

## Fifth—Sixth Grade Lesson Sequence

(From 61 to 82 Lessons)

If students have been taught for understanding in earlier grades and if they have understood what they have learned, the concepts understood need not be retaught in the upper grades. Concepts understood are best retained through use.

Understanding of concepts taught may be assessed by asking the student how he or she would explain the concept to a person who did not understand. We may ask assessment questions like: "How would you explain the concept of adding or subtracting in a different base to a person new to our school?" If the explanations indicate that our students understand, we need not repeat the teaching of the concepts again this year. Instead, we provide opportunities to use the skills that insures the understanding is maintained.

In the lesson sequence below, lessons most likely to have been learned and understood in earlier years are indicated. The topics covered in earlier years are not repeated here. If it is necessary to teach understanding not provided in an earlier year, the topics from Third-Fourth Lesson Sequence may be used.

### Patterns and Connections

- Lesson Four** Learn to look for patterns in numbers. Students look at number charts for patterns and describe the patterns they see.
- Topic 00-99 matrix.
  - Topic 25 X 25 multiplication matrix.
  - Topic Pascal's triangle.
  - Topic 12 month calendar, with all months visible
- Lesson Five** Learn to extend pattern searches beyond the period set aside for math. Learn to connect mathematics to art. Students create pattern designs for themselves.
- Topic String designs.
  - Topic Other patterned art we might choose to use.

### Beginning Number

- Lesson Seven** The families of addition facts. Assess to see if concepts taught in earlier years have been retained. If the concepts are not yet understood, then teach the topics listed in the Third-Fourth Lesson Sequence.

### Sorting, Classifying, Expanding Language

- Lesson One** Learn to sort by attributes. Students sort objects into groups in a variety of ways. Teacher or students list the ways.
- Topic Each new material produces a variation of the basic lesson. Lists are used again in Lesson Seven.
  - Topic Sorting buttons, making lists.
  - Topic Sorting keys, making lists.
  - Topic Sorting whatever else is available in quantity.
  - Topic Students trade lists and add to others' written lists.
- Lesson Three** Use sorting and classification knowledge to create informal definitions of words. We sort shapes, words, or objects as students create definitions for the sorts.
- Topic We use this lesson when we have a definition we wish to teach.
  - Topic Shapes—quadrilaterals, triangles, squares, etc.
  - Topic Nouns, verbs, prepositions, adjectives, etc.
  - Topic What other definitions might we choose to use?
- Lesson Four** Learn to find relationships between different shapes. Students play games with Attribute Blocks that focus on thinking logically and systematically.
- Topic Pattern sorts, teacher at the overhead.
  - Topic Identify the missing piece.
  - Topic Which piece does not belong?
  - Topic Which piece is missing from the matrix?
  - Topic Add a piece to the line that is one different than the piece before.

- Topic If one-different is understood, try two different, then three.
- Lesson Five** Learn to see the attributes in ourselves. Students describe themselves using attributes that define them as unique. For students who can write, we read their descriptions aloud.
- Topic Students descriptions of themselves are read aloud.  
Topic Each new set of descriptions is a separate topic.
- Lesson Six** Learn the process of using individual attributes to categorize. Students use their attributes to develop categories that uniquely describe everyone in class.
- Topic The teacher leads the class in sorting itself into successively smaller sub-categories.  
Topic New categories are used on subsequent days.
- Lesson Seven** Expand the process of learning to use individual attributes to categorize. Students use lists of attributes developed in Lesson One to describe objects sorted earlier.
- Topic Buttons on a sorting tree.  
Topic Buttons sorted differently.  
Topic Keys on a sorting tree.  
Topic Keys sorted differently.  
Topic Whatever other materials we have.
- Lesson Eight** Learn the meaning of selected words. In a lesson that takes five minutes now and then, we ask the class as a whole to demonstrate the meaning of selected words. The lesson can be extended with Attribute Blocks.
- Topic Both-and.  
Topic If-then.  
Topic Either-or, neither-nor.  
Topic Other words we choose.  
Topic "Show me..." with Attribute Blocks.
- Lesson Nine** Learn how to communicate more effectively with words. We and our students give instructions and discuss how we can make the instructions clearer.
- Topic Building and instructing with Pattern Blocks.  
Topic Building and instructing with Power Blocks.  
Topic Attribute Blocks, geoboards, wooden cubes, Unifix Cubes, or other materials with which to build. Each new material is a topic.  
Topic Students write out the instructions to be followed.

