Lesson Summaries

Chapter 2

Free Exploration and Creative Learning

Lesson One Summary

Goals for our students:

Learn the rules for using materials in our class.
Explore freely the potential uses of the materials we present.
Learn that creativity and inventiveness are allowed.
Learn that interacting with one another is part of learning in our room.

How to reach the goals:

Present the materials with care to the students in our room who are ready to learn.
Convert our observations into questions, which serve as catalysts.

What is important to assess:

Proper use of the materials by our students.
Student creativity.
Student interaction.
Our ability to convert our observations into questions that motivate investigations and inquiries.
Our ability to learn from watching as our students work.

The lesson:

The lesson is within the children and the materials themselves.
There is nothing for us to teach. There are only things for us to learn.

Measure to see if the goals have been reached:

We will know when the rules for using the materials have been learned.
We do not measure when students reach the goals of exploring freely and of interacting with one's peers. They are goals for all of life.

Chapter 3

Patterns and Connections

Lesson One Summary

Goals for our students:

Learn what we mean by *pattern.*
(A pattern is something that helps us know what is coming next or what has gone before.)
Learn the A-B language for describing patterns.

How to reach the goals:

Use the students in our class to model A-B patterns.
Have our students use their own actions to model A-B patterns.
Have students modify the patterns offered by others in class.

What is important to assess:

Student replication of the patterns that we model.
Student ability to create patterns that match the model.

The lesson:

We model A-B patterns and their extensions.
Students create their own models to share.
Students learn to search for A-B patterns everywhere.

Measure to see if the goals have been reached:

The goals have been reached when:
Students can replicate a basic pattern.
Students can generate a pattern of their own.
We measure by observing our students at work.

Keep on teaching until everyone understands:

We group students who understand with students who do not.
The groups work together to create patterns to share.

Lesson Two Summary

Goals for our students:

Learn to make A-B patterns with manipulative materials.
Learn how to check each other's work, to ensure that everyone understands.

How to reach the goals:

Model A-B patterns with materials.

Have students create A-B patterns with materials.
Have students share the patterns they create with everyone in class.
Teach our students the rules for checking their neighbor's work.

What is important to assess:

Students' ability to extend A-B patterns with materials.
Students' ability to create their own patterns.
Students' ability to check their neighbors' work diplomatically.

The lesson:

We model A-A-B patterns with Pattern Blocks, Power Blocks and other materials.
Students copy and extend our examples.
Students create their own examples to share.
We teach our students how to check each other's work.

Measure to see if the goals have been reached:

The goals have been reached when:
Students can replicate a basic pattern.
Students can generate a pattern of their own.
Students check each other's work and offer positive feed back to one another.
We measure by observing our students at work.

Lesson Three Summary

Goals for our students:

Learn how to record patterns.
(Mathematics and recording go hand in hand.)
Learn to reproduce and extend patterns that others have made.

How to reach the goals:

We add representational materials for recording the A-B patterns created with the concrete materials already in use.
Students create pattern cards to share.

What is important to assess:

Students' ability to duplicate A-B patterns on paper.
Students' ability to create patterns uniquely their own.

The lesson:

Introduce recording and we model recording techniques.
Have students create pattern cards.
Have students exchange cards and use materials to copy and then extend the patterns on the cards.

Measure to see if the goals have been reached:

Can our students record with reasonable accuracy the patterns they have made?
Can our students copy and extend patterns made by others?

Lesson Four Summary

Goals for our students:

To know there are patterns in numbers.
To know there may be more patterns to see every time they look.

How to reach the goals:

We ask our students to look at charts of numbers and describe the patterns that they see.

What is important to assess:

Students' continually growing ability to see and describe patterns in numbers.

The lesson:

Introduce each number matrix.
Ask, "What patterns can you see?"
Have students describe the patterns found.
Leave the chart(s) posted throughout the year.

Measure to see if the goals have been reached:

No formal measuring is done. The goals are reached immediately and the goals are never reached at all.

Lesson Five Summary

Goals for our students:

Learn that learning is not confined to a lesson.
Learn that creativity is allowed.

How to reach the goals:

We use art as a vehicle for encouraging creativity and discovery.
We present selected art lessons that focus on pattern.

What is important to assess:

We do not assess. We share what students create.

The lesson:

Provide our students with the materials necessary for them to create patterns and designs in art.
Ask questions that encourage and promote creativity.

Measure to see if the goals have been reached:

Learning does not always have to be measured. Learning can sometimes just be admired.

Chapter 4

Beginning Number

Lesson One Summary

Goal for our students:

Learn how to count up and back by ones.

How to reach the goal:

Set up situations that require counting.
Count with our students until they can count without us.
Have our students count with each other's help until they can count alone.

What is important to assess:

No formal assessment is needed.
We watch to see who can count.

The lesson:

Lesson One is not a lesson. It is a counting environment.
We look for counting opportunities.
We ask, "How many?"
We and our students count.

Measure to see if the goal has been reached:

Measuring takes place every time we ask our students to count.

Keep on teaching until everyone understands:

Opportunities for counting are as limitless as grains of sand upon a beach.

Lesson Two Summary

Goals for our students:

Learn to look for patterns in the counting numbers.
Learn to listen for patterns in the counting numbers.

How to reach the goals:

Post a list of the counting numbers and ask our students to describe to us the patterns they see.
Point to the numbers as we say them and ask our students to tell us the patterns they hear.

What is important to assess:

We are teaching a framework for looking. No formal assessment is needed.
We watch to see who can see and hear patterns.

The lesson:

We post a 0-100 column of numbers.
Our students tell us the patterns they see.
We say the numbers in the column.
Our students tell us the patterns they hear.

Measure to see if the goals have been reached:

The framework we are teaching now will be used again in place value.
We wait until we do the lessons in different number bases before we measure the learning done.

Lesson Three Summary

Goals for our students:

Learn cardinal numbers: 1, 2, 3...
Learn ordinal numbers: 1st, 2nd, 3rd...

How to reach the goals:

Use the numbers as we use language.

