Lessons Per Chapter

Free Exploration and Creative Learning—Chapter 2

Lesson One

Purpose	Learn the rules, establish the environment.
Summary	Students experience new materials on their own. Our questions guide their thinking.
Materials	Pattern Blocks, Power Blocks, Geoblocks, Unifix Cubes, toothpicks, or any other available materials.
Topic	Pattern Blocks.
Topic	Power Blocks.
Topic	Other materials that we have.

Patterns and Connections—Chapter 3

Lesson One

Purpose	Learn what is meant by "pattern." Learn the A-B system of describing patterns.
Summary	The whole class invents patterns, then smaller groups devise ways to share. Pattern searches extend to the environment.
Materials	Students in the room. Patterns in the world.
Topic	A-A-B.
Topic	A-A-A-B.
Topic	A-B, A-B.
Topic	A-B-C and more.
Topic	Five minutes now and then.
Homework	We send the search for patterns home.

Lesson Two

Purpose	Learn to relate A-B patterns to materials.
Summary	Students create and share A-B patterns. Students also learn to check their neighbors to ensure that everybody understands.
Materials	Pattern Blocks, Power Blocks and any other material we might select.
Topic	Pattern Blocks and A-A-B.
Topic	Power Blocks and A-A-B.
Topic	Pattern Blocks and A-A-A-B.
Topic	Power Blocks and A-A-A-B.
Topic	Other materials. and A-B patterns.
Homework	We encourage our students to make and share A-B patterns with materials available at home.

Lesson Three

Purpose	Learn how to record patterns.
Summary	Students record patterns and use the recordings of others to reproduce and extend the patterns.

Materials	Pattern Blocks and Pattern Block shapes or stencils; Power Blocks.
Topic	Pattern Block A-B patterns recorded.
Topic	Pattern Block A-B patterns copied and extended.
Topic	Power Block A-B patterns recorded.
Topic	Power Block A-B patterns copied and extended.
Homework	What materials for recording do our students have at home? What materials might we send?

Lesson Four

Purpose	Learn to look for patterns in numbers.
Summary	Students look at number charts for patterns and describe the patterns they see.
Materials	Blacklines for assorted number charts and the number charts themselves.
Topic	00-99 matrix.
Topic	1 -100 number strip.
Topic	10 X 10 multiplication matrix.
Topic	25 X 25 multiplication matrix.
Topic	Pascal's triangle.
Topic	12 month calendar, with all months visible.
Homework	We send number charts home for viewing, too.

Lesson Five

Purpose	Learn to extend pattern searches beyond the period set aside for math. To connect mathematics to art.
Summary	Students create pattern designs for themselves.
Materials	Art supplies of various kinds. Drawing paper, graph paper, crayons, colored pencils and so on.
Topic	Pattern Block walls.
Topic	Pattern Block mosaic designs.
Topic	String designs.
Topic	Graph paper patterns.
Topic	Other patterned art that we might choose to use.

Beginning Number—Chapter 4

Lesson One

Purpose	Learn to count up and back by ones.
Summary	We establish a counting environment.
Materials	Anything available to count.
Topic	Counting up.
Topic	Counting back.
Topic	Counting every time there are things to count.
Homework	We encourage parents to count with their children at home.

Lesson Two

Purpose	Learn to look for patterns in the counting numbers.
Summary	We post numbers for students to search for patterns.
Materials	0-100 column of numbers.
Topic	Search the numbers from 0 to 100 for patterns.
Topic	Learn to say the names of numbers.
Topic	Look at the 00-99 matrix once again.
Homework	We send numbers home so the search for patterns may be shared.

Lesson Three

Purpose	Learn the difference between 1, 2, 3 and 1st, 2nd, 3rd.
Summary	We use numbers in language to convey meaning.
Materials	The language we use.
Topic	Language use is not lesson bound. The use of cardinal and ordinal numbers is done consciously everyday.
Homework	We send parents a list of words to emphasize at home.

Lesson Four

Purpose	Learn the fiveness of five.
Summary	We surround our students with the concept of numbers from three to as far as we decide to go.
Materials	Power Block squares or tiles, toothpicks, wooden cubes, Pattern Blocks.
Topic	3 with squares.
Topic	3 with tooth picks.
Topic	3 with wooden cubes.
Topic	3 with Pattern Blocks.
Topic	4 with squares.
Topic	4 with tooth picks.
Topic	4 with wooden cubes.
Topic	4 with Pattern Blocks.
Topic	5 and more with each material, in turn.
Homework	We send the number stations home to be done with any material present in the house.

Lesson Five

Purpose	Record the number concepts learned in Lesson Four.
Summary	Students learn a different way to record each material. The recordings have a use in Lesson Six.
Materials	Power Block squares or tiles, toothpicks, wooden cubes, Pattern Blocks, a different kind of recording paper for each material, paste or glue. Blackline masters for recording cube shapes.
Topic	The lesson focus is on recording.
Topic	Recording 3 with squares.
Topic	Recording 3 with tooth picks.
Topic	Recording 3 with Pattern Blocks.
Topic	Recording 3 with wooden cubes (upper grades).
Topic	Recording 4 with squares.
Topic	Recording 4 with tooth picks.
Topic	Recording 4 with Pattern Blocks.
Topic	Recording 4 with wooden cubes (upper grades).
Topic	Recording 5 and more with each material, in turn.
Homework	Homework for Lesson Five waits for Lesson Six.

Lesson Six

Purpose	Record in words and numbers while learning to envision what the words and numbers mean.
Summary	Students imagine then write words and numbers to describe numeric designs.
Materials	Recordings from Lesson Five, paper for writing words, separate paper for writing numbers. Number lines. Number line templates. Spelling Notebook introduced.
Topic	Writing words for recordings of 3 squares.
Topic	Writing numbers for recordings of 3 squares.
Topic	Creating square designs for numbers.
Topic	Writing words for recordings of 3 toothpicks.
Topic	Writing numbers for recordings of 3 toothpicks.
Topic	Creating toothpick designs for numbers.
Topic	Writing words for recordings of 3 Pattern Blocks.
Topic	Writing numbers for recordings of 3 Pattern Blocks.
Topic	Creating Pattern Block designs for numbers.
Topic	Repeating the three step writing and creating cycle for the numbers 4 and 5 and maybe 6.
Homework	We send the word and number books home to be read by our students to any audience that can be found.