### **Geometry, Shapes, Relationships and Constructions**

- Lesson One** Provide a background in geometry equally for boys and girls, rich and poor while exploring shapes in geometry. Students build as our questions focus their discoveries.
- Topic Today is building day, let's see what you can build.  
Topic Lego blocks, Tinker Toys, Geoblocks, Pattern Blocks, Power Blocks, straws, toothpicks and clay. Each material used for building is a topic.
- Lesson Three** Learn to recognize reflective symmetry in shapes. Students explore lines of symmetry with materials and mirrors.
- Topic Free exploration with mirrors.  
Topic Pattern Blocks and mirrors - exploring symmetry.  
Topic Power Blocks and mirrors - exploring symmetry.  
Topic Lines of symmetry in the room - make a list.  
Topic Free exploration with hinged mirrors.  
Topic Symmetry with Pattern Blocks and hinged mirrors.  
Topic Symmetry with Power Blocks and hinged mirrors.  
Topic Kaleidoscope - three mirror exploration.
- Lesson Four** Learn which polygons tessellate the plane, while discovering more kinds of symmetry. Students explore shapes that tessellate and shapes that do not. They create their own tessellating shapes and turn them into Escher-like designs.
- Topic Power Blocks, Pattern Blocks, tag board cutouts, or templates—which polygons tessellate the plane.  
Topic Patterns for the polygons that tessellate.

Topic	Cutting tessellating polygons Escher style.
Topic	The evolution is from math to art.
<b>Lesson Five</b>	Learn that math and art are not separate subjects. We teach art as we always do. In Patterns & Connections, Lesson Five, we pointed out the patterns to be seen. We now point to the geometric connections to be made, as well.
Topic	Extending the tessellations from Lesson Four.
Topic	3-D straw constructions.
Topic	What math can we see in other art that is a part of the art we teach?
<b>Lesson Six</b>	Learn to draw three-dimensional shapes from Geoblocks or other 3-D shapes. Students learn to draw three-dimensional shapes so that the drawn shape identifies the shape.
Topic	Draw one block. Others identify the block drawn.
Topic	Draw two blocks. Others identify the blocks drawn.
Topic	Draw three and more blocks. Others identify the blocks drawn.
<b>Lesson Seven</b>	Learn what an angle is and how to measure it. Students learn to use angles in giving instructions. They learn to measure angles and use a protractor as a measuring device.
Topic	Each new question asked or material explored is like a lesson of its own.
Topic	Students direct each other using paces and turns.
Topic	Students find right angles in the room.
Topic	Angles are measured with straws and sticks, as lists of successively larger angles are made.
Topic	Protractors are explored.
Topic	360°
<b>Lesson Eight</b>	Learn to use a compass, protractor and straightedge to explore geometric properties. We ask questions and pose challenges for our students that guide their explorations with compass, protractor and straightedge.
Topic	See what you can make.
Topic	Make two or more circles the same size.
Topic	Use a compass to compare the lengths of lines.
Topic	Make two angles that are the same size.
Topic	Copy an angle.
Topic	Divide a line in half.
<b>Lesson Nine</b>	Learn to be aware of the geometry in our lives. We ask our students to look more closely at what they already see.
Topic	What shall we look for today? Why are the things that we see the shape that they are?
Topic	What angles, shapes, lines in a bicycle?
<b>Lesson Ten</b>	The purpose is a teacher purpose. Our assignment is to find the opportunities. We make ourselves aware of the opportunities for geometric experiences that exist. We use the opportunities that we find.
Topic	Opportunities that we find.

### **Beginning Addition and Subtraction**

<b>Lesson Two</b>	Learn to apply the skills of addition. We give our students problem-solving questions and number patterns to explore with squares and Unifix Cubes.
Topic	Problems in the middle of a stream.
Topic	Start with, go by.
Topic	Start with, go by, both.
Topic	Consecutive whole numbers.
Topic	Odd and even numbers.
<b>Lesson Four</b>	Learn to apply skills of subtraction. We give our students problem-solving questions and number patterns to explore.
Topic	Starting with 100.
Topic	Problems in a stream, a negative flow.

