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</table>

Lesson total: 99

Free Exploration and Creative Learning—Chapter 2

Lesson One

Purpose: Learn the rules, establish the environment.
Summary: Students experience new materials on their own. Our questions guide their thinking.
Materials: Pattern Blocks, Power Blocks, Geoblocks, Unifix Cubes, toothpicks, or any other available materials.
Topic: Pattern Blocks.
Topic: Power Blocks.
Topic: Other materials that we have.

Patterns and Connections—Chapter 3

Lesson One

Purpose: Learn what is meant by "pattern." Learn the A-B system of describing patterns.
Summary: The whole class invents patterns, then smaller groups devise ways to share. Pattern searches extend to the environment.
Materials: Students in the room. Patterns in the world.
Topic: A-A-B.
Topic: A-B, A-B.
Topic: A-B-C and more.
Topic: Five minutes now and then.
Homework: We send the search for patterns home.

Lesson Two

Purpose: Learn to relate A-B patterns to materials.
Summary: Students create and share A-B patterns. Students also learn to check their neighbors to ensure that everybody understands.
Materials: Pattern Blocks, Power Blocks and any other material we might select.
Topic: Pattern Blocks and A-A-B.
Topic: Power Blocks and A-A-B.
Topic: Other materials and A-B patterns.
Homework: We encourage our students to make and share A-B patterns with materials available at home.

Lesson Three

Purpose: Learn how to record patterns.
Summary: Students record patterns and use the recordings of others to reproduce and extend the patterns.
Lesson Four

Purpose Learn to look for patterns in numbers.
Summary Students look at number charts for patterns and describe the patterns they see.
Materials Blacklines for assorted number charts and the number charts themselves.
Topic 00-99 matrix.
Topic 1-100 number strip.
Topic 10 X 10 multiplication matrix.
Topic 25 X 25 multiplication matrix.
Topic Pascal's triangle.
Topic 12 month calendar, with all months visible.
Homework We send number charts home for viewing, too.

Lesson Five

Purpose Learn to extend pattern searches beyond the period set aside for math. To connect mathematics to art.
Summary Students create pattern designs for themselves.
Materials Art supplies of various kinds. Drawing paper, graph paper, crayons, colored pencils and so on.
Topic Pattern Block walls.
Topic Pattern Block mosaic designs.
Topic String designs.
Topic Graph paper patterns.
Topic Other patterned art that we might choose to use.

Beginning Number—Chapter 4

Lesson One

Purpose Learn to count up and back by ones.
Summary We establish a counting environment.
Materials Anything available to count.
Topic Counting up.
Topic Counting back.
Topic Counting every time there are things to count.
Homework We encourage parents to count with their children at home.

Lesson Two

Purpose Learn to look for patterns in the counting numbers.
Summary We post numbers for students to search for patterns.
Materials 0-100 column of numbers.
Topic Search the numbers from 0 to 100 for patterns.
Topic Learn to say the names of numbers.
Topic Look at the 00-99 matrix once again.
Homework We send numbers home so the search for patterns may be shared.

Lesson Three

Purpose Learn the difference between 1, 2, 3 and 1st, 2nd, 3rd.
Summary We use numbers in language to convey meaning.
Materials The language we use.
Topic Language use is not lesson bound. The use of cardinal and ordinal numbers is done consciously everyday.
Homework We send parents a list of words to emphasize at home.
Lesson Four

Purpose Learn the fiveness of five.
Summary We surround our students with the concept of numbers from three to as far as we decide to go.
Materials Power Block squares or tiles, toothpicks, wooden cubes, Pattern Blocks.
Topic 3 with squares.
Topic 3 with tooth picks.
Topic 3 with wooden cubes.
Topic 3 with Pattern Blocks.
Topic 4 with squares.
Topic 4 with tooth picks.
Topic 4 with wooden cubes.
Topic 4 with Pattern Blocks.
Topic 5 and more with each material, in turn.
Homework We send the number stations home to be done with any material present in the house.

Lesson Five

Purpose Record the number concepts learned in Lesson Four.
Summary Students learn a different way to record each material. The recordings have a use in Lesson Six.
Materials Power Block squares or tiles, toothpicks, wooden cubes, Pattern Blocks, a different kind of recording paper for each material, paste or glue. Blackline masters for recording cube shapes.
Topic The lesson focus is on recording.
Topic Recording 3 with squares.
Topic Recording 3 with tooth picks.
Topic Recording 3 with Pattern Blocks.
Topic Recording 3 with wooden cubes (upper grades).
Topic Recording 4 with squares.
Topic Recording 4 with tooth picks.
Topic Recording 4 with Pattern Blocks.
Topic Recording 4 with wooden cubes (upper grades).
Topic Recording 5 and more with each material, in turn.
Homework Homework for Lesson Five waits for Lesson Six.

Lesson Six

Purpose Record in words and numbers while learning to envision what the words and numbers mean.
Summary Students imagine then write words and numbers to describe numeric designs.
Materials Recordings from Lesson Five, paper for writing words, separate paper for writing numbers. Number lines. Number line templates. Spelling Notebook introduced.
Topic Writing words for recordings of 3 squares.
Topic Writing numbers for recordings of 3 squares.
Topic Creating square designs for numbers.
Topic Writing words for recordings of 3 toothpicks.
Topic Writing numbers for recordings of 3 toothpicks.
Topic Creating toothpick designs for numbers.
Topic Writing words for recordings of 3 Pattern Blocks.
Topic Writing numbers for recordings of 3 Pattern Blocks.
Topic Creating Pattern Block designs for numbers.
Topic Repeating the three step writing and creating cycle for the numbers 4 and 5 and maybe 6.
Homework We send the word and number books home to be read by our students to any audience that can be found.

Lesson Seven

Purpose Learn the families of addition facts.
Summary Students look for ways to make number combinations with Unifix Cubes using two different sets of rules.

Lesson Eight

Purpose: Learn to link number to area. Learn to prove answers found.

Summary: Students prove Power Block areas before creating shapes on their geoboards and proving areas of their created shapes. Adds to the wooden cube experiences from Lessons Four and Five. Provides more beginning number experiences for the older child.