Lesson Seven

Purpose	Learn the families of addition facts.
Summary	Students look for ways to make number combinations with Unifix Cubes using two different sets of rules.

Materials	Unifix Cubes, A-B cube paper.
Topic	The first set of rules leads to flash cards that go home.
Topic	Creating number combinations with Unifix Cubes and Rule One.
Topic	Creating flash cards to accompany Rule One.
Topic	Explore number patterns with Unifix Cubes and Rule Two.
Homework	We send the flash cards home.

Lesson Eight

Purpose	Learn to link number to area. Learn to prove answers found.
Summary	Students prove Power Block areas before creating shapes on their geoboards and proving areas of their created shapes. Adds to the wooden cube experiences from Lessons Four and Five. Provides more beginning number experiences for the older child.
Materials	Power Blocks, Geoboards and Blacklines for three levels of recording sheets. (One, four, twelve.)
Topic	Power Block S-1 square has an area of one, what are the areas of all the other shapes?
Topic	Geoboards—ways to make 2, without then with recording.
Topic	Reviewing other people's "2's".
Topic	Ways to make 3.
Topic	Ways to make 4 or more, while proving areas found.
Homework	If geoboards can be sent home, explorations are continued there.

Sorting, Classifying, Expanding Language—Chapter 5

Lesson One

Purpose	Learn to sort by attributes.
Summary	Students sort objects into groups in a variety of ways. Teacher or students list the ways.
Materials	Buttons, keys, or any other common objects available in quantity; individual chalkboards.
Topic	Each new material produces a variation of the basic lesson. Lists created by older students are used again in Lesson Seven.
Topic	Sorting buttons, making lists.
Topic	Sorting keys, making lists.
Topic	Sorting whatever else is available in quantity.
Homework	What groups of things do children have at home that they might sort?

Lesson Two

Purpose	Learn to be aware of attributes everywhere.
Summary	Students take sorting walks to learn to see what is already there.
Materials	The world outside our class.
Topic	Sorting walks. Older students keep written lists of what is seen. Younger students may draw instead of write.
Topic	Each walk taken is a different element.
Homework	Sorting walks can take place while coming to or going home from school each day.

Lesson Three

Purpose	Use sorting and classification knowledge to create informal definitions of words.
Summary	We sort shapes, words, or objects as students create definitions for the sorts.
Materials	Whatever relates to the definition we wish to teach.
Topic	We use this lesson when we have a definition we wish to teach.
Topic	Shapes—quadrilaterals, triangles, squares, etc.
Topic	Nouns, verbs, prepositions, adjectives, etc.
Topic	What other definitions might we choose to use?

Lesson Four

Purpose	Learn to find relationships between different shapes.
Summary	Students play games with Attribute Blocks that focus on thinking logically and systematically.

Materials	Attribute Blocks, overhead set, blackline recording sheet.
Topic	Pattern sorts, teacher at the overhead.
Topic	Identify the missing piece.
Topic	Which piece does not belong?
Topic	Which piece is missing from the matrix?
Topic	Add a piece to the line that is one different than the piece before.
Topic	If one-different is understood, try two different, then three.

Lesson Five

Purpose	To learn to see the attributes in ourselves.
Summary	Students describe themselves using attributes that define them as unique. For students who can write, we read their descriptions aloud.
Materials	Students in class, writing paper.
Topic	Students descriptions of themselves are read aloud.
Topic	Each new set of descriptions is a separate element.

Lesson Six

Purpose	Learn the process of using individual attributes to categorize.
Summary	Students use their attributes to develop categories that uniquely describe everyone in class.
Materials	Students in class.
Topic	The teacher leads the class in sorting itself into successively smaller sub-categories.
Topic	New categories are used on subsequent days.

Lesson Seven

Purpose	Expand the process of learning to use individual attributes to categorize.
Summary	Students use lists of attributes developed in Lesson One to describe uniquely objects sorted earlier.
Materials	Objects used and lists created in Lesson One: sorting tree blackline.
Topic	Buttons on a sorting tree.
Topic	Buttons sorted differently.
Topic	Keys on a sorting tree.
Topic	Keys sorted differently.
Topic	Whatever other materials we have.

Lesson Eight

Purpose	Learn the meaning of selected words.
Summary	In a lesson that takes five minutes now and then, we ask the class as a whole to demonstrate the meaning of selected words. The lesson can be extended with Attribute Blocks.
Materials	Students in class, Attribute Blocks.
Topic	Both-and.
Topic	If-then.
Topic	Either-or, neither-nor.
Topic	Other words we choose.
Topic	"Show me..." with Attribute Blocks.

Lesson Nine

Purpose	Learn how to communicate more effectively with words.
Summary	We and our students give instructions and discuss how we can make the instructions clearer.
Materials	Pattern Blocks, shield behind which to build, Power Blocks, Attribute Blocks, geoboards, wooden cubes, Unifix Cubes, or other materials with which to build.
Topic	At the youngest ages, the teacher is the only builder in the room.
Topic	Building and instructing with Pattern Blocks.
Topic	Building and instructing with Power Blocks.
Topic	Each new material is a topic.

Geometry, Shapes, Relationships and Constructions—Chapter 6

Lesson One

Purpose	Provide a background in geometry equally for boys and girls, rich and poor while exploring shapes in geometry.
Summary	Students build as our questions focus their discoveries.
Materials	Building materials off all kinds: Lego blocks, Tinker Toys, Geoblocks, Pattern Blocks, Power Blocks, straws, toothpicks and clay, kindergarten building blocks.
Topic	Today is building day, let's see what you can build.
Topic	Each material used for building is a topic.
Homework	We encourage parents to provide building materials at home for boys and girls alike.

