Fifth—Sixth Grade Lesson Sequence  
(From 61 to 82 Lessons)

If students have been taught for understanding in earlier grades and if they have understood what they have learned, the concepts understood need not be retaught in the upper grades. Concepts understood are best retained through use.

Understanding of concepts taught may be assessed by asking the student how he or she would explain the concept to a person who did not understand. We may ask assessment questions like: “How would you explain the concept of adding or subtracting in a different base to a person new to our school?” If the explanations indicate that our students understand, we need not repeat the teaching of the concepts again this year. Instead, we provide opportunities to use the skills that insures the understanding is maintained.

In the lesson sequence below, lessons most likely to have been learned and understood in earlier years are indicated. The topics covered in earlier years are not repeated here. If it is necessary to teach understanding not provided in an earlier year, the topics from Third-Fourth Lesson Sequence may be used.

**Patterns and Connections**

**Lesson Four** Learn to look for patterns in numbers. Students look at number charts for patterns and describe the patterns they see.

- **Topic** 00-99 matrix.
- **Topic** 25 X 25 multiplication matrix.
- **Topic** Pascal’s triangle.
- **Topic** 12 month calendar, with all months visible

**Lesson Five** Learn to extend pattern searches beyond the period set aside for math. Learn to connect mathematics to art. Students create pattern designs for themselves.

- **Topic** String designs.
- **Topic** Other patterned art we might choose to use.

**Beginning Number**

**Lesson Seven** The families of addition facts. Assess to see if concepts taught in earlier years have been retained. If the concepts are not yet understood, then teach the topics listed in the Third-Fourth Lesson Sequence.

**Sorting, Classifying, Expanding Language**

**Lesson One** Learn to sort by attributes. Students sort objects into groups in a variety of ways.

- **Topic** Each new material produces a variation of the basic lesson. Lists are used again in Lesson Seven.
- **Topic** Sorting buttons, making lists.
- **Topic** Sorting keys, making lists.
- **Topic** Sorting whatever else is available in quantity.
- **Topic** Students trade lists and add to others’ written lists.

**Lesson Three** Use sorting and classification knowledge to create informal definitions of words. We sort shapes, words, or objects as students create definitions for the sorts.

- **Topic** We use this lesson when we have a definition we wish to teach.
- **Topic** Shapes—quadrilaterals, triangles, squares, etc.
- **Topic** Nouns, verbs, prepositions, adjectives, etc.
- **Topic** What other definitions might we choose to use?

**Lesson Four** Learn to find relationships between different shapes. Students play games with Attribute Blocks that focus on thinking logically and systematically.

- **Topic** Pattern sorts, teacher at the overhead.
- **Topic** Identify the missing piece.
- **Topic** Which piece does not belong?
- **Topic** Which piece is missing from the matrix?
- **Topic** Add a piece to the line that is one different than the piece before.
Lesson Five  
Learn to see the attributes in ourselves. Students describe themselves using attributes that define them as unique. For students who can write, we read their descriptions aloud.

Lesson Six  
Learn the process of using individual attributes to categorize. Students use their attributes to develop categories that uniquely describe everyone in class.

Lesson Seven  
Expand the process of learning to use individual attributes to categorize. Students use lists of attributes developed in Lesson One to describe objects sorted earlier.

Lesson Eight  
Learn the meaning of selected words. In a lesson that takes five minutes now and then, we ask the class as a whole to demonstrate the meaning of selected words. The lesson can be extended with Attribute Blocks.

Lesson Nine  
Learn how to communicate more effectively with words. We and our students give instructions and discuss how we can make the instructions clearer.

Geometry, Shapes, Relationships and Constructions

Lesson One  
Provide a background in geometry equally for boys and girls, rich and poor while exploring shapes in geometry. Students build as our questions focus their discoveries.

Lesson Three  
Learn to recognize reflective symmetry in shapes. Students explore lines of symmetry with materials and mirrors.

Lesson Four  
Learn which polygons tessellate the plane, while discovering more kinds of symmetry. Students explore shapes that tessellate and shapes that do not. They create their own tessellating shapes and turn them into Escher-like designs.

Lesson Five
Lesson Six
Lesson Seven
Lesson Eight
Lesson Nine
Lesson Ten

**Beginning Addition and Subtraction**
Lesson Five  The families of addition facts above ten. Assess to see if concepts taught in earlier years have been retained. If the concepts are not yet understood, then teach the topics listed in the Third-Fourth Lesson Sequence.

Lesson Seven  Learn to create and solve story problems. Students create their own stories to go along with numbers. First the teacher provides numbers, then numbers are taken from student lives.