Materials: Power Blocks, Geoboards and Blacklines for three levels of recording sheets. (One, four, twelve.)

Topic: Power Block S-1 square has an area of one, what are the areas of all the other shapes?

Topic: Geoboards—ways to make 2, without then with recording.

Topic: Reviewing other people’s ”2’s”.

Topic: Ways to make 3.

Topic: Ways to make 4 or more, while proving areas found.

Homework: If geoboards can be sent home, explorations are continued there.

Sorting, Classifying, Expanding Language—Chapter 5

Lesson One

Purpose: Learn to sort by attributes.

Summary: Students sort objects into groups in a variety of ways. Teacher or students list the ways.

Materials: Buttons, keys, or any other common objects available in quantity; individual chalkboards.

Topic: Each new material produces a variation of the basic lesson. Lists created by older students are used again in Lesson Seven.

Topic: Sorting buttons, making lists.

Topic: Sorting keys, making lists.

Topic: Sorting whatever else is available in quantity.

Homework: What groups of things do children have at home that they might sort?

Lesson Two

Purpose: Learn to be aware of attributes everywhere.

Summary: Students take sorting walks to learn to see what is already there.

Materials: The world outside our class.

Topic: Sorting walks. Older students keep written lists of what is seen. Younger students may draw instead of write.

Topic: Each walk taken is a different element.

Homework: Sorting walks can take place while coming to or going home from school each day.

Lesson Three

Purpose: Use sorting and classification knowledge to create informal definitions of words.

Summary: We sort shapes, words, or objects as students create definitions for the sorts.

Materials: Whatever relates to the definition we wish to teach.

Topic: We use this lesson when we have a definition we wish to teach.

Topic: Shapes—quadrilaterals, triangles, squares, etc.

Topic: Nouns, verbs, prepositions, adjectives, etc.

Topic: What other definitions might we choose to use?

Lesson Four

Purpose: Learn to find relationships between different shapes.

Summary: Students play games with Attribute Blocks that focus on thinking logically and systematically.
<table>
<thead>
<tr>
<th>Materials</th>
<th>Attribute Blocks, overhead set, blackline recording sheet.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Topic</td>
<td>Pattern sorts, teacher at the overhead.</td>
</tr>
<tr>
<td>Topic</td>
<td>Identify the missing piece.</td>
</tr>
<tr>
<td>Topic</td>
<td>Which piece does not belong?</td>
</tr>
<tr>
<td>Topic</td>
<td>Which piece is missing from the matrix?</td>
</tr>
<tr>
<td>Topic</td>
<td>Add a piece to the line that is one different than the piece before.</td>
</tr>
<tr>
<td>Topic</td>
<td>If one-different is understood, try two different, then three.</td>
</tr>
</tbody>
</table>

**Lesson Five**

<table>
<thead>
<tr>
<th>Purpose</th>
<th>To learn to see the attributes in ourselves.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Summary</td>
<td>Students describe themselves using attributes that define them as unique. For students who can write, we read their descriptions aloud.</td>
</tr>
<tr>
<td>Materials</td>
<td>Students in class, writing paper.</td>
</tr>
<tr>
<td>Topic</td>
<td>Students descriptions of themselves are read aloud.</td>
</tr>
<tr>
<td>Topic</td>
<td>Each new set of descriptions is a separate element.</td>
</tr>
</tbody>
</table>

**Lesson Six**

<table>
<thead>
<tr>
<th>Purpose</th>
<th>Learn the process of using individual attributes to categorize.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Summary</td>
<td>Students use their attributes to develop categories that uniquely describe everyone in class.</td>
</tr>
<tr>
<td>Materials</td>
<td>Students in class.</td>
</tr>
<tr>
<td>Topic</td>
<td>The teacher leads the class in sorting itself into successively smaller sub-categories.</td>
</tr>
<tr>
<td>Topic</td>
<td>New categories are used on subsequent days.</td>
</tr>
</tbody>
</table>

**Lesson Seven**

<table>
<thead>
<tr>
<th>Purpose</th>
<th>Expand the process of learning to use individual attributes to categorize.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Summary</td>
<td>Students use lists of attributes developed in Lesson One to describe uniquely objects sorted earlier.</td>
</tr>
<tr>
<td>Materials</td>
<td>Objects used and lists created in Lesson One: sorting tree blackline.</td>
</tr>
<tr>
<td>Topic</td>
<td>Buttons on a sorting tree.</td>
</tr>
<tr>
<td>Topic</td>
<td>Buttons sorted differently.</td>
</tr>
<tr>
<td>Topic</td>
<td>Keys on a sorting tree.</td>
</tr>
<tr>
<td>Topic</td>
<td>Keys sorted differently.</td>
</tr>
<tr>
<td>Topic</td>
<td>Whatever other materials we have.</td>
</tr>
</tbody>
</table>

**Lesson Eight**

<table>
<thead>
<tr>
<th>Purpose</th>
<th>Learn the meaning of selected words.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Summary</td>
<td>In a lesson that takes five minutes now and then, we ask the class as a whole to demonstrate the meaning of selected words. The lesson can be extended with Attribute Blocks.</td>
</tr>
<tr>
<td>Materials</td>
<td>Students in class, Attribute Blocks.</td>
</tr>
<tr>
<td>Topic</td>
<td>Both-and.</td>
</tr>
<tr>
<td>Topic</td>
<td>If-then.</td>
</tr>
<tr>
<td>Topic</td>
<td>Either-or, neither-nor.</td>
</tr>
<tr>
<td>Topic</td>
<td>Other words we choose.</td>
</tr>
<tr>
<td>Topic</td>
<td>“Show me...” with Attribute Blocks.</td>
</tr>
</tbody>
</table>

**Lesson Nine**

<table>
<thead>
<tr>
<th>Purpose</th>
<th>Learn how to communicate more effectively with words.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Summary</td>
<td>We and our students give instructions and discuss how we can make the instructions clearer.</td>
</tr>
<tr>
<td>Materials</td>
<td>Pattern Blocks, shield behind which to build, Power Blocks, Attribute Blocks, geoboards, wooden cubes, Unifix Cubes, or other materials with which to build.</td>
</tr>
<tr>
<td>Topic</td>
<td>At the youngest ages, the teacher is the only builder in the room.</td>
</tr>
<tr>
<td>Topic</td>
<td>Building and instructing with Pattern Blocks.</td>
</tr>
<tr>
<td>Topic</td>
<td>Building and instructing with Power Blocks.</td>
</tr>
<tr>
<td>Topic</td>
<td>Each new material is a topic.</td>
</tr>
</tbody>
</table>

**Geometry, Shapes, Relationships and Constructions—Chapter 6**
### Lesson One

**Purpose**

Provide a background in geometry equally for boys and girls, rich and poor while exploring shapes in geometry.