Lesson Two

Purpose	Expand the exploration of shape.
Summary	Students explore the properties of shapes guided by the questions that we ask.
Materials	Power Blocks, Geoboards, recording paper.
Topic	Power Blocks, what shapes make other shapes?
Topic	Geoboards, make shapes with 3 sides. 4 sides. 5 sides. More sides.
Topic	Which shapes can be made with Power Blocks & duplicated on a geoboard? Which shapes cannot?
Homework	If geoboards can be sent home, explorations are continued there.

Lesson Three

Purpose	Learn to recognize reflective symmetry in shapes.
Summary	Students explore lines of symmetry with materials and mirrors.
Materials	Mirrors, hinged and regular, Pattern Blocks, Power Blocks, paper for tracing shapes, colored construction paper, lined paper.
Topic	Free exploration with mirrors.
Topic	Pattern Blocks and mirrors - exploring symmetry.
Topic	Power Blocks and mirrors - exploring symmetry.
Topic	Lines of symmetry in the room - make a list.
Topic	Free exploration with hinged mirrors.
Topic	Symmetry with Pattern Blocks and hinged mirrors.
Topic	Symmetry with Power Blocks and hinged mirrors.
Topic	Kaleidoscope - three mirror exploration.
Homework	The search for symmetry is sent home.

Lesson Four

Purpose	Learn which polygons tessellate the plane, while discovering more kinds of symmetry.
Summary	Students explore shapes that tessellate and shapes that do not. They create their own tessellating shapes and turn them into Escher-like designs.
Materials	Power Blocks, Pattern Blocks, tag board cutouts, or templates. Blackline for tessellating shapes with angles written in and the shapes labeled.
Topic	Which polygons tessellate the plane.
Topic	Patterns for the polygons that tessellate.
Topic	Cutting tessellating polygons Escher style.
Topic	The evolution is from math to art.
Homework	Escher cutouts made at school go home for further exploration.

Lesson Five

Purpose	Learn that math and art are not separate subjects.
Summary	We teach art as we always do. In Patterns & Connections, Lesson Five, we pointed out the patterns to be seen. We now point to the geometric connections to be made, as well.
Materials	Assorted art supplies.
Topic	Extending the tessellations from Lesson Four.
Topic	Name symmetry.
Topic	Snow flakes.
Topic	3-D straw constructions.
Topic	What math can we see in other art that is a part of the art we teach?

Homework	Children who have art materials at home can continue their art work there.

Lesson Six

Purpose	Learn to draw three-dimensional shapes.
Summary	Students learn to draw three-dimensional shapes so that the drawn shape identifies the shape.
Materials	Geoblocks or other 3-D shapes, drawing paper.
Topic	Draw one block. Others identify the block drawn.
Topic	Draw two blocks. Others identify the blocks drawn.
Topic	Draw three and more blocks. Others identify the blocks drawn.

Lesson Seven

Purpose	Learn what an angle is and how to measure it.
Summary	Students learn to use angles in giving instructions. They learn to measure angles and use a protractor as a measuring device.
Materials	Students in the class, paper, protractors, straws, sticks, calculators.
Topic	Each new question asked or material explored is like a lesson of its own.
Topic	Students direct each other using paces and turns.
Topic	Students find right angles in the room.
Topic	Angles are measured with straws and sticks, as lists of successively larger angles are made.
Topic	Protractors are explored.
Topic	360°

Lesson Eight

Purpose	Learn to use a compass, protractor and straightedge to explore geometric properties.
Summary	We ask questions and pose challenges for our students that guide their explorations with compass, protractor and straightedge.
Materials	Compass, protractor, straightedge, paper.
Topic	See what you can make.
Topic	Make two or more circles the same size.
Topic	Use a compass to compare the lengths of lines.
Topic	Make two angles that are the same size.
Topic	Copy an angle.
Topic	Divide a line in half.

Lesson Nine

Purpose	Learn to be aware of the geometry in our lives.
Summary	We ask our students to look more closely at what they already see.
Materials	Paper for writing lists.
Topic	Make a list of rectangular shapes. What other shapes can we list?
Topic	List the tessellations that you can find.
Topic	What shall we look for today? Why are the things that we see the shape that they are?
Topic	What angles, shapes, lines in a bicycle?
Homework	This is a lesson on looking and wondering. Looking and wondering questions can be sent home everyday.

Lesson Ten

Purpose	The purpose is a teacher purpose. Our assignment is to find the opportunities.
Summary	We make ourselves aware of the opportunities for geometric experiences that exist. We use the opportunities that we find.
Materials	Our own awareness.
Topic	Opportunities that we find.
Homework	What opportunities can our students find at home?

Beginning Addition and Subtraction—Chapter 7

Lesson One

Purpose	Learn to create and check addition problems.
Summary	Students create addition problems that they can check by counting.
Materials	Squares or tiles, chalkboards, calculators, paper.
Topic	Creating addition problems with handsful of squares.
Topic	Creating addition problems with handsful of squares and checking the answers with calculators.
Homework	Once the process is understood, our students create addition problems at home.

Lesson Two

Purpose	Learn to apply the skills of addition.
Summary	We give our students problem-solving questions and number patterns to explore.
Materials	Squares, Unifix Cubes, calculators, lined paper.
Topic	Start with, go bys.
Topic	Start with, go bys, both.
Topic	Problems in the middle of a stream.
Topic	Consecutive whole numbers.
Topic	Odd and even numbers.
Homework	What is understood may be continued at home.

Lesson Three

Purpose	Learn to create and check subtraction problems.
Summary	Students create subtraction problems that they can check by counting.
Materials	Squares or tiles, chalkboards, calculators, paper.
Topic	Creating subtraction problems with handsful of squares.
Topic	Creating subtraction problems with handsful of squares and checking the answers with calculators.
Homework	Once the process is understood, our students create subtraction problems at home.