- Lesson Five** The families of addition facts above ten. Assess to see if concepts taught in earlier years have been retained. If the concepts are not yet understood, then teach the topics listed in the Third-Fourth Lesson Sequence.
- Lesson Seven** Learn to create and solve story problems. Students create their own stories to go along with numbers. First the teacher provides numbers, then numbers are taken from student lives.
- Topic Students create addition stories to share.  
 Topic Stories are shared before the next addition creations are produced.  
 Topic Students create subtraction stories to share.  
 Topic Stories are shared before the next subtraction creations are produced.  
 Topic Addition and subtraction are mixed.
- Lesson Eight** Learn to think about the reasonableness of answers. Students learn to ask: "Does the answer I have found make sense?"
- Topic How did you get your answer and how do you know it is reasonable?  
 Topic We pose problems to help our students understand "reasonable".
- Lesson Nine** Learn to connect school math to life. We look for problems that exist around us for our students to solve.
- Topic Twenty problems or just one.

### **Graphing, Probability and Statistics**

- Lesson One** Learn to use graphing as a tool for finding answers to questions. Students learn to turn their curiosity into data to graph. Graphs made now will be used again in Lesson Four.
- Topic Students make graphs in response to questions asked or curiosity expressed that leads to numbers that can be represented pictorially.
- Lesson Two** Learn how to display information in a variety of ways. Students invent more ways to graph data than they had thought to use before.
- Topic Examples of different kinds of graphs are shared as students think of ways to graph they have not used before.
- Lesson Three** Asking questions for a graph. Assess to see if concepts taught in earlier years have been retained. If the concepts are not yet understood, then teach the topics listed in the Third-Fourth Lesson Sequence.
- Lesson Four** Ask questions for graphs that students make and see. Assess to see if concepts taught in earlier years have been retained. If the concepts are not yet understood, then teach the topics listed in the Third-Fourth Lesson Sequence.
- Lesson Five** Learn a beginning framework for connecting probability to graphs. Students toss cardboard squares, graph the outcomes and predict what future outcomes might occur.
- Topic One square toss and graph.  
 Topic Two square toss and graph.  
 Topic Three square toss and graph.  
 Topic Four square toss and graph.  
 Topic Five square toss and graph.
- Lesson Six** Learn a connection between ways possible and ways that actually occur. Students roll dice, graph results, and learn to predict likely outcomes in advance.
- Topic One die toss and graph.  
 Topic Two dice, chart the ways, toss and graph.  
 Topic Three dice, chart the ways, toss and graph.  
 Topic Four dice, chart the ways, toss and graph.  
 Topic One die, chart the ways.
- Lesson Seven** Learn to apply the probability from Lessons Five and Six to graphs made or to be made. Students review graphs in newspapers and old graphs they have made and engage in one-die/four-dice wondering.

Topic The focus of the lesson is on building a frame of reference for viewing past and future data more analytically.

### **Measurement, Estimation and Time**

**Lesson One** Learn that measurement is a part of everything we do. We create a measuring environment in our room by making measurements a tool for finding out.

Topic Measurement is in the environment we create.  
Topic Measurement is in the questions that we ask.  
Topic Measurement is in "Is taller than...".

**Lesson Two** Learn to make and read maps. Students make maps and use and critique maps made by others to refine their own map making techniques.

Topic Build and describe with cubes on graph paper.  
Topic Repeatedly making and critiquing maps to get from here to there.  
Topic Drawing and critiquing maps of class.

**Lesson Three** Learn to make and use balances. Students learn to make and use different kinds of balances for weighing. Each balance is explored for longer than a day.

Topic Board and fulcrum balances.  
Topic Bowl balances.  
Topic Double-arm balances.  
Topic Student-made balances.  
Topic Weighing skills developed are used.

**Lesson Four** We teach time by using it. Specific time concepts can be conveyed with materials like Unifix Cubes.

Topic Miles per hour represented with cubes.

**Lesson Five** Learn how to make good estimates. Students estimate length, surface area, weight and volume while learning what estimation means, but we do not teach estimation and then move on. Estimation is a thinking skill our students use and use. We make estimation a part of all the lessons we teach by the questions that we ask.

