Student: Half a class.
Student: Half a pizza.
Student: Half the cookies in the box.
Student: Half an hour.
Teacher: How many different fractions can you think of that are equivalent to one-half? Two quarters is equivalent to a half. There are two quarters in half a football game. There are two quarters in half a dollar. What other equivalencies are real?

For what other fractions can we ask these questions?

Teacher: List examples of when we add or subtract fractions whose denominators are not the same.

What examples might we find? Half hours and quarter hours, periods of games, ratios, discounts at a sale, coins that make up change, recipes, measurements of all kinds. Our list is added to each day as our students find more examples. The more our students find, the more we and they become aware of how frequently we use equivalencies in our lives.

Teacher: How would you explain to a new student in our room how to add or subtract fractions whose denominators are not the same?

Explanations can be given orally. Explanations can be written, too.

What's the point?...

Add 1/2 and 1/4 on a calculator that can add fractions and 3/4 flashes on the screen. Enter 1/6 and 1/7 and 13/42 flashes just as fast. What's the point of teaching children how to add or subtract unlike fractions? Calculators get the answer faster than we can. Awareness is the point. We want our students to be aware that:

We are capable of understanding how numbers work.
We do not have to memorize to know the rules.
Fractions whose denominators are not the same cannot be added or subtracted as they are.
The factor pattern has a use.

Add 1/2 to 1/4 on a calculator that adds fractions. The answer is 3/4. Press the button on the calculator that converts the fraction to a decimal. The answer is .75. Press the button once again to convert the decimal back into a fraction. Now the calculator says the answer is 75/100, not 3/4.

Factors and equivalencies help our students understand the relationship between fractions and decimals. We want our students to be aware of why 3/4 and .75 and 75/100 are all the same. We do not need lists of rules. We need an understanding that fractions make sense. We need to know that the rules for using fractions are rules we can discover for ourselves.

Lesson Seven

Purpose	Learn to see fractions as a part of measurement. Learn to estimate fractions of a length.
Summary	Students use strips of paper to estimate, then measure. We teach techniques for determining fractional lengths.
Materials	18" strips of paper, objects in the room.
Topic	Estimate lengths. Calculate the fractions involved.
Topic	Estimating and calculating techniques are improved with practice and with time.
Homework	Estimating can be done at home.

Measurements...

In *Measurement, Estimation and Time* we asked our students to make estimates. We said:

> Find an object in the room to measure that is not right next to where you are. Make a pile of Unifix Cubes that you think will snap together to be that length. Snap the cubes together and see how close your estimate was.

Estimation is a thinking skill our students use and use. Fractions are a thinking skill, as well.

The teacher cuts strips from construction paper or newsprint 18 inches long and 1 or 2 inches wide.

Teacher: Here are some strips of paper I have cut. Find an object in the room that is not right next to where you are sitting. Estimate how many strips and/or fractional strips of your paper you think will equal the object's length or height or width. Write down your estimate. Then use your strip of paper to measure the object, so that you can see how close your estimate was. Estimate and measure as many different objects as time allows. You may work with a friend if you wish. If you need help writing words, bring me your spelling notebook turned to the correct page.

One is what we say it is. In Lesson Five, our students folded a piece of paper we called one to find the fractions in the whole. Now, the paper strips we give our students are called one. Our students use their paper-folding knowledge to make their measurements in whole units and fractions, too. Measurements are a part of every child's life. Fractions are a part of measurement.

If our students need assistance with how to measure fractions with their strip of paper, we provide the guidance that they need.

(illustration 11-7-1)
(Paper strip laid next to an object that is about two-fifths of the length of the paper.)

Teacher: What fraction of this measuring paper is this object?
Student: About a half.
Teacher: Let's see how close it is to being a half.

(illustration 11-7-2)
(Paper strip folded in half, laid next to the object.)

Teacher: It's smaller than a half. I think it's closer to a third. I'll fold a paper strip into thirds to see if I am right.

(illustration 11-7-3)
(New strip of paper folded into thirds, laid next to the object. The old strip is still visible.)

Teacher: It's bigger than a third and smaller than a half. I'll try fourths. No, I know that won't work. A fourth is smaller than a third and two-fourths is the same as a half. I'll try fifths. It won't be one-fifth, but it might be two.

(illustration 11-7-4)
(New strip of paper folded into fifths, laid next to the object. The old strips are still visible.)

Teacher: It looks like this object is about two-fifths.

Folding strips of paper into halves and thirds and fifths consumes a lot of paper. We can suggest a paper-saving way our students may use:

Teacher: When I was trying to measure my object, I folded a strip in half, then I folded another strip into thirds and then I folded another strip into fifths. I think I could have used a single strip of paper for all the different measurements.

(illustration 11-7-5)
(Strip folded in half, with 1/2 written at the mid-point. Same strip folded in thirds, with 1/3, 2/3 written at the appropriate points. Same strip folded in fourths, with 1/4, 2/4. 3/4 written at the appropriate points. The 2/4 is written beneath the 1/2. Same strip folded in fifths, with 1/5, 2/5, 3/5, 4/5 written at the appropriate points.)

Teacher: You can use a different strip for each new fraction, or you can keep track of more than one kind of fraction on a strip. It's up to you which way you use.

Linear measurements provide our students an opportunity to use fractions naturally. Measured fractions teach our students that fractions are not just slices of a pie or shares of cookies on a tray. Fractions can be distances from a starting point or marks upon a line.

Variations...

Teacher: Can you find a way to record fractional lengths on your strip of paper without folding the strip. You may use any materials in our room that you think might help. Work with a friend or two to see what way you can invent.

Students might use wooden cubes or centimeter cubes or Power Blocks or rulers. They might use estimating as their aide. The more freely our students invent, the more strategies they will find.

Teacher: Can you find an object in the room that is half the length of your strip? Can you find an object that is a third? A fourth? Can you find an object that is two-thirds? Two-fourths? Two-fifths? For how many different fractions can you find objects?

Other questions we might ask:

How does your strip of paper compare to the rulers in our class?
What do all the ruler markings mean?
Why does a yardstick have 36 inches on it? Does this relate to the 360° from geometry? (*Geometry*, page 000)
Why do meter sticks have a hundred markings? Why not thirty-six, like yards?

Whether our students use a different strip of paper for every fraction they measure or a single strip on which they write the numbers for all the different fractions that they find, they are learning about rulers. A ruler is a measuring strip with marks already on it. Students who have folded and marked strips of paper have a better understanding of the marks on the rulers that they use.

Assessment...

Assessment is not a complicated task. We assess by asking what we want to know:

Find a third of this piece of string. Prove that its a third.
Give me some examples of when you see or use fractions in your lives.
What is a fraction, anyway?

Lesson Eight

Purpose	Learn what a ratio is. Learn to see practical applications of ratios and equivalencies.
Summary	We use opportunities that exist or that we create to give our students practical experiences in finding ratios.
Materials	Calculators, bouncing balls, shadows.
Topic	Shadow questions.
Topic	Shadow ratios.
Topic	Bouncing balls.
Topic	Furthest jumps.
Topic	Diagonals of rectangles.
Topic	Other opportunities that arise.
Homework	Ratios explored at school can be explored at home.

Ratios...