Lesson Four

Purpose	Learn to apply skills of subtraction.
Summary	We give our students problem-solving questions and number patterns to explore.
Materials	Squares, calculators, lined paper.
Topic	Starting with 100.
Topic	Problems in a stream, a negative flow.
Homework	What is understood may be continued at home.

Lesson Five

Purpose	Learn the families of addition facts above ten.
Summary	Students practice number facts, through flash cards and number games. There are no tests of speed.
Materials	Blackline for flash cards; games like Blackjack, dominoes and shaker dice.
Topic	Flash cards.
Topic	Blackjack.
Topic	Dominoes.
Topic	Shaker dice.
Homework	Flash cards are sent home. Games to play at home are recommended, too.

Lesson Six

Purpose	Learn to solve word problems that the teacher creates.
Summary	Students learn to draw the necessary numbers from the stories that we tell.
Materials	Students and materials available in class, chalkboards.
Topic	Word problems for adding.
Topic	Word problems for adding, extraneous information added in.
Topic	Word problems for subtracting.
Topic	Word problems for subtracting, extraneous information added in.
Topic	Adding and subtracting mixed.

Lesson Seven

Purpose	Learn to create and solve story problems.
Summary	Students create their own stories to go along with numbers. First the teacher provides numbers, then numbers are taken from student lives.
Materials	Writing paper, drawing paper.
Topic	Students create addition stories to share.
Topic	Stories are shared before the next addition creations are produced.
Topic	Students create subtraction stories to share.
Topic	Stories are shared before the next subtraction creations are produced.
Topic	Addition and subtraction are mixed.
Homework	Look at home for number problems to bring to school.

Lesson Eight

Purpose	Learn to think about the reasonableness of answers.
Summary	Students learn to ask: "Does the answer I have found make sense?"
Materials	The materials used depend on the questions asked. Calculators.
Topic	How did you get your answer and how do you know it is reasonable?
Topic	We pose problems to help our students understand "reasonable".
Homework	Where might a use for "reasonableness" be found at home?

Lesson Nine

Purpose	Learn to connect school math to life.
Summary	We look for problems that exist around us for our students to solve.
Materials	Problems from ours and our students' lives.
Topic	Twenty problems or just one.
Homework	What problems can our students find at home?

Graphing, Probability and Statistics—Chapter 8

Lesson One

Purpose	Learn to use graphing as a tool for finding answers to questions.
Summary	Students learn to turn their curiosity into data to graph. Graphs made now will be used again in Lesson Four.
Materials	Materials depend on the questions students ask.
Topic	Students make graphs in response to questions asked or curiosity expressed that leads to numbers that can be represented pictorially.
Homework	Ideally, our students learn to use graphing as a tool for displaying information at or from home.

Lesson Two

Purpose	Learn how to display information in a variety of ways.
Summary	Students invent more ways to graph data than they had thought to use before.
Materials	Examples of a variety of graphs, materials in the room, and student creativity and inventiveness.
Topic	Examples of different kinds of graphs are shared as students think of ways to graph they have not used before.
Homework	The search for different kinds of graphs is continued at home.

Lesson Three

Purpose	Learn how to ask questions for a graph.
Summary	We assemble unseen graphs to guide students in learning how to ask what it is they want to know.
Materials	Shield for the graphs, cut-off milk carton boxes to create the hidden graphs.
Topic	A graph is assembled behind a shield as students ask questions about data that remains unseen.

Lesson Four

Purpose	Learn to ask questions for the graphs that students make and see.

Summary	Students learn to add written questions to their graphs. The lesson on asking questions is also a lesson on learning to speak math and learning to ask math questions.
Materials	Graphs from Lesson One, cut-off milk carton boxes.
Topic	Students add questions to graphs already made.
Homework	What math questions can our students bring from home?

Lesson Five

Purpose	Learn a beginning framework for connecting probability to graphs.
Summary	Students toss cardboard squares, graph the outcomes and predict what future outcomes might occur.
Materials	Cardboard squares and graph, lined, or plain paper.
Topic	One square toss and graph.
Topic	Two square toss and graph.
Topic	Three square toss and graph.
Topic	Four square toss and graph.
Topic	Five square toss and graph.

Lesson Six

Purpose	Learn a connection between ways possible and ways that actually occur.
Summary	Students roll dice, graph results, and learn to predict likely outcomes in advance.
Materials	Dice and graph, lined, or plain paper.
Topic	One die toss and graph.
Topic	Two dice, chart the ways, toss and graph.
Topic	Three dice, chart the ways, toss and graph.
Topic	Four dice, chart the ways, toss and graph.
Topic	One die, chart the ways.

Lesson Seven

Purpose	Learn to apply the probability from Lessons Five and Six to graphs made or to be made.
Summary	Students review graphs in newspapers and old graphs they have made and engage in one-die/four-dice wondering.
Materials	Past and future graphs.
Topic	The focus of the lesson is on building a frame of reference for viewing past and future data more analytically.
Homework	Look for graphs at home to think about and analyze.

Measurement, Estimation and Time—Chapter 9

Lesson One

Purpose	Learn that measurement is a part of everything we do.
Summary	We create a measuring environment in our room by making measurements a tool for finding out.
Materials	Depends on the environment that exists in our room or that we create. Anything from rice tables and water play, to the questions that we ask the and materials available to support the opportunities that arise, to paper for "Is taller than...".
Topic	Measurement is in the environment we create.
Topic	Measurement is in the questions that we ask.
Topic	Measurement is in "Is taller than...".
Homework	We provide parents with examples of the kinds of measuring questions they can ask at home.

Lesson Two

Purpose	Learn to make and read maps.
Summary	Students make maps and use and critique maps made by others to refine their own map making techniques.
Materials	Geoboards, geoboard recording paper blacklines, wooden cubes, graph paper, paper of all kinds.
Topic	Record geoboard shapes on paper.
Topic	Build and describe with cubes on graph paper.

