Chapter 17

Parental Involvement and Homework

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The basics...

According to survey information, the American public wants schools to give students a solid grounding in the "basics." This desire for basic education for all children is matched by skepticism about the many innovations in curriculum. In math, this skepticism is directed towards early use of calculators, mixed ability groupings and an emphasis on the process of solving problems over speed.

The public's desire for a basic education for all children combined with its skepticism of the benefits of innovative curriculum transcends all racial, religious and economic lines. The pervasive public view is that too many children today are leaving school without having mastered the most basic skills. Many people wonder why, if they as children learned these same basics so well, and the present generation is not, we don't return to the successful basic methods of the past?

We teachers are the public, too. Many of us are also parents. Our concern for the learning of the children in our homes is no different from the concern we have for the students in our class. As teachers and as parents we want every child to have a basic education. Why, then, does the public feel that the education we provide is not the basic education that our children need? What is a "basic" education, anyway?

In the context of education, the meaning of the word *basic* changes over time. A hundred years ago, basic meant being prepared to work on a factory floor or to add sums by hand in a bank's back room. What will basic mean a hundred years from now? Would the public expect children a hundred years from now to be taught as children were when Model Ts were rolling off Ford's assembly line? We know that the public would not. Why, then, is it difficult for people to see that the changes we are making now are a necessary part of changing from what was to what will be?

The difficulty lies not in the public's understanding of the need for change. The difficulty lies in our ability as teachers to convey the nature and the value of the changes we make. The public uses the phrase "back to basics" to express the direction we should take. But "forward to the basics" is more descriptive of the need. Our responsibility is to share the understanding that we have with the public and to give each parent an understanding of the need.

Three elements...

We teach understanding to the parents of the children in our class by making parent education a part of our curriculum. Parents are as important to teach as are the children in our class.

Our approach has three elements:

1. We teach the parents the kinds of activities that they may do at home that make math a conscious part of every child's life.

2. We use homework to move math from school to home.
3. We use parent meeting times and communications that we send home to explain our philosophical approach.

First Element

Speaking math again...

In *Graphing, Probability and Statistics* we talked about speaking math (page 171). We can teach the parents of our students to speak math, as well. Here is a dialog heard often in the family car:

Child: How much longer until we are there?
Parent: Five minutes sooner than the last time that you asked!

Here is the same dialog with the parent speaking math:

Child: How much longer until we are there?
Parent: It is 8:30 now. We will be at your cousin's house in about an hour. Will you please let me know periodically how much longer it will be before we are there.

The child's question is the same. What different kinds of thinking can the adult's response produce?

To encourage parents to speak math, we can send a list of words home that we suggest parents use in conversations with their children. Our word list might include:

Large, larger, largest.
Small, smaller, smallest.
Tall, taller, tallest.
Short, shorter, shortest.
Long, longer, longest.
Wide, wider, widest.
Close, closer, closest.
Near, nearer, nearest.
Far, farther, farthest.
More than, less than, same as.
Words that express similarities
Words that express differences.
Words that describe attributes.
Comparison words of all kinds.

Toys...

In the chapter on free exploration and creative learning we said, "We know it is through playing at home that children accomplish much of their learning before coming to school. For the child, playing and learning go hand in hand."

Playing for a child takes many forms. Hide and seek, jumping rope, bouncing balls, rolling down a hill, climbing up a tree. Toys are part of playing, too. Most parents choosing toys for their children are aware that playing time is learning time. Are they aware of the mathematics in toys? Although the toys we recommend may already be common in most homes, it is useful to recommend them anyway. Our recommendations heighten parental awareness of the mathematics in toys.

Mathematical toys are:

Bouncing balls and throwing balls.
Building blocks.
Play cooking sets.
Sand boxes with plastic pails and shovels—anything that scoops, pours, fills, or empties out.
Plastic animals of all kinds: Prehistoric, farm, jungle, sea, forest, domestic household pets, insects. Anything with a name to learn. Anything that sorts into categories.
Anything to sew, knit, or weave.
Crayons, paint, chalk and paper.
Mirrors, magnifying glasses, telescopes and binoculars.
Puzzles and maps.
Teeter-totters.
Musical instruments of all kinds.
Swings.
Jump ropes.

Play money and play clocks.
Play items for a play store.
Play dishes for a play house.
Models to build.
Beads to string.

Mathematics is more than the arithmetic that we learned in school. Mathematics is:

Shape	Size
Relationships	Patterns
Numbers of all kinds	Measurements
Estimations	Approximations
Likelihoods	Space and time
Building up and tearing down	Creativity
Inventiveness	Thinking of all kinds

Any toy or game that introduces these concepts naturally is one we can recommend.

A dialog overheard...

Parent: You are really doing well at following the instructions for that Lego car. Did you know that as you build, you are using math?

Child: No.

Parent: Well, you are. Builders cannot build without math. Look at how you use size and shape to see if the piece in your hand is the same as the piece on the instruction page. Look at all the counting you are doing to see where the pieces go. Following instructions means thinking logically. Logical thinking is a part of math. Anytime you build, you are using math.

Child: Wow! I'm getting pretty good at math!

Parent: Yes you are.

The parents of our students may not conduct this dialog with their child, but any dialog about math is our goal. It is not enough for a parent to know that math is everywhere if the knowledge stays silently inside the parent's head.

Second Element

Establishing a structure...

How do we match the homework to the child? In a room of 30 children, there are 30 different assignments that we prepare each night. Can we establish a structure that allows us to do 30 different things at once?

The most basic skill a child learns in school is reading. Therefore, each night's homework should involve reading of some kind. Reading can mean being read to, or reading to a younger sibling, or reading quietly to oneself. Whatever form it takes, reading is the basic homework every night.

With parent help, we establish the amount of reading to be done at home. If a half-hour or an hour each night is to be devoted to homework, then, unless we send another assignment home, the half-hour or the hour is to be spent reading. The reading can be reported to us on a daily, weekly, or monthly log or by any other method we, the parent, and the child devise.

Once the basic homework is established, we change the assignment for a child only when the change serves a specific learning purpose for that child. We introduce collective changes to the assignment when the change matches our class's current need. The added assignments may sometimes call for writing, or mathematics, or social studies, or science—whatever that day's needs dictate.