Fractions compare two numbers with each other. A ratio is a fraction, too. A ratio compares or expresses the relationship of two numbers to each other through a division of one number by the other. The ratio of 3 to 1 is 3. The ratio of 30 to 10 is 3. Ratios are equivalencies.

Ratios are:

How we adjust a recipe to fit the family's size: for homemade maple syrup, add 2 cups of sugar and 1 teaspoon of Mapoline to 1 cup of boiling water for each new pitcher of syrup.
Proportions in geometry: the Golden Rectangle's 1.618 ratio of length to width or the Golden Ratio used by ancient Greeks to build the Parthenon atop the Acropolis.
Pi (π): the ratio between the diameter and the circumference of every circle.
The background our students need for making sense of algebra: the value of x in $x/6 = 12/24$.
How we draw our maps to scale.

Calculators give us ratios effortlessly. But calculators cannot teach our students how to think or feel. Just as digital clocks cannot give our students a sense of time, calculators cannot give our students a sense of ratios.

Science or mathematics...

Shadows are useful teaching tools for introducing our students to ratios and equivalencies. Before we introduce shadows for ratios, we give our students shadow questions to explore.

How much do our students know about shadows? Do they know that shadows are formed when an object blocks the sun? Do they know that shadows are sometimes used for marking time? Do they know what makes shadows shrink or grow?

Teacher: This morning before school, I made a small x with masking tape on the window. Can any one find the shadow of the x on the classroom wall or floor?
Students: (Shadow found)
Teacher. Okay. Its nine o'clock right now. Tell me where you think the shadow will be at ten o'clock. Will it still be where it is or will it move? If you think it will have moved, can you predict where you think it will move to?

What path will the shadow of our school building take if we mark its shadow every hour of the hours we spend in class? Will the shadow move? We know it will. Will the shadow move predictably? Will its movement be the same each hour?

We use measurements of time to know when to mark the shadow's position each hour. We use measurements of length to know how far a shadow moves. Whether we call the time our students spend exploring shadows science or mathematics, shadows are connections waiting to be made.

Look at the shadow that a hand makes on the ground. Does the shadow of our hand grow larger or smaller as we move our hand closer to or farther from the ground? Or does the shadow's size remain the same regardless of our hand's distance from the ground? Does the shadow of our hand cast by the sun act differently than the hand shadow cast by an electric light? We do not need to know the answers to all the questions that we ask. We can explore the possibilities as our students do.

Shadow ratios...

As our students explore shadows, we can ask them to explore shadow ratios. For ratio explorations, we use the shadows cast by the sun and not those cast by the lights in our room.

Teacher: How many cubes long is the shadow of a stack of Unifix Cubes ten cubes high?

What questions follow from the first?

How long is the shadow of a stack 20 cubes high? 30 high? 40?
Is there a relationship between the height of the cube stack and the length of its shadow?
Can we use the relationship to predict shadow lengths for sticks of cubes we have yet to make?
Does the time of day have an effect on the length of the shadow?
Does the relationship between each stick and its shadow apply to your body's shadow? Could you use your shadow's length to tell how tall you are?

(illustration 11-8-1)
(A chart of recorded shadow lengths for different lengths of cube sticks. Show the relationships written as equivalent fractions.)

We pose the questions. Our students seek answers and discover the solutions. We can give a hint or two. We may suggest using calculators to divide length of shadow by length of stick to see if the ratio is the same for each new stick. Does it make a difference if we divide length of stick by length of shadow instead? We may divide either way, as long as the way we choose is the way we use for each new length of stick.

If our students find patterns in the ratios of shadows lengths to stick lengths, can they use these ratios to find heights that are beyond their reach?

If you can tell your height from your shadow's length, can you tell the height of taller things from the shadows that they cast?
Can you measure heights of trees or buildings using the relationships you have found?

(illustration 11-8-2)
(A drawing of a stick and its shadow with the measurements included. Along side the first drawing, a drawing of a building and its shadow with only the measurement of the shadow included. The height of the building is indicated by the letter X. For the discussion below, it is assumed that the numbers in this illustration are: Stick = 2. Stick shadow = 1. Building = X. Building shadow = 20.)

There will be a time in middle school or high school algebra when our students learn to solve for x in fractional equations such as 1/2 = x/20. The ratio problems now are readiness for algebra.

By the time students confront the building-shadow problem, they have already explored equivalent fractions, seen patterns for sticks and shadows, and used calculators. What ways might students with this background invent to find the heights of objects that are beyond their reach?

(illustration 11-8-3)
(Examples of student solutions to the problem of finding the building's height. Include examples of drawing to scale on graph paper, solving for equivalent fractions and using a calculator to figure the ratio. Include the algebraic way, as well. The algebraic way means multiplying both sides of the equation by 20.)

Shadows give our students the opportunity to see that equivalent fractions have a use. Shadows also give our students the opportunity to learn about ratios naturally. Mathematics is patterns waiting to be seen and connections waiting to be made.

Bouncing balls...

Drop a ball from a height. How high does it bounce? Does a higher drop cause a higher bounce? Is there a ratio between height of drop and height of bounce that will let us predict a ball's bounce from any height? Do all balls bounce the same?

There are many problems that our students need to solve for bouncing balls besides calculating ratios. Where is drop height measured from? From the bottom of the ball or the top? Or should we measure from the middle? How is height of bounce to be measured? A ball does not wait conveniently at its highest point of bounce for us to measure it. What measuring units are best to use? Practical applications do not always produce results that are precise.

Diagonals of rectangles...

(illustration 11-8-4)
(Series of rectangles of successively larger size, all drawn with their lower left-hand corner in common, all sharing the same diagonal extending from their common lower left corner through their upper right corners.)

If rectangles share the same diagonal, are their sides proportional? Do their lengths and widths grow by the same ratio? Do other shapes that grow proportionally share the same diagonals? Are there shapes that grow proportionally that do not have diagonals in common? If the sides grow proportionally, do the areas also grow proportionally?

Jumping farthest...

Who can jump the farthest? Who can jump the farthest compared to his or her height? Does the height of jumper/distance jumped ratio change with the size of the person? What are the jumping ratios of the man and woman who hold the world records for a jump? How do human jumping ratios compare to the jumping ratios of an elephant or a flea? How far could we jump if we had the jumping ratio of a flea?

Other opportunities...

What other opportunities let our students use their skills at defining problems, seeking answers and discovering solutions? What are the possibilities that we see? Can our students find examples of ratios in their lives?

Lesson Nine

Purpose	Learn what rates and rate tables are for.
Summary	Students search for examples of rates, learn to make rate tables and learn to use rate tables to answer questions.
Materials	Local newspapers, calculators.
Topic	Search the newspaper for rate examples.
Topic	Create rate tables for gas consumed and cost.
Topic	Create rate tables of various kinds.
Homework	Gather rate table data from home.

Rates...

Fractions express relationships between two numbers. The share of cookies each individual receives is related to the cookies available to share. The area of the square that is shaded in is related to the total area of the square. The measurement made is related to the unit of measurement used. Cookies to cookies, area to area, centimeters to meters—the numbers in these fractions compare parts to wholes.