Topic How much?  
Topic How high?  
Topic How many?  
Topic How far?  
Topic How soon?  
Topic What else?

### **Beginning Multiplication and Division**

**Lesson One** Create and record (individually and in matrices) multiplication problems. Assess to see if concepts taught in earlier years have been retained. If the concepts are not yet understood, then teach the topics listed in the Third-Fourth Lesson Sequence.

**Lesson Two** Create multiplication and division problems, with and without remainders. Assess to see if concepts taught in earlier years have been retained. If the concepts are not yet understood, then teach the topics listed in the Third-Fourth Lesson Sequence.

**Lesson Four** The multiplication number facts. Assess to see if concepts taught in earlier years have been retained. If the concepts are not yet understood, then teach the topics listed in the Third-Fourth Lesson Sequence.

**Lesson Five** Learn to think about what the numbers in multiplication and division problems represent. We create multiplication and division problems that our students solve, as they identify what the numbers in their answers represent.

Topic Multiplication word problems created with people and objects in the room.  
Topic Division word problems created with people and objects in the room.

**Lesson Six** Learn to create stories for multiplication and division problems. Learn to see the stories in numbers everywhere. Students write or draw stories for numbers we provide. Students look for number stories in their own lives.

- Topic Students write stories or draw illustrations to accompany multiplication and division problems.
- Topic Selected stories written one day are read as creative inspiration the next.
- Topic Students write stories or draw illustrations to accompany multiplication and division problems that they provide.
- Topic If we choose to, we select some student stories as problems for the class to solve.

- Lesson Seven** Learn that problems to be solved are everywhere around. Students seek the multiplication and division problems that already exist in their lives.
- Topic Students describe number situations that exist. Finding the answers to the situations is not required yet.
- Topic Selected problems from the situations found are solved by the class.

### **Fractions, Ratios, Money, Decimals and Percent**

- Lesson One** The words to say and the numbers to write for fractions. Assess to see if concepts taught in earlier years have been retained. If the concepts are not yet understood, then teach the topics listed in the Third-Fourth Lesson Sequence.
- Lesson Two** Finding and proving areas of shapes on geoboards. Assess to see if concepts taught in earlier years have been retained. If the concepts are not yet understood, then teach the topics listed in the Third-Fourth Lesson Sequence.
- Lesson Three** Adding and subtracting simple fractions. Assess to see if concepts taught in earlier years have been retained. If the concepts are not yet understood, then teach the topics listed in the Third-Fourth Lesson Sequence.
- Lesson Four** Learn to be aware of fractions in life. We ask our students to think about the sharing, cutting and dividing fractions in their lives.
  - Topic Teacher lead discussion on the sharing, cutting and dividing that lead to fractions in our lives.
- Lesson Five** Generating lists of equivalencies. Assess to see if concepts taught in earlier years have been retained. If the concepts are not yet understood, then teach the topics listed in the Third-Fourth Lesson Sequence.
- Lesson Six** Using equivalencies to find factors. Assess to see if concepts taught in earlier years have been retained. If the concepts are not yet understood, then teach the topics listed in the Third-Fourth Lesson Sequence.
- Lesson Seven** Learn to see fractions as a part of measurement. Learn to estimate fractions of a length. Students use strips of paper to estimate, then measure. We teach techniques for determining fractional lengths.
  - Topic Estimate lengths. Calculate the fractions involved.
  - Topic Estimating and calculating techniques are improved with practice and with time.
- Lesson Eight** Learn what a ratio is. Learn to see practical applications of ratios and equivalencies. We use opportunities that exist or that we create to give our students practical experiences in finding ratios.
  - Topic Shadow questions.
  - Topic Shadow ratios.
  - Topic Bouncing balls.
  - Topic Furthest jumps.
  - Topic Diagonals of rectangles.
  - Topic Other opportunities that arise.
- Lesson Nine** Learn what rates and rate tables are for. Students search for examples of rates, learn to make rate tables and learn to use rate tables to answer questions.
  - Topic Search the newspaper for rate examples.
  - Topic Create rate tables for gas consumed and cost.
  - Topic Create rate tables of various kinds.