  Topic  Students create addition stories to share.
  Topic  Stories are shared before the next addition creations are produced.
  Topic  Students create subtraction stories to share.
  Topic  Stories are shared before the next subtraction creations are produced.
  Topic  Addition and subtraction are mixed.

Lesson Eight  Learn to think about the reasonableness of answers. Students learn to ask: "Does the answer I have found make sense?"

  Topic  How did you get your answer and how do you know it is reasonable?
  Topic  We pose problems to help our students understand "reasonable".

Lesson Nine  Learn to connect school math to life. We look for problems that exist around us for our students to solve.

  Topic  Twenty problems or just one.

Graphing, Probability and Statistics

Lesson One  Learn to use graphing as a tool for finding answers to questions. Students learn to turn their curiosity into data to graph. Graphs made now will be used again in Lesson Four.

  Topic  Students make graphs in response to questions asked or curiosity expressed that leads to numbers that can be represented pictorially.

Lesson Two  Learn how to display information in a variety of ways. Students invent more ways to graph data than they had thought to use before.

  Topic  Examples of different kinds of graphs are shared as students think of ways to graph they have not used before.

Lesson Three  Asking questions for a graph. Assess to see if concepts taught in earlier years have been retained. If the concepts are not yet understood, then teach the topics listed in the Third-Fourth Lesson Sequence.

Lesson Four  Ask questions for graphs that students make and see. Assess to see if concepts taught in earlier years have been retained. If the concepts are not yet understood, then teach the topics listed in the Third-Fourth Lesson Sequence.

Lesson Five  Learn a beginning framework for connecting probability to graphs. Students toss cardboard squares, graph the outcomes and predict what future outcomes might occur.

  Topic  One square toss and graph.
  Topic  Two square toss and graph.
  Topic  Three square toss and graph.
  Topic  Four square toss and graph.
  Topic  Five square toss and graph.

Lesson Six  Learn a connection between ways possible and ways that actually occur. Students roll dice, graph results, and learn to predict likely outcomes in advance.

  Topic  One die toss and graph.
  Topic  Two dice, chart the ways, toss and graph.
  Topic  Three dice, chart the ways, toss and graph.
  Topic  Four dice, chart the ways, toss and graph.
  Topic  One die, chart the ways.

Lesson Seven  Learn to apply the probability from Lessons Five and Six to graphs made or to be made. Students review graphs in newspapers and old graphs they have made and engage in one-die/four-dice wondering.
<table>
<thead>
<tr>
<th>Topic</th>
<th>The focus of the lesson is on building a frame of reference for viewing past and future data more analytically.</th>
</tr>
</thead>
</table>

### Measurement, Estimation and Time

**Lesson One**  
Learn that measurement is a part of everything we do. We create a measuring environment in our room by making measurements a tool for finding out.  
Topic Measurement is in the environment we create.  
Topic Measurement is in the questions that we ask.  
Topic Measurement is in "Is taller than...".  

**Lesson Two**  
Learn to make and read maps. Students make maps and use and critique maps made by others to refine their own map making techniques.  
Topic Build and describe with cubes on graph paper.  
Topic Repeatedly making and critiquing maps to get from here to there.  
Topic Drawing and critiquing maps of class.  

**Lesson Three**  
Learn to make and use balances. Students learn to make and use different kinds of balances for weighing. Each balance is explored for longer than a day.  
Topic Board and fulcrum balances.  
Topic Bowl balances.  
Topic Double-arm balances.  
Topic Student-made balances.  
Topic Weighing skills developed are used.  

**Lesson Four**  
We teach time buy using it. Specific time concepts can be conveyed with materials like Unifix Cubes.  
Topic Miles per hour represented with cubes.  

**Lesson Five**  
Learn how to make good estimates. Students estimate length, surface area, weight and volume while learning what estimation means, but we do not teach estimation and then move on. Estimation is a thinking skill our students use and use. We make estimation a part of all the lessons we teach by the questions that we ask.  
Topic How much?  
Topic How high?  
Topic How many?  
Topic How far?  
Topic How soon?  
Topic What else?  

### Beginning Multiplication and Division

**Lesson One**  
Create and record (individually and in matrices) multiplication problems. Assess to see if concepts taught in earlier years have been retained. If the concepts are not yet understood, then teach the topics listed in the Third-Fourth Lesson Sequence.  

**Lesson Two**  
Create multiplication and division problems, with and without remainders. Assess to see if concepts taught in earlier years have been retained. If the concepts are not yet understood, then teach the topics listed in the Third-Fourth Lesson Sequence.  