What is important to assess:

Knowledge of the language of number.

The lesson:

We make cardinal and ordinal numbers a part of the language that we use.

Measure to see if the goals have been reached:

How do we measure when language is learned? We listen to the language students use.

Lesson Four Summary

Goal for our students:
Learn the fiveness of five.
How to reach the goal:
We create an environment for knowing numbers by using the following materials in succession: Power Block squares, toothpicks, wooden cubes and Pattern Blocks.
We surround our students with the concept of numbers for the numbers three through ten.
What is important to assess:
No formal assessment is needed.
The surrounding environment that we create ensures that all our students will eventually learn.
The lesson:
We present each new number from three through ten with four different materials, one material at a time.
Our class explores ways to make numbers, creating a sea of numbers for everyone to see.
Our students check the work of their neighbors, so that each of our students has many more teachers than one.
Measure to see if the goal has been reached:
When does a child finally learn the meaning of the word *car*? Do we ever test to see when the meaning is clear?

Lesson Five Summary

Goal for our students:
Learn how to record number designs for the four materials students use.
How to reach the goal:
Provide our students with recording materials for Power Block squares, toothpicks, Pattern Blocks and wooden cubes.
Teach our students to use the recording materials we provide.
What is important to assess:
Do the recorded designs reflect accurately the designs they record?
The lesson:
We present the numbers and the materials from Lesson Four with recording added in this time.
We teach a different recording technique for each material used.
Measure to see if the goal has been reached:
We observe to see if the recording techniques are understood.

Lesson Six Summary

Goals for our students:
Learn to describe and record designs in words and in numbers.
Learn to look at numbers and envision the designs they might represent.
Learn to see the number patterns in their designs.
Understand the concept-connecting-symbolic link.
Understand the symbolic-connecting-concept link, as well.
How to reach the goals:
Have students imagine and then write words to describe their designs.
Have students see and then write numbers that their designs can represent.
What is important to assess:
We do not assess the imagination in the words our students write, we encourage it.
For the numbers that our students write, we look to see if the numbers written represent the designs they are to match.
The lesson:
We teach our students to write word descriptions for their designs.
We make design books from our students' words and designs.
We teach our students to write number descriptions for their designs.
We make design books from our students' numbers and designs.
Measure to see if the goals have been reached:
The measure is within every design book of words or numbers that our students create. We can see the imagination and the understanding present on every page.

Lesson Seven Summary

Goals for our students:
Learn to describe Unifix Cube number designs with the letters A and B.
Learn to look for patterns in ways to make numbers with Unifix Cubes.
Learn families of facts.
How to reach the goals:
Have students use Unifix Cubes to find ways to make the numbers from one to ten.

Have students use two different rules.
Have students use the numbers found to create flash cards for the families of addition facts.

What is important to assess:

Can our students find the ways that numbers can be made with cubes?
Can our students see the connections between the ways to make one number and the next?

The lesson:

We use cube sticks made with two colors or less to find the different ways that numbers can be put together.
Rule one: Two or fewer colors snapped together any way.
Rule two: Two or fewer colors, colors cannot be separated.
We look for patterns that help us know when all the ways are found.
We record the cubes in numbers.
We link patterns found in Unifix Cubes with families of facts.

Measure to see if the goals have been reached:

We observe to see if our students, working in pairs and sharing with everyone around, can find all the ways there are to find.

Lesson Eight Summary

Goals for our students:

Learn to make shapes on the geoboard with areas of two and beyond.
Learn to find areas of simple and not-so-simple geometric shapes.
Learn to prove answers through logic and arithmetic.
Learn to record geoboard shapes accurately on paper in different scales.

How to reach the goals:

Create an environment for knowing numbers by using geoboards.
Teach our students to create shapes on their geoboards and prove the areas of the shapes they make.
Teach our students procedures for accurately recording their shapes.

What is important to assess:

No formal assessment is needed. Geoboards show us our students' thinking.
We look to see if our students can record their shapes accurately on paper.
We look to see if our students can prove the areas that they find.

The lesson:

Lesson Eight teaches the twoness of two and the fiveness of five to students who may feel they are too old to learn such simple things.
Our students make shapes with areas of two square units on their geoboards.
We teach our students techniques for proving the areas that they find.
We teach our students techniques for recording the shapes they create.
As we and our students are able to make and prove shapes for areas of two we ask our students to make shapes with areas of three and more.

Measure to see if the goals have been reached:

The assessments we make are within everything we watch our students do.

Chapter 5

Sorting, Classifying, Expanding Language

Lesson One Summary

Goals for our students:

Learn to see attributes in objects.
Learn to create and then use rules for sorting objects into exclusive groups.
Learn to use the creativity and imagination of their classmates to expand their own thinking.

How to reach the goals:

Provide common objects for our students to sort.
Model the sorting process with our students.
Have students make lists of all the sorting ways they find.
Each time our students believe they have found all the sorting ways they can, ask them to think of more ways again.

What is important to assess:

Our students' understanding of what we mean by attribute.
Students' ability to find ways that divide the objects they are sorting into exclusive groups by rules that they can make clear to us.
The ability of everyone in each group to contribute to the list of sorting ways.

The lesson:

We sort objects on the overhead and ask our students to figure out our rule.

Our students work together to sort objects into groups.
We (or they) compile lists of all the ways that anyone can find.
Our students exchange lists and add to them, so that our students may create more ways than they think they can.

Measure to see if the goals have been reached:

We observe the groups at work and the lists each group creates.

Lesson Two Summary

Goal for our students:

Learn to be aware of the attributes in objects all around.

How to reach the goal:

Make the search for attributes more than just a lesson for class.
Create awareness of attributes by the questions that we ask.
Take our students on sorting walks to teach them how to see what is already there.
Have our students write lists or draw pictures of the attributes for everything they can find.

What is important to assess:

Students' ability to find attributes in the world outside our room.

The lesson:

Lesson Two is a lesson in awareness. The lesson never ends.
We ask questions that make our students more aware of the attributes that already exist.
We take our students on sorting walks and talk about the attributes in everything we find.
We ask our students to notice attributes in things they have at home.
We have our students draw or write to illustrate the attributes they find.