**Summary**

Students build as our questions focus their discoveries.

**Materials**

Building materials off all kinds: Lego blocks, Tinker Toys, Geoblocks, Pattern Blocks, Power Blocks, straws, toothpicks and clay, kindergarten building blocks.

**Topic**

Today is building day, let’s see what you can build.

**Homework**

Each material used for building is a topic.

We encourage parents to provide building materials at home for boys and girls alike.

### Lesson Two

**Purpose**

Expand the exploration of shape.

**Summary**

Students explore the properties of shapes guided by the questions that we ask.

**Materials**

Power Blocks, Geoboards, recording paper.

**Topic**

Power Blocks, what shapes make other shapes?

**Topic**

Geoboards, make shapes with 3 sides. 4 sides. 5 sides. More sides.

**Topic**

Which shapes can be made with Power Blocks & duplicated on a geoboard? Which shapes cannot?

**Homework**

If geoboards can be sent home, explorations are continued there.

### Lesson Three

**Purpose**

Learn to recognize reflective symmetry in shapes.

**Summary**

Students explore lines of symmetry with materials and mirrors.

**Materials**

Mirrors, hinged and regular, Pattern Blocks, Power Blocks, paper for tracing shapes, colored construction paper, lined paper.

**Topic**

Free exploration with mirrors.

**Topic**

Pattern Blocks and mirrors - exploring symmetry.

**Topic**

Power Blocks and mirrors - exploring symmetry.

**Topic**

Lines of symmetry in the room - make a list.

**Topic**

Free exploration with hinged mirrors.

**Topic**

Symmetry with Pattern Blocks and hinged mirrors.

**Topic**

Kaleidoscope - three mirror exploration.

**Homework**

The search for symmetry is sent home.

### Lesson Four

**Purpose**

Learn which polygons tessellate the plane, while discovering more kinds of symmetry.

**Summary**

Students explore shapes that tessellate and shapes that do not. They create their own tessellating shapes and turn them into Escher-like designs.

**Materials**

Power Blocks, Pattern Blocks, tag board cutouts, or templates. Blackline for tessellating shapes with angles written in and the shapes labeled.

**Topic**

Which polygons tessellate the plane.

**Topic**

Patterns for the polygons that tessellate.

**Topic**

Cutting tessellating polygons Escher style.

**Topic**

The evolution is from math to art.

**Homework**

Escher cutouts made at school go home for further exploration.

### Lesson Five

**Purpose**

Learn that math and art are not separate subjects.

**Summary**

We teach art as we always do. In Patterns & Connections, Lesson Five, we pointed out the patterns to be seen. We now point to the geometric connections to be made, as well.

**Materials**

Assorted art supplies.

**Topic**

Extending the tessellations from Lesson Four.

**Topic**

Name symmetry.

**Topic**

Snow flakes.

**Topic**

3-D straw constructions.

**Topic**

What math can we see in other art that is a part of the art we teach?
Homework  Children who have art materials at home can continue their art work there.

Lesson Six

Purpose  Learn to draw three-dimensional shapes.
Summary  Students learn to draw three-dimensional shapes so that the drawn shape identifies the shape.
Materials  Geoblocks or other 3-D shapes, drawing paper.
Topic  Draw one block. Others identify the block drawn.
Topic  Draw two blocks. Others identify the blocks drawn.
Topic  Draw three and more blocks. Others identify the blocks drawn.

Lesson Seven

Purpose  Learn what an angle is and how to measure it.
Summary  Students learn to use angles in giving instructions. They learn to measure angles and use a protractor as a measuring device.
Materials  Students in the class, paper, protractors, straws, sticks, calculators.
Topic  Each new question asked or material explored is like a lesson of its own.
Topic  Students direct each other using paces and turns.
Topic  Students find right angles in the room.
Topic  Angles are measured with straws and sticks, as lists of successively larger angles are made.
Topic  Protractors are explored.
Topic  360°

Lesson Eight

Purpose  Learn to use a compass, protractor and straightedge to explore geometric properties.
Summary  We ask questions and pose challenges for our students that guide their explorations with compass, protractor and straightedge.
Materials  Compass, protractor, straightedge, paper.
Topic  See what you can make.
Topic  Make two or more circles the same size.
Topic  Use a compass to compare the lengths of lines.
Topic  Make two angles that are the same size.
Topic  Copy an angle.
Topic  Divide a line in half.

Lesson Nine

Purpose  Learn to be aware of the geometry in our lives.
Summary  We ask our students to look more closely at what they already see.
Materials  Paper for writing lists.
Topic  Make a list of rectangular shapes. What other shapes can we list?
Topic  List the tessellations that you can find.
Topic  What shall we look for today? Why are the things that we see the shape that they are?
Topic  What angles, shapes, lines in a bicycle?
Homework  This is a lesson on looking and wondering. Looking and wondering questions can be sent home everyday.

Lesson Ten

Purpose  The purpose is a teacher purpose. Our assignment is to find the opportunities.
Summary  We make ourselves aware of the opportunities for geometric experiences that exist. We use the opportunities that we find.
Materials  Our own awareness.
Topic  Opportunities that we find.
Homework  What opportunities can our students find at home?

