out of clay or centimeter cubes.
Make a standard weight out of clay or cubes two times as heavy as the first.

Make a standard weight out of clay or cubes four times as heavy as the first.
Make a standard weight out of clay or cubes eight times as heavy as the first.
Make a standard weight out of clay or cubes sixteen times as heavy as the first.
How many different weights can you measure using the five clay or cube weights that you have made? 1+2+4+8+16=31. Are there any weight amounts between 1 and 31 that you cannot equal with your five weights?
Where have you seen the 1, 2, 4, 8, 16 doubling pattern before?
Could you use your Power Blocks as weights in place of the clay or cube weights you have made?

Power Blocks are designed around the powers of two, which means successive pieces of the same shape have areas or masses of 1, 2, 4, 8, 16. As students are doubling their clay or cube weights, they are duplicating the pattern built into their Power Blocks. Students can create a table that shows which cube or block weights are needed to weigh objects with weights from 0 to 31.

(illustration 9-3-4)
(For the eventual illustration, complete this matrix. Label the top row Power Block squares. Indicate that the * mean which squares are used to produce the weight indicated in the left-hand column. In the caption, ask what patterns are to be seen in the columns beneath each square. Also add the question: What weights could be made if the T-1 triangle were added to the chart?)

	S-1	**S-2**	**S-3**	**S-4**	**S-5**
0					
1	*				
2		*			
3	*	*			
4			*		
5	*		*		
6		*	*		
7	*	*	*		
8				*	

Double-arm balance...
Equipment:

Pegboard strip (balance arm)
Balance stand
Paper clips
Bits of clay to act as equalizing weights for boards not perfectly cut.
Standard weights

(illustration 9-3-5)
(A simple drawing of a pegboard strip on a balance stand.)

Questions to ask:

Can you balance the pegboard strip with nothing else hooked on?
How do you know when the balance arm is balanced?
(Bits of clay may be added to one side of the balance arm or the other as levelers.)
What happens if the balance arm is hooked into a different hole?
Is there only one hole you can use to balance the strip, or are there several that will work?
Using paper clips as hooks, can you hang weights on both sides of the balance arm and still keep the arm in balance?
Can you balance the arm with three weights hooked on it? Four? Five?
How many ways can you balance the arm with different numbers of weights hooked on?
Does it make any difference how the weights are hooked on the paper clip?

(illustration 9-3-6)
(Include a drawing (and not a photograph) of a double arm balance to illustrate the meaning of the following question.

Does a weight hung at the bottom of a chain of paper clips have the same effect as one hung at the top?

We ask our students to predict what will balance and what will not. Some students may predict with reason and with accuracy. Some may predict with intuition or with guesses. Mathematics is a way of thinking. We can help our students organize their thinking by the questions that we ask.

The fourth balance...

Teacher: You have worked with fulcrum balances, bowl balances and double-arm balances. Now I want to see what kind of balance you can make for yourselves.
Student: Do we have to make one of those three?
Teacher: You may make any kind of balance that you wish. It can be one of the three you have already used, or it can be a balance that is not like anything we have used in class before.

We can expand our students balance-making horizons by bringing in examples of balances they have not yet seen.

(illustration 9-3-7)
(Coat hanger balance. Pan balance with a milk carton base. Straw balance from kitchen physics.)

What our students actually make for balances depends partly on their imaginations and partly on what may be available in our room or in their homes.

The kinds of items we might make available are:

Tin cans	Bowls	Pie tins
Metal springs	Rubber bands	Coat hangers
String	Yarn	Rope
Rulers	Dowels	Straws
Blocks of wood	Milk cartons	Scraps of lumber
Nails	Tape	Building tools
Glass jars	Cardboard containers	Hooks and weights
Any other materials already in the room		

For each new balance a student or a team creates, the questions are:

Does it balance?
If so, why?
If not, what changes will you make so that it does?

Yesterday's hamster...