Making better spellers...

Reading is the most important skill a child learns in the elementary grades. Writing is a close second. Mathematics is in third, but mathematics as a way of thinking is also part of one and two.

If reading, math and writing are the most important skills, why does spelling consume so much homework time in many schools? Does the spelling sent home each night really help a child learn to spell? Did any poor speller ever become a good one as a result of all the hours spent studying at home for the endless end-of-week tests, or is spelling homework going home just because it always has?

We achieve our homework goals by using our imaginations and our problem-solving skills to match the work that we send home to the needs of every child. If our goal is making better spellers of our students, then how might we reach this goal? We might consider the following:

What kind of words do our students need to spell as they write about what interests them or about their solutions and their proofs to the problems they solve in mathematics?
Is this list of words the same for every child?
Should we have the children who already know these words study them as if they did not?
Are not words learned best in the context of writing? If this is so, then how much writing should we require of our students to give context to their spelling?
If words are learned best in the context of writing, then might we consider placing the emphasis on writing frequently, and not send any separate spelling homework home at all?

Making better mathematicians...

With reading as the basic homework assignment, we also send home assignments that make better mathematicians of our students. Following are suggestions from each chapter for the kinds of work we might send home to help us reach our goal.

Free Exploration and Creative Learning:

1. Encourage parents to provide building toys for their boys and girls. If there is not enough money in the home for Lego Blocks or Tinker Toys, boxes of all kinds and sizes are good building materials.
2. Offer suggestions of the kinds of computer games that are the most beneficial for learning.
3. Stress the importance of reading nightly to or with the child regardless of the child's grade.

Patterns and Connections:

1. Ask students to look for A-A-B patterns at home. As sophistication grows, they record the patterns that they find.
2. Have students teach their parents and their siblings what it means to look for A-A-B so that everyone at home can join the hunt.
3. What designs are used at home in floors, walls, ceiling tiles, roofs, garage doors, fences, furniture upholstery and so on?
4. What other patterns are in places outside of school?
5. Send number charts home for viewing. A copy of the 00-99 matrix can be on every child's bedroom wall or refrigerator door. Will members of the family see patterns in the chart not yet seen by members of the class?

Beginning Number:

1. Send the counting home to help parents make counting a conscious part of daily life.
2. Send home examples of the cardinal and ordinal numbers we want parents to emphasize.
3. Send home the fiveness of five. As a child is working on the ways to make a number at school with toothpicks, squares, or wooden cubes, we ask the parent to provide the child similar opportunities at home with available materials. The focus is on a number and all the variations of that number that can be found.
4. Number books created by the class, go home to be read out loud by the child.
5. Send home flash cards for the families of number facts. Teach parents how to help their child use the cards as a learning tool.
6. If possible, send geoboards home with older children so they may share their search for ways to make two with members of their family. Family members are encouraged to join the search.

Sorting, Classifying, Expanding Language:

In Lesson Two, we listed samples of the kinds of questions we might sent home. The questions were:

1. How many different kinds of cars can you see between here and when you reach your home? What shall we mean by *different*?
2. How many different kinds of houses, fences, doors, or trees are in your neighborhood? Dogs, cats, birds, or pets?
3. What colors or shapes are in your home?
4. What do students think we should sort?

In addition to these questions, we can suggest activities like the following and inform the parents that these activities are all mathematically based:

Putting away toys. What toys go where and why?
Sorting laundry.
Pairing sock.
Storing and hanging different kinds of clothes after they are clean.

What is the sorting plan before washing the dishes?
What goes where in the dishwasher and why?
Putting away all the different dishes.
Sorting the silverware into the drawer.
Putting away pots and pans, matching lids with pans.
Nesting mixing bowls.
Grocery shopping. How does the grocery store decide where to put the goods on the shelf?
What is the plan used by the owner of any kind of store for the shopping that we do? Is there a pattern to where the items are, or is everything put just anywhere?
Putting the groceries away at home. What goes on the pantry or the cupboard shelf? What goes in the freezer or refrigerator? Why?
Sorting leaves or rocks or shells or sticks collected on a walk.

Geometry, Shapes, Relationships and Constructions:

1. If we did not have the opportunity to stress the importance of building at home as a part of free exploration, then we take the opportunity in geometry.
2. Increase students' and parents' awareness of the geometry in their lives by asking them to search for attributes, looking for different kinds of shapes in the environment.
3. What lines of symmetry can our students find outside of school? With parental help, they can make a list of their discoveries.
4. Ask students and parents to search for tessellating shapes.
5. Students take home their Escher cutouts for continued exploration.

Beginning Addition and Subtraction:

1. Ask students to create addition and subtraction problems at home, substituting Match Box cars or Lego Blocks or tea bags for the squares they use at school. Ask students to think along with us the kinds of objects they have at home that they might use.
2. Add the families of facts for 11 through 20 to the flash cards at home.
3. Send subtraction flash cards home, as well.
4. Remind parents that games like Blackjack, shaker dice and dominoes use number facts.
5. Ask students to look for number stories to bring with them to school. Parents can join the search.
6. Ask parents to explain to children how to find answers to the addition and subtraction problems that they encounter at home and how they know the answers are reasonable.

Graphing, Probability and Statistics:

1. As we graph at school, ask students to gather data from home for their graphs.
2. Encourage students to put their graphing skills to use in gathering and displaying data while at home. Home graphs are brought to school to share.
3. Ask students to search for examples of graphs in newspapers, magazines, TV shows, or any other place they find. Our students' parents are partners the search.

Measurement, Estimation and Time:

1. Provide parents with examples of the kinds of measuring questions they can ask at home.
2. Ask students to make a blue print of their home or a map of how they get from home to school.
3. Encourage parents to teach their children about telling time by talking time.
4. Expand parents' notion of telling time from reading a clock's face accurately to understanding time.
5. Send home examples of the kinds of estimation questions a parent might ask a child: estimating time, distances, capacities, weights, heights and so on.
6. Encourage parents to wonder with their children so that wondering aloud is something the family shares. Inform parents that once a week each child is to bring to school a subject their family has wonder about together.