- Lesson Ten** Learn what kind of fraction decimals are. Students explore base-ten blocks, geoboards, graph paper rectangles and calculators as they learn about decimal fractions. This lesson is presented after Advanced Multiplication and Division.
- Topic One is what we say it is for base-ten blocks.
- Topic Think about where the decimal goes in answers to addition and subtraction problems.
- Topic Calculator generated lists of equivalencies.
- Topic Search for patterns for where the decimal goes when multiplying decimals.
- Topic Demonstrate why the fraction becomes smaller when we multiply.
- Topic 10 X 10 paper for multiplying decimals.
- Lesson Eleven** Learn the difference between decimals and percents. Learn to understand the questions that percent can represent. We talk about the meaning of percent. We give our students practice using percent.
- Topic As we talk about percent with our students, what our students say guides what we do next.
- Lesson Twelve** We use real money and real money situations to teach our students how to find real money answers. Money is taught best at home.
- Topic Milk money, restaurant menus, classroom store, fundraising events: any opportunities that arise provide the framework for the money lessons that we teach.

### **Advanced Addition and Subtraction**

- Lesson One** Search for number patterns in bases other than ten. Assess to see if concepts taught in earlier years have been retained. If the concepts are not yet understood, then teach the topics listed in the Third-Fourth Lesson Sequence.
- Lesson Two** Adding and subtracting numbers greater than one in different bases. Assess to see if concepts taught in earlier years have been retained. If the concepts are not yet understood, then teach the topics listed in the Third-Fourth Lesson Sequence.
- Lesson Three** The concept of place value. Assess to see if concepts taught in earlier years have been retained. If the concepts are not yet understood, then teach the topics listed in the Third-Fourth Lesson Sequence.
- Lesson Four** Creating addition and subtraction problems in any base three through ten. Assess to see if concepts taught in earlier years have been retained. If the concepts are not yet understood, then teach the topics listed in the Third-Fourth Lesson Sequence.
- Lesson Five** Recognizing place-value patterns within and between bases. Assess to see if concepts taught in earlier years have been retained. If the concepts are not yet understood, then teach the topics listed in the Third-Fourth Lesson Sequence.
- Lesson Six** Names for larger numbers. Assess to see if concepts taught in earlier years have been retained. If the concepts are not yet understood, then teach the topics listed in the Third-Fourth Lesson Sequence.
- Lesson Seven** Learn that addition and subtraction are tools for finding out. Learn to apply math skills to problems in real life. We work with our students to find and then solve real problems that use the skills that our students possess. Students keep a written record of their work.
- Topic Problems drawn from stories.
- Topic Problems drawn from questions.
- Topic Problems drawn from life.
- Lesson Eight** Learn to look for patterns in numbers everywhere. Students look for patterns between bases, in palindromes and in ordinary events.
- Topic Compare plus one strips from different bases.
- Topic Palindromes.
- Topic Examine situations for patterns.
- Lesson Nine** Learn that creativity and inventiveness are problem-solving tools. Our students use their inventiveness to solve problems that we give and share their individual or collective inventiveness with everyone in class.

Topic Kids in class, kids in school.  
Topic Thinking and mental arithmetic. Reasonableness.

### Advanced Multiplication and Division

- Lesson One** Seeing multiplication patterns within and between bases. Assess to see if concepts taught in earlier years have been retained. If the concepts are not yet understood, then teach the topics listed in the Third-Fourth Lesson Sequence..
- Lesson Two** Learn how to represent concretely a multiplication problem larger than a matrix shows. Learn how to multiply in any base. We give our students larger multiplication problems to solve. We help our students apply what they already know to the new problems that they face. Once they understand the process, they use dice to create problems of their own to solve.
- Topic Multiplication problems in base five.  
Topic Multiplication problems in different bases.
- Lesson Three** Learn techniques for finding answers to the multiplication problems likely to be on the end-of-year standardized test. If students will not be permitted to use calculators or materials on the year-end test, we teach them how to calculate answers for the test.
- Topic An algorithm for multiplication is taught.
- Lesson Four** Seeing patterns in multiplying numbers with zeros at the end. Assess to see if concepts taught in earlier years have been retained. If the concepts are not yet understood, then teach the topics listed in the Third-Fourth Lesson Sequence.
- Lesson Five** Learn to apply multiplication skills. With our students, we look for real problems to solve that use multiplication.
- Topic A problem a day is enough to pose. A problem is posed, small groups of students discuss and write down ways it might be solved, then students share their ways with the class.
- Topic Problem posed, discussed, solved, solutions shared.  
Topic Another problem is posed.
- Lesson Six** Dividing in any base. Assess to see if concepts taught in earlier years have been retained. If the concepts are not yet understood, then teach the topics listed in the Third-Fourth Lesson Sequence.
- Lesson Seven** Learn to use the arithmetic skills we have. We look for real problems that use division and any other arithmetic skill. Our students look, as well, and keep a written record of the problem solving steps they use.
- Topic A single sandwich.  
Topic The daily life of a child.  
Topic Analysis.  
Topic Averages of all kinds.