Topic	Repeatedly making and critiquing maps to get from here to there.
Topic	Drawing and critiquing maps of class.
Homework	Students make maps of home and of routes to get from here to there.

Lesson Three

Purpose	Learn to make and use balances.
Summary	Students learn to make and use different kinds of balances for weighing. Each balance is explored for longer than a day.
Materials	Materials for making three different kinds of balances. Pegboard strips, washer and centimeter cube weights, fulcrums, balance stands, paper bowls and string, Power Blocks, plastic bag weights.
Topic	Board and fulcrum balances.
Topic	Bowl balances.
Topic	Double-arm balances.
Topic	Student-made balances.
Topic	Weighing skills developed are used.
Homework	Balance explorations can be carried on at home.

Lesson Four

Purpose	Learn about time.
Summary	In general, we teach time buy using it. Specific time concepts can be conveyed with materials like Unifix Cubes.
Materials	Digital clock, chalkboard, analog clock, "hand" clock, Unifix Cubes, timer, assorted time-measuring devices.
Topic	Time is an experience. It is taught all day long, all the time.
Topic	Miles per hour represented with cubes.
Homework	We provide parents with examples of talking time at home.

Lesson Five

Purpose	Learn how to make good estimates.
Summary	Students estimate length, surface area, weight and volume while learning what estimation means, but we do not teach estimation and then move on. Estimation is a thinking skill our students use and use. We make estimation a part of all the lessons we teach by the questions that we ask.
Materials	Unifix Cubes, Power Block S-1 squares, bowl balances and weights.
Topic	How much?
Topic	How high?
Topic	How many?
Topic	How far?
Topic	How soon?
Topic	What else?
Homework	Measuring questions asked for school can be asked for home, as well.

Beginning Multiplication and Division—Chapter 10

Lesson One

Purpose	Learn what it means to multiply. Learn to search for patterns in multiplication arrays.
Summary	Students learn to create and record (individually and in matrices) multiplication problems. Matrices are searched for patterns.
Materials	Squares or tiles, blank 12x12 matrix blackline, Unifix Cubes, geoboards, multiplication wall matrix, lined paper.
Topic	Making and recording rectangles with squares.
Topic	Recording rectangle patterns on matrices.
Topic	Examining matrices for patterns.
Topic	Unifix matrices made and searched for patterns.
Topic	Geoboard matrices made and searched for patterns.
Topic	Add the matrix to the wall charts for pattern searches endlessly.
Topic	If needed, more matrices are made.
Homework	The search for patterns on the multiplication matrix can be shared at home.

Lesson Two

Purpose	Learn to create multiplication and division problems, with and without remainders.
Summary	Students create and solve problems in a sideways L and answer three basic questions that we ask.
Materials	Squares, sideways L, chalkboards, paper.
Topic	Create problems, ignore remainder. Create problems for each other. Create problems, remainder not ignored. Remainder recorded as a fraction.
Topic	Create times problems with hands full of squares, remainders are ignored.
Topic	Answer three questions for the sideways L, no remainders yet.
Topic	Remainders added in.
Topic	Three questions asked again.
Topic	Remainders recorded as fractions.
Homework	Once the process is understood, our students create multiplication problems at home.

Lesson Three

Purpose	Learn to look for non numeric patterns. Connect the non numeric patterns to numeric patterns seen before.
Summary	Students create patterns with Unifix Cubes, break the patterns apart and study the "break-aparts" for patterns. Students connect the cube patterns to patterns in multiplication and on the 0-99 chart.
Materials	Unifix Cubes, Crayons, blacklines for recording break-aparts, 0-99 and multiplication matrices.
Topic	Break-apart pattern searches.
Topic	Look again at the matrices on the wall for patterns.

Lesson Four

Purpose	Learn the multiplication number facts.
Summary	Students use flash cards to commit number facts to memory and examine a matrix for patterns to reduce the number of facts to be learned from 100 to 36.
Materials	Blackline for flash cards, shaded 10 x 10 multiplication matrix.
Topic	Flash cards are used to put the multiplication facts in each student's head.
Homework	Flash cards are sent home.

Lesson Five

Purpose	Learn to think about what the numbers in multiplication and division problems represent.
Summary	We create multiplication and division problems that our students solve, as they identify what the numbers in their answers represent.
Materials	Students and objects in our room, chalkboards, blackline for a 12 x 12 blank matrix (optional).
Topic	Multiplication word problems created with people and objects in the room.
Topic	The problems can be recorded in matrix form.
Topic	Division word problems created with people and objects in the room.
Topic	The problems can be recorded in matrix form.
Homework	The process of creation can go home.

Lesson Six

Purpose	Learn to create stories for multiplication and division problems. Learn to see the stories in numbers everywhere.
Summary	Students write or draw stories for numbers we provide. Students look for number stories in their own lives.
Materials	Writing and drawing paper.
Topic	Students write stories or draw illustrations to accompany multiplication and division problems.
Topic	Selected stories written one day are read as creative inspiration the next.
Topic	Students write stories or draw illustrations to accompany multiplication and division problems that they provide.
Topic	If we choose to, we select some student stories as problems for the class to solve.
Homework	Numbers found at home can be brought to school to share.

Lesson Seven

Purpose	Learn that problems to be solved are everywhere around.
Summary	Students seek the multiplication and division problems that already exist in their lives.
Materials	Everything around.
Topic	Students describe number situations that exist. Finding the answers to the situations is not required yet.
Topic	Selected problems from the situations found are solved by the class.
Homework	The search for number situations extends to home.

Fractions, Ratios, Money, Decimals and Percent—Chapter 11

Lesson One

Purpose	Learn about fractions informally. Learn the words to say and the numbers to write.
Summary	Students explore Power Blocks, guided by the questions that we ask. We teach words and numbers that describe the fractions.
Materials	Power Blocks, recording paper, blackline matrix.
Topic	Freely exploring, with learning directed through questions asked.
Topic	Which pieces make into other pieces?
Topic	S-1 = 1, T-1 = 1.
Topic	S-5 = 1.
Topic	Matrix for everything = 1.
Topic	Learning the words to say for fractions.