Beginning Addition and Subtraction—Chapter 7

Lesson One
Lesson One

Purpose Learn to create and check addition problems.
Summary Students create addition problems that they can check by counting.
Materials Squares or tiles, chalkboards, calculators, paper.
Topic Creating addition problems with handful of squares.
Homework Once the process is understood, our students create addition problems at home.

Lesson Two

Purpose Learn to apply the skills of addition.
Summary We give our students problem-solving questions and number patterns to explore.
Materials Squares, Unifix Cubes, calculators, lined paper.
Topic Start with, go bys.
Topic Odd and even numbers.
Homework What is understood may be continued at home.

Lesson Three

Purpose Learn to create and check subtraction problems.
Summary Students create subtraction problems that they can check by counting.
Materials Squares or tiles, chalkboards, calculators, paper.
Topic Creating subtraction problems with handful of squares.
Homework Once the process is understood, our students create subtraction problems at home.

Lesson Four

Purpose Learn to apply skills of subtraction.
Summary We give our students problem-solving questions and number patterns to explore.
Materials Squares, calculators, lined paper.
Topic Starting with 100.
Topic Problems in a stream, a negative flow.
Homework What is understood may be continued at home.

Lesson Five

Purpose Learn the families of addition facts above ten.
Summary Students practice number facts, through flash cards and number games. There are no tests of speed.
Materials Blackline for flash cards; games like Blackjack, dominoes and shaker dice.
Topic Flash cards.
Topic Blackjack.
Topic Dominoes.
Topic Shaker dice.
Homework Flash cards are sent home. Games to play at home are recommended, too.

Lesson Six

Purpose Learn to solve word problems that the teacher creates.
Summary Students learn to draw the necessary numbers from the stories that we tell.
Materials Students and materials available in class, chalkboards.
Topic Word problems for adding.
Topic Word problems for subtracting.
Topic Word problems for adding, extraneous information added in.
Topic Word problems for subtracting, extraneous information added in.
Topic Adding and subtracting mixed.

Lesson Seven
### Lesson Eight

**Purpose**
Learn to think about the reasonableness of answers.

**Summary**
Students learn to ask: "Does the answer I have found make sense?"

**Materials**
The materials used depend on the questions asked. Calculators.

**Topic**
How did you get your answer and how do you know it is reasonable?

**Homework**
Where might a use for "reasonableness" be found at home?

### Lesson Nine

**Purpose**
Learn to connect school math to life.

**Summary**
We look for problems that exist around us for our students to solve.

**Materials**
Problems from ours and our students' lives.

**Topic**
Twenty problems or just one.

**Homework**
What problems can our students find at home?

### Graphing, Probability and Statistics—Chapter 8

#### Lesson One

**Purpose**
Learn to use graphing as a tool for finding answers to questions.

**Summary**
Students learn to turn their curiosity into data to graph. Graphs made now will be used again in Lesson Four.

**Materials**
Materials depend on the questions students ask.

**Topic**
Students make graphs in response to questions asked or curiosity expressed that leads to numbers that can be represented pictorially.

**Homework**
Ideally, our students learn to use graphing as a tool for displaying information at or from home.

#### Lesson Two

**Purpose**
Learn how to display information in a variety of ways.

**Summary**
Students invent more ways to graph data than they had thought to use before.

**Materials**
Examples of a variety of graphs, materials in the room, and student creativity and inventiveness.

**Topic**
Examples of different kinds of graphs are shared as students think of ways to graph they have not used before.

**Homework**
The search for different kinds of graphs is continued at home.

#### Lesson Three

**Purpose**
Learn how to ask questions for a graph.

**Summary**
We assemble unseen graphs to guide students in learning how to ask what it is they want to know.

**Materials**
Shield for the graphs, cut-off milk carton boxes to create the hidden graphs.

**Topic**
A graph is assembled behind a shield as students ask questions about data that remains unseen.

#### Lesson Four

**Purpose**
Learn to ask questions for the graphs that students make and see.
<table>
<thead>
<tr>
<th>Summary</th>
<th>Students learn to add written questions to their graphs. The lesson on asking questions is also a lesson on learning to speak math and learning to ask math questions.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Materials</td>
<td>Graphs from Lesson One, cut-off milk carton boxes.</td>
</tr>
<tr>
<td>Topic</td>
<td>Students add questions to graphs already made.</td>
</tr>
<tr>
<td>Homework</td>
<td>What math questions can our students bring from home?</td>
</tr>
</tbody>
</table>

**Lesson Five**

<table>
<thead>
<tr>
<th>Purpose</th>
<th>Learn a beginning framework for connecting probability to graphs.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Summary</td>
<td>Students toss cardboard squares, graph the outcomes and predict what future outcomes might occur.</td>
</tr>
<tr>
<td>Materials</td>
<td>Cardboard squares and graph, lined, or plain paper.</td>
</tr>
<tr>
<td>Topic</td>
<td>One square toss and graph.</td>
</tr>
<tr>
<td>Topic</td>
<td>Two square toss and graph.</td>
</tr>
<tr>
<td>Topic</td>
<td>Three square toss and graph.</td>
</tr>
<tr>
<td>Topic</td>
<td>Four square toss and graph.</td>
</tr>
<tr>
<td>Topic</td>
<td>Five square toss and graph.</td>
</tr>
</tbody>
</table>

**Lesson Six**

<table>
<thead>
<tr>
<th>Purpose</th>
<th>Learn a connection between ways possible and ways that actually occur.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Summary</td>
<td>Students roll dice, graph results, and learn to predict likely outcomes in advance.</td>
</tr>
<tr>
<td>Materials</td>
<td>Dice and graph, lined, or plain paper.</td>
</tr>
<tr>
<td>Topic</td>
<td>One die toss and graph.</td>
</tr>
<tr>
<td>Topic</td>
<td>Two dice, chart the ways, toss and graph.</td>
</tr>
<tr>
<td>Topic</td>
<td>Three dice, chart the ways, toss and graph.</td>
</tr>
<tr>
<td>Topic</td>
<td>Four dice, chart the ways, toss and graph.</td>
</tr>
<tr>
<td>Topic</td>
<td>One die, chart the ways.</td>
</tr>
</tbody>
</table>