Rates are fractions, too. Rates compare numbers that are not parts of wholes. Rates convert dollars to yen and Fahrenheit to Celsius. Rates tell us that two rolls for 25¢ is a better buy than 3 rolls for 39¢. Rates help us compare the fuel efficiency of cars. Rates convert times at bat and times the ball is hit to patterns of success. Rates tell us how quickly or how slowly a quantity is growing or being used. Rates are exchanges, conversions and comparisons.

Teacher: A rate is something that compares two numbers to one another. Does anyone know how many miles your family car gets on a gallon of gas?
Student: My mother said it was 26 the last time we got gas.
Teacher: Miles per gallon is the rate at which a car uses gas. The two numbers being compared or related are miles and gallons. Does anyone know what the rate of speed is when the needle on your car's speedometer is pointing to 55?
Student: You're going 55 miles an hour.
Teacher: Miles per hour is another example of a rate. What are the two kinds of numbers being compared?
Student: Miles and hours?

We teach our students the meaning of the word *rate* much as we would teach a toddler the meaning of the word *car*. The toddler learns the word *car* by seeing many different kinds of cars and being told that each is car. Our students learn from us and from each other what a rate is.

Teacher: I have some copies of the newspaper here. What I would like you and your workmate to do is look through the paper and make a list of all the examples of rates that you can find. In a little while, I'll ask you to share examples of rates that you have found.

The examples that our students find may or may not be rates. Our students refine their definition of rates as we discuss their findings as a class. We may not always be so sure ourselves. The question that we ask is: Which two numbers do the rates represent?

Examples of rates that might be found or that we might point out:

Comparison shopping: quantities of food at so much a package, pound, bunch, or head.
Two-for-one clothing sales.
Wages paid by the hour, week, month, or year.
Prices for a share of stock.
Rates of exchange from one currency to another.
Interest rates at banks on savings, loans and mortgages.
Apartment rents.
Batting averages in baseball, throwing accuracy for quarterbacks, shooting percents in basketball.
Graduation or dropout rates.
Tax rates, inflation rates, growth of the economy.
Rainfall per hour or per day.
Rates charged for all the ads.
The lowest rates in town.

A single tank of gas...

Teacher: If we know a car gets 26 miles per gallon, what else would we have to know to figure out how far the car could travel on a single tank of gas?
Student: How much gas the tank will hold.
Teacher: My car holds 20 gallons in its tank. If I know how far my car goes on a gallon of gas, how can I find out how far my car will go on two, three, or four gallons?

Do we need to tell our students how to find the number of miles traveled for each gallon of gas used? Or are they wise enough collectively to solve this problem for themselves?

Teacher: Let's make a rate table to see how far my car could travel if it got 26 miles for each gallon used.

Gallons	1	2	3	4	5	...
Miles	26	52	78	104	130	...

Teacher: When the warning light comes on for the gas gauge in my car, it means I have about two gallons left in my tank. How many miles at most can I go while I am searching for a gas station?

What other questions might give our students practice in using the rate table that we make? What questions might give our students practice in making rate tables for themselves?

Teacher: When you get home from school today, ask your parents if they know how many miles per gallon their car gets. Then ask how many gallons their gas tank holds. Can you make a rate table for your parents' car? If you can, will the rate table that you make allow you to predict how many miles your parents' car might travel before your parents fill it up again?

If parents do not know how many miles the family car travels per gallon, students may need to calculate the rate to use. Do the students in our class know that cars have odometers that can tell the miles traveled on a tank of gas? Some cars have computers that calculate miles per gallon automatically.

Will the rate table that our students make for the family car be useful for predicting the mileage driven on a tank of gas? Our students' parents may tell them that the miles a car travels on each gallon of gas is not a number they can guarantee. Mileage for running errands around town is not the same as mileage for a lengthy freeway drive. Miles per gallon is an average rate. Does an average rate allow for all the different kinds of driving that exist?

Teacher: The gas I buy costs $1.49 per gallon. If I am getting 26 miles per gallon, how much does gas cost for each mile that I drive?

We can teach our students how to find the cost of gas for each mile driven, or we can let them use their calculators and their collective reasoning to solve this problem for themselves. Which way produces more learning for the students in our class?

How long to reach Los Angeles?...

Teacher: If I drove my car at an average speed of 55 miles per hour, how long would it take me to reach Los Angeles?

Once the students find how far it is from where they are to Los Angeles, how do they go about finding how long it would take to drive to L. A.? They can divide the total miles to L. A. by 65. They can also make a rate table.

Hours	1	2	3	4	5	...
Miles	65	130	195	260	325	...

Teacher: If I am traveling at 65 miles per hour, how far can I travel on a tank of gas? What would the table look like if I traveled at 55 miles per hour instead?

Rate tables...

Is two rolls for 25¢ a better buy per roll than three rolls for 39¢?

Roll	1	2	3	4	5	...
Cost	12.5¢	25¢	37.5¢	50¢	67.5¢	...

Roll	1	2	3	4	5	...
Cost	13¢	26¢	39¢	52¢	65¢	...

Income-tax forms come with rate tables to tell us the tax we owe. Before electronic cash registers did the arithmetic automatically, rate charts for sales tax were at every sales person's side. Some people carry rate cards for figuring restaurant tips. There are rate tables that tourists carry for converting prices in yen, pounds, or pesos to dollars.

Rates are ratios extended over time. Rate tables can include equivalent fractions, common multiples and even decimals and percents. We introduce rates and rate tables to the students in our class as one more example of how we use the mathematics in our lives.

Lesson Ten

Purpose	Learn what kind of fraction decimals are.

Summary	Students explore base-ten blocks, geoboards, graph paper rectangles and calculators as they learn about decimal fractions. This lesson is presented after *Advanced Multiplication and Division.*
Materials	Base-ten blocks, calculators, geoboards, 10 X 10 and 100 X 100 blacklines, paper, chalkboards.
Topic	One is what we say it is for base-ten blocks.
Topic	Think about where the decimal goes in answers to addition and subtraction problems.
Topic	Calculator generated lists of equivalencies.
Topic	Search for patterns for where the decimal goes when multiplying decimals.
Topic	Demonstrate why the fraction becomes smaller when we multiply.
Topic	10 X 10 paper for multiplying decimals.

Focus on an understanding...

We do not teach our students about decimals in kindergarten or first grade. We wait until they are older—when they understand place value, fractions and equivalencies for the formal lessons to begin. The longer we wait, the more our students learn about decimals while waiting for our teaching to begin.

Decimals are as much a part of every child's life as the language that we speak. Decimals are found in:

The prices on everything that children buy, from food to toys to clothes. Money amounts use decimal points to tell the dollars from the cents.
Metric measures.
Sports reports. Batting averages. Shooting percentages. Team and player data for the season.
Scoreboard clocks. Stop watches. Digital timers.
News reports. Numbers in headlines. Rainfall amounts.
Averages and statistics of all kinds.
Body temperatures.
How we find an FM station on the radio.

The decimal lessons that we teach do not need to focus on existence or on use. Our lessons acknowledge the background that our students have developed through the years. The decimal lessons that we teach focus on an understanding of the decimals students see.

One is what we say it is, again...

The materials and experiences most useful for teaching decimals are also used in *Advanced Addition & Subtraction* (pages 000-000). We present this decimal lesson when our students have the background that the lesson requires to succeed.

(illustration 11-10-1)
(Show a base-ten block, flat, long and cube.)