Lesson Two

Purpose	Learn to find areas of shapes on geoboards. Learn to prove the areas of found shapes.
Summary	Students make shapes on their geoboards and learn specific techniques for proving areas. Areas of triangles are searched specifically for patterns.
Materials	Geoboards, geoboard recording paper; paper triangles and squares or Power Blocks for proofs.
Topic	Make shapes with areas of 2 1/2 and prove areas.
Topic	Make shapes with areas of 3 1/2 and prove areas.
Topic	Make shapes with areas of any size and prove areas.
Topic	Make right triangles and prove areas.
Topic	Find areas for and search for patterns in triangles with bases on the bottom row.
Homework	If geoboards can be sent home, finding areas of shapes is continued there.

Lesson Three

Purpose	Learn that fractions are special numbers describing part/whole relationships. Learn to add and subtract simple fractions.
Summary	Students learn to use people in the room to create simple fractions, then addition and subtraction problems. They also learn to create stories to accompany fractional numbers.
Materials	Students in the room, chalkboards, paper.
Topic	Fractions are created with people in the class.
Topic	Students create their own addition problems.
Topic	Students create their own subtraction problems.
Homework	What problems can our students create with the people in their home?

Lesson Four

Purpose	Learn to be aware of fractions in life.
Summary	We ask our students to think about the sharing, cutting and dividing fractions in their lives.
Materials	None.
Topic	Teacher lead discussion on the sharing, cutting and dividing that lead to fractions in our lives.
Homework	Students bring examples from home to school to share.

Lesson Five

Purpose	Learn about equivalencies.
Summary	Students use paper folding and Power Blocks to generate lists of equivalencies, which they then search for patterns.
Materials	Paper, Power Blocks, chalkboards.
Topic	Folding paper, recording the equivalencies formed.
Topic	Finding and recording Power Block equivalencies.
Homework	Paper folding can be shared at home.

Lesson Six

Purpose	Learn to use equivalencies to find factors.
Summary	Students use Unifix Cube sticks to generate lists of equivalencies which, combined with Start with, go bys, they search for patterns to help them make sense of factors.
Materials	Unifix Cubes, butcher paper, Start with, go by chart, Power Blocks.
Topic	Unifix equivalencies are recorded and explored.
Topic	Finding common denominators to use to add unlike fractions. Specific techniques taught depend on the patterns that the students see and the sense they make out of their equivalency experiences.

Lesson Seven

Purpose	Learn to see fractions as a part of measurement. Learn to estimate fractions of a length.
Summary	Students use strips of paper to estimate, then measure. We teach techniques for determining fractional lengths.
Materials	18" strips of paper, objects in the room.
Topic	Estimate lengths. Calculate the fractions involved.
Topic	Estimating and calculating techniques are improved with practice and with time.
Homework	Estimating can be done at home.

Lesson Eight

Purpose	Learn what a ratio is. Learn to see practical applications of ratios and equivalencies.
Summary	We use opportunities that exist or that we create to give our students practical experiences in finding ratios.
Materials	Calculators, bouncing balls, shadows.
Topic	Shadow questions.
Topic	Shadow ratios.
Topic	Bouncing balls.
Topic	Furthest jumps.
Topic	Diagonals of rectangles.
Topic	Other opportunities that arise.
Homework	Ratios explored at school can be explored at home.

Lesson Nine

Purpose	Learn what rates and rate tables are for.
Summary	Students search for examples of rates, learn to make rate tables and learn to use rate tables to answer questions.
Materials	Local newspapers, calculators.
Topic	Search the newspaper for rate examples.
Topic	Create rate tables for gas consumed and cost.
Topic	Create rate tables of various kinds.
Homework	Gather rate table data from home.

Lesson Ten

Purpose	Learn what kind of fraction decimals are.
Summary	Students explore base-ten blocks, geoboards, graph paper rectangles and calculators as they learn about decimal fractions. This lesson is presented after *Advanced Multiplication and Division*.
Materials	Base-ten blocks, calculators, geoboards, 10 X 10 and 100 X 100 blacklines, paper, chalkboards.
Topic	One is what we say it is for base-ten blocks.

Topic	Think about where the decimal goes in answers to addition and subtraction problems.
Topic	Calculator generated lists of equivalencies.
Topic	Search for patterns for where the decimal goes when multiplying decimals.
Topic	Demonstrate why the fraction becomes smaller when we multiply.
Topic	10 X 10 paper for multiplying decimals.

Lesson Eleven

Purpose	Learn the difference between decimals and percents. Learn to understand the questions that percent can represent.
Summary	We talk about the meaning of percent. We give our students practice using percent.
Materials	Calculators.
Topic	As we talk about percent with our students, what our students say guides what we do next.
Homework	What percent examples can be brought from home?

Lesson Twelve

Purpose	Learn about money from using it.
Summary	We use real money and real money situations to teach our students how to find real money answers.
Materials	Money.
Topic	Milk money, restaurant menus, classroom store, fundraising events: any opportunities that arise provide the framework for the money lessons that we teach.
Homework	Money is taught best at home.

Advanced Addition and Subtraction—Chapter 12

Lesson One

Purpose	Learn to search for patterns in bases other than ten.
Summary	Student record and examine plus one and minus one patterns in different bases.
Materials	Trading boards, squares, cups, bowls, blacklines for recording strips.
Topic	Squares and cups: base 4, base 5, base 6, if needed base 3. Then squares, cups and bowls as the base cycle is repeated. Then Base ten.
Topic	+ and - 1 with squares and cups, base 4.
Topic	+ and - 1 with squares and cups, base 5.
Topic	+ and - 1 with squares and cups, base 6.
Topic	+ and - 1 with squares and cups, base 3.
Topic	+ and - with squares, cups and bowls, base 4.
Topic	+ and - with squares, cups and bowls, base 5.
Topic	+ and - with squares, cups and bowls, base 6.
Topic	+ and - with squares, cups and bowls, base 3.
Topic	+ and - with squares, cups and bowls, base ten.
Homework	If students need more practice, cups and bowls can be sent home, but only with children who understand the basic process.