Measure to see if the goal has been reached:

Awareness is a lifelong goal.

Lesson Three Summary

Goal for our students:

Learn to use their knowledge of sorting and classification to create informal definitions of words, as they already do outside of school.

How to reach the goal:

Sort and ask our students to predict the pattern of our sort.
Provide the words that describe the pattern of our sort.

What is important to assess:

Our ability to provide the sorting experiences that help our students define the words we wish to teach. There is more for us to assess about ourselves than about the learning of our students.

The lesson:

We sort shapes or words or objects on the overhead that represent the words we wish to teach.
We ask our students to figure out the rule by which we sort.
We provide the word that represents the pattern of our sort.

Measure to see if the goal has been reached:

We judge how well we do at finding sorts that teach the words we want students to learn.

Lesson Four Summary

Goals for our students:

Learn that sorting is a part of the thinking that we do.
Learn to use a simplified mathematical model to find relationships between differing shapes.

How to reach the goals:

Control the number of attributes students are to study.
Teach our students attribute games that require them to think in systematic ways.

What is important to assess:

Our students' understanding of the rules for each new sorting game.

The lesson:

We ask our students to sort the attribute blocks by rules that we announce.
We sort the attribute blocks and have our students guess our rules.
We have our students look at rows of blocks and decide which block is missing or which block should not be there.
We ask our students to add a block one different to the block already there.
After adding a one-different block, we ask students to add two-different and three-different blocks.
If we feel our students are ready, we ask them to create matrices of differences.

Measure to see if the goals have been reached:

We observe as our students play attribute games with one another.

Lesson Five Summary

Goal for our students:
Learn to see attributes in themselves.
How to reach the goal:
Describe our students to our students and have our students learn to describe themselves.
What is important to assess:
Our students' ability to write descriptions of themselves that separate themselves from everybody else.
The lesson:
We describe students in our room one attribute at a time, to model for our students the process of describing that they will use.
Our students write attribute descriptions of themselves.
We read selected descriptions aloud to see if the description's student author is the only child left standing at the end.
Measure to see if the goal has been reached:
We use each student's written list of attributes to assess the understanding of each child.

Lesson Six Summary

Goal for our students:
Learn the process of using individual attributes to categorize.
How to reach the goal:
Have students use the attributes they developed for themselves in Lesson Five to describe everyone in class uniquely.
What is important to assess:
Our students' ability to find a way to describe everyone in class uniquely.
The lesson:
We ask our students to use attributes to divide themselves into successively smaller groups.
The object of the divisions is to discover a list of attributes that uniquely describes every child in class.
Measure to see if the goal has been reached:
The goal is reached when our students find a list of attributes that separates everyone in class from everybody else.

Lesson Seven Summary

Goal for our students:
Learn the process of using attributes of objects to categorize.
How to reach the goal:
Have students use the lists of attributes they generated in Lesson One to describe the materials they sort uniquely.
What is important to assess:
Our students find a way to describe the items that they sort uniquely.
The lesson:
We ask our students to use the attributes they found for objects they sorted in Lesson One to divide the same objects into successively smaller groups.
We ask our students to discover a list of attributes that uniquely describes each object in a group.
Measure to see if the goal has been reached:
The goal is reached when our students find a list of attributes that separates each object from every other object in their group.

Lesson Eight Summary

Goal for our students:
Learn the meaning of some adjectives and conjunctions in use in math.
How to reach the goal:
Ask our class as a whole to demonstrate their understanding of the words we select for use.
What is important to assess:
Our students' understanding of the words.
The lesson:
We select the words we wish to teach.
We use the words in sentences and ask our students to respond by standing or sitting as they think the words direct them to do.
We discuss with our students how they interpreted our words.
Measure to see if the goal has been reached:
We observe and note who stands who should be sitting.
We observe and note who sits when standing is required.

Lesson Nine Summary

Goals for our students:

Learn how to communicate more effectively with words.
Learn the meaning of words like parallel and perpendicular.
Learn to work together to achieve the goal.
Learn how to write directions well.

How to reach the goals:

We and our students give instructions and discuss how we can make the instructions clearer.

What is important to assess:

How well each group understands every student in the group.
How well each group writes instructions that are understood by all.

The lesson:

We build behind a shield, describing as we build.
Our students follow our instructions as best they can.
We all discuss together how instructions might be made clearer.
Our students take turns building behind shields and describing what they are building to the members of their group.
Groups discuss among themselves how instructions can be made clearer.
If we judge our students ready, groups try writing out instructions to be followed by everyone in class.

Measure to see if the goals have been reached:

We measure groups, not individuals.
The goals involve communicating clearly: A group communicates to us when it has reached the goal.

Chapter 6

Geometry, Shapes, Relationships and Constructions

Lesson One Summary

Goal for our students:

Provide a balanced background in geometry equally for boys and girls, rich and poor.

How to reach the goal:

Have students build with anything around.
As our students build, ask questions to focus their discoveries.
Teach words to increase our students' geometric vocabulary.

What is important to assess:

Building is not a lesson we assess. Our students experience building for themselves.

The lesson:

We provide as wide a variety of building materials as we can.
Our students build.
We often ask our students to build and then describe.

Measure to see if the goal has been reached:

Students reach the goal as we provide opportunities for them to build, during math or any other time that we provide.

Lesson Two Summary

Goal for our students:

Learn to analyze the geometric properties of shapes more systematically.

How to reach the goal:

Provide students with geoboards for exploring the properties of shapes.

What is important to assess:

Our students' ability to see the similarities and differences between geometric shapes.

The lesson:

We ask our students questions that encourage them to explore shapes on their geoboards.

Measure to see if the goal has been reached:

Measuring to see if our students understand the properties of shapes is like trying to count the words a five year old knows. We do not measure words, we keep on talking and listening to the five year old. We do not measure understanding of properties, we continually assist the endless learning.

Lesson Three Summary

Goals for our students:

Learn to recognize reflective symmetry in geometric shapes.
Learn that patterns in mathematics do not have to be numerical.