Teacher: When the cube has a value of one, what are the values of the long and flat and block?

Block	**Flat**	**Long**	**Cube**
1,000	**100**	**10**	**1**

If I say the long is one, what are the values of the cube and flat and block?

Block	**Flat**	**Long**	**Cube**
100	**10**	**1**	**1/10**

If I say the flat is one, what are the values of the other pieces now?

Block	**Flat**	**Long**	**Cube**
10	**1**	**1/10**	**1/100**

If I say the block is one, what values do the other pieces have?

Block	**Flat**	**Long**	**Cube**
1	**1/10**	**1/100**	**1/1000**

What fraction is this?

1/10

Student: One-tenth.
Teacher: Decimals are the tenth, hundredth and thousandth fractions with their denominators removed. When I write one-tenth as a decimal, its fraction name remains the same.

1/10 = .1

Base ten blocks provide a visual representation of the kind of fractions decimals are. Decimals are powers of ten fractions with the denominators gone:

1/10 = .1
1/100 = .01
1/1,000 = .001

Block	Flat	Long	Cube
1,000	100	10	1
100	10	1	.1
10	1	.1	.01
1	.1	.01	.001

Where the decimal goes...

(illustration 11-10-2)
(A trading board for base-ten blocks. First row: 1 block, 2 flats, 0 longs, 4 cubes. Second row: 2 blocks, 3 flats, 5 longs, 8 cubes.)

Teacher: Please record this base-ten block problem on your chalkboards. Please record the answer, too.

$$\begin{array}{r} 1{,}204 \\ \underline{2{,}354} \\ 3{,}558 \end{array}$$

Teacher: Please read me the answer that you found.
Student: Three blocks, five flats, five longs and eight cubes.
Teacher: Now read the answer in thousands, hundreds, tens and ones.
Student: Three thousand, five hundred, fifty-eight.
Teacher: How would you write the problem and the answer if I say the long is one? Where do you think the decimal point would go?

$$\begin{array}{r} 120.4 \\ \underline{235.4} \\ 355.8 \end{array}$$

Teacher: And, how would you read the answer now?

When we say the long is one, some students may understand where to place the decimal. Some students may even know how to read the answer with the decimal in. Most students may need our help in contemplating where the decimal goes and how to read the number with the decimal in.

What we say and do depends on the assistance that our students need. Our students may need to see again the visual representation of the tenths, hundredths and thousandths that the blocks can represent. We may need to post a chart of fraction-to-decimal equivalencies.

Base ten blocks are a tool we use to make the meaning of the decimal clear. One is what we say it is, for Power Blocks, cube sticks, and base-ten blocks. The base-ten blocks help our students see that decimals are only fractions with the denominators gone.

Teacher: How would you write the problem and the answer if I say the flat is one? Where would the decimal point go? How would you read the answer now?

The process of creating and recording problems for the base-ten blocks is the same as that for multibase blocks in *Advanced Addition and Subtraction*, Lesson Five (page 310). In that lesson, the cube is one. Now, one can be a long, a flat, or a block. The value of the blocks and flats and longs may change. The adding and subtracting is the same.

Teacher: How would you write the problem and the answer if I say the block is one? Where does the decimal go? Can you read the answer to me now?

New equivalencies...

Once our students have the framework that the base-ten blocks provide, they use their calculators to make lists of equivalencies. How we put the calculators to use depends on the tricks the calculators know. Some calculators have a single key for converting a fraction to its decimal equivalent. Others may require a step or two before the equivalent appears.

Teacher: Write the decimal value of the fraction five-tenths on your chalk board.
Students: (Written) .5
Teacher: Write the decimal value of the fraction one-half on your boards.

It is not likely that our students will be as quick to write .5 on their boards as the decimal equivalent for a half as they would be to write .5 as the decimal equivalent for five-tenths. Knowing that decimals are fractions with the denominators gone only helps when the denominator that disappears is a ten, hundred, thousand, or greater power of ten.

Teacher: Calculators can help us convert fractions to their decimal equivalencies. To find the decimal value for a fraction, we divide the numerator by the denominator. Use your calculator to divide the numerator by the denominator for the fraction five-tenths. Divide five by ten and show me what you get.
Students: (Written) .5
Teacher: Now divide the numerator by the denominator for the fraction one-half. Divide one by two and show me what you find.
Students: (Written) .5
Teacher: I know why .5 is the decimal equivalent of five-tenths. But why is .5 also the decimal equivalent for one-half?

Can our students explain this phenomenon to us? Do they remember that one-half is equivalent to five-tenths? Is an equivalency-chart for fractions still on our wall? Can any student model with Unifix Cubes, base-ten blocks, Power Blocks, or our butcher paper chart of equivalencies that five-tenths and one half are the same? We use calculators to find equivalencies. We use our knowledge of fractions and equivalencies to prove the calculator answers are correct.

Other equivalency questions we can ask:

Is there a pattern for the decimal equivalencies that we find?
Is .5 the answer for all the fractions that are equivalent to a half? Do 2/4 and 3/6 and 4/8 all have .5 as their decimal equivalency?
What are the fractions that are equivalent to 1/4? Do all these fractions share a decimal equivalency, as well?
Look at the records of equivalent fractions you made for paper folding (Lesson Five, page 000) and Power Blocks (Lesson Six, page 000). Do each of the common equivalencies that you found have common decimal equivalencies, too? Are there any fractions that are equivalent to one another that do not share a decimal equivalency? Why do you think what you say is so?
What else can you find out about fractions and the decimals they become?
Make a list of what you find.
What patterns can you see?
Earlier (Lesson Six, page 000), we added 1/2 to 1/4 by finding common denominators. Can you add 1/2 to 1/4 by converting each fraction to its decimal equivalent instead?
What other fractions can you add this way?

With our questions as their guide, our students' explorations help them learn that they can understand what decimals are and what decimal numbers mean.

A rule for multiplying...

In *Advanced Multiplication and Division*, Lesson Four, we ask our students to see if they can discover patterns for multiplying numbers. The lesson asks:

What is any number times 1? Times 10? Times 100, 1,000 and 10,000?
What happens when you multiply by 2s, 20s, 200s and 2,000s?
Keep a record of the numbers that you multiply and the answers that you find.

Can you see any patterns that might tell you what numbers to expect for each new set of numbers?
Does counting all the zeros in the numbers give you a hint of the pattern to expect?
Is the pattern the same for any set of 1, 10, 100, 1,000, 10,000 number(s) that you multiply?
Are there any numbers for which your pattern does not work?

Calculators easily provide the numbers for these pattern searches. Calculators give our students decimal answers, too. What patterns can our students find for multiplying decimals? This lesson asks:

What is any number times .1? Times .01? Times .001? Times .0001?
Where does the decimal in the answer go?
What happens when you multiply by .2s, .02s, .002s and .0002s? Where is the answer's decimal now?
Keep a record of the numbers that you multiply and the answers that you find.
Can you see a pattern that tells you what numbers to expect for each new set of numbers?
Do you see a pattern that tells you where to place the decimal?
Does seeing where the decimal is in the number that you multiply give you any hint?
Is the pattern that you see the same for any set of .1, .01, .001, .0001 numbers that you multiply?
What happens to your pattern if both numbers have decimals in them? What happens when you start with 2.34 as the number times .1, then .01, then .001, then .0001? Where is the decimal in the answer now?
Does counting places to the left of the decimal in both numbers give you any hint?