Balance is a part of measurement. Measurement is a part of math. Mathematics is a part of life. What kind of questions might we ask a student that makes use of balancing skills?

Does the hamster in our class weigh more today than it did yesterday? How can you put yesterday's hamster in a balance bowl to measure against the hamster of today?
How much food does the hamster eat each day? Do we weigh the hamster or do we weigh the food?
What is the weight of the water we drink in one long drink at the fountain?
What is the weight of the water we drink each day?
What are the things we have in class that weigh exactly one pound or one kilogram?
Can we make a balance or a scale for our class that will weigh heavier items than those we have fit into our balance bowls?
How much does each child's backpack weigh when the backpack is filled with books and homework to go home?
Can we make a balance that will weigh the children in our room?
Could we put the people in our class in order by weight?
Can we calculate everybody's weight in pounds or kilograms?
What does everybody in our class weigh altogether? Do we weigh a ton?
If we chose to, could we calculate the weight of everything in our room? Would we have to weigh each item separately to know the total weight?
How much trash do we dump out each day? What does our trash weigh? How much do we recycle?

Balances are meant to be used. What other questions can our students think to ask and then find answers for, now that they can weigh?

Lesson Four

Purpose	Learn about time.
Summary	In general, we teach time buy using it. Specific time concepts can be conveyed with materials like Unifix Cubes.
Materials	Digital clock, chalkboard, analog clock, "hand" clock, Unifix Cubes, timer, assorted time-measuring devices.
Topic	Time is an experience. It is taught all day long, all the time.
Topic	Miles per hour represented with cubes.
Homework	We provide parents with examples of talking time at home.

Time is...

Time is:

Past, present, future.
Seconds, weeks, days, hours, years.
Bedtime.
Dinnertime.
Show time.
The time since we were born.
The time since dinosaurs roamed the earth.
The time until we reach our destination.
The time before the next activity starts.
Pulse beats in a minute.
Birthdays in a life.
The speed of light.
The speed of sound.
The speed of sprinters in a race.
The space between the lightning's flash and the thunder's boom.
Time spent waiting in a line.
Time spent lying in the sun.
Something to be scheduled.
Something to be wasted or well spent.

Time is:

Something we experience.
Something we draw attention to.
Something learned, as language is learned, through use.

Both kinds of clocks...

The numbers on a digital clock and the hands on an analog clock help our students read time in different ways. The numbers on a digital display show the hours, minutes and seconds consecutively in a 12- or 24-hour pattern. Moving in real time, the hands of an analog clock point to the numbers in a circular pattern and, because all the numbers are visible at once, we can see time coming and time past.

Teacher: It is nine o'clock now. I want you to work at each reading station for fifteen minutes. When will you change stations the first time?

The teacher asks the question even if the students are too inexperienced with time to answer 9:15. Whether anyone answers or not, the teacher says:

Teacher: You will change stations at nine-fifteen.

(illustration 9-4-1)
(Analog clock. Cardboard clock with adjustable hands beneath the analog clock. Digital clock beside the analog clock. Chalk board beneath the digital clock. The analog and digital clock each read 9:00. The hands on the cardboard clock below the analog clock are set to 9:15. The chalkboard beneath the digital clock has 9:15 written on it.)

The teacher may rely on the students in the class to realize when 9:15 has come or use a timer that buzzes when fifteen minutes has passed. When the time to rotate the reading groups comes, the teacher asks:

Teacher: What time is it now?
Students: 9:15.

Teacher: What time will it be for the next rotation, fifteen minutes from now?

The teacher asks the questions for each rotation and shows the future times below the clocks.

Reading time. Time till recess. Time to go to lunch. Time the school day ends. Any time our students need to know can be shown with hands on the cardboard clock and numbers written on a board. The times are shown two different ways for two different kinds of clocks. The times are said aloud as well. Students can learn to tell time as naturally as they learn to speak. We do not wait for formal lessons to teach language. We do not wait for formal lessons to teach time.