### Algebra

- Lesson One** Learn to write symbolic equations or formulas for familiar patterns or experiences. Students look at geoboard, Power Block, and wooden cube patterns and use letters to record the patterns seen.
- Topic Squares and rectangles made with S-1 squares, lengths, widths and areas recorded and searched for patterns.
- Topic Boxes made with wooden cubes, with the formula for volume sought.  
Topic Geoboard formulas for area and Pick's theorem.  
Topic Algebraic relationships between the Power Blocks.
- Lesson Two** Learn to plot coordinate points on a graph. We play coordinate tic-tac-toe with our students as a class. Students play the game of Battleships with themselves.
- Topic Students play coordinate tic-tac-toe as a class.  
Topic Students play Battleships in small groups.



<b>Lesson Three</b>	Learn a framework for understanding addition, subtraction and multiplication of signed numbers. We present Letter Carrier stories to our students to teach them rules for arithmetic operations with positive and negative numbers.
Topic	Letter Carrier stories for + and —.
Topic	Letter Carrier stories for x.
Topic	Students create stories for numbers we provide.
<b>Lesson Four</b>	Learn to write tables for pairs of numbers that are related in a patterned way. Learn to write equations or formulas for the patterns. Students use a "Magic Box" function machine to predict from numbers going in, the numbers coming out.
Topic	Teacher creates the rules for the numbers going in and coming out.
Topic	Students create the rules for the numbers going in and coming out.
Topic	Write formulas for the rules.
<b>Lesson Five</b>	Learn to graph functions and the equations they represent. Students plot the data from the tables in Lesson Four and other functional relationships on coordinate graphs and write equations to accompany their graphs.
Topic	$x = y = 10$ .
Topic	Tables from Lesson Four.
Topic	Graphing the area formula, $a = lw$ .
Topic	Graphing circumference, $c = \pi d$ .
Topic	Graphing multiplication facts, $y = 2x$ , $y = 3x$ , etc.
Topic	Graphing toothpick patterns.
<b>Lesson Six</b>	Learn to recognize the slope and intercept of an equation to be graphed. Students graph equations in the $y = mx$ or $y = mx + b$ formats and look for slope and intercept patterns in the resulting lines.
Topic	$y = mx$ .
Topic	$y = mx + b$ .
<b>Lesson Seven</b>	Learn to recognize the effect that exponents have on a graph. Students graph equations in the $y = mx^2 + b$ or $y = mx^3 + b$ formats and look for patterns in the resulting lines.
Topic	$y = mx^2 + b$ .
Topic	$y = mx^3 + b$ .
<b>Lesson Eight</b>	Learn to multiply $(x + y)(x + y)$ equations. Learn to connect the multiplying to concepts already understood. We show our students how to use drawings to solve equations like $(x + y)(x + y)$ . We connect the algebra to arithmetic.
Topic	$(x + y)(x + y)$ .
Topic	$(x + y)(x + z)$ .
Topic	$(x + y)(z + w)$ .
Topic	What comes next depends.
<b>Lesson Nine</b>	Learn that the ratios, rates, proportions and equivalencies from the fractions chapter are functional relationships. Our students graph data they developed in their fraction lessons and use their graphs to create algebraic formulas.
Topic	Ratios.
Topic	Rates.
Topic	Proportions.
Topic	Equivalencies.
<b>Lesson Ten</b>	Learn to use algebra. Learn to connect algebra in school to algebra in real life. We create an algebra environment in our room by making algebra a tool for finding out. We use opportunities already present in our student's lives.
Topic	Algebra is in the environment we create.
Topic	Algebra is in the questions that we ask.