A calculator cannot do our thinking for us, but the many answers that it gives helps us do much more thinking for ourselves.

What the answers mean...

1/3 x 1/4 = 1/12
.1 x .1 = .01

The dictionary says multiplying means to increase in number. Why then, when we multiply fractions or decimals together, is the answer smaller than either of the numbers that we started with? We can find the answers by applying all the rules or by pressing calculator keys, but what do the answers mean?

What do we know? From *Beginning Multiplication and Division* (page 215) comes the knowledge that geoboards can be used to find answers to multiplication problems. From *Fractions* (page 253) comes the knowledge that one is what we say it is. From people fractions (page 247) comes the knowledge that there can be more than one *one* at once. From *Algebra* (page 363) comes the knowledge that the patterns for the areas of shapes can be recorded in formulas.

(illustration 11-10-3)
(Geoboard with a three by four rectangle on it. The rectangle has its lower left hand corner in the lower left hand corner of the Geoboard.)

Teacher: What multiplication problem does this rectangle represent? Remember, we count the spaces, not the nails.
Students: Three times four equals twelve.
Teacher: Remember from our lessons with Power Blocks and cube sticks, one is what we say it is. If I say the bottom of the rectangle is one, what fraction would each of the spaces be?
Students: A third
Teacher: If I say the side of this rectangle is one, what fraction would each of the spaces be now?
Students: A fourth.
Teacher: What is the area of this shape?
Students: Twelve square units.
Teacher: What fraction would each square unit be if I said the whole rectangle had an area of one?
Students: One-twelfth.
Teacher: What was the pattern for predicting the area of rectangular shapes on our geoboards?
Students: Length times width equals area.
Teacher: One is what we say it is. For this rectangle on our geoboard, I will say the length is one length, the width is one width and the area is one area.
Teacher: This rectangle represents the problem 1 x 1 = 1, or one width times one length equals one area. What multiplication problem does this fractional piece of the whole represent?

(illustration 11-10-4
(Geoboard 3 x 4 rectangle from above with a one by one square in the lower left hand corner.)

The teacher guides the students into seeing that the small square in the larger shape represents 1/3 x 1/4 = 1/12. (A third of the width times a fourth of the length equals a twelfth of the area.)

Calculators that display fractions can show our students that 1/3 times 1/4 equals 1/12. Geoboards can show our students why.

There are several different *ones* in this example. Can our students grasp the logic of so many ones at once? Can we? Were our students confused when we had each row of people in class represent one? Each student represented a different fractional amount depending on the row or group that he or she was in. One is what we say it is. There can be more ones in a problem than just one.

Teacher: Let's see what other fraction problems we can find answers to from the rectangle on my geoboard:

(illustration 11-10-5)
(Show the geoboard shapes that go with each problem.)

1/3 X 2/4 = 2/12
1/3 X 3/4 = 3/12
1/3 X 4/4 = 4/12
2/3 X 1/4 = 2/12
2/3 X 2/4 = 4/12
2/3 X 3/4 = 6/12
2/3 X 4/4 = 8/12
3/3 X 1/4 = 3 12
3/3 X 2/4 = 6/12
3/3 X 3/4 = 9/12
3/3 X 4/4 = 12/12

Teacher: What kind of fraction problems can we find for a different rectangle?

(illustration 11-10-6)
(Geoboard with a two by three rectangle on it. The rectangle has its lower left hand corner in the lower left hand corner of the Geoboard. Fraction problems written underneath. Show the geoboard shapes that go with each problem.)

1/2 X 1/3 = 1/6
1/2 X 2/3 = 2/6
1/2 X 3/3 = 3/6
2/2 X 1/3 = 2/6
2/2 X 2/3 = 4/6
2/2 X 3/3 = 6/6

Teacher: Can you find a pattern that will allow you to predict the answers to the multiplication problems we have made just by looking at the fractions to multiply?

What patterns might our students find for multiplying fractions? Can they discover on their own that numerator times numerator and denominator times denominator produces the same answers as the rectangles on the geoboard? Yes, they can.

Teacher: Why is it that when we multiply fractions together, the answer is a smaller fraction than either of the fractions that we started with?

Why, indeed.

Ten by ten...

(illustration 11-10-7)
(Ten by ten square of graph paper.)

Teacher: If I say each side of this square is one, what fraction would each space on each side be?
Students: One-tenth.
Teacher: What decimal would that be?
Students: One-tenth.
Teacher: There are ten small squares on each side of this large square shape. How many small squares are there altogether in the shape? What is ten times ten?
Students: One hundred.

Teacher: If the large square has an area of one, what fraction is each small square?
Students: One-hundredth.
Teacher: What is the decimal value of each small square?
Students: One-hundredth.
Teacher: What multiplication problem does this shaded area represent?

(illustration 11-10-8)
(The ten by ten square of graph paper from the illustration above with a two by three rectangle in the lower left hand corner shaded in.)

.2 X .3 = .06

What other multiplication problems for decimals can we create on our ten-by-ten square? Do the answers that we find match the answers that our calculators produce? Are there patterns in the problems and the answers that tell us the answers for problems not yet solved? What kind of answers might we get if we used a bigger piece of graph paper and marked off a 100-by-100 square? Can we see why multiplying decimals produces answers smaller than the decimals that we started with?

Lesson Eleven

Purpose	Learn the difference between decimals and percents. Learn to understand the questions that percent can represent.
Summary	We talk about the meaning of percent. We give our students practice using percent.
Materials	Calculators.
Topic	As we talk about percent with our students, what our students say guides what we do next.
Homework	What percent examples can be brought from home?

Not equivalent...

Percent is from the Latin *per centum*, which means "per hundred." If there are one hundred people in the room, then each person is one-hundredth of the group, or one percent. If only four people are in the room, then each person individually is one-fourth of the group. We use the same equivalencies we used for decimals to find percents. One-fourth is equivalent to 25/100, so one-fourth is equivalent to .25 and to 25%. Are decimals equivalent to percents?

An overheard conversation in a restaurant:

First person: I'll get the check.
Second person: No, let me.
First person: I invited you. I insist.
Second person: Let me at least pay half.
First person: Why don't you just get the tip?
Second person. Okay. But you'll have to tell me what is. I'm terrible with percents.
First person: What do you mean, you're terrible with percents. You use decimals at work.
Second person: Decimals aren't percents. I don't have any problem with decimals. I just don't understand percents.

Is the second person justified in claiming to be comfortable with decimals and not percents? What happens when we add the decimal ten-hundredths to, and then subtract it from 1.00?

1.00 + .10 = 1.10 1.10 — .10 = 1.00

What happens when we add ten percent to, and then subtract it from 1.00?

1.00 + 10% = 1.10 1.10 — 10% = 0.99

Decimals are not equivalent to percents. Decimals are straightforward—adding means to add; subtracting means to subtract—no tricks. Percents are decimals, but percents are also something else. Adding a percent means multiplying first, then adding. Subtracting a percent means multiplying first, then subtracting. Percents involve tricks with words that we never think to use for decimals.

The difference...