Beginning Multiplication and Division:

1. Send home flash cards for multiplication number facts. Remind the parents that learning facts is not a test of speed.
2. Send home squares and a sideways L for practice in creating multiplication and division problems.
3. Ask students to look for multiplication or division problems at home. Parents join the search.

Fractions, Ratios, Money, Decimals and Percent:

1. What people-fraction questions can our students find at home?
2. Geoboards go home so that students can share their search for areas of shape with family members.
3. Ask students to look for examples of fractions at home, cut them out, write up, draw them, or carry them in their heads to school. Parents help their children search.

4. Ask students and their parents to make a list of all the equivalencies they can find at home. The list can be extended to include where a parent works, where the family shops, where vacations are spent, or any other place outside of school where equivalencies might be found.
5. Ask parents to join their child in searching for examples of rates in newspapers and in magazines.
6. On separate days, the newspaper and magazine searches can be for decimals or percents.
7. Use Lesson Twelve's list of questions to help parents learn to use the money opportunities that arise at home (page 277).

Advanced Addition and Subtraction:

1. As we create real problems at school, ask students to search for real uses for addition and subtraction at home. Parents and other family members join the search. Students bring their examples to school to share.
2. Give parents the list of questions from the home section of Lesson Seven that they may ask their children to contemplate (page 321).
3. Encourage parents to do their mathematics thinking out loud.

Advanced Multiplication and Division:

1. Repeat the familiar pattern of asking students to search at home for multiplication and division problems. Parents and other family members are always invited to join the search. What is found is brought to school to share.

Algebra:

1. The work to send home for algebra depends on the work we have elected to assign in class. If we are exploring (a+b)(a+b) relationships in class, then (a+b)(a+b) relationships can also be explored at home. If we are studying slope/intercept or area in class, then the explorations can be shared with family members after school. If our students are using their geoboards to discover Pick's theorem, any family member who enjoys the challenge can join in the effort. Any graphing opportunity that occurs at home which involves coordinates is a use of algebra. We pick and choose what goes home from the activities that flow most naturally from the teaching in our class.

Third Element

Parental support...

Regardless of the socioeconomic background of our students, parents want their children to receive the best education possible. The conflicts that arise between parents and teachers are usually based on the methods in use to educate their children. If the curriculum presented to their children deviates too much from what they encountered as students, parents may become concerned that a new or experimental method will be bad for their children. Unless we are careful to explain why we are using a particular method, parents have no basis for understanding why what we are doing is an appropriate and effective way to educate their children.

Parental support is extremely important for the success of any educational program. Therefore, regardless of the programs or methods we select for our students, it is essential that we communicate with parents the reasons for our choice. Parents will support us if they believe in what we are doing. But parents cannot develop much confidence in a program if the program remains a mystery.

The SCANS basics...

What is it that we would explain to parents in a parent meeting? Can we put it into words? Or, is it just a feeling that we have of what is right to teach? To make our students ready for a future of wondrous possibilities and disconcerting uncertainties, we provide them with a solid grounding in the basics of a changing world.

But what are the basics that our students and our children need? The U. S. Department of Labor's publication *What Work Requires of School - A SCANS Report for America 2000* lists five areas of competence that it says encompass the talents that will be needed by <u>all</u> skilled workers in the future. The five areas of competency are: resources, interpersonal, information, systems and technology.

1. **Resources:** Identifies, organizes, plans and allocates resources.
   a. Allocates time - Selects goal-relevant activities, ranks them, allocates time and prepares and follows schedules.
   b. Allocates money - Uses or prepares budgets, makes forecasts, keeps records and makes adjustments to meet objectives.
   c. Allocates material and facility resources - Acquires, stores, allocates and uses materials or space efficiently.

d. Allocates human resources - Assesses skills and distributes work accordingly: evaluates performance and provided feedback.

2. **Interpersonal:** Works with others.
   a. Participates as a member of a team - Works cooperatively with others and contributes to group with ideas, suggestions and effort.
   b. Teaches others new skills - Helps others learn.
   c. Serves client/customers - Works and communicates with clients and customers to satisfy their expectations.
   d. Exercises leadership - Communicates thoughts, feelings and ideas to justify a position: encourages, persuades, convinces or otherwise motivates an individual or groups, including responsibly challenging existing procedures and policies.
   e. Negotiates - Works toward agreements involving exchanging specific resources or resolving divergent interests.
   f. Works with diversity - Works well with men and women from diverse backgrounds.

3. **Information:** Acquires and uses information.
   a. Acquires and evaluates information - Identifies need for data, obtains it from existing sources or creates it and evaluates its relevance and accuracy.
   b. Organizes and maintains information - Organizes, processes and maintains written or computerized records and other forms of information in a systematic fashion.
   c. Interprets and communicates information - Selects and analyzes information and communicates the results to others using oral, written, graphic, pictorial, or multi-media methods.
   d. Uses computers to process information - Employs computers to acquire, organize, analyze and communicate information.

4. **Systems:** Understands complex interrelationships.
   a. Understands systems - Knows how social, organizational and technological systems work and operates effectively with them.
   b. Monitors and corrects performance - Distinguishes trends, predicts impact of actions on system operations, diagnoses deviations in the function of a systems/organization and takes necessary action to correct performance.
   c. Improves or designs systems - Makes suggestions to modify existing systems to improve products and services; develops new or alternative systems.

5. **Technology:** Works with a variety of technologies.
   a. Selects technology - Judges which set of procedures, tools, or equipment, including computers and their programs, will produce the desired result.
   b. Applies technology to task - Understands the overall intent and proper procedures for setting up and operating equipment, including computers and their programming systems.
   c. Maintains and troubleshoots technology - Prevents, identifies, or solves problems in machines, computers and other technologies.

The *SCANS Report* also lists a three-part foundation that it says are part of each of the five competencies: basic skills, thinking skills, and personal qualities.