We give our students lessons in the mysteries of telling time in analog as the time is told. We talk about the little hand and how it points to hours. Hours are the numbers written all around the clock. The big hand points to minutes. To know the minutes, spaces must be counted. There are no minutes written in. The numbers for the hours only help us know the minutes if we can multiply or count by fives. Mathematics is patterns and connections. The number patterns from the addition squares and cups; the start-with, go-bys; or the rows and columns from beginning multiplication are quite useful when it comes to telling time.

The eighteen year old...

The eighteen year old had been working part-time after school at the same place of business since she was a sophomore in high school. She had worked three afternoons a week during the school year and four or five days a week each summer. The business was not large. She knew her boss well. The following conversation took place the summer before she left for college:

Eighteen year old chatting with her boss at work: Look at what my godmother gave me for my high school graduation. I can't believe she would give me something like this!
Boss: What did she give you?
Eighteen year old: This watch!
Boss: It looks like a pretty nice watch to me. What's wrong with it?
Eighteen year old: My godmother knows I can't tell time when the clock has hands on it.
Boss: What do you mean, you can't tell time?
Eighteen year old: I never learned how.
Boss: But you could always tell time at work. Our clock has hands on it. And you always know when to leave. How could you know when to go home if you can't tell time? School clocks all have hands on them, too. How could you know when to change classes or when lunch time was over or anything without being able to tell time?
Eighteen year old: If I want to know what time it is at work all I do is ask someone to tell me. I just say, "I can't see what time it is from here, would you please tell me?" Bells ring all the time at school. You don't need a clock to tell when it's time for things to begin or end at school. Besides, if you ever need to know, all you have to do is ask someone what time it is. They always tell you. Anybody with a watch. My clock at home is digital. I can read that kind of clock. I just can't read the ones with hands.
Boss: But how can you not tell time? I've never known anyone who couldn't tell time.
Eighteen year old: My mother says it's because I missed the days in school when learning how to read a clock was being taught. I had measles for a few weeks during first grade and that was the time that everybody else learned to read the hands.
Boss: You mean, your mother says you did not learn to tell time because you missed the clock lessons in school?
Eighteen year old: Yes. She was really mad at the school for not teaching me.
Boss: But, your mother is a first-grade teacher!
Eighteen year old: I know.
Boss: Then couldn't she have taught you?
Eighteen year old: She said it was the school's job to do.

What does the story mean? Choose an answer from the four below:

1. Children should not get measles in first grade.
2. First-grade teachers should teach telling time for more than just a week or two.
3. Learning to tell the time is a lesson only meant for school.
4. Some children (even eighteen year olds) will benefit from the two kinds of clocks we put on our classroom wall and the hands and numbers we position underneath.

Teaching time...

We do not teach time by teaching time. We teach time by using time. Time is not a unit we present to our class like "Early Explorers" or "Animals of the Desert". Time is an experience.

We provide our students with the tools:

Clocks and watches:
- The clock on the classroom wall.
- The digital alarm clock on the teacher's desk.
- The wristwatch a child might receive as a Christmas present.
- The wristwatch that the teacher wears.

Stop watches:
- Digital watches with seconds and hundredths flashing by.
- Analog watches with long, thin, sweeping hands.

Timers:
- An electronic timekeeper with numbers that count down.
- The ticking, ringing, set-by-hand kitchen timer.
- A sand-filled hour glass.
- A three-minute egg timer.
- A metronome to sound out seconds as pendulums swing or candles burn in science class.

Other measurements of time:
- The pocket chart on the wall for placing the names for yesterday, today and tomorrow.
- A calendar of all the days and dates each month.
- The timeline on the wall we use to count the days we spend in school.