There is a difference between a decimal and a percent, but the difference is not the value of the fraction that the numbers represent. Twenty-five parts of one hundred can be written as .25 or 25%. The difference is that a decimal reports the value of the number—.25 has a value of twenty-five hundredths. Whether we add a decimal number or subtract it, its meaning will not change. By itself, 25% has no meaning—the value of 25% changes as the value of the number that it acts upon increases or decreases.

2.00 + .25 = 2.25	**2.25 - .25 = 2.00**
2.00 + 25% = 2.50	**2.50 - 25% = 1.875**
200.00 + .25 = 200.25	**200.25 - .25 = 200.00**
200.00 + 25% = 250.00	**250.00 - 25% = 187.50**

The difference between a decimal and a percent is not the fraction that the numbers represent. Percent is an application of knowledge we already possess. If we are asked to add .10 to anything, we add .10. The number to add is present in the question. If we are asked to add 10% to anything, the number to add is not present in the question. To know the number to add, we must first ask, "10% of what?" Percents have no value by themselves. Percents are part of something else. To understand percents, we need to understand the questions that percents answer.

How do we convey this difference to our students? They know about decimals. How do we teach them about percents? First we talk about the meaning of percent. Then we look for the times in our students' lives when they encounter percents and make them into problems to solve.

Teacher: If there are one hundred people in the room and twenty of them go outside, what percent have gone outside?

What definition of percent do our students carry in their minds? The word *percent* is a part of every student's vocabulary. We ask questions to learn how our students define the word. Our questions are our assessment of what our students know. Their answers guide the questions we ask next.

Teacher: Percent means per hundred. If twenty people in a group of one hundred are wearing red, then twenty percent of the people in the group are wearing red.
If only fifty people are in the group, and I tell you that ten percent are wearing blue, how many in the group would that be?

We draw every question from the answer to the question that preceded it. If our students cannot collectively reason that 10% of 50 is 5, because 10% is equivalent to 10/100 and 10/100 is equivalent to 5/50, then we connect the present question to our students' knowledge of equivalencies. Mathematics is patterns and connections. Sometimes the patterns that our students see help them make the connections for themselves. Sometimes our students need a little teacher guidance to see the connections to be made.

Calculator key...

Nearly every calculator for use beyond the earliest grades has a key that is marked %. This key is meant to take away the trauma of working with percent. Add 10% to 1.00 using the % key and the calculator gives us 1.1. Subtract the same 10% from 1.1 using the % key and the calculator gives us 0.99.

Calculators are useful tools. Every child in our room should have access to one for all the lessons that we teach. But pressing keys on a calculator does not give meaning to the answers that the keys produce. The calculator does not explain to us why, when we add and then subtract a number that appears to be the same, we end up with a smaller number than we started with.

Calculators perform number operations for us rapidly, but calculators do not teach us understanding. Teaching understanding is what teachers do.

Knowledge we can refine...

Percents are as much a part of every child's life as decimals are. Percents are found in:

Sales signs where anything is sold: Percents indicate how much the prices have been reduced.
Sales tax on items bought.
Statistics in sports: Percents indicate success or failure rates for shooting baskets, hitting balls, or slapping pucks.
News reports of deviations for nearly everything that has a norm: Percents help us analyze housing sales for the month, business profits for the year, the rise or fall in the economy, inflation rate, unemployment rate, this winter's rainfall compared to last.
Restaurant tips.

Comparisons of all kinds.

Our students do not come to us ignorant of the world in which they live—they bring knowledge with them that we can refine. Questions we ask help us see what our students think they know:

What examples can you think of where percents are used?
What do you think your examples mean?
We can draw a picture of what a decimal means. Can we draw a picture of what we mean by percent? What is the difference between the two?
Do you ever have a reason to use percents?

We tie our lessons to the uses children have and the understanding children need.

Tipping:

Teacher: How many of you have ever eaten dinner with your family and seen your father or mother leave a tip for the waiter?
Do you know how your parents calculate the tip?
Could you calculate a tip of 15% in your head?

Do we calculate tip amounts in our heads? Or are we like the second person in the restaurant who said that he could not? The question to ask when we calculate a tip is, "What is 15%, or 15/100ths, of the total bill?"

If we want to add a 15% tip to a restaurant bill, we can use our knowledge of equivalencies and percents to do the arithmetic in our heads. If the bill is $34.75, we give our heads a break and round the 34.75 up to 35.00 to make the numbers easier. We give our heads another break by using 10% instead of 15%, since the patterns for multiplying by 10 are easier than the patterns for 15. We can add the missing 5% back again once we find an answer for the 10: 10% of 35.00 is the same as .10 times 35.00. We use the pattern for multiplying decimals we learned in Lesson Ten. We need only move the decimal point one place to the left. .1 x 35.00 = 3.50.

Now we add the missing 5% back again. Five is half of ten. If 10% of 35.00 is 3.50, then 5% is half of 3.50. To give our heads another break, we can round 3.50 up to 4.00 and take half of that. Half of 4.00 is 2.00; 5% and 10% together are 3.50 and 2.00. A tip of 15% on a bill of 34.75 is 5.50, more or less. Or, if the service was good enough, we can give our heads a bigger break and reward the waiter even more. A tip of 20% is 10% added twice. Once we understand the question, we can make the arithmetic easier by rounding up or down.

Shopping:

Teacher: Have you ever been to a store that had signs with percents on them advertising a sale?
Do you know how to figure out how much you would be saving when you see the percent marked down?
Can you calculate the savings in your head?

What is the question being asked? For a tip, we calculated the percent and added the percent onto the bill. For a sale, we calculate the percent and then take that number from the starting price. In both cases, we round the numbers up or down to make them easier to do inside our heads.

For an item marked down 25%, we use our knowledge of equivalencies to know that 25% is 25/100 or 1/4. If the pair of pants sells for $19.95, we round the 19.95 to 20.00 and take one-fourth of that. One-fourth of 20.00 is 5.00. We take the 5.00 away from 20.00 and find a sale price of $15.00.

If the pants cost $16.95, we would round the 16.95 to 17.00. How do we find one-fourth of 17.00? We round 17.00 up or down to a number easier to do inside our heads. 16.00 is an easier number to find fourths for. One-fourth of 16.00 is 4.00. We take the 4.00 away from 16.00 and find the sales price is $12.00, plus a little more. If we want to know the exact amount, we take one-fourth of the dollar that we dropped from 17.00. One-fourth of 1.00 is .25. If we take .25 away from 1.00, we find the actual sales price is closer to $12.25.

Sports:

What are examples of percents in sports?

Shooting percents in basketball.
Pass completion percents in football.
Power-play scoring percents in hockey.
Percentages of ace serves in tennis.

Percentages of goalie save in water polo.
Winning percents in league standings.
Percentages that summarize performances in the game.
Statistics to compare one team or player to another or to compare a player to himself or herself.

Percentages are connected to the fraction and the decimal knowledge that our students have. Tipping at a restaurant adds a fraction to the total of the bill. Calculating savings for on-sale goods subtracts a fraction from the starting price. Sports percentages are most often obtained by dividing the successes (numerator) by the total of the tries (denominator). This is the same technique our students learned in finding decimal equivalents for fractions (page 270). The difference now is that the decimal found is called percent.