1. **Basic skills:** Reads, writes, performs arithmetic and mathematical operations, listens and speaks.
   a. Reading - Locates, understands and interprets written information in prose and in documents such as manuals, graphs and schedules.
   b. Writing - Communicates thoughts, ideas, information and messages in writing; creates documents such as letters, directions, manuals, reports, graphs and flow charts; records information completely and accurately.
   c. Arithmetic - Performs basic computations; uses basic numerical concepts such as whole numbers and percentages in practical situations; makes reasonable estimates of arithmetic results without a calculator; and uses tables, graphs, diagrams and charts to obtain or convey quantitative information.
   d. Mathematics - Approaches practical problems by choosing appropriately from a variety of mathematical techniques; uses quantitative data to construct logical explanations for real world situations; expresses mathematical ideas and concepts orally and in writing; and understands the role of chance in the occurrence and prediction of events.
   e. Listening - Receives, attends to, interprets and responds to verbal messages and other cues.
   f. Speaking - Organizes ideas and communicates oral messages appropriate to the listeners and the situations; participates in conversations, discussions and group presentations;

selects an appropriate medium for conveying a message; understands and responds to listener feedback; asks questions when necessary.

2. **Thinking Skills:** Thinks creatively, makes decisions, solves problems, visualizes, knows how to learn and reasons.
    a. Creative thinking - Generates new ideas, uses imagination freely, combines ideas or information in new ways, makes connections between seemingly unrelated ideas and reshapes goals in ways that reveal new possibilities.
    b. Decision making - Specifies goals and constraints, generates alternatives, considers risks and evaluates and chooses best alternative.
    c. Problem solving - Recognizes that a problem exists (i.e., there is a discrepancy between what is and what should or could be), identifies possible reasons for the discrepancy and devises and implements plan of action to resolve it; evaluates and monitors progress and revises plan as indicated by findings.
    d. Seeing things in the mind's eye - Organizes and processes symbols, pictures, graphs, objects and other information; for example, sees a building from a blueprint, a system's operations from schematics, the flow-of-work activities from narrative descriptions, or the taste of food from a recipe.
    e. Knowing how to learn - Uses efficient learning techniques to acquire and apply new knowledge and skills; recognizes and can use learning techniques to apply and adapt new knowledge and skills in both familiar and changing situations; involves being aware of learning tools, such as personal learning styles (visual, aural, etc.), formal learning strategies (note taking or clustering items that share some characteristics) and informal learning strategies (awareness of unidentified false assumptions that may lead to faulty conclusions).
    f. Reasoning - Discovers a rule or principle underlying the relationship between two or more objects or events and applies it to solving a problem; for example, uses logic to draw conclusions from available information, extracts rules or principals from a set of objects or written text, applies rules and principles to a new situation, or determines which conclusions are correct when given a set of facts and a set of conclusions.

3. **Personal qualities:** Displays responsibility, self-esteem, sociability, self-management, integrity and honesty.
    a. Responsibility - Exerts a high level of effort and perseveres toward goal attainment; works hard to become excellent at doing tasks by setting high standards, paying attention to details, working well and displaying a high level of concentration, even when assigned an unpleasant task; displays high standards of attendance, punctuality, enthusiasm, vitality and optimism in approaching and completing tasks.
    b. Self-esteem - Believes in own self-worth and maintains a positive view of self; demonstrates knowledge of own skills and abilities; is aware of impact on others; and knows own emotional capacity and needs and how to address them.
    c. Sociability - Demonstrates understanding, friendliness, adaptability, empathy and politeness in new and ongoing group settings; asserts self in familiar and unfamiliar social situations; relates well to others; responds appropriately as the situation requires and takes an interest in what others say and do.
    d. Self-management - Assesses own knowledge, skills and abilities accurately; sets well-defined and realistic personal goals; monitors progress toward goal attainment and motivates self through goal achievement; exhibits self-control and responds to feedback unemotionally and non-defensively; is a self-starter.
    e. Integrity/Honesty - Can be trusted; recognizes when faced with making decisions or exhibiting behavior that may break with commonly held personal or social values; understands the impact of violating these beliefs and codes on an organization, self and others; chooses an ethical course of action.

    What work Requires of Schools - A SCANS Report for America 2000, The Secretary's Commission on Achieving Necessary Skills (SCANS), U. S. Department of Labor, June, 1991.

Agencies that contributed members to the SCANS commission:

Aetna Life and Casualty
Aguirre International
American Association of Community and Junior Colleges
Applied Behavioral and Cognitive Sciences, Inc.
Board of Education, Lauderdale County, Alabama
Citizens and Southern Corporation
Communications Workers of America

Cross Timber Oil Company
Gannett Company, Inc.
General Electric Company
Greater Southeast Community Hospital
Gregory Forest Products
GTE Corporation
High School Redirection
International Business Machines
Iowa Department of Commerce
Los Angeles Unified School District
MCI Communications
Motorola, Inc.
National Education Association
New York State AFL/CIO
RJR Nabisco Foundation
Secretary of Labor
TGI Fridays, Inc.
The Brock Group
UAW/Chrysler National Training Center
United Food and Commercial Workers
University of Pittsburgh
Wayne County Regional Educational Service Agency
Wisconsin Department of Health and Social Services

What does the *SCANS Report* mean to the parents of the children in our class? It means there is more our children need to know to succeed in a changing world than how to add or spell.

Everybody Counts...

The *SCANS Report* tells us what employers will be looking for as the children that we teach graduate from school or drop out. The *SCANS Report* says what. *Everybody Counts* says why.

> *Everybody Counts, A Report to the Nation on the Future of Mathematics Education,* National Research Council, National Academy Press, Washington, D.C., 1989.

If we could require homework of the parents of our students, the first assignment might be to read *Everybody Counts*. The 1989 copyright date makes the book seem older year by year, but the book's message remains fresh. If we cannot require the reading of the book, we might choose to send home excerpts over time.

Samples of excerpts we might use:

Mathematics is:

Mathematics is a living subject which seeks to understand patterns that permeate both the world around us and the mind within us. Mathematics is much more than arithmetic. Mathematics is a diverse discipline that deals with data, measurements and observations; with inference, deduction and proofs; and with major mathematical models of natural phenomena, of human behavior and of social systems. The process of "doing" mathematics is far more than just calculation or deduction; it involves observation of patterns, testing of conjectures and estimation of results.

Mathematics is an area in which even young children can solve a problem and have confidence that the solution is correct - not because the teacher says it is, but because its inner logic is so clear. More than most other school subjects, mathematics offers special opportunities for children to learn the power of thought as distinct from the power of authority.