Lesson Two

Purpose	Learn about adding or subtracting numbers greater than one in different bases.
Summary	Students play racing-up and racing-back place-value trading games.
Materials	Trading boards, squares, cups, bowls, dice.
Topic	Racing up in base 4, winner is the first up.
Topic	Racing up in base 5, winner is the last up.
Topic	Racing up in base 6, winner is the first up.
Topic	Racing up in base 3, winner is the last up.
Topic	Racing up in base ten, winner is either the first or last up, decided in advance.
Topic	Racing back in base 4, winner is the first back.
Topic	Racing back in base 5, winner is the last back.
Topic	Racing back in base 6, winner is the first back.
Topic	Racing back in base 3, winner is the last back.
Topic	Racing back in base ten, winner is either the first or last up, decided in advance.

Lesson Three

Purpose	Learn the concept of place value.
Summary	Students learn a game that teaches them the value places have.
Materials	Trading boards, squares, cups, bowls, recording paper, dice.
Topic	Place value game in base 4, most wins.
Topic	Place value game in base 5, least wins.
Topic	Place value game in base 6, most wins.
Topic	Place value game in base 3, least wins.
Topic	Place value game in base ten, either most or least wins, decided in advance.

Lesson Four

Purpose	Learn to create addition and subtraction problems in any base three through ten.
Summary	Students learn how to create addition and subtraction problems and how to find and check the answers to the problems they create.
Materials	Trading boards, squares, cups, bowls, addition and subtraction recording paper blackline, dice, calculators.
Topic	Addition problems are created in base 4.
Topic	Addition problems are created in base 5.
Topic	Addition problems in any base 3 to 6.
Topic	Subtraction problems are created in base 4.
Topic	Subtraction problems are created in base 5.
Topic	Subtraction problems in any base 3 to 6.
Topic	Addition problems are created in base ten.
Topic	Subtraction problems are created in base ten.
Homework	If students need more practice, dice and cups and bowls can be sent home, but only with children who understand the basic process.

Lesson Five

Purpose	Learn to see place-value patterns within and between bases. Learn to use materials to prove answers.
Summary	Students record values for different bases on a single matrix and examine the matrix for patterns. Then students use the multibase blocks (if available) to create, solve, and then check problems.
Materials	Squares, cups, bowls, big bowls, blank matrix, calculators, multibase blocks (if available).
Topic	Looking for patterns in 1000, 100, 10, 1.
Topic	Addition problems using multibase blocks.
Topic	Subtraction problems using multibase blocks.

Lesson Six

Purpose	Learn the names for larger numbers.
Summary	Students learn to read large numbers.
Materials	None.
Topic	Learn to read large numbers.

Lesson Seven

Purpose	Learn that addition and subtraction are tools for finding out. Learn to apply math skills to problems in real life.
Summary	We work with our students to find and then solve real problems that use the skills that our students possess. Students keep a written record of their work.
Materials	Paper.
Topic	Problems drawn from stories.
Topic	Problems drawn from questions.
Topic	Problems drawn from life.
Homework	We send a list of questions home for parents to contemplate with their children.

Lesson Eight

Purpose	Learn to look for patterns in numbers everywhere.
Summary	Students look for patterns between bases, in palindromes and in ordinary events.

Materials	Palindrome blackline. Plus-one strips from Lesson One.
Topic	Compare plus one strips from different bases.
Topic	Palindromes.
Topic	Examine situations for patterns.
Homework	Pattern situations are sought at home.

Lesson Nine

Purpose	Learn that creativity and inventiveness are problem-solving tools.
Summary	Our students use their inventiveness to solve problems that we give and share their individual or collective inventiveness with everyone in class.
Materials	None outside our heads.
Topic	Kids in class, kids in school.
Topic	Thinking and mental arithmetic. Reasonableness.
Homework	We ask parents to do their mental arithmetic aloud.

Advanced Multiplication and Division—Chapter 13

Lesson One

Purpose	Learn what it means to multiply in any base. Learn to search for multiplication patterns within and between bases.
Summary	Students learn to make multiplication matrices for bases three through ten as they search for patterns in each new matrix.
Materials	Squares, blacklines for 11 X 11 blank matrices.
Topic	Base-three matrix made by the full class. Pairs of students work in pairs to make matrices fir bases four and five. For base six, students look at the previous bases for patterns that they can anticipate will appear. The process is repeated for bases seven, eight and nine. How much of base ten can be filled in from all the patterns seen in three through nine?
Topic	Base-four matrix started by the full class, finished working in pairs.
Topic	Base-five matrix in pairs, information shared collectively.
Topic	Base-six matrix made by looking at bases three, four and five.
Topic	Bases seven, eight and nine and, for some, sixteen.
Topic	Patterns is bases three through nine are used to fill in a matrix for base ten.

Lesson Two

Purpose	Learn how to represent concretely a multiplication problem larger than a matrix shows. Learn how to multiply in any base.
Summary	We give our students larger multiplication problems to solve. We help our students apply what they already know to the new problems that they face. Once they understand the process, they use dice to create problems of their own to solve.
Materials	Chalkboards, squares, cups, bowls, trading boards, dice, multibase blocks (if available).
Topic	Multiplication problems in base five.
Topic	If available, multibase blocks and multiplication problems in different bases.

Lesson Three

Purpose	Learn techniques for finding answers to the multiplication problems likely to be on the end-of-year standardized test.
Summary	If our students will not be permitted to use calculators or materials on their year-end test, we teach them how to calculate answers for the test.
Materials	Paper.
Topic	An algorithm for multiplication is taught.

Lesson Four

Purpose	Learn to look for patterns in multiplying numbers with zeros at the end. Learn how to find reasonable answers for multiplication problems that are large.
Summary	Students use their calculators to fill in worksheets that they then examine for patterns for multiplying numbers with zeros on the right. Students use the patterns to learn to estimate reasonable answers for large problems.