What else?

Where else do percents appear in our students' lives?

Percent of sales tax on what we buy.
Percent we are paid as interest on a savings account.
Percent of the various declines or gains reported in the daily news.
Percent of the likelihood that tomorrow it will rain or snow.
Percent of active ingredients on bottles or tubes of medicine.

What else can we think of that makes use of percent? We have thirty students in our room to help us with our search.

For each use of percentages that we find, the focus of our lesson is understanding the question that the percentage represents. To know the answer to the question asked, we must identify the whole from which the percentage is to come. When we know the whole, we must reason what to do with the percentage that we find. For tips, we add. For bargain sales, we subtract. For sports, we are more likely to convert fractions to equivalencies. For each example, we find the answer by reasoning what the numbers mean. Reasoning is a skill we encourage our students to develop when they use percents.

Lesson Twelve

Purpose	Learn about money from using it.
Summary	We use real money and real money situations to teach our students how to find real money answers.
Materials	Money.
Topic	Milk money, restaurant menus, classroom store, fundraising events: any opportunities that arise provide the framework for the money lessons that we teach.
Homework	Money is taught best at home.

Fast finishers...

The students in the second-grade class were to complete a workbook page of coin problems. The teacher sat at her desk, checking papers as the students turned them in. The students had paper coins as problem solving tools, but not a single child used the coins.

The fast finishers were sent to parent helpers in the class to work more money problems on another workbook page until the time for recess came. One parent helper asked the fast finishers in her group to solve a problem that was not on the page.

Parent to the six fast finishers in her group: Please tell me the total value of the paper coins you each were given today. You may help each other calculate the amount.
Students: (After helping each other calculate each student's separate amount) $1.50.
Parent: There are six people in our group. You each have $1.50 in paper coins. How much money will you have when we put all your paper coins together?
Student: (Without counting any coins) $6.50.
Parent: Does anyone have a different answer he or she would like to share?
Other students: No. $6.50 is the total for all of us.
Parent: Does everyone agree?
All the other students, each in turn: Yes. It's $6,50.
Parent: $6.50 is the number you predict as the answer we will find. Now, let's try adding all your coins together.

If you can make a dollar with the coins you have, that's fine. Give me any dollars that you make. If you have any coins left over, then see how many dollars you can make sharing with the others in our group.
(After students have finished working) How much money did you find you had altogether?
Students: $9.00.
Parent: Not $6.50?
Students: No.

The fast finishers had answered every problem right on the page of money problems that their teacher gave to them. Yet, not a single child could see that an answer of $6.50 was logically absurd. Numbers correctly filled in on a page are no measure of what a student understands. Solutions that are learned by rote do not encourage children to reason out what makes sense. We learn about money by using it, not by memorizing rules.

Vending machine...

When the child at the vending machine had previously asked his mother for money for the machine, she had held out a handful of change and asked him to select the right amount of coins to use. He knew by now the exact amount of nickels, dimes and quarters to put into the machine. This time the mother changed the rules.

Son: Can I have a candy bar from the vending machine?
Mother: How much is it?
Son: Sixty-five cents.
Mother: Here are three quarters. What change will you get back?
Son: (Puzzled look)
Mother: Put your three quarters on the table.
Son: (Puts three quarters on the table)
Mother: (Holds out a handful of change) Look at the coins in my hand and pick out the ones you would need to make sixty-five cents.
Son: (Selects two quarters a dime and a nickel)
Mother: Put the sixty-five cents on the table, but don't mix it up with your three quarters.
Son: (Places additional coins on the table near, but not next to, the three quarters)
Mother: How many quarters in your sixty-five cents?
Son: Two.
Mother: Okay, look at your first three quarters on the table. Give one of them back to me and I'll change it into dimes and nickels.
Son: (Changes a quarter for two dimes and a nickel)
Mother. Put the coins you need to make sixty-five cents next to your first two quarters.
Son: (Adds a dime and nickel to the two quarters left on the table)
Mother: What do you have left in your hand?
Son: A dime.
Mother: What do you think your change will be when you put three quarters in the machine?
Son: A dime?
Mother: What is a dime worth?
Son: Ten cents.
Mother: Put your three quarters in the machine and see what you get back.

Do we know to teach the parents?...

What kinds of questions might a parent ask once a child knows that the change from 75¢ spent on a 65¢ candy bar is a dime?

Can we use all dimes to buy the candy bar? How many dimes would it take? What change would we get back then?
Could we use all nickels?
What change would we get back if we put a paper dollar in the dollar slot?
Why doesn't the vending machine take pennies, too?

What kinds of money questions come in the months and years before a parent asks about the change?

Which coins are which?
What are their names?
What is each coin worth?
Can you pick the exact amount of money that you will need from the coins I have in my hand?
Are there different coins that you can use that will give you the same amount?

We learn language by hearing it and speaking it. We learn money by using it. We know to teach our students about money as the opportunities arise in school. But the opportunities to use money arise most often outside of class. What kinds of questions might we teach parents to ask?

Can you sort all the coins that are alike into piles?
Can you stack the coins by size?
How many coins are in each stack? (If the child cannot count the stacks, then count the stacks as the child counts along.)
Can you name the kinds of coins there are?
Which combinations of coins equal which others?
How many different ways can you group coins so that the total value of the coins is twenty-five?
What is the monetary value of the coins we have in front of us?
What kinds of paper money are there? Which coins add up to the paper?
Where is money spent in small amounts?
Where is money spent in larger quantities?
What will an allowance buy?
How much will a savings account save and for how long?
For every trip to a movie, a sporting event, a store, a local arcade, a fast-food restaurant, an amusement park, or the zoo, a set of money questions is waiting to be asked.

If a calculator is available for any question that a parent asks, then the child can use a calculator to find or check the answer.

Milk money...

Teacher in a kindergarten class: Let's find out how much money we have collected for milk today. First, let's see if we can sort the coins by size.

Do the students already know the names for all the coins? If not all know the names, then the lesson is a lesson in vocabulary. If most know, the rest will learn from hearing their classmates speak the words as they sort the coins.

Teacher: Let's put twenty nickels in each cup for as many different cups as the nickels use. Twenty nickels is the same as one-hundred cents. Twenty nickels is also one dollar.
Let's put ten dimes in each cup for as many different cups as the dimes will fit. Ten dimes is the same as one-hundred cents. Ten dimes is also one dollar.
Let's put four quarters in each cup for as many cups as there are quarters for. Four quarters is one-hundred cents. Four quarters is also one dollar.
If you have any coins left over, see how many cups of one-hundred cents you can make by adding different coins together. How many pennies make a nickel? How many nickels make a dime? What coins make a quarter?

We adjust the difficulty of the problems to match our students' skills. Some questions may be beyond our students' understanding at the start. Is that a reason for not asking? When our language is beyond an infant's understanding, should we stop talking?

Teacher: Does anyone know why 25 cents is called a quarter?

Cash is such a useful learning tool. Wherever there is money that we can count, there is a lesson waiting to be taught.

Restaurant lunch...

Teacher: Raise your hand if you have eaten at the Trattoria Restaurant in town?
Students: (Many raised hands)
Teacher: What kind of food do they serve?
Student: Italian.
Teacher: Here are copies of the lunch menu from the Trattoria. I would like you to plan a meal that you might order there. Write the items you would order and the prices for each one. Then figure out what your meal would cost.