**Lesson Four**  
The multiplication number facts. Assess to see if concepts taught in earlier years have been retained. If the concepts are not yet understood, then teach the topics listed in the Third-Fourth Lesson Sequence.  

**Lesson Five**  
Learn to think about what the numbers in multiplication and division problems represent. We create multiplication and division problems that our students solve, as they identify what the numbers in their answers represent.  
Topic Multiplication word problems created with people and objects in the room.  
Topic Division word problems created with people and objects in the room.  

**Lesson Six**  
Learn to create stories for multiplication and division problems. Learn to see the stories in numbers everywhere. Students write or draw stories for numbers we provide. Students look for number stories in their own lives.
<table>
<thead>
<tr>
<th>Lesson</th>
<th>Topic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Seven</td>
<td>Students write stories or draw illustrations to accompany multiplication and division problems. If we choose to, we select some student stories as problems for the class to solve.</td>
</tr>
<tr>
<td>One</td>
<td>The words to say and the numbers to write for fractions. Assess to see if concepts taught in earlier years have been retained. If the concepts are not yet understood, then teach the topics listed in the Third-Fourth Lesson Sequence.</td>
</tr>
<tr>
<td>Two</td>
<td>Finding and proving areas of shapes on geoboards. Assess to see if concepts taught in earlier years have been retained. If the concepts are not yet understood, then teach the topics listed in the Third-Fourth Lesson Sequence.</td>
</tr>
<tr>
<td>Three</td>
<td>Adding and subtracting simple fractions. Assess to see if concepts taught in earlier years have been retained. If the concepts are not yet understood, then teach the topics listed in the Third-Fourth Lesson Sequence.</td>
</tr>
<tr>
<td>Four</td>
<td>Learn to be aware of fractions in life. We ask our students to think about the sharing, cutting and dividing fractions in their lives.</td>
</tr>
<tr>
<td>Five</td>
<td>Generating lists of equivalencies. Assess to see if concepts taught in earlier years have been retained. If the concepts are not yet understood, then teach the topics listed in the Third-Fourth Lesson Sequence.</td>
</tr>
<tr>
<td>Six</td>
<td>Using equivalencies to find factors. Assess to see if concepts taught in earlier years have been retained. If the concepts are not yet understood, then teach the topics listed in the Third-Fourth Lesson Sequence.</td>
</tr>
<tr>
<td>Seven</td>
<td>Learn to see fractions as a part of measurement. Learn to estimate fractions of a length. Students use strips of paper to estimate, then measure. We teach techniques for determining fractional lengths.</td>
</tr>
<tr>
<td>Eight</td>
<td>Learn what a ratio is. Learn to see practical applications of ratios and equivalencies. We use opportunities that exist or that we create to give our students practical experiences in finding ratios.</td>
</tr>
<tr>
<td>Nine</td>
<td>Learn what rates and rate tables are for. Students search for examples of rates, learn to make rate tables and learn to use rate tables to answer questions.</td>
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**Fractions, Ratios, Money, Decimals and Percent**

*Lesson One* The words to say and the numbers to write for fractions. Assess to see if concepts taught in earlier years have been retained. If the concepts are not yet understood, then teach the topics listed in the Third-Fourth Lesson Sequence.

*Lesson Two* Finding and proving areas of shapes on geoboards. Assess to see if concepts taught in earlier years have been retained. If the concepts are not yet understood, then teach the topics listed in the Third-Fourth Lesson Sequence.

*Lesson Three* Adding and subtracting simple fractions. Assess to see if concepts taught in earlier years have been retained. If the concepts are not yet understood, then teach the topics listed in the Third-Fourth Lesson Sequence.

*Lesson Four* Learn to be aware of fractions in life. We ask our students to think about the sharing, cutting and dividing fractions in their lives.

*Lesson Five* Generating lists of equivalencies. Assess to see if concepts taught in earlier years have been retained. If the concepts are not yet understood, then teach the topics listed in the Third-Fourth Lesson Sequence.

*Lesson Six* Using equivalencies to find factors. Assess to see if concepts taught in earlier years have been retained. If the concepts are not yet understood, then teach the topics listed in the Third-Fourth Lesson Sequence.

*Lesson Seven* Learn to see fractions as a part of measurement. Learn to estimate fractions of a length. Students use strips of paper to estimate, then measure. We teach techniques for determining fractional lengths.

*Lesson Eight* Learn what a ratio is. Learn to see practical applications of ratios and equivalencies. We use opportunities that exist or that we create to give our students practical experiences in finding ratios.