Materials	Blackline worksheet for 1, 10, 100, 1000; chalkboards; calculators.
Topic	Multiplying by 2, 20, 200, 2000 and so on. Patterns sought.
Topic	Reasonableness for answers to large problems.

Lesson Five

Purpose	Learn to apply multiplication skills.
Summary	With our students, we look for real problems to solve that use multiplication.
Materials	Writing paper.
Topic	A problem a day is enough to pose. A problem is posed, small groups of students discuss and write down ways it might be solved, then students share their ways with the class.
Topic	Problem posed, discussed, solved, solutions shared.
Topic	Another problem is posed.
Homework	Our students search at home for multiplication problems that occur naturally. Parents join in the search.

Lesson Six

Purpose	Learn what it means to divide in any base.
Summary	We teach dividing techniques. Our students then use dice to create division problems for themselves.
Materials	Squares, cups, bowls, trading boards, dice, calculators.
Topic	Divide in base five, then base four and base six.
Topic	Divide in base ten. Calculators check answers.
Topic	If available, divide with multibase blocks in different bases.

Lesson Seven

Purpose	Learn to use the arithmetic skills we have.
Summary	We look for real problems that use division and any other arithmetic skill. Our students look, as well, and keep a written record of the problem solving steps they use.
Materials	The materials relate to the problems found.
Topic	A single sandwich.
Topic	The daily life of a child.
Topic	Analysis.
Topic	Averages of all kinds.
Homework	Our students search at home for division problems that occur naturally. Parents join in the search.

Algebra—Chapter 14

Lesson One

Purpose	Learn to write symbolic equations or formulas for familiar patterns or experiences.
Summary	Students look at geoboard, Power Block, and wooden cube patterns and use letters to record the patterns seen.
Materials	Power Blocks, geoboards, wooden cubes.
Topic	Squares and rectangles made with S-1 squares, lengths, widths and areas recorded and searched for patterns.
Topic	Boxes made with wooden cubes, with the formula for volume sought.
Topic	Geoboard formulas for area and Pick's theorem.
Topic	Algebraic relationships between the Power Blocks.
Homework	Pattern searches leading to Pick's Theorem are continued at home for everyone to share.

Lesson Two

Purpose	Learn to plot coordinate points on a graph.
Summary	We play coordinate tic-tac-toe with our students as a class. Students play the game of Battleships with themselves.
Materials	Graph paper.
Topic	Students play coordinate tic-tac-toe as a class.
Topic	Students play Battleships in small groups.

Homework	Students may continue playing Battleships at home.

Lesson Three

Purpose	Learn a framework for understanding addition, subtraction and multiplication of signed numbers.
Summary	We present Letter Carrier stories to our students to teach them rules for arithmetic operations with positive and negative numbers.
Materials	Chalk boards.
Topic	Letter Carrier stories for + and —.
Topic	Letter Carrier stories for x.
Topic	Students create stories for numbers we provide.

Lesson Four

Purpose	Learn to write tables for pairs of numbers that are related in a patterned way. Learn to write equations or formulas for the patterns.
Summary	Students use a "Magic Box" function machine to predict from numbers going in, the numbers coming out.
Materials	"Magic Box" function machine, teacher-made and student-made "magic" cards.
Topic	Teacher creates the rules for the numbers going in and coming out.
Topic	Students create the rules for the numbers going in and coming out.
Topic	Write formulas for the rules.

Lesson Five

Purpose	Learn to graph functions and the equations they represent.
Summary	Students plot the data from the tables in Lesson Four and other functional relationships on coordinate graphs and write equations to accompany their graphs.
Materials	Graph paper, circular objects, toothpicks.
Topic	$x = y = 10$.
Topic	Tables from Lesson Four.
Topic	Graphing the area formula, $a = lw$.
Topic	Graphing circumference, $c = \pi d$.
Topic	Graphing multiplication facts, $y = 2x$, $y = 3x$, etc.
Topic	Graphing toothpick patterns.

Lesson Six

Purpose	Learn to recognize the slope and intercept of an equation to be graphed.
Summary	Students graph equations in the $y = mx$ or $y = mx + b$ formats and look for slope and intercept patterns in the resulting lines.
Materials	Graph paper.
Topic	$y = mx$.
Topic	$y = mx + b$.

Lesson Seven

Purpose	Learn to recognize the effect that exponents have on a graph.
Summary	Students graph equations in the $y = mx^2 + b$ or $y = mx^3 + b$ formats and look for patterns in the resulting lines.
Materials	Graph paper blackline—squares small enough to permit graphing of x^2 and x^3.
Topic	$y = mx^2 + b$.
Topic	$y = mx^3 + b$.

Lesson Eight

Purpose	Learn to multiply $(x + y)(x + y)$ equations. Learn to connect the multiplying to concepts already understood.
Summary	We show our students how to use drawings to solve equations like $(x + y)(x + y)$. We connect the algebra to arithmetic.
Materials	Graph paper.
Topic	$(x + y)(x + y)$.

Topic	$(x + y)(x + z)$.
Topic	$(x + y)(z + w)$.
Topic	What comes next depends.

Lesson Nine

Purpose	Learn that the ratios, rates, proportions and equivalencies from the fractions chapter are functional relationships.
Summary	Our students graph data they developed in their fraction lessons and use their graphs to create algebraic formulas.
Materials	Tables of data from earlier mathematical experiences.
Topic	Ratios.
Topic	Rates.
Topic	Proportions.
Topic	Equivalencies.

Lesson Ten

Purpose	Learn to use algebra. Learn to connect algebra in school to algebra in real life.
Summary	We create an algebra environment in our room by making algebra a tool for finding out. We use opportunities already present in our student's lives.
Materials	Materials in our room and/or materials we find outside of class.
Topic	Algebra is in the environment we create.
Topic	Algebra is in the questions that we ask.
Homework	The work that we send home depends upon the work we do in class.