The questions that we ask focus our students attention on the experiences:

What time is it right now?
What time do we change activities?
What time will it be when you get home from school today?
How long from here to there?
Who lives the most minutes from the school?
Keep a log of the minutes you spend each day:

eating	sleeping	driving to and from
playing	working	studying hard in school
watching TV	toiletries	visiting with friends

What time is it in New York?
What time is it in Oregon?
Are the times the same?
What time is it on MTV or CNN?
What time is it on the moon?
Do you watch your microwave at home as it counts the seconds down?
Use a stop watch to find out how long it takes you to do the things you do.
Use a timer to tell you when your time is up.
How old are you?
Who is older?
Who is younger?
By how much?
How do you know?
How old were your parents when you were born?
Do you know anybody who is that age now?
How many years ago did this or that thing happen?
Who do you know who might have been alive back then?
What year is this?
What year is it in China?
What year is it in Saudi Arabia?
Are the years in China or Saudi Arabia the same length as ours?
(Teacher note: The Saudi Arabian calendar is lunar, not solar, so it has eleven fewer days each year than the solar calendar.)
What year was it when the dinosaurs roamed?
What year will it be when you are old enough to drive a car?
Can you tell me when a minute has passed without looking at a clock?
What were ways to measure time before calendars and clocks existed?
Can you make a device that measures time? What kind of time would your device measure?

What might we learn...

What might we learn by watching two parents and an infant enjoying the infant's first steps? One parent is the launching pad, sending the infant to the other parent's outreached hands. The other parent turns the infant right around, relaunched to take a second step.

The infant's wobbly step is small in distance, but distance is not the measure of accomplishment. The parents know what follows from that first step. Walking, then running, then going off to school, then growing up and moving out, then having infants of one's own, then watching over that same first step again, in twenty or thirty years.

What might we learn from watching? That learning is as natural as life. The infant learns to walk because it is strong enough and old enough to walk. It learns to walk with confidence because, if needed, there is always a parent's finger to hold. It learns to walk with pride and joy because every little step is accepted as a an accomplishment worthy of praise. Enough time is given to the infant without ever making him or her feel she or he should have done more.

What does walking have to do with how we teach our students time? Learning about time is as natural as learning how to walk or speak. No timed worksheets or lessons make time a part of every child's life. The child learns to tell time and to use time because the child is old enough and wise enough to absorb the concepts. The child learns to talk about time with confidence because, if needed, we are always there to explain what is not understood. The child learns to keep on feeling good about the learning because all the learning is accepted as worthy of praise—the child has no sense of having failed. Enough time is given to the child to understand the meaning of time.

Minutes, miles and meaning...

The more a child knows about time, the more a child is ready to know. As our students grow older, we ask questions to encourage their expanding knowledge. Sometimes the questions occur quite naturally—how many more minutes will it be until lunch? Sometimes the questions are ones that we create—what does it mean to travel at a speed of fifty-five miles an hour? We ask the questions. We provide suggestions for how to find the answers.

Teacher: How many minutes in an hour?
Student: (Assuming somebody knows) Sixty.

(illustration 9-4-2)
(Sixty Unifix Cubes snapped together in a line.)

Teacher: There are sixty cubes in this stack. If each cube stands for a minute, how many hours does this stack represent?

Minutes can be represented by little spaces on a clock or by numbers changing every sixty seconds on a digital display. Minutes can be represented by cubes as well. The cubes do not help us tell the time, but they can help us understand the arithmetic involved.

Students: One hour.
Teacher: If I can walk two miles in an hour, how far can I walk in half an hour?
Student: Four miles.
Student: Sixty miles.
Student: Thirty miles.
Student: Two miles.
Student: One mile.
Students: (Many, or most, or some, with no idea at all.)
Teacher: This stick of cubes is sixty minutes, or one hour. If it took me one hour to walk a mile, then this stick represents how long it would take me to walk a mile. I walked one mile in sixty cubes.
If I walked two miles in an hour, that's the same as saying I walked two miles in sixty minutes or sixty cubes. If I walked two miles in sixty cubes, how many cubes would represent how long it would take me to walk just one mile?
Student: Sixty?