What questions might we ask next? What else might we want to know?

Do we ask our students to add in the sales tax?
Shall we have our students calculate the tip?
With calculators available, what else would our students need to know to figure tax and tip?
Should we ask what the total for the bill would be if we all went there together?
Could we find a way to raise the money to take ourselves to the Trattoria one day for lunch?

How close would our students' calculations of the total price be to the actual amount we would pay?
How old would our students have to be before we ask them to figure out a restaurant meal's cost?
What other places in our town do our students go with their parents to spend money?
What other money-spending problems can we use?

Stores...

Teacher: Let's build a store.

What questions might come next?

Are we really going to build it, or are we only going to make a plan?
What does it take to build?
If we build it, where will it stand? How much room will it take? How much might it cost?
What kind of store will it be?
Can we build a store that sells only the groceries that children might want to buy?
What kinds of groceries would our students want to stock?
Would a children's toy store be a good store to construct?
What kind and how much inventory would we stock?
What would we have to spend to stock the items in our store?
What prices would we charge?
If we make a profit, what shall we use the profit for?
Is there a fundraising event for which we might build and stock and staff a store?
Shall we advertise?
At what age group would we have children learn about shopping in a store? Is this a kindergarten activity or one for an upper grade?

Is building a store just a lesson on money? Or is a lesson on money just another way of connecting all the many different kinds of math there are?

Think money...

We teach money by using money. The money lessons that we teach are as real as we can make them. To know the experiences we might use in school, we have to think about the instances in or out of school that involve money. We have to *think* money.

When does money come into our lives in school?

Milk money at snack time.
Lunch money collected in advance.
P.T.A. memberships.
Fundraising of all kinds.
Field-trip fees.
Money from home to spend on the field trip.
Scholastic book club purchases.

When is money already there?

The cost of all the paper and the pencils that we use.
The cost of books.
The cost of furniture.
The cost of the lunch the cafeteria provides: Is the cost to make it different than the cost to buy it?
Stories that we read: Where is money mentioned in the books?
The daily newspaper we might bring with us to school: Where in the news is money mentioned? Where is it not mentioned!
Parties in our class: What is the cost of all the food the students bring in? How much of that total was spent for each child?

What kinds of money situations can we bring in?

We can teach our students the basics of comparison shopping. Which food or packaged good on a grocery shelf is the best buy? Is the best buy always measured by the lowest price? Which toys last? Which break right away? Is durability something that we should factor in?
We can help our students keep records of money collected and/or spent. Do we need to be the ones recording milk money turned in or lunch money paid to the cafeteria? At what age can a child learn to keep track?
We can volunteer our students to manage a fundraising booth at a P.T.A. event. We can volunteer ourselves to help them with making change and totaling the day's receipts.

We can bring blank checks to our class and set up a fictitious bank. Writing checks, balancing a checkbook and understanding a bank statement are all part of money math. We can add in charge cards, too, and even learn to process loans. If we have a spreadsheet program on our class computer, we can send out statements that summarize the transactions we invent.

For every kind of situation, either natural or created, our students can check answers by using their calculators. If no one but the teacher can use a calculator well, then we project an overhead calculator on the screen and let everyone watch as we check answers electronically.

Assessments...

Each money situation that arises in class is an assessment of what our students know. As we watch our students count milk money into cups or make change at a booth, we judge who understands and who needs more experience. The assessments that we make guide us in the questions that we ask.

If we want to know if our students understand money, we ask them what they know about money. If we want to know if our students understand which coins are equivalent to each other, we ask them which coins are equivalent. Assessment means knowing what we want to know and asking to find out.

We may wish to write our assessments in anecdotal style. Sometimes, however, we may want students to record what they understand in their own hand.

Teacher: Using only nickels, dimes and quarters how many different ways can you find to make fifty cents? Write or draw the ways that you can find. Can you prove that there are no other ways than the ones that you have found? Draw or write an explanation of your proof. If you want to, you may work with a friend or two.
Yen is the money people use in Japan. There are no pennies or dollars there. If you had a family visiting you from Japan, how would you explain our money to them? How would you teach them about our coins? How would you teach them about the paper money that we use? Draw and write something that would help them know which kinds of money are equal to each other? If you want to, you may work with a friend or two.
I have twelve coins in my pocket. Altogether, they add up to a dollar. At least one of the coins is a quarter. Can you tell me what my coins might be? Keep a written record of all the different possibilities that you find. If you want to, you may work with a friend or two.

If we want to know, we ask. If we want it written down, we ask that, too.

Summary

Twelve lessons...

Lessons are a means we have of presenting what it is we want our students to learn. Twelve lessons in a chapter. Twelve separate lessons to be taught. As we teach each lesson, we keep in mind that students who learn skills in isolation often struggle to apply these same skills in real life. Lessons are our vehicle for organizing the knowledge that we present. But the purpose of our lessons is to ensure that what is taught is learned and what is learned stays learned.

Our instruction breaks down the artificial barriers that exist between school curriculum and the world outside our class. We teach fractions, ratios, decimals and percents because our students use fractions, ratios, decimals and percents in their lives. We encourage connections between what is learned in school and what is learned in life. Knowledge learned in isolation stays isolated in the child's mind. Knowledge that is connected to the child's life finds continued use.

Questions from Teachers

1. How do we assess for fractions? What kind of assessments might we use?

The answer to any fraction question that we ask is an assessment of what our students know. But not every question assesses what our students understand.

(illustration 11-Q-1)
(Geoboard five dot array (five dots by five dots). A shaded triangle divides the array in half diagonally.)

Teacher: If this whole shape has an area of one, what is the area of the shaded triangle?
Student: One-half.

The answer to our question tells us that the student knows the area is one-half. But does the answer tell us what the student understands?

(illustration 11-Q-2)
(Two different geoboard five by five dot arrays placed side by side. The first array contains a shaded triangle that divides the array in half diagonally. The second array contains a shaded square formed by connecting the mid points of each of the four sides of the array. The arrays are labeled figure A and figure B respectively.)

Teacher: A friend of mine told me that he thinks these two shaded figures have equal areas. Do you agree or disagree with him? Please write or draw your proof of why you think my friend is right or wrong.

If our students prove the areas are the same, we can extend the question by another that we ask:

Teacher: Please draw four different figures that have the same areas as figures A and B.

Our assessments come from the questions that we ask. Questions that can be answered with an number like 1/2 tell us what our students know. Questions that require our students to explain or prove the answers that they give tell us what our students understand.

2. In Lesson Ten, for the multiplication problems on the geoboard, the fractions in the answers were not reduced. Why not? Isn't the teaching of the reducing of fractions an extension of equivalencies?

Reducing fractions is an activity required by school. It is not a skill we need in real life. When we add 1/4 and 1/2 on a calculator, the answer that the calculator gives is 3/4. When we add 2/8 and 4/8, the calculator says the answer is 6/8. The calculator does not reduce the 6/8 to 3/4. Why should we teach our students rules their calculators do not care about?

Equivalent means equal in amount. If the fractions are equivalent, why should we say 3/4 is somehow better than 6/8? If the fractions are equivalent, what difference does it make which one we use?