**Lesson Seven**

<table>
<thead>
<tr>
<th>Purpose</th>
<th>Learn to apply the probability from Lessons Five and Six to graphs made or to be made.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Summary</td>
<td>Students review graphs in newspapers and old graphs they have made and engage in one-die/four-dice wondering.</td>
</tr>
<tr>
<td>Materials</td>
<td>Past and future graphs.</td>
</tr>
<tr>
<td>Topic</td>
<td>The focus of the lesson is on building a frame of reference for viewing past and future data more analytically.</td>
</tr>
<tr>
<td>Homework</td>
<td>Look for graphs at home to think about and analyze.</td>
</tr>
</tbody>
</table>

*Measurement, Estimation and Time—Chapter 9*

**Lesson One**

<table>
<thead>
<tr>
<th>Purpose</th>
<th>Learn that measurement is a part of everything we do.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Summary</td>
<td>We create a measuring environment in our room by making measurements a tool for finding out.</td>
</tr>
<tr>
<td>Materials</td>
<td>Depends on the environment that exists in our room or that we create. Anything from rice tables and water play, to the questions that we ask the and materials available to support the opportunities that arise, to paper for “Is taller than...”.</td>
</tr>
<tr>
<td>Topic</td>
<td>Measurement is in the environment we create.</td>
</tr>
<tr>
<td>Topic</td>
<td>Measurement is in the questions that we ask.</td>
</tr>
<tr>
<td>Topic</td>
<td>Measurement is in “Is taller than...”.</td>
</tr>
<tr>
<td>Homework</td>
<td>We provide parents with examples of the kinds of measuring questions they can ask at home.</td>
</tr>
</tbody>
</table>

**Lesson Two**

<table>
<thead>
<tr>
<th>Purpose</th>
<th>Learn to make and read maps.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Summary</td>
<td>Students make maps and use and critique maps made by others to refine their own map making techniques.</td>
</tr>
<tr>
<td>Materials</td>
<td>Geoboards, geoboard recording paper blacklines, wooden cubes, graph paper, paper of all kinds.</td>
</tr>
<tr>
<td>Topic</td>
<td>Record geoboard shapes on paper.</td>
</tr>
<tr>
<td>Topic</td>
<td>Build and describe with cubes on graph paper.</td>
</tr>
</tbody>
</table>
Topic
Repeatedly making and critiquing maps to get from here to there.

Purpose
Learn to make and use balances.

Lesson Three

Summary
Students learn to make and use different kinds of balances for weighing. Each balance is explored for longer than a day.

Materials
Materials for making three different kinds of balances. Pegboard strips, washer and centimeter cube weights, fulcrums, balance stands, paper bowls and string, Power Blocks, plastic bag weights.

Topic
Board and fulcrum balances.

Lesson Four

Topic
Bowl balances.

Materials
Digital clock, chalkboard, analog clock, "hand" clock, Unifix Cubes, timer, assorted time-measuring devices.

Topic
Double-arm balances.

Homework
We provide parents with examples of talking time at home.

Lesson Five

Materials
Student-made balances.

Topic
Student-made balances.

Homework
Weighing skills developed are used.

Beginning Multiplication and Division—Chapter 10

Lesson One

Materials
Unifix Cubes, Power Block S-1 squares, bowl balances and weights.

Topic
Making and recording rectangles with squares.

Homework
The search for patterns on the multiplication matrix can be shared at home.
Lesson Two

Purpose
Learn to create multiplication and division problems, with and without remainders.

Summary
Students create and solve problems in a sideways L and answer three basic questions that we ask.

Materials
Squares, sideways L, chalkboards, paper.

Topic
Create problems, ignore remainder. Create problems for each other. Create problems, remainder not ignored. Remainder recorded as a fraction.

Topic
Create times problems with hands full of squares, remainders are ignored.

Topic
Answer three questions for the sideways L, no remainders yet.

Topic
Remainders added in.

Topic
Three questions asked again.

Topic
Remainders recorded as fractions.

Homework
Once the process is understood, our students create multiplication problems at home.

Lesson Three

Purpose
Learn to look for non numeric patterns. Connect the non numeric patterns to numeric patterns seen before.

Summary
Students create patterns with Unifix Cubes, break the patterns apart and study the "break-aparts" for patterns. Students connect the cube patterns to patterns in multiplication and on the 0-99 chart.

Materials
Unifix Cubes, Crayons, blacklines for recording break-aparts, 0-99 and multiplication matrices.

Topic
Break-apart pattern searches.

Topic
Look again at the matrices on the wall for patterns.

Lesson Four

Purpose
Learn the multiplication number facts.

Summary
Students use flash cards to commit number facts to memory and examine a matrix for patterns to reduce the number of facts to be learned from 100 to 36.

Materials
Blackline for flash cards, shaded 10 x 10 multiplication matrix.

Topic
Flash cards are used to put the multiplication facts in each student's head.

Homework
Flash cards are sent home.

Lesson Five

Purpose
Learn to think about what the numbers in multiplication and division problems represent.

Summary
We create multiplication and division problems that our students solve, as they identify what the numbers in their answers represent.

Materials
Students and objects in our room, chalkboards, blackline for a 12 x 12 blank matrix (optional).

Topic
Multiplication word problems created with people and objects in the room.

Topic
The problems can be recorded in matrix form.

Topic
Division word problems created with people and objects in the room.

Topic
The problems can be recorded in matrix form.

Homework
The process of creation can go home.

Lesson Six

Purpose
Learn to create stories for multiplication and division problems. Learn to see the stories in numbers everywhere.

Summary
Students write or draw stories for numbers we provide. Students look for number stories in their own lives.

Materials
Writing and drawing paper.

Topic
Students write stories or draw illustrations to accompany multiplication and division problems.

Topic
Selected stories written one day are read as creative inspiration the next.

Topic
Students write stories or draw illustrations to accompany multiplication and division problems that they provide.

Topic
If we choose to, we select some student stories as problems for the class to solve.

Homework
Numbers found at home can be brought to school to share.
Lesson Seven

Purpose
Learn that problems to be solved are everywhere around.