Just because we ask the question does not mean our students will understand it. How can cubes be minutes? How do stacks of minutes relate to miles? Minutes are the sixtieth part of an hour. An hour is the twenty-fourth part of a day. A day is how long it takes the earth to spin once around its axis. Are there minutes on the moon?

If the jogger runs for twenty-five minutes and covers three miles, how fast is she running in miles per hour? If a car is driving at fifty-five miles per hour on the freeway, how many minutes before the car reaches the town fifteen miles away? Which numbers do we multiply or divide on our calculator? How do we know if the answer we get is right? We may hear the phrase "miles per hour" frequently, but hearing the phrase does not guarantee we feel comfortable with the numbers involved.

Teacher: When it took me sixty cubes to walk one mile, one stack of sixty cubes represented the cubes for each mile, or cubes per mile. When I walk two miles in sixty cubes, I need to make one stack for each mile. How many miles?
Students: Two.
Teacher: Yes, but I still have to use the sixty cubes, because I only walked for one hour. How can I get two stacks of cubes from the sixty?
Student: Break it in two.
Teacher: The two stacks together is the hour it took me to walk two miles. Each stack separately is how long it took me to walk one of the miles. How many cubes in each stack?
Students: Thirty.
Teacher: Then it took me thirty cubes or thirty minutes to walk one mile.
If I walked three miles in an hour, that's the same as saying I walked three miles in sixty minutes or sixty cubes. If I walk three miles in sixty cubes, how many stacks of cubes would I need to represent three miles?
Students: Three.
Teacher: How many cubes or minutes it would take me to walk each of the miles?
Student: Twenty.
Teacher: Let's try walking four miles in an hour.

Students work together with a partner and find the cubes per minute for four miles per hour, then five, then six, then seven and so on. Students keep a record of what they find, looking for patterns that help them know what hey might find next.

m.p.h.	cubes
1	60
2	30
3	20
4	15
5	12
6	10
7	8 4/7
8	7 4/8
9	6 6/9
etc.	etc.

Questions we can ask our students when cubes represent minutes and miles:

At how many cubes per mile are we running, not walking?
If cubes can be minutes, what else can cubes be?
Can we use cubes to show the speed of a car?
How many cubes for each mile is a car traveling if the car's speed is sixty miles per hour?
How many cubes for each mile is a jet traveling if the jet's speed is six-hundred miles per hour?
How fast does a rocket or a satellite travel? Can we measure satellite speed with the cubes?
Can we make a stack of cubes that represents the minutes we are in school for a day?
How do we spend these minutes in school?
Do we have enough cubes to show all the minutes in a 24-hour day?
Can we use our cubes to show how much time we spend sleeping compared to when we are awake?
Can we compare the time we spend at school with the time we are at home?
Can we see from the cubes how long we spend watching TV?
What other ways do we use minutes that cubes can help us to see?
How many hours are in a day?
How many hours are in a week?
How many hours do we sleep in a week?
How many hours do we sleep in a year?
How many days are the hours we sleep in the year?
If cubes are days, what does a year look like?
How many days are we at school in the year?
Do we have more days in school or out of school?
If cubes can be hours or days, can cubes also stand for years?
If we can live to be one-hundred years, how old are we now compared to how old we can live to be?
How old are our parents now compared to how old we are?
How old were our parents when we were born?
How old will they be when we are their age now?
Do we have enough cubes to show how old our country is? Our state? Our town?

How long ago did the first person land on the moon?
What kind of timeline can we make with cubes?
If cubes can stand for one year, can they stand for ten or one hundred or one thousand years?
What is the speed of sound?
What is the speed of light?
If the speed of light is 186,000 miles each second and the speed of sound is about one mile every five seconds, can we use the different speeds to know how far away a flash of lightning is from where we stand?