How to reach the goals:

Provide students with mirrors that they use to explore reflective symmetry.

What is important to assess:

Our students' knowledge of what reflective symmetry is.

The lesson:

We give our students mirrors (plain and hinged) and blocks with which to explore reflective symmetry.

We have our students search out examples of symmetry in their environment.

Measure to see if the goals have been reached:

We observe to see if our students can construct shapes with blocks that demonstrate reflective symmetry.

We observe to see if our students can find examples of reflective symmetry in the world outside of mirrors.

Lesson Four Summary

Goals for our students:

To learn which polygons and other shapes tessellate the plane.

To learn that there are more kinds of symmetry than reflective.

To learn that the beauty of nature is mathematical.

How to reach the goals:

We provide our students with materials to create tessellations and the instructions they need to tessellate.

What is important to assess:

Our students' understanding of the word *tessellate*.

Our students' ability to create shapes that tessellate.

The lesson:

We teach our students the meaning of the word *tessellate*.

We have our students create tessellations with Power Blocks and Pattern Blocks.

We have our students create tessellations with cardboard cutouts of polygons.

We have our students invent their own shapes for creating tessellations.

Measure to see if the goals have been reached:

The measuring is in the work our students create. We see which students can create their own tessellations. We help the ones who have difficulty until they can create as well.

Lesson Five Summary

Goals for our students:

Learn that math and art are not separate subjects. Dividing lines for subjects are distinctions mostly made in school.

Learn that tessellations blur a line between math and art that should be blurred.

How to reach the goals:

Have students connect math to art.

What is important to assess:

Our students' ability to see the connection between math and art.

Our ability to see the connection beyond the tessellation lesson that we teach.

The lesson:

We teach our art lessons as we always do and point out the connections to be made.

Measure to see if the goals have been reached:

Our measurement is of our own awareness of the mathematics in art.

Lesson Six Summary

Goals for our students:

To draw objects well enough so that someone else can identify the objects drawn.

To identify geometric objects from drawings done by someone else.

How to reach the goals:

Provide students with practice in drawing and identifying three-dimensional geometric shapes.

What is important to assess:

Our students' ability to capture on paper enough of the properties of a three-dimensional shape to allow the shape to be correctly identified.

Our students' ability to identify the real shapes represented in drawings made by someone else.

The lesson:

We demonstrate the process of using two-dimensional drawings to represent three-dimensional blocks.

Our students draw blocks. They then trade with other students to see which drawings accurately depict the blocks drawn.

We discuss methods to improve techniques and begin drawing again.

As our students improve their skills at drawing and identifying, we increase the complexity of the drawings we require.

Measure to see if the goals have been reached:

We observe to see whose drawings communicate their message clearly. We work with the students who need help in communicating their shapes until all our students can draw and then identify.

Lesson Seven Summary

Goal for our students:

Learn what an angle is and how to measure it.

How to reach the goal:

Teach our students about angles and turns.

Have our students give instructions involving turns.

Have our students measure angles informally, then formally.

What is important to assess:

Students' ability to measure angles.

The lesson:

We have our students give instructions that involve turns to one another.

We ask our students to compare angles in the room with angles from folded paper or straws and sticks.

We help our students learn to use protractors as an angle-measuring device.

Measure to see if the goal has been reached:

We observe to see if our students know how to measure angles.

Lesson Eight Summary

Goals for our students:

Learn to use a compass, protractor and straightedge.

Learn to draw geometric shapes.

How to reach the goals:

Introduce the materials with the statement: "See what you can make."

Ask questions and pose challenges to guide the explorations.

What is important to assess:

Our students' ability to use the materials comfortably—nothing more.

The lesson:

We teach our students how to use a compass safely and well.

We give our students a compass, protractor and straightedge and the freedom to explore.

We pose questions in the form of challenging shapes to draw.

We demonstrate possible ways that the shapes may be drawn.

Measure to see if the goals have been reached:

We observe the shapes our students draw.

Lesson Nine Summary

Goals for our students:

Learn to be more aware of the geometry in their own lives.

Learn that awareness and wonder are a part of mathematics.

How to reach the goals:

Ask students to look more carefully at what they already see.

What is important to assess:

As we listen to the informal conversation of our students, evidence that they are more aware of the math in their lives.

As we prepare for each new day in school, our awareness of the math everywhere.

The lesson:

We give our students assignments in looking.

We ask questions to focus our students' attention on what they might see but may not notice.

We ask, "Why?"

Measure to see if the goals have been reached:

Not all goals we teach have a measure to make.

Lesson Ten Summary

Goals for our students:

The goals for this lesson are goals for ourselves:

See and use the opportunities that present themselves for using geometry.

Create opportunities whenever we can.

How to reach the goals:

Make ourselves aware of the opportunities that exist.

What is important to assess:
Our ability to seize the opportunities that arise.
The lesson:
Lesson Ten is not a lesson that is taught.
Measure to see if the goals has been reached:
The measure is within our own growing awareness of geometry.

Chapter 7

Beginning Addition and Subtraction

Lesson One Summary

Goals for our students:
Learn to create and check their own addition problems.
Learn to prove the answers that they find.
Learn that they can know when an answer is right without our help.
How to reach the goals:
Teach our students to create problems with squares.
Have students use their counting skills to find the answers to the problems they create.
What is important to assess:
Students' ability to use their counting skills to find answers to their problems.
Their ability to count carefully.
The lesson:
We model the process of creating and recording addition problems.
Our students record and check the problems they create with squares.
We allow students to use calculators to double-check the accuracy of their answers.
Measure to see if the goals have been reached:
We check to see that our students can count and record accurately. The answers do not come from us.
Our students prove their own answers.
If we are not sure what our students know, we find out with Problems out the door... in Lesson One.
Keep on teaching until everyone understands:
When every student can count and record what he or she has counted, we are ready to move on.