Summary
Students seek the multiplication and division problems that already exist in their lives.

Materials
Everything around.

Topic
Students describe number situations that exist. Finding the answers to the situations is not required yet.

Topic
Selected problems from the situations found are solved by the class.

Homework
The search for number situations extends to home.

Fractions, Ratios, Money, Decimals and Percent—Chapter 11

Lesson One

Purpose
Learn about fractions informally. Learn the words to say and the numbers to write.

Summary
Students explore Power Blocks, guided by the questions that we ask. We teach words and numbers that describe the fractions.

Materials
Power Blocks, recording paper, blackline matrix.

Topic
Freely exploring, with learning directed through questions asked.

Topic
Which pieces make into other pieces?

Topic
S-1 = 1, T-1 = 1.

Topic
S-5 = 1.

Topic
Matrix for everything = 1.

Topic
Learning the words to say for fractions.

Lesson Two

Purpose
Learn to find areas of shapes on geoboards. Learn to prove the areas of found shapes.

Summary
Students make shapes on their geoboards and learn specific techniques for proving areas. Areas of triangles are searched specifically for patterns.

Materials
Geoboards, geoboard recording paper; paper triangles and squares or Power Blocks for proofs.

Topic
Make shapes with areas of 2 1/2 and prove areas.

Topic
Make shapes with areas of 3 1/2 and prove areas.

Topic
Make right triangles and prove areas.

Topic
Find areas for and search for patterns in triangles with bases on the bottom row.

Homework
If geoboards can be sent home, finding areas of shapes is continued there.

Lesson Three

Purpose
Learn that fractions are special numbers describing part/whole relationships. Learn to add and subtract simple fractions.

Summary
Students learn to use people in the room to create simple fractions, then addition and subtraction problems. They also learn to create stories to accompany fractional numbers.

Materials
Students in the room, chalkboards, paper.

Topic
Fractions are created with people in the class.

Topic
Students create their own addition problems.

Topic
Students create their own subtraction problems.

Homework
What problems can our students create with the people in their home?

Lesson Four

Purpose
Learn to be aware of fractions in life.

Summary
We ask our students to think about the sharing, cutting and dividing fractions in their lives.

Materials
None.

Topic
Teacher lead discussion on the sharing, cutting and dividing that lead to fractions in our lives.

Homework
Students bring examples from home to school to share.

Lesson Five
Lesson Six

Purpose Learn to use equivalencies to find factors.
Summary Students use Unifix Cube sticks to generate lists of equivalencies which, combined with Start with, go bys, they search for patterns to help them make sense of factors.
Materials Unifix Cubes, butcher paper, Start with, go by chart, Power Blocks.
Topic Unifix equivalencies are recorded and explored.
Topic Finding common denominators to use to add unlike fractions. Specific techniques taught depend on the patterns that the students see and the sense they make out of their equivalency experiences.
Homework Estimating can be done at home.

Lesson Seven

Purpose Learn to see fractions as a part of measurement. Learn to estimate fractions of a length.
Summary Students use strips of paper to estimate, then measure. We teach techniques for determining fractional lengths.
Materials 18” strips of paper, objects in the room.
Topic Estimate lengths. Calculate the fractions involved.
Topic Estimating and calculating techniques are improved with practice and with time.
Homework Estimating can be done at home.

Lesson Eight

Purpose Learn what a ratio is. Learn to see practical applications of ratios and equivalencies.
Summary We use opportunities that exist or that we create to give our students practical experiences in finding ratios.
Materials Calculators, bouncing balls, shadows.
Topic Shadow questions.
Topic Shadow ratios.
Topic Bouncing balls.
Topic Furthest jumps.
Topic Diagonals of rectangles.
Topic Other opportunities that arise.
Homework Ratios explored at school can be explored at home.

Lesson Nine

Purpose Learn what rates and rate tables are for.
Summary Students search for examples of rates, learn to make rate tables and learn to use rate tables to answer questions.
Materials Local newspapers, calculators.
Topic Search the newspaper for rate examples.
Topic Create rate tables for gas consumed and cost.
Topic Create rate tables of various kinds.
Homework Gather rate table data from home.

Lesson Ten

Purpose Learn what kind of fraction decimals are.
Summary Students explore base-ten blocks, geoboards, graph paper rectangles and calculators as they learn about decimal fractions. This lesson is presented after Advanced Multiplication and Division.
Materials Base-ten blocks, calculators, geoboards, 10 X 10 and 100 X 100 blacklines, paper, chalkboards.
Topic One is what we say it is for base-ten blocks.
Topic: Think about where the decimal goes in answers to addition and subtraction problems.
Topic: Calculator generated lists of equivalencies.
Topic: Search for patterns for where the decimal goes when multiplying decimals.
Topic: Demonstrate why the fraction becomes smaller when we multiply.
Topic: 10 X 10 paper for multiplying decimals.

Lesson Eleven
Purpose: Learn the difference between decimals and percents. Learn to understand the questions that percent can represent.
Summary: We talk about the meaning of percent. We give our students practice using percent.
Materials: Calculators.
Topic: As we talk about percent with our students, what our students say guides what we do next.
Homework: What percent examples can be brought from home?

Lesson Twelve
Purpose: Learn about money from using it.
Summary: We use real money and real money situations to teach our students how to find real money answers.
Materials: Money.
Topic: Milk money, restaurant menus, classroom store, fundraising events: any opportunities that arise provide the framework for the money lessons that we teach.
Homework: Money is taught best at home.

Advanced Addition and Subtraction—Chapter 12

Lesson One
Purpose: Learn to search for patterns in bases other than ten.
Summary: Student record and examine plus one and minus one patterns in different bases.
Materials: Trading boards, squares, cups, bowls, blacklines for recording strips.
Topic: Squares and cups: base 4, base 5, base 6, if needed base 3. Then squares, cups and bowls as the base cycle is repeated. Then Base ten.
Topic: + and - 1 with squares and cups, base 4.
Topic: + and - 1 with squares and cups, base 5.
Topic: + and - 1 with squares and cups, base 6.
Topic: + and - 1 with squares and cups, base 3.
Topic: + and - with squares, cups and bowls, base 4.
Topic: + and - with squares, cups and bowls, base 5.
Topic: + and - with squares, cups and bowls, base 3.
Topic: + and - with squares, cups and bowls, base ten.
Homework: If students need more practice, cups and bowls can be sent home, but only with children who understand the basic process.