Mathematics is the key to opportunity. No longer just the language of science, mathematics now contributes in direct and fundamental ways to business, finance, health and defense.

Mathematics and the work force:

Communication has created a world economy in which working smarter is more important than merely working harder. Jobs that contribute to this world economy require workers who are prepared to absorb new ideas, to adapt to change, to cope with ambiguity, to perceive patterns and to solve unconventional problems. It is these needs, not just the need for calculation (which is now done mostly by machines) that make mathematics a prerequisite to so many jobs. More than ever before, Americans need to think to make a living; more than ever before, they need to think mathematically.

Increasingly, mathematics plays a major role in determining the strength of the nation's work force. Over 75% of all jobs require proficiency in simple algebra and geometry, either as a prerequisite to a training program or as part of a licenser examination. However, three out of four Americans stop studying mathematics before completing career or job prerequisites. Most students leave school without sufficient preparation in mathematics to cope with either on-the-job demands for problem solving or college expectations for mathematical literacy. When one compares the potential return on investment in education with the consequences of inaction, it becomes clear that we as a nation have no choice: we must improve the ways our children learn mathematics.

Today's world is more mathematical than yesterday's and tomorrow's will be more mathematical than today's. While arithmetic proficiency may have been "good enough" for many in the middle of the century, anyone whose mathematical skills are limited to computation has little to offer today's society that is not done better by an inexpensive machine.

Mathematics and the schools:

The great majority of American children spend most of their school mathematics time learning only practical arithmetic. Adults who determine policy in mathematics education often measure the mathematical needs of today's students by their own outdated mathematical accomplishments. From the faulty premise that most students "can't do math", they rationalize that expectations should be limited to basic levels. The result is lowered expectations, in which poor performance in mathematics has become socially acceptable.

Public attitudes about mathematics are shaped primarily by adults' childhood school experiences. Consequently, mathematics is seen not as something that people actually use, but as a best forgotten (and often painful) requirement of school. Mathematics is the worst villain in driving students to failure in school.

U. S and the world:

Only in the United States do people believe that learning mathematics depends on special ability. In other countries, students, parents and teachers all expect that most students can master mathematics if only they work hard enough. The record of accomplishment in these countries shows that most students can learn much more mathematics than is commonly assumed in this country.

Average students in other countries often learn as much mathematics as the best students in the U. S. The mathematics performance of the top 5% of U. S. students is matched by the top 50% of students in Japan. All U. S. students, whether below, at, or above average, can and must learn more mathematics.

Since 1970, the percentage of Americans studying mathematics in graduate school has declined steadily so that now Americans are frequently a minority in U. S. graduate schools. As Americans drop out of mathematics, international students converge on the U. S. to study mathematics based subjects. What our own students see as a burden, students from other countries see as opportunity. As in engineering, fewer than half the mathematics doctorates awarded by U. S. universities go to U. S. citizens.

To participate fully in the world of the future, America must tap the power of mathematics. Today's children all over the world are using mathematical training as a platform on which to build up their lives. America's children deserve the same chance.

Present school curriculum:

As technology has "mathematicized" the work place and as mathematics has permeated society, a complacent America has tolerated under-achievement as the norm for mathematical education. We have inherited a mathematical curriculum conforming to past, blind to the future, and bound by a tradition of minimum expectations.

Curricula and instruction in our schools and colleges are years behind the times. They reflect neither the increased demand for higher-order thinking skills, nor the greatly expanded uses of the mathematical sciences, nor what we know about the best ways for students to learn mathematics. The subject moves on, yet the curriculum is stagnant.

Evidence from many sources shows that the least effective mode of mathematics learning is the one that prevails in most American classrooms: lecturing and listening. Students simply do not retain for long what they learn by imitation from lectures, worksheets or routine homework. Presentation and repetition help students do well on standardized tests and lower-order skills, but they are ineffective as teaching strategies for long-term learning, for higher-order thinking and for versatile problem solving.

Teachers almost always present mathematics as an established doctrine to be learned just as it was taught. This leads students to expect that mathematics is about right answers rather than about clear creative thinking. In the early grades, arithmetic becomes the stalking horse for this authoritarian model of learning, sowing seeds of expectation that dominate student attitudes all the way through college.

As children become socialized by school and society, they begin to view mathematics as a rigid system of externally dictated rules governed by standards of accuracy, speed and memory. Their view of mathematics shifts gradually from enthusiasm to apprehension, from confidence to fear.

Only in America do adults openly proclaim their ignorance of mathematics as if it were some sore of merit badge. Only in mathematics is poor school performance socially acceptable.

Research on learning:

Research on learning shows that most students cannot learn mathematics effectively by only listening and imitating, yet most teachers teach mathematics just this way. Research in learning shows that students actually construct their own understanding based on new experiences that enlarge the intellectual framework in which ideas can be created. Much of the failure in school mathematics is due to a tradition of teaching that is inappropriate to the way most students learn.

Educational research offers compelling evidence that students learn mathematics well only when they construct their own mathematical understanding. To understand what they learn, they must enact for themselves verbs that permeate the mathematics curriculum: examine, represent, transform, solve, apply, prove, communicate. This happens most readily when students work in groups, engage in discussion, make presentations and in other ways take charge of their own learning. In reality, no one can *teach* mathematics. Effective teachers are those who can stimulate students to *learn* mathematics.

Curriculum as it should be:

The ability of individuals to cope with mathematics wherever it arises in their later lives, whether as wage earners, parents, or citizens, depends on the attitudes towards mathematics conveyed in schools.

Virtually all young children like mathematics. They do mathematics naturally, discovering patterns and making conjectures based on observation. Natural curiosity is a powerful teacher, especially for mathematics. Elementary school mathematics should reinforce a child's natural curiosity about patterns.

When students explore mathematics on their own, they construct strategies that bear little resemblance to the examples presented in standard textbooks. Even in the absence of calculators, neither children nor adults make much use of the specific arithmetic techniques taught in school. School children do need to learn how to use mathematics for common tasks - making change, measuring quantities (food, lumber, fabric), planning schedules, estimating chances - but the particular means that they use must be appropriate to the task.