Lesson Two Summary

Goals for our students:
Learn to apply skills in addition.
Learn to seek and find patterns in the answers to problems in addition.
Learn to see addition as a problem-solving, pattern-searching tool.
Learn to use calculators as a pattern-checking tool.
How to reach the goals:
Give students problem-solving questions to explore.
Show students that patterns can be found in numbers anytime numbers are used sequentially or systematically.
Ask students to create problems for themselves.
What is important to assess:
Students' ability to see the patterns in the answers that they find.
Students' ability to use the patterns to anticipate the answers.
The lesson:
We present our students with number patterns to explore.
We ask our students to create problems for themselves.
Our students look for patterns every time.
We allow students to use calculators as answer-checking tools.
Measure to see if the goals have been reached:
We watch our students work.
We share in and marvel at the discoveries that they make.

Lesson Three Summary

Goals for our students:
Learn to create and check subtraction problems.
Learn to prove the answers.
How to reach the goals:
Teach students to create problems with squares.
Have students use their counting skills to find the answers to the problems they create.

Ask students to create problems for themselves.

What is important to assess:

Students' ability to use their counting skills to find answers to their problems.
Students' ability to count carefully and know what to count.

The lesson:

We model the process of creating and recording subtraction problems.
Our students record and check the problems they create with squares.
They use calculators to double-check the accuracy of answers.
We ask our students to create problems for themselves.

Measure to see if the goals have been reached:

We check to see that our students can count and record accurately.
Our students prove their own answers.
If we are not sure what our students know, we find out with Problems out the door... in Lesson One.

Keep on teaching until everyone understands:

When every student can count and record what he or she has taken away or has left, we are ready to move on.

Lesson Four Summary

Goals for our students:

Learn to apply skills in subtraction.
Learn to seek and find patterns in the answers to problems in subtraction.
Learn to use calculators as a pattern-checking tool.

How to reach the goals:

Give students problem-solving questions to explore.

What is important to assess:

Students' ability to see the patterns in the answers they find.
Students' ability to use the patterns to anticipate answers.

The lesson:

We present our students with number patterns to explore.
Our students look for patterns.
They use calculators as answer-checking tools.

Measure to see if the goals have been reached:

We watch our students work.
We observe the discoveries that they make.

Lesson Five Summary

Goal for our students:

Learn the families of addition and subtraction facts above ten.

How to reach the goal:

Provide students with opportunities to learn the number facts for quicker recall without the pressure of time or speed.

What is important to assess:

Students' ability to provide answers without having to count out the answer every time.

The lesson:

We provide flash cards and the opportunity to practice without speed.
We teach our students games to play that use the basic facts they need to know for school.

Measure to see if the goal has been reached:

We continue to provide every child in our room the opportunities to practice during class and at home, until every child can provide answers without counting on either hand.

Lesson Six Summary

Goals for our students:

Learn to accept responsibility for the answers that they find.
Learn to sort out extraneous information in solving problems.
Learn ask for information missing in the problems.
Learn that problems can be created using the children in class.

How to reach the goals:

Accept class answers for the problems we create, right or wrong.
Give our students problems containing more or less information than they need.

What is important to assess:

Whether we trust our students' ability to find their own answers to problems once we have provided them with basic problem-solving techniques.

The lesson:

We create addition and subtraction problems for our students using the children and materials available in our class.
We create problems that have too much or too little information added in.
We accept the answers written most commonly on our students' chalkboards as the answer to the problems we create.

Measure to see if the goals have been reached:

Students reach the goals when they can accurately record the problems we create on their chalkboards.
We reach our goal for ourselves is when we can accept as "right" the answers that our students find, unless our students vote them "wrong."

Lesson Seven Summary

Goals for our students:

Learn to create stories for addition and subtraction problems.
Learn to construct meanings for what symbols represent.
Learn to see the stories in all numbers.
Learn that writing is a part of math.

How to reach the goals:

Ask our students to write and draw stories for the problems we provide.
Help our students look for number stories in their own lives.

What is important to assess:

Whether our students know what numbers mean.

The lesson:

We provide simple addition and subtraction problems to our students.
We ask our students to write or draw stories that show they understand what the numbers in our problems mean.
We encourage our students to be inventive in the stories they create.
We read the most inventive stories to encourage even more creative efforts than we might have received so far.
We ask our students to look at their own lives for math stories.

Measure to see if the goals have been reached:

Every story a child writes or draws is a measure of what that child understands.

Lesson Eight Summary

Goals for our students:

Learn to think about whether answers make sense.
Learn to use thinking to judge the answers.

How to reach the goals:

Ask students to think about answers to questions for which the answer cannot be known.
Ask students to transfer this thinking to answers that they think they know are right.

What is important to assess:

Students' ability to explain the reasonableness of the answers they give as approximations of answers they cannot find.
Students' ability to explain the reasonableness of answers that they find.

The lesson:

We teach our students the meaning of the word *reasonableness*.
We give our students problems that do not have a number answer they can know for sure.
We ask our students to defend with reason the answers they produce.
For every problem that they face, we teach our students to ask, "Does the answer I have found make sense?"

Measure to see if the goals have been reached:

Our students provide us with proof for all the answers they find.
We judge to see if the proofs are reasonable.

Lesson Nine Summary

Goals for our students:

Learn to connect the mathematics in school to the mathematics in life.
Learn that mathematics is something we do, just as language is something we speak.

How to reach the goals:

Draw the problems that we ask our students to solve from our lives and the lives of our students.

What is important to assess:

Whether we have made mathematics a part of every child's thinking.
Students' ability to use the mathematics that we teach when we have not told them to.

The lesson:

We have our students walk a mile, instead of Mr. Smith.
We look for the mathematics that is already all around and use this math for the problems we ask our students to solve.
We draw the questions that we ask from the questions that we ask.

Measure to see if the goals have been reached:

We measure by watching to see if our students use math when we have not told them to.
We observe whether our students use math to solve problems from life and know they are using math.

Chapter 8

Graphing, Probability and Statistics

Lesson One Summary

Goals for our students:

Learn that graphing is a tool for finding answers to our questions.
Learn that the ideas for our graphs come from what we would like to know.