Lesson Two
Purpose: Learn about adding or subtracting numbers greater than one in different bases.
Summary: Students play racing-up and racing-back place-value trading games.
Materials: Trading boards, squares, cups, bowls, dice.
Topic: Racing up in base 4, winner is the first up.
Topic: Racing up in base 5, winner is the last up.
Topic: Racing up in base 6, winner is the first up.
Topic: Racing up in base 3, winner is the last up.
Topic: Racing up in base ten, winner is either the first or last up, decided in advance.
Topic: Racing back in base 4, winner is the first back.
Topic: Racing back in base 5, winner is the last back.
Topic: Racing back in base 6, winner is the first back.
Topic: Racing back in base 3, winner is the last back.
Topic: Racing back in base ten, winner is either the first or last up, decided in advance.

Lesson Three
Purpose: Learn the concept of place value.
Summary: Students learn a game that teaches them the value places have.
Materials: Trading boards, squares, cups, bowls, recording paper, dice.
Topic: Place value game in base 4, most wins.
Topic: Place value game in base 5, least wins.
Topic: Place value game in base 6, most wins.
Topic: Place value game in base 3, least wins.
Topic: Place value game in base ten, either most or least wins, decided in advance.

**Lesson Four**

Purpose: Learn to create addition and subtraction problems in any base three through ten.
Summary: Students learn how to create addition and subtraction problems and how to find and check the answers to the problems they create.
Materials: Trading boards, squares, cups, bowls, addition and subtraction recording paper blackline, dice, calculators.
Topic: Addition problems are created in base 4.
Topic: Addition problems are created in base 5.
Topic: Addition problems are created in any base 3 to 6.
Topic: Subtraction problems are created in base 4.
Topic: Subtraction problems are created in base 5.
Topic: Subtraction problems are created in any base 3 to 6.
Topic: Addition problems are created in base ten.
Topic: Subtraction problems are created in base ten.
Homework: If students need more practice, dice and cups and bowls can be sent home, but only with children who understand the basic process.

**Lesson Five**

Purpose: Learn to see place-value patterns within and between bases. Learn to use materials to prove answers.
Summary: Students record values for different bases on a single matrix and examine the matrix for patterns. Then students use the multibase blocks (if available) to create, solve, and then check problems.
Materials: Squares, cups, bowls, big bowls, blank matrix, calculators, multibase blocks (if available).
Topic: Looking for patterns in 1000, 100, 10, 1.
Topic: Addition problems using multibase blocks.
Topic: Subtraction problems using multibase blocks.

**Lesson Six**

Purpose: Learn the names for larger numbers.
Summary: Students learn to read large numbers.
Materials: None.
Topic: Learn to read large numbers.

**Lesson Seven**

Purpose: Learn that addition and subtraction are tools for finding out. Learn to apply math skills to problems in real life.
Summary: We work with our students to find and then solve real problems that use the skills that our students possess. Students keep a written record of their work.
Materials: Paper.
Topic: Problems drawn from stories.
Topic: Problems drawn from questions.
Topic: Problems drawn from life.
Homework: We send a list of questions home for parents to contemplate with their children.

**Lesson Eight**

Purpose: Learn to look for patterns in numbers everywhere.
Summary: Students look for patterns between bases, in palindromes and in ordinary events.
Materials        Palindrome blackline.  Plus-one strips from Lesson One.
Topic          Compare plus one strips from different bases.
Topic          Palindromes.
Topic          Examine situations for patterns.
Homework       Pattern situations are sought at home.

**Lesson Nine**

**Purpose**       Learn that creativity and inventiveness are problem-solving tools.
**Summary**       Our students use their inventiveness to solve problems that we give and share their individual or collective inventiveness with everyone in class.
**Materials**     None outside our heads.
**Topic**         Kids in class, kids in school.
**Topic**         Thinking and mental arithmetic.  Reasonableness.
**Homework**      We ask parents to do their mental arithmetic aloud.

**Advanced Multiplication and Division—Chapter 13**

**Lesson One**

**Purpose**       Learn what it means to multiply in any base.  Learn to search for multiplication patterns within and between bases.
**Summary**       Students learn to make multiplication matrices for bases three through ten as they search for patterns in each new matrix.
**Materials**     Squares, blacklines for 11 X 11 blank matrices.
**Topic**         Base-three matrix made by the full class.  Pairs of students work in pairs to make matrices for bases four and five.  For base six, students look at the previous bases for patterns that they can anticipate will appear.  The process is repeated for bases seven, eight and nine.  How much of base ten can be filled in from all the patterns seen in three through nine?
**Topic**         Base-four matrix started by the full class, finished working in pairs.
**Topic**         Base-five matrix in pairs, information shared collectively.
**Topic**         Base-six matrix made by looking at bases three, four and five.
**Topic**         Bases seven, eight and nine and, for some, sixteen.
**Topic**         Patterns is bases three through nine are used to fill in a matrix for base ten.

**Lesson Two**

**Purpose**       Learn how to represent concretely a multiplication problem larger than a matrix shows.  Learn how to multiply in any base.
**Summary**       We give our students larger multiplication problems to solve.  We help our students apply what they already know to the new problems that they face.  Once they understand the process, they use dice to create problems of their own to solve.
**Materials**     Chalkboards, squares, cups, bowls, trading boards, dice, multibase blocks (if available).
**Topic**         Multiplication problems in base five.
**Topic**         If available, multibase blocks and multiplication problems in different bases.

**Lesson Three**

**Purpose**       Learn techniques for finding answers to the multiplication problems likely to be on the end-of-year standardized test.
**Summary**       If our students will not be permitted to use calculators or materials on their year-end test, we teach them how to calculate answers for the test.
**Materials**     Paper.
**Topic**         An algorithm for multiplication is taught.