*Lesson Nine* Learn what rates and rate tables are for. Students search for examples of rates, learn to make rate tables and learn to use rate tables to answer questions.
Lesson Ten  Learn what kind of fraction decimals are. Students explore base-ten blocks, geoboards, graph paper rectangles and calculators as they learn about decimal fractions. This lesson is presented after Advanced Multiplication and Division.

- **Topic** One is what we say it is for base-ten blocks.
- **Topic** Think about where the decimal goes in answers to addition and subtraction problems.
- **Topic** Calculator generated lists of equivalencies.
- **Topic** Search for patterns for where the decimal goes when multiplying decimals.
- **Topic** Demonstrate why the fraction becomes smaller when we multiply.
- **Topic** 10 X 10 paper for multiplying decimals.

Lesson Eleven  Learn the difference between decimals and percents. Learn to understand the questions that percent can represent. We talk about the meaning of percent. We give our students practice using percent.

- **Topic** As we talk about percent with our students, what our students say guides what we do next.

Lesson Twelve  We use real money and real money situations to teach our students how to find real money answers. Money is taught best at home.

- **Topic** Milk money, restaurant menus, classroom store, fundraising events: any opportunities that arise provide the framework for the money lessons that we teach.

Advanced Addition and Subtraction

Lesson One  Search for number patterns in bases other than ten. Assess to see if concepts taught in earlier years have been retained. If the concepts are not yet understood, then teach the topics listed in the Third-Fourth Lesson Sequence.

Lesson Two  Adding and subtracting numbers greater than one in different bases. Assess to see if concepts taught in earlier years have been retained. If the concepts are not yet understood, then teach the topics listed in the Third-Fourth Lesson Sequence.

Lesson Three  The concept of place value. Assess to see if concepts taught in earlier years have been retained. If the concepts are not yet understood, then teach the topics listed in the Third-Fourth Lesson Sequence.

Lesson Four  Creating addition and subtraction problems in any base three through ten. Assess to see if concepts taught in earlier years have been retained. If the concepts are not yet understood, then teach the topics listed in the Third-Fourth Lesson Sequence.

Lesson Five  Recognizing place-value patterns within and between bases. Assess to see if concepts taught in earlier years have been retained. If the concepts are not yet understood, then teach the topics listed in the Third-Fourth Lesson Sequence.

Lesson Six  Names for larger numbers. Assess to see if concepts taught in earlier years have been retained. If the concepts are not yet understood, then teach the topics listed in the Third-Fourth Lesson Sequence.

Lesson Seven  Learn that addition and subtraction are tools for finding out. Learn to apply math skills to problems in real life. We work with our students to find and then solve real problems that use the skills that our students possess. Students keep a written record of their work.

- **Topic** Problems drawn from stories.
- **Topic** Problems drawn from questions.
- **Topic** Problems drawn from life.

Lesson Eight  Learn to look for patterns in numbers everywhere. Students look for patterns between bases, in palindromes and in ordinary events.

- **Topic** Compare plus one strips from different bases.
- **Topic** Palindromes.
- **Topic** Examine situations for patterns.

Lesson Nine  Learn that creativity and inventiveness are problem-solving tools. Our students use their inventiveness to solve problems that we give and share their individual or collective inventiveness with everyone in class.
<table>
<thead>
<tr>
<th>Topic</th>
<th>Kids in class, kids in school.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Topic</td>
<td>Thinking and mental arithmetic. Reasonableness.</td>
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</table>

**Advanced Multiplication and Division**

**Lesson One**  Seeing multiplication patterns within and between bases. Assess to see if concepts taught in earlier years have been retained. If the concepts are not yet understood, then teach the topics listed in the Third-Fourth Lesson Sequence.

**Lesson Two**  Learn how to represent concretely a multiplication problem larger than a matrix shows. Learn how to multiply in any base. We give our students larger multiplication problems to solve. We help our students apply what they already know to the new problems that they face. Once they understand the process, they use dice to create problems of their own to solve.

**Topic** Multiplication problems in base five.

**Topic** Multiplication problems in different bases.

**Lesson Three**  Learn techniques for finding answers to the multiplication problems likely to be on the end-of-year standardized test. If students will not be permitted to use calculators or materials on the year-end test, we teach them how to calculate answers for the test.

**Topic** An algorithm for multiplication is taught.

**Lesson Four**  Seeing patterns in multiplying numbers with zeros at the end. Assess to see if concepts taught in earlier years have been retained. If the concepts are not yet understood, then teach the topics listed in the Third-Fourth Lesson Sequence.