How to reach the goals:

Help our students learn to turn questions for which they would like to know the answers into information that they can graph.

What is important to assess:

Students' ability to turn their natural curiosity into topics for their graphs.

The lesson:

We ask our students what they would like to find out.
We ask the same of ourselves.
We help our students turn their curiosity into data that they can graphed.
We use each graph our students make as the basis for more graphs.

Measure to see if the goals have been reached:

The measure is of the ideas for graphs our students create. No one number of graphs needs to be reached. We observe the students' interest.

Lesson Two Summary

Goals for our students:

Learn to expand the questions that students ask beyond parroting back the questions we have asked of them.
Learn how to graph information in a variety of ways.

How to reach the goals:

Ask students to use their own creativity as they learn to graph.

What is important to assess:

Students' ability to find ways to graph that are not the same each time.

The lesson:

We begin with a basic graphing situation, like a collection of favorite fruit.
We ask our students to think of different ways to display the information.
We focus on finding many ways to graph one set of data rather than on graphing the data once and moving on.

Measure to see if the goals have been reached:

We check to see if our students can invent more ways to display the data than they could before the lesson began.

Lesson Three Summary

Goals for our students:

Learn how to ask questions about graphs.
Learn the art of asking what they really want to know.
Learn to see graphs as a source of questions to be asked and answers to be found.

How to reach the goals:

Create a graphing situation in which the only way to know what the graph reveals is by asking the right questions.

What is important to assess:

Students' ability to ask questions that give them the answers they are looking for.

The lesson:

We assemble a graph that our students cannot see.
Our students ask us questions about our unseen graph.
We answer only the questions that we are asked.
Once the questions are answered, we disassemble the graph.
Our students learn that the way the structure their questions determines what they will find out.

Measure to see if the goals have been reached:
We watch our students improve their questioning as we assemble and then disassemble each new graph.

Lesson Four Summary

Goals for our students:
Learn to ask questions about the graphs they make and the graphs they see.
Learn to write questions for graphs.
Learn to ask questions in math.
How to reach the goals:
Ask our students to ask questions about graphs they see.
Ask our students to write questions for the graphs they make.
What is important to assess:
Students' ability to think of questions they can ask.
The lesson:
We assemble a graph that our students can see.
We record the questions that our students ask.
We ask our students to add questions to the graphs they make.
We help with all the writing and the spelling required.
Measure to see if the goals have been reached:
Learning to ask questions in math is as important for ourselves as for our students. As we learn to ask questions in math, our students follow our example.

Lesson Five Summary

Goals for our students:
Learn a beginning framework for understanding why we might predict from some graphs, but not from all.
Learn a beginning framework for connecting probability to the graphs that they have made.
How to reach the goals:
Have students graph the results of square tosses.
Ask them to think about why they get the results that they do.
What is important to assess:
Students' ability to anticipate likely outcomes for five-square tossing. If they cannot, we ask them to try tossing six.
The lesson:
Our students toss one square as we compile their cumulative results.
We ask, "Why?" for any results they find.
Our students toss two, then three, then four squares as we record their cumulative results.
We continue to ask, "Why?" for any results they find.
Before our students begin their five-square tossing, we ask them to anticipate what they expect to find.
Our students toss their squares to find out if what they anticipated came to pass.
Measure to see if the goals have been reached:
Our measure is our students' ability to anticipate the results for five-square tossing.

Lesson Six Summary

Goals for our students:
Learn a frame of reference for making connections between graphs.
Learn to see a new kind of pattern in events.
How to reach the goals:
Help our students make a connection between ways that are possible and ways that actually occur.
What is important to assess:
Students' ability to provide an explanation for the most frequently rolled number combinations as they create their four-dice graphs.
The lesson:
Our students roll one die as we compile their cumulative results.
We ask, "Why?" for any results they find.
Before our students roll two dice, we help them make a list of all the possible combinations for a two-dice roll.
Our students then roll their dice as we compile their cumulative results.
Students repeat the listing, rolling and compiling for three dice.
We ask our students to predict the likely winning combinations for their four-dice graph before they even start to roll.
Measure to see if the goals have been reached:

The goals have been reached if our students can tell us why some numbers never win, why some win more times than not and what the reasons are.

Lesson Seven Summary

Goals for our students:

Learn to apply the one-die, four-dice frame of reference to the graphs that they have made or that they will make.

Learn how to tell which graphs may be useful for predicting and which may not.

How to reach the goals:

Ask students to examine the graphs they have already made to determine what kinds of graphs they are.

What is important to assess:

Students' ability to see that some graphs are like one-die graphs, useless to predict. Others are like four-dice graphs, replicable.

The lesson:

We ask, "I wonder which graphs we have already made are like rolling one die and which graphs are like rolling four dice at once?"

Measure to see if the goals have been reached:

We measure our students understanding of the one-die, four-dice concept of analyzing graphs by listening to our students talk about their graphs and what the data from their graphs might mean.

Chapter 9

Measurement, Estimation and Time

Lesson One Summary

Goals for our students:

Learn that measurement is a part of nearly everything we do in school.

Learn that measurement is a part of life.

How to reach the goals:

Create an environment in our classroom in which measurement is a natural part of nearly everything we do.

Connect measurement to life.

What is important to assess:

There is no one thing our students need to know.

The lesson:

We create an environment in our classroom that uses measurement as a tool for finding out.

Measurement is in the questions that we ask.

Measurement is in the opportunities that arise.

Measure to see if the goals have been reached:

How do we measure a state of mind?

Lesson Two Summary

Goals for our students:

Learn to make and read maps.

Learn to refine measuring skills.

Learn that mathematics is meant to be used.

How to reach the goals:

Ask students to make maps.

Help students develop their mapmaking skills.

What is important to assess:

Whether our students' skills grow with experience and with time.

The lesson:

We teach our students to record geoboard shapes on paper.

We teach our students to build and describe with wooden cubes.

We ask our students to make maps that describe how to get from here to there.

We gradually blend more measuring skills into the maps that our students make.

We ask the questions that lead mapping into measurement.