Student need an environment for learning mathematics that provides generous room for trial and error. In the long run, it is not memorization of mathematical skills that is particularly important - without constant use, skills fade rapidly - but the confidence that one knows how to find and use mathematical tools whenever they become necessary. There is no way to build this confidence except through the process of creating, constructing and discovering mathematics.

Mathematics instruction must not reinforce the common impression that the only problems amenable to mathematical analysis are those that have unique correct answers. Even more, it must not leave the impression that mathematical ideas are the product of authority or wizardry. Mathematics is a natural mode of human thought, better suited to certain types of problems than to others, yet always subject to confirmation and checking with other types of analysis. There is no place in a proper curriculum for mindless mimicry mathematics.

Despite massive effort, relatively little is accomplished by remediation programs. No one knows how to reverse a consistent early pattern of low achievement and failure. Repetition rarely works; more often than not, it simply reinforces previous failure. The best time to learn mathematics is when it is first taught; the best way to teach it is to teach it well the first time.

Calculators and technology:

Those who use mathematics in the work place rarely use paper and pencil procedures any more. Electronic spreadsheets, numerical analysis packages, symbolic computer systems and sophisticated computer graphics have become the power tools of mathematics in industry. In spite of the intimate intellectual link between mathematics and computing, school mathematics has responded hardly at all to the curricular changes implied by the computer revolution. Curriculum, texts, tests and teaching habits - but not the students - are all products of a pre-computer age.

Many adults fear that early introduction of calculators will prevent children from learning basic arithmetic "properly", as their parents learned it. The experiences during the last fifteen years shows that this fear is unfounded. Students who use calculators learn traditional arithmetic as well as those who do not use calculators, and emerge from elementary school with better problem solving skills and much better attitudes about mathematics. Although mindless calculation can be as destructive as mindless arithmetic, proper use of calculators can stimulate growth of a realistic and productive number sense in each child.

Using calculators intelligently is an integral part of number sense. Children should use calculators throughout their lives. More important, children must learn when to use them and when not to do so. They must learn when to estimate and when to seek exact answers. They must also learn how to estimate answers to verify the plausibility of calculator results. Calculators create whole new opportunities for ordering the curriculum and integrating mathematics into science.

Tests and assessment:

Commonly used achievement tests stress simple skills rather than sophisticated tasks, not because such skills are more important, but because they are easier to measure. In America (but not in other countries), objective, multiple-choice tests are the norm; they are efficient, economical and seemingly fair. Nonetheless, multiple-choice tests lead to widespread abuses which the public rarely recognizes:

- Tests become ends in themselves. Teachers often teach to the tests and not to the curriculum or to the children. What is tested is what gets taught.
- Tests stress lower, rather than higher-order thinking skills.
- Test scores are sensitive to special coaching.
- Tests reinforce the narrow image of mathematics.
- Timed tests stressing speed inhibit learning for many students.
- Normed tests ignore the vast differences in rates at which children learn.
- Tests provide snapshots of performance under the most stressful environment for students, rather than continuous information about performance in a supportive atmosphere.
- Poor test scores lead students to poor self-images, destroying rather than building confidence in ability.
- Minimal competency testing often leads to minimal performance, where the floor becomes the ceiling.

We must ensure that tests measure what is of value, not just what is easy to test. If we want students to investigate, explore and discover, assessment must not measure just mimicry mathematics. By making testing more important than learning, present practice holds today's students hostage to yesterday's mistakes.

Assessment should be an integral part of teaching. It is the mechanism whereby teachers can learn *how* students think about mathematics as well as *what* students are able to accomplish. To assess development of a student's mathematical power, a teacher needs to use a mixture of means: essays, homework, projects, short answers, quizzes, blackboard work, journals, oral reviews and group projects. Only broad-based assessment can reflect fairly the important, higher-order objectives of mathematics curriculum.

The N.C.T.M. Standards...

What we feel is right may not always be enough to convince a principal or a parent that our judgment is as sound as theirs. For parents who need to hear from experts besides ourselves, we quote the *SCANS Report* and *Everybody Counts*. For principals or teachers down the hall, the N.C.T.M. *Curriculum and Evaluation Standards* are the outside authority that we hold up. If some parents would benefit from a more detailed rationale for why we teach the way we do, we make the *Standards* available to these parents, too.

Meeting times...

Just as our goal is to teach every child, our goal is to meet with every parent. Not every child is easy to teach, but we know that every child in our room will learn. Not every parent is easy to meet, but we know that every parent will be met.

A special parent night allows us to meet with as many parents at once as we can entice to school. Our reward for producing a large turnout is having fewer individual meetings to conduct. We can reduce the number of separate meetings even further by arranging collective appointments with groups of parents who miss the larger gathering. We meet with every parent, no matter how many separate meetings it may take.

We use meeting times at school to present our philosophy of education to every parent that we can. We also use meeting times to share teaching techniques that parents can use at home.

Words are not enough...

Words are nice, but words are not enough. At least one meeting should be a time of learning by doing. Our parallel goals for the "doing" meeting are to share our philosophy and to share our teaching techniques—the two are intertwined.

A parent lesson that we teach should be a lesson that:

1. We are comfortable teaching.
2. Involves the parents actively.
3. Makes the points that we want made.

The specific lesson that we choose is not important, so long as it meets our lesson goals. A favorite lesson from our daily teaching may be the one we choose, or we may select a lesson designed more specifically for adults. The Penny Lesson from the *Math Their Way Summary Newsletter* (pages 2.5-2.7) is a lesson we might choose. Teachers who have attended the Math Their Way or Math a Way of Thinking Workshops sponsored by the Center for Innovation in Education may choose to use the State Math Text Book lesson that begins these workshops. Or, we may elect to use the Power Block lesson described below.

Power Block Lesson...

Materials:

Power Blocks at workstations.
Overhead and overhead marking pens, paper towel, water.
Newsprint.

Lesson:

Teacher: Tonight we are going to do a math lesson just like the children do, so you can experience for yourselves the kinds of learning that take place in our class.
The blocks on your desk are called Power Blocks.
Work with the other parents in your group to find out how many different pieces are in the set in front of you. By different, I mean different sizes and different shapes.
When your group thinks it knows the number, call it out.