**Lesson Five**  Learn to apply multiplication skills. With our students, we look for real problems to solve that use multiplication.

**Topic** A problem a day is enough to pose. A problem is posed, small groups of students discuss and write down ways it might be solved, then students share their ways with the class.

**Topic** Problem posed, discussed, solved, solutions shared.

**Topic** Another problem is posed.

**Lesson Six**  Dividing in any base. Assess to see if concepts taught in earlier years have been retained. If the concepts are not yet understood, then teach the topics listed in the Third-Fourth Lesson Sequence.

**Lesson Seven**  Learn to use the arithmetic skills we have. We look for real problems that use division and any other arithmetic skill. Our students look, as well, and keep a written record of the problem solving steps they use.

**Topic** A single sandwich.

**Topic** The daily life of a child.

**Topic** Analysis.

**Topic** Averages of all kinds.

**Algebra**

**Lesson One**  Learn to write symbolic equations or formulas for familiar patterns or experiences. Students look at geoboard, Power Block, and wooden cube patterns and use letters to record the patterns seen.

**Topic** Squares and rectangles made with S-1 squares, lengths, widths and areas recorded and searched for patterns.

**Topic** Boxes made with wooden cubes, with the formula for volume sought.

**Topic** Geoboard formulas for area and Pick’s theorem.

**Topic** Algebraic relationships between the Power Blocks.

**Lesson Two**  Learn to plot coordinate points on a graph. We play coordinate tic-tac-toe with our students as a class. Students play the game of Battleships with themselves.

**Topic** Students play coordinate tic-tac-toe as a class.

**Topic** Students play Battleships in small groups.
<table>
<thead>
<tr>
<th>Lesson</th>
<th>Description</th>
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</table>
| **Lesson Three** | Learn a framework for understanding addition, subtraction and multiplication of signed numbers. We present Letter Carrier stories to our students to teach them rules for arithmetic operations with positive and negative numbers.  
   Topic | Letter Carrier stories for + and −.  
   Topic | Letter Carrier stories for x.  
   Topic | Students create stories for numbers we provide. |
| **Lesson Four** | Learn to write tables for pairs of numbers that are related in a patterned way. Learn to write equations or formulas for the patterns. Students use a "Magic Box" function machine to predict from numbers going in, the numbers coming out.  
   Topic | Teacher creates the rules for the numbers going in and coming out.  
   Topic | Students create the rules for the numbers going in and coming out.  
   Topic | Write formulas for the rules. |
| **Lesson Five** | Learn to graph functions and the equations they represent. Students plot the data from the tables in Lesson Four and other functional relationships on coordinate graphs and write equations to accompany their graphs.  
   Topic | x = y = 10.  
   Topic | Tables from Lesson Four.  
   Topic | Graphing the area formula, a = lw.  
   Topic | Graphing circumference, c = πd.  
   Topic | Graphing multiplication facts, y = 2x, y = 3x, etc.  
   Topic | Graphing toothpick patterns. |
| **Lesson Six** | Learn to recognize the slope and intercept of an equation to be graphed. Students graph equations in the y = mx or y = mx + b formats and look for slope and intercept patterns in the resulting lines.  
   Topic | y = mx.  
   Topic | y = mx + b. |
| **Lesson Seven** | Learn to recognize the effect that exponents have on a graph. Students graph equations in the y = mx² + b or y = mx³ + b formats and look for patterns in the resulting lines.  
   Topic | y = mx² + b.  
   Topic | y = mx³ + b. |
| **Lesson Eight** | Learn to multiply (x + y)(x + y) equations. Learn to connect the multiplying to concepts already understood. We show our students how to use drawings to solve equations like (x + y)(x + y). We connect the algebra to arithmetic.  
   Topic | (x + y)(x + y).  
   Topic | (x + y)(x + z).  
   Topic | (x + y)(z + w).  
   Topic | What comes next depends. |
| **Lesson Nine** | Learn that the ratios, rates, proportions and equivalencies from the fractions chapter are functional relationships. Our students graph data they developed in their fraction lessons and use their graphs to create algebraic formulas.  
   Topic | Ratios.  
   Topic | Rates.  
   Topic | Proportions.  
   Topic | Equivalencies. |
| **Lesson Ten** | Learn to use algebra. Learn to connect algebra in school to algebra in real life. We create an algebra environment in our room by making algebra a tool for finding out. We use opportunities already present in our student’s lives.  
   Topic | Algebra is in the environment we create.  
   Topic | Algebra is in the questions that we ask. |