Measure to see if the goals have been reached:

The measurement is the maps that our students make.

Maps that are usable by someone else are maps that have reached the goals.

Lesson Three Summary

Goal for our students:

Learn what being in balance means.

How to reach the goal:

Provide the balancing materials and the questions.

What is important to assess:

Students' ability to use balances to find weights.

Students' ability to anticipate what will balance and what will not.

The lesson:

We provide our students with: board and fulcrum balances, bowl balances and double-arm balances.

We ask questions that guide our students' discoveries.

We allow the pacing and direction of the lesson to be governed by our students' interest and discoveries.

Measure to see if the goal has been reached:

We ask our students to make their own balances.

We observe to see that our students can use a balance to measure weight.

Lesson Four Summary

Goal for our students:

Learn that learning about time is as natural as learning about life.

How to reach the goal:

We make time a conscious part of every child's life.

What is important to assess:

Our students' awareness of time.

The lesson:

We teach time like we teach language.

We teach time by using it.

Measure to see if the goal has been reached:

It is more important to measure our ability to make time a conscious part of every child's life than to see what specific knowledge students have gained today.

Lesson Five Summary

Goals for our students:

Learn how to make good estimates.

Learn that making good estimates involves connecting past experiences to present situations.

How to reach the goals:

Add estimating questions to the questions that we ask.

What is important to assess:

Estimation itself is an assessment of what our students understand.

The lesson:

Estimation is a question that we add to the questions we have asked before.

We ask how many, how high, how far, how soon and on and on.

The most important part of teaching estimation is encouraging its use.

Measure to see if the goals have been reached:

We ask our students if the answers they find make sense.

Can they also answer why or why not?

Chapter 10

Beginning Multiplication and Division

Lesson One Summary

Goals for our students:

Learn what it means to multiply.

Learn that multiplication is a record of a pattern for numbers added over and over again.

Learn to make and use a multiplication matrix.

How to reach the goals:

Have students record answers to multiplication problems on matrices.

Ask students to search the matrices for the patterns to be seen.

What is important to assess:

Students' ability to see that the matrix they are making is one they have already made.

The lesson:

We have our students create and record multiplication problems.

We show our students how to record their answers on a matrix.

Our students look at each matrix they create for the patterns they can find.

We ask our students to use the matrices they create to answer questions to problems that we pose.

Measure to see if the goals have been reached:

When our students can recognize that each new matrix they make is not new at all, they have learned what they need to know.

Keep on teaching until everyone understands:

Our students keep on making matrices until they see that the matrix they are making is one they have already made.

Lesson Two Summary

Goal for our students:

Learn to create multiplication and division problems, with and without remainders.

How to reach the goal:

Teach students to create and record multiplication and division problems with squares.

What is important to assess:

Students' ability to answer three different questions for each problem they create.

The lesson:

We have our students create and record problems with squares in a sideways L.

The problems become either multiplication or division based on the three questions that we ask.

Our students create their own problems without, then with, remainders.

Measure to see if the goal has been reached:

The goal has been reached when our students can answer the three questions that we ask.

Lesson Three Summary

Goals for our students:

Learn to look for nonnumeric patterns.

Learn to connect nonnumeric patterns to the patterns in number matrices.

How to reach the goals:

Show students how to make and break Unifix patterns and look at the patterns they create.

What is important to assess:

Can our students see the patterns to be found in the Unifix Cube patterns they create.

Whether the Unifix Cube patterns help our students expand the patterns they can see in the matrices on the wall.

The lesson:

We have our students create A,B trains with Unifix Cubes.

Our students break the trains apart and look for patterns in the cubes.

We ask our students to use the patterns that they see to predict the patterns they will see in trains not yet broken.

We ask our students to look anew at the matrices on the wall to see what new relationships they might discover.

Measure to see if the goals have been reached:

The goals have been reached when our students can anticipate the pattern in the next Unifix Cube break apart.

Lesson Four Summary

Goal for our students:

Learn the answers to the number facts for multiplication.

How to reach the goal:

Provide students with stacks of cards, or have them create their own, so that they can learn to say the answers to simple problems automatically.

What is important to assess:

That we have been careful not to equate the learning of basic multiplication facts with tests of speed.

The lesson:

We present our students with sets of flash cards and try to use them imaginatively. We reduce the number of facts to be learned by using patterns from a multiplication matrix to identify uniqueness.

Measure to see if the goal has been reached:

Our students measure themselves. They know when they know. They keep working over time until every fact is in their heads.

Keep on teaching until everyone understands:

The flash cards are available to our students until everyone has learned.

Lesson Five Summary

Goals for our students:

Learn to think about what the numbers in multiplication and division problems mean.

Learn that the recording method students use should be the one most suitable for the problem at hand.

Learn that problems do not have to come from a book.

How to reach the goals:

Create multiplication and division problems for students and ask them to add descriptive words to the answers they find.

What is important to assess:

Students' ability to identify what the numbers in the problem represent.

The lesson:

We create multiplication and division problems for our students to solve using people and objects in our class.

We ask our students to identify what the numbers in the answers represent.

If we choose, we present the problems we create in patterns that can be recorded on matrices.

Measure to see if the goals have been reached:

The measure of the goals comes in Lesson Six, when we ask our students to create problems for themselves.

Lesson Six Summary

Goals for our students:

Learn to create stories for multiplication and division problems.

Learn to construct meanings for what symbols represent.

Learn to see stories in the numbers that are all around.

Learn that writing is a part of math.

How to reach the goals:

Have students write and draw stories for the problems we provide.

Help students to look for number stories in their own lives.

What is important to assess:

Whether students know what the numbers in their problems mean.

The lesson:

We provide simple multiplication and division problems to our students.

We ask our students to write or draw stories that show they understand what the numbers in our problems mean.

We encourage our students to be inventive in the stories they create.

We read the most inventive stories aloud to encourage even more creative efforts.

We ask our students to look at their own lives to find math stories.

If we choose to, we select some student stories as problems for the class to solve.

Measure to see if the goals have been reached:

Every story a child writes or draws is a measure of what that child understands.