After working for awhile, the groups begin to call out numbers. If the groups are a little shy about calling out, we wait awhile and ask the groups specifically for the numbers each has found.

Group one: There are eighteen different pieces.
Group two: We have twenty-two.
Group three: We have twenty-four.
Group four: We have twenty-eight.
Teacher: You are free to get up and move about. If you say there are eighteen or twenty-two or twenty-four, go to the tables of groups that say they have found more pieces than you have. See what pieces they have found that you have not.

The number they find is not important. The purpose of the question is to acquaint the parents with the different pieces in the set. The purpose is also to allow the parents to see how answers to a question can be found by sharing information with each other. We do not need to be the one to tell them that the number of different pieces in a set is twenty-four.

Teacher: Which shapes fit together to make other shapes? Save the different ways you and the others in your group find, so I can see your ways as I walk around the room.

Parents put the pieces together to make other pieces in their set as we walk around the room.

Teacher: Now I'm going to ask you some more specific questions about the pieces in your set. Look at the pieces I have placed on the overhead.

(illustration 17-0-1)
(Power Blocks T-1 through T-5 on the overhead. One of each piece.)

Teacher: How many T-1s to make T-2?
Parents: Two.
Teacher: Show me.

(illustration 17-0-2)
(Two T-1s making a T-2.)

Teacher: How many T-2s to make a T-3?
Parents: Two.
Teacher: Show me.

The pattern to the questions is the same. For each new shape, the parents are asked how many it will take to make the next size. The pattern to the parents' answers is the same, as well. The answer to the question is "two."

(illustration 17-0-3)
(Show the ways to make each shape using two of the previous size shapes, T-1 through T-5.)

Teacher: Power Blocks are a "proving set." You can say it, but back your words with proof. How many T-1s to make T-3? You can tell me, but you must be able to show me, too.
Parents: Four.
Teacher: How many T-1s to make T-4?
Parents: Eight.
Teacher: What question will I ask you next?
Parents: How many T-1s to make T-5.
Teacher: And what will the answer be?
Parents: Sixteen.
Teacher: The pattern for making all the shapes from T-1 to T-5 using T-1s is 1, 2, 4, 8, 16. Let's see what the pattern is for P-1 through P-4. How many P-1s does it take to make each of the other parallelograms in your set?

(illustration 17-0-4)
(Show the P-1 through P-4 shapes. Below each shape show it made with P-1s. Write the 1, 2, 4, 8 numbers underneath.)

Teacher: The pattern for parallelograms is 1, 2, 4, 8. If we had a P-5 piece, I would imagine it could be made with 16. What might the pattern for the square shapes be? Starting with S-1, prove the areas of the S-2 through S-5 square shapes.

We know from our work in class and from reading this book (page 241) that the ways to make each succeeding size from the S-1 square are not as evident as the ways to make the triangles or parallelograms. If the parents are like most adults, they may not discover any method of indirect proof on their own. Their background in understanding shapes may be as poor as that of our students before we began to teach them. A parent who finds a way, shares it with all. If no parent can prove S-2 is equivalent to two S-1s, we demonstrate the proof on the overhead.

(illustration 17-0-5)
(Show S-1 as two T-1s. Then show four T-1s making the S-2 shape. Then show the four T-1s equaling two S-1s.)

Teacher: Power Blocks are a proving set. Some problems can be proved directly, like two T-1s equal T-2. Some problems have proofs that are indirect, like demonstrating that two S-1s are equivalent to S-2. Proofs of all kinds are an important part of math.
Here is a different problem for you to explore.

(illustration 17-0-6)
(S-5 square, labeled underneath.)

Teacher: What is the smallest number of pieces it would take to make the S-5 shape if we could not use another S-5 shape? Prove the answer that you give.
Parents: Two.

(illustration 17-0-7)
(S-5 shape made with two T-5s or two R-4s.)

Teacher: What is the largest number of pieces we might use?

Parent: Sixteen.
Parent: No, thirty-two.
Teacher: Which thirty-two pieces would you use?
Parent: The thirty-two T-1s that make the sixteen S-1s that make the S-5.
Teacher: Two is the smallest. Thirty-two is the largest. Now, working together with the others in your group, can you find all the in-between ways? Can you find a way using three pieces? Or four? Or thirty-one? Can you find ways for every number between two and thirty-two?

As we walk around watching the parents work, we will observe that, perhaps with no exceptions at all, they are finding ways randomly. We give them a few minutes on their own before we intercede.

Teacher: In our class, we teach the children to look for patterns in everything they do. Let's see if we can find a pattern in what I have asked you to do that might make your search proceed more systematically.
You told me how to make the S-5 square with thirty-two T-1s. If you start with thirty-two T-1s, is there a way to get from thirty-two to thirty-one?
Parent: Use an S-1 square instead of two T-1s.

If no parent thinks to mention this possibility, we suggest it ourselves.

(illustration 17-0-8)
(Show the 32 triangles used to make the S-5 square. Next to it, show an S-1 square substituted for two T-1 triangles.)

Teacher: Is there a way to get from thirty-one to thirty?
Parent: Use an S-1 square in place of two more T-1s.
Teacher: To get from thirty to twenty-nine?
Parents: Use another S-1 square instead of two T-1s.
Teacher: Is there a pattern to what you are doing?
Parent: Using S-1s in place of T-1s to make the number of pieces one smaller each time.
Teacher: I wonder how far the pattern might carry you in finding ways. I also wonder what other patterns we might find that would help us find new ways.
You showed me how to make the S-5 square with two pieces. If you start with the two T-5 triangles, could you change one of the T-5 triangles into two? Remember the ways you made each triangle from two smaller ones?
Parent: We can use two T-4s.

(illustration 17-0-9)
(Show S-5 made with two T-5s. Next to it, show the S-5 made with one T-5 and two T-4s.)

Teacher: That's a way to make the S-5 with three pieces. What pattern might we use to make the S-5 with four?
Parent: Substitute two more T-4s for the other T-5.
Teacher: Can we use the same kind of pattern to find a way to make the S-5 with five?
Parent: Use two T-3s instead of one of the T-4s.

If no parent sees this as a possibility, we make the suggestion ourselves.