Time exists whether we learn to say what time it is or not. Night passes into day. Winter becomes spring. Babies grow into mothers and fathers who bring babies into this world. The earth orbited around the sun before people decided that the time for each orbit is one year. Time is a daily part of everybody's life. Learning about time is a natural part of learning about life.

Lesson Five

Purpose	Learn how to make good estimates.
Summary	Students estimate length, surface area, weight and volume while learning what estimation means, but we do not teach estimation and then move on. Estimation is a thinking skill our students use and use. We make estimation a part of all the lessons we teach by the questions that we ask.
Materials	Unifix Cubes, Power Block S-1 squares, bowl balances and weights.
Topic	How much?
Topic	How high?
Topic	How many?
Topic	How far?
Topic	How soon?
Topic	What else?
Homework	Measuring questions asked for school can be asked for home, as well.

Estimation questions...

Question: What is an estimate?
Answer: An estimate is a judgment or an opinion. In mathematics, judgments do not have to be exact. Opinions do not always have to be precise. Estimations use words like: about, nearly, close to, between, a little less than, and approximately.

Question: What is the difference between an estimation and a guess?
Answer: Experience.

Question: Does an estimate differ from knowing what is reasonable?
Answer: Reasonableness looks at an answer and decides if it makes sense. Estimation thinks about the answer in advance. Estimation is reasonableness before an answer has been found.

Question: Do we teach estimation by guessing jelly beans in the jar?
Answer: If we live in Candy Town. Or, if guessing jelly beans is a skill for which we have a use.

Question: When do we use estimation?
Answer: Estimations and approximations are the most common use we make of math. A partial list:

The time we leave from home to catch the airplane flight.
The answer to the traveling child's lament: When will we be there?
The cost of dinner before the bill arrives.
The tip for service on the meal.
Clothes size when shopping for someone not with us that day.
The number of suitcases our car trunk will hold.
How much luggage we think we'll need to hold the clothes.
How long the food we have on hand will last.
How far we can travel on the gas remaining in the tank.
The number of errands we can run in the hour we have for lunch.
The monthly payments we can afford.
How many people are watching the parade.

The number of days it will take for the letter to get from here to there.
The length of time it will take for the check to clear.
How long until the movie starts.
How many relatives the turkey will serve.
How much fish food we should feed the fish.
How much the vacation will cost.
How much longer it will be until dark.
How many buckets of paint we need to paint the room.
How long the painting will take.
How long the candle will burn.
How much laundry soap we need to add.
How high the ceiling and whether the Christmas tree will fit in here.
How much it will cost for our class party.
How long it will take to finish this book.

We use estimations whenever "close enough" is good enough for now.

Sometimes we estimate and refine the estimate through experience:

When should we leave to get to work on time?
How far is it from here to there?

Sometimes an estimate is all we have:

How many leaves are on the tree?
How many stars are in the universe?

Estimation is...

Estimation is more than guessing. Estimation is patterns and connections. Good estimates are estimates that are reasonable. Reasonable estimates are made by connecting past experience to the present situation. Knowing which experiences to connect means knowing what patterns apply. Estimation is not a lesson that we teach for a week and then move on. Estimation is a thinking skill our students use and use. Estimation is a question that we add to all the questions we have asked before.

A question:

Teacher: As you walk to the cafeteria today, count the paces that you take.

A question with an estimation question added:

Teacher: As you walk to the cafeteria today, count the paces that you take. How many paces do you think there will be?

The first day that students predict paces, their predictions are guesses. On future days, the number of paces counted out the days before provides a basis for predictions.

Teacher: How many paces do you think there are between our classroom and the library?

Predictions for distances not yet paced and counted are made from distances already measured. Is the distance to the library farther than or closer than the distance to the cafeteria? Should the paces each student predicts be a bigger or a smaller number? Each measurement we make adds to the knowledge we can use for future estimations. The experiences we have separate an estimation from a guess. Estimation questions help our students connect what they know now to what they will know next.