**Lesson Four**

**Purpose**       Learn to look for patterns in multiplying numbers with zeros at the end.  Learn how to find reasonable answers for multiplication problems that are large.
**Summary**       Students use their calculators to fill in worksheets that they then examine for patterns for multiplying numbers with zeros on the right.  Students use the patterns to learn to estimate reasonable answers for large problems.
Materials: Blackline worksheet for 1, 10, 100, 1000; chalkboards; calculators.
Topic: Multiplying by 2, 20, 200, 2000 and so on. Patterns sought.
Topic: Reasonableness for answers to large problems.

Lesson Five

Purpose: Learn to apply multiplication skills.
Summary: With our students, we look for real problems to solve that use multiplication.
Materials: Writing paper.
Topic: A problem a day is enough to pose. A problem is posed, small groups of students discuss and write down ways it might be solved, then students share their ways with the class.
Topic: Problem posed, discussed, solved, solutions shared.
Topic: Another problem is posed.
Homework: Our students search at home for multiplication problems that occur naturally. Parents join in the search.

Lesson Six

Purpose: Learn what it means to divide in any base.
Summary: We teach dividing techniques. Our students then use dice to create division problems for themselves.
Materials: Squares, cups, bowls, trading boards, dice, calculators.
Topic: Divide in base five, then base four and base six.
Topic: If available, divide with multibase blocks in different bases.

Lesson Seven

Purpose: Learn to use the arithmetic skills we have.
Summary: We look for real problems that use division and any other arithmetic skill. Our students look, as well, and keep a written record of the problem solving steps they use.
Materials: The materials relate to the problems found.
Topic: A single sandwich.
Topic: The daily life of a child.
Topic: Analysis.
Topic: Averages of all kinds.
Homework: Our students search at home for division problems that occur naturally. Parents join in the search.

Algebra—Chapter 14

Lesson One

Purpose: Learn to write symbolic equations or formulas for familiar patterns or experiences.
Summary: Students look at geoboard, Power Block, and wooden cube patterns and use letters to record the patterns seen.
Materials: Power Blocks, geoboards, wooden cubes.
Topic: Squares and rectangles made with S-1 squares, lengths, widths and areas recorded and searched for patterns.
Topic: Boxes made with wooden cubes, with the formula for volume sought.
Topic: Geoboard formulas for area and Pick’s theorem.
Topic: Algebraic relationships between the Power Blocks.
Homework: Pattern searches leading to Pick’s Theorem are continued at home for everyone to share.

Lesson Two

Purpose: Learn to plot coordinate points on a graph.
Summary: We play coordinate tic-tac-toe with our students as a class. Students play the game of Battleships with themselves.
Materials: Graph paper.
Topic: Students play coordinate tic-tac-toe as a class.
Topic: Students play Battleships in small groups.
Homework Students may continue playing Battleships at home.

Lesson Three

Purpose Learn a framework for understanding addition, subtraction and multiplication of signed numbers.
Summary We present Letter Carrier stories to our students to teach them rules for arithmetic operations with positive and negative numbers.
Materials Chalk boards.
Topic Letter Carrier stories for + and —.
Topic Letter Carrier stories for x.
Topic Students create stories for numbers we provide.

Lesson Four

Purpose Learn to write tables for pairs of numbers that are related in a patterned way. Learn to write equations or formulas for the patterns.
Summary Students use a "Magic Box" function machine to predict from numbers going in, the numbers coming out.
Materials "Magic Box" function machine, teacher-made and student-made "magic" cards.
Topic Teacher creates the rules for the numbers going in and coming out.
Topic Students create the rules for the numbers going in and coming out.
Topic Write formulas for the rules.

Lesson Five

Purpose Learn to graph functions and the equations they represent.
Summary Students plot the data from the tables in Lesson Four and other functional relationships on coordinate graphs and write equations to accompany their graphs.
Materials Graph paper, circular objects, toothpicks.
Topic \( x = y = 10 \).
Topic Tables from Lesson Four.
Topic Graphing the area formula, \( a = lw \).
Topic Graphing circumference, \( c = \pi d \).
Topic Graphing multiplication facts, \( y = 2x, y = 3x \), etc.
Topic Graphing toothpick patterns.

Lesson Six

Purpose Learn to recognize the slope and intercept of an equation to be graphed.
Summary Students graph equations in the \( y = mx \) or \( y = mx + b \) formats and look for slope and intercept patterns in the resulting lines.
Materials Graph paper.
Topic \( y = mx \).
Topic \( y = mx + b \).

Lesson Seven

Purpose Learn to recognize the effect that exponents have on a graph.
Summary Students graph equations in the \( y = mx^2 + b \) or \( y = mx^3 + b \) formats and look for patterns in the resulting lines.
Materials Graph paper blackline—squares small enough to permit graphing of \( x^2 \) and \( x^3 \).
Topic \( y = mx^2 + b \).
Topic \( y = mx^3 + b \).

Lesson Eight

Purpose Learn to multiply \((x + y)(x + y)\) equations. Learn to connect the multiplying to concepts already understood.
Summary We show our students how to use drawings to solve equations like \((x + y)(x + y)\). We connect the algebra to arithmetic.
Materials Graph paper.
Topic \((x + y)(x + y)\).
Lesson Nine

Purpose  Learn that the ratios, rates, proportions and equivalencies from the fractions chapter are functional relationships.
Summary  Our students graph data they developed in their fraction lessons and use their graphs to create algebraic formulas.
Materials  Tables of data from earlier mathematical experiences.
Topic  Ratios.
Topic  Rates.
Topic  Proportions.
Topic  Equivalencies.

Lesson Ten

Purpose  Learn to use algebra. Learn to connect algebra in school to algebra in real life.
Summary  We create an algebra environment in our room by making algebra a tool for finding out.
We use opportunities already present in our student’s lives.
Materials  Materials in our room and/or materials we find outside of class.
Topic  Algebra is in the environment we create.
Topic  Algebra is in the questions that we ask.
Homework  The work that we send home depends upon the work we do in class.