Teacher: With six?
Parents: Two more T-3s.
Teacher: Would these patterns have made it easier to find all the ways to make S-5 from two to thirty-two?
Parents: Yes.
Teacher: Patterns in mathematics are everywhere. When we see them, problems that seem hard at first become much easier. The children in this class are being trained to look. How many of us were taught to look for patterns when we were in school?
Now, let's do a little geometry. Remember when we learned geometry in high school? Remember axioms and theorems?
I am holding up an S-1 shape. The rule for the geometric proofs that we will do next is that S-1 has an area of one. That is our axiom. An axiom is something that is true because the teacher says it is. S-1 is one because I say it is. A theorem is what you can prove with the axiom.
If S-1 has an area of one, what is the area of the R-1 shape?
Parent: Two.
Teacher: Power Blocks are proving blocks. You have to use the axiom to prove your theorem is true. Show me that R-1 equals two S-1s.

(illustration 17-0-10)
(Two S-1s placed in the same shape as an R-1, next to an R-1.)

Teacher: Okay, starting with the axiom that S-1 has an area of one, work with a partner or partners to find the areas of as many of the other pieces as you can. Once you find the area of a shape, trace the shape on the paper on your table and write its area. Then on the shape you trace, also trace how you proved the area. You must be able to prove to me any area you find.

We give an example on the overhead of what it means to trace the shape and trace the proof.

Teacher: The areas of some shapes in your set can be proved directly. Some proofs are indirect. Direct means that you can actually lay the S-1 shapes on the piece to find its area. Indirect means that you may have to find the area of another shape first and use your reasoning abilities to figure out what its S-1 area might be. Reasoning is an important part of math.

We walk around observing the proofs the parents find and trace. We clarify our instructions as we see the need.

Teacher: Remember, an axiom is true because the teacher says it is. Because I'm the teacher, I'm going to change the axiom for you. Now, S-5 is the piece with an area of one. If S-5 has an area of one, what is the area of the T-5 shape?
Parent: One-half.
Teacher: Remember, Power Blocks are proving blocks. You have to use the axiom to prove your theorem is true. Show me that T-5 is half of S-5.

(illustration 17-0-11)
(Two T-5s placed next to an S-5, in the same shape as the S-5.)

Teacher: Okay, starting with the axiom that S-5 has an area of one, work with your partner or partners to find the areas of as many of the other pieces as you can. Once you find the area of a shape, trace the shape on the paper on your table and write its area. Also trace how you proved the area.

As the parents work, we walk around the room observing.

Teacher: Now I would like to talk to you about what we learned tonight.
We covered more ground tonight than we do each day in class. But the kinds of learning that you were doing here are those we do in class.
When I asked you to find the different pieces in your set, you were using your sorting and classifying skills. You were looking for similarities and differences as you studied each shape. Sorting and classifying skills are used in nearly every field. Remember all the classifications we had to learn in high school biology? Imagine how hard it would be to find a book in the library if someone had not thought of a way to sort them out.
As you were checking with each other to find the pieces you could not find, you were learning to rely on your own collective thinking ability. You checked your work yourselves. You did not need the answer to come from me. The answer was in the materials in front of you. All you had to do was work together to find it out.
When I asked you to find which shapes fit together to make other shapes, you were building the basis for understanding fractions. You were also learning basic geometry.
When I asked you how many T-1s made T-2, how many T-2s made T-3 and so on, you were looking for patterns that you later used to see the number of pieces needed to make successively larger sizes of other shapes. Seeing patterns and using them is a most essential part of mathematics. The particular pattern that you found (1, 2, 4, 8, 16) is called "the powers of two." The powers-of-two pattern, or doubling pattern, is the pattern designed into the Power Blocks. It is the number pattern used most commonly in computer programming and computer chip design.
When you found the ways to make all the other squares from the S-1 piece, you were learning how to prove the answers that you found. Initially, children have no idea of what it means to prove something or how to go about a proof. Power Blocks give our children the opportunity to see what it means to prove something in math.
You were also using indirect proofs, just like our children do in class. Proving things indirectly means using what you know in one situation to help you reason answers in another. It is hard to find textbooks or workbooks at this grade level that require proofs of any kind.
The amount of help we needed in understanding how to prove that the S-2 shape really was the same as two S-1s is a measure of how little we were allowed to experience shapes and their

relationships when we were in school. There were no indirect proofs required of us until high school geometry, and the proofs required then were abstract, not concrete.

When you found all the ways to make the S-5 shape from two to thirty-two, you learned how much easier work goes when you look for patterns along the way. You also learned to think about the problem logically. You will recognize the emphasis we place on the search for patterns in this class from the assignments that your children will be bringing home.

We used the blocks to experience geometry and algebra naturally. We used axioms to prove theorems. As the axioms changed, the numbers in the answers changed, but the ways we proved the areas remained the same. We traced the proofs. We could have written equations for the proofs as well. We can write T-5 plus T-5 equals S-5 like this:

$$T5 + T5 = S5$$

T-5 plus T-5 equals S-5 is true regardless of what the teacher's axiom says the value of S-5 is. No matter what the numbers are, it still takes two T-5s to make S-5. This is beginning algebra.

You all used one material. You all worked in teams. You all learned together. Because you all worked together, there was help from everyone. You communicated well with one another. Communication is an important skill in school and in all of life.

We use the Power Blocks in class for mathematics of all kinds, but the blocks are only one of the many different mathematical materials we use.

Not all the lessons that I teach use materials. The materials are not the goal—learning is. The ways we will be teaching this year allow every child to understand what we learn in math, regardless of the amount of math the child has learned before.

You've seen a little bit of how we reach our learning goals at school. Now I'd like to give you a few suggestions for the kinds of things that you can do at home to help...

Summary

Number-one responsibility...

Our students are our number-one responsibility. Their learning and their welfare are our number-one concerns. Our responsibility to our students may sometimes cause us to come in to conflict with administrators or school boards or even politicians, if any or all of these groups propose actions that place political concerns above the needs of the students in our room.

Our greatest ally in meeting our responsibilities to our students is the one group whose concern for our students is equal to or greater than our own—the parents of the children in our room. When we communicate with parents positively and frequently, we encourage parents to communicate with us, as well. Quality communication is the key to building an alliance and keeping the alliance strong.