Estimation is not a contest or a competition. We do not keep track of whose estimate is the closest. Every student makes an estimate. Every student counts the paces. Every student has a separate answer. Paces are all different. Every student gains experience in estimation from every estimation that is made.

Simple activity...

Teacher: Find an object in the room to measure that is not right next to where you are sitting, like the height of my desk or the width of the door. Now, make a pile of Unifix Cubes on your desk that you think will snap together to be the right length. After you have made your pile, snap all the cubes together to measure the object so you can see how close your estimate was.

Simple activity, infinite variations.

Teacher: Find a surface area in the room to measure, like the top of your desk or the cover of your binder. Now, make a pile of Power Block S-1 squares on your desk that you think will cover the same area. After you have made your pile, cover the area with the Power Block S-1 squares so you can see how close your estimate was.

Another variation.

Teacher: Find an object in the room to weigh that you have not already weighed before, like a book from the shelf or one of your shoes. Now, make a pile of weights on your desk that you think will weigh the same as the object you have picked. After you have made your pile, then use your bowl balance to find how close your estimate was.

The difference between an estimation and a guess is experience. Experience comes from making estimates.

Questions from Teachers

1. How do we assess measurement activities?

Assessment is not a separate activity. In measurement, as in all the other areas of mathematics, assessment is a part of each assignment that we give. As we pose problems, we observe our students as they problem-solve. Our assessment is in the observations that we make.

Assessment is made more formal when:

We keep a written record of our observations.
We save the written records that our students make.

Here are assessment problems we might pose:

How many scoops of rice will fill the jar?
Make a map from our school to where you live.
Balance the pegboard strip on the fulcrum. How do you know it is balanced?
What time is it now? What time will it be for the next rotation fifteen minutes from now?
How far is it from here to there?

These assessments are from Lessons One through Five. Assessment is a part of each assignment that we give.

Our assessment for ourselves:

Do our students have a use for what we teach?
Is the use something more than, "They'll need to know it for the test!"
Will the skills we teach help our students in their lives?
Are we creating an environment for measurement?
Are we creating an environment for mathematics?
Have we managed not to leave anyone behind?

2. How do we know what to do in measurement for each grade level when the lessons do not say what grade level they are for?

The eighteen year old who could not tell time was once five years old, then six, then seven, then eight. When was the right grade level to teach her to tell time? When was the right grade level to make sure she had learned?

We learn from our students what they know already and what gaps in knowledge they may have. We move as quickly or as slowly through the lessons as our students teach us that we should. We cannot learn what to teach the children in our room from a list in a book. Our students teach us what to teach.

3. I am not sure what it is we are supposed to teach our students about measurement. Are they just supposed to problem solve and make maps, or are there particular measuring skills to teach, like knowing how to use a ruler?

There are two basic ways to measure:

Direct comparison: Holding the objects to be measured up against one another, as when children stand back to back to see who is taller.
Indirect comparison: Measuring with units, like cubes or weights, and then counting the units. The measurement is made by comparing the units and not the objects themselves. Eventually, indirect comparisons lead to standardized units, such as inches, centimeters, or gram weights.

Almost all measuring involves indirect comparisons. We choose a unit, compare the unit to the object to be measured and then report the number of units for the measurement. The measuring skill we teach is making indirect comparisons.

We begin with nonstandard units, like scoops of rice, lengths of string, or paces on the ground. We use nonstandard units to develop an appreciation for the convenience that standard units bring. We replace nonstandard with standard units that are already familiar to the students in our class. The first standard units our students use for measuring are objects in our class, like cubes, blocks and dowels.

When students understand the use of standard units we introduce the standards that are used outside our class: inches, centimeters, miles, kilometers, ounces, grams, quarts, liters and so on.

When students understand the process of measuring, is it any harder to measure how far across the desk in inches or in cubes? Is it any more difficult to put metal washers in the balance bowl than gram weights? Measuring understood is measuring that can be used whenever it is required.