Lesson Seven Summary

Goals for our students:

Learn that problems are everywhere around.

Learn to seek and find mathematics everywhere students go.

How to reach the goals:

We ask our students to find the multiplication and division problems that already surround us all.

What is important to assess:

Students' ability to find the multiplication and division problems already there.

The lesson:

We ask our students to write down the multiplication problems that window patterns represent.

We ask our students to find other designs that represent multiplication in their lives.

We expand our students' search to include division as well.

We learn to ask the multiplication and division questions already present in much of what we do.

Measure to see if the goals have been reached:

The goals have been reached when our students do not need a book to tell them when or what to multiply and divide.

Chapter 11

Fractions, Ratios, Money, Decimals and Percent

Lesson One Summary

Goals for our students:

Learn about fractions informally, the way students do before they come to school.

Learn to say fraction words and to write fraction numbers.

How to reach the goals:

Have students use Power Blocks to make fraction discoveries on their own.

Guide students' learning by asking questions.
Teach students to say fraction words and to write fraction numbers as they discover fractions with their Power Blocks.

What is important to assess:

Students' ability to find the value of the different Power Block pieces once we tell our students which piece is "one."
Students' ability to describe in words and numbers the fractions that they find.

The lesson:

Our students free explore Power Blocks as we guide their explorations with our questions.
We teach our students the words and numbers to use to record the fractions that they find.

Measure to see if the goals have been reached:

We say, "The S-5 square has an area of one. What are the areas of the other pieces?"
Our assessment is in the answers to the questions.

Keep on teaching until everyone understands:

The Power Blocks will be with our students all year long. The opportunity to learn will be with them all year long, as well.

Lesson Two Summary

Goals for our students:

Learn to find the areas of shapes on geoboards.
Learn that the knowledge and the proofs students developed for Power Blocks connect to their work with geoboards.

How to reach the goals:

Have students make shapes on their geoboards with and without fractional areas.
Teach students the techniques they need to prove the areas of the shapes they make.
Have students search for patterns in the areas that they find.

What is important to assess:

Students' ability to find the areas of shapes on their geoboards.
Students' ability to prove the areas that they find.

The lesson:

Our students make shapes on their geoboards with areas that we dictate to them.
Students must prove the area of each shape.
We teach a variety of techniques for proving the areas that our students find.
We expand the range of shapes as our students' skills progress.
We focus our students' attention on the areas for triangles and ask them to search for patterns in the areas that they find.

Measure to see if the goals have been reached:

We ask students to prove in words or pictures the areas that they find.

Keep on teaching until everyone understands:

We keep on teaching until every student can find areas and prove them.

Lesson Three Summary

Goals for our students:

Learn that fractions are special numbers that describe a relationship between parts and wholes.
Learn that people can be wholes and fractions, too.
Learn to add and subtract simple fractions.

How to reach the goals:

Use the people in our room to create problems for students to solve.

What is important to assess:

Students' ability to describe themselves as fractions.
Students' ability to add and subtract themselves as fractions.

The lesson:

We use the people in our room to create fractions.
We teach our students how to record the fractions that we create.
We use people fractions to create addition and subtraction problems for our students to record and solve.
Our students create addition and subtraction problems for themselves.

Measure to see if the goals have been reached:

We ask our students to create stories to go with numbers that we provide.
Our measure is whether our students can create an explanation for the numbers that they see.

Keep on teaching until everyone understands:

We keep on teaching until every student's chalkboard contains a correctly written answer to the people problems we create.

Lesson Four Summary

Goal for our students:

Learn to be aware of the fractions in our lives.

How to reach the goal:

Talk about fractions and fraction situations with the students.

What is important to assess:

Students' ability to describe fraction situations in their lives at school and at home.

The lesson:

We ask our students to think about the kinds of fractions that may or may not be created from sharing, cutting and dividing.

We ask our students to search their homes for fraction-creating situations.

Measure to see if the goal has been reached:

We ask our students to write down everything they think they know about fractions.

Keep on teaching until everyone understands:

The awareness that we create is a part of our continued goal of connecting math in school to math in life.

Lesson Five Summary

Goals for our students:

Learn that some fractions have the same value as other fractions.

Learn about equivalencies.

How to reach the goals:

Use paper folding and Power Blocks to help our students make lists of equivalent fractions to examine for patterns.

What is important to assess:

Students' ability to see a relationship between the numerators and denominators of equivalent fractions that might help them predict what an equivalent fraction might be.

The lesson:

We have our students fold paper to generate lists of equivalent fractions.

They examine their lists for patterns they see.

We have our students use Power Blocks to generate new lists of equivalent fractions.

They examine their new lists for patterns they see.

As our students grow in knowledge, we ask them to predict the numbers they will find before the Blocks tell them what the numbers are.

Measure to see if the goals have been reached:

We observe to see if our students can use the patterns they see to help them predict in advance the equivalent fractions they will find with their Power Blocks.

Lesson Six Summary

Goals for our students:

Learn to use equivalencies to find answers for adding and subtracting unlike fractions.

Learn that factors are patterns that students can discover for themselves.

Develop an awareness of factors and equivalencies that will help students understand later on when learning the relationship between fractions and decimals.

How to reach the goals:

Use cube sticks to help students generate lists of fractions to examine for patterns.

Have students use Start with, go bys to know which cube sticks to use to solve the fraction problems that they face.

What is important to assess:

Students' ability to add and subtract unlike fractions.

Students' ability to prove the correctness of their answers.

The lesson:

We teach our students how to use cube sticks to generate lists of fractions.

Our students search their lists of cube-stick fractions for patterns.

What happens next in the lesson depends on what happens when we give our students fraction problems to solve.

Our students will likely that learn to use Start with, go bys to help them know which cube sticks are the best to solve the fraction problems.

Measure to see if the goals have been reached:

We use the materials and the situations in our class to create problems that have meaning for the students.

We observe to see that our students can explain their answers.

Keep on teaching until everyone understands:

We do not leave the lesson until every child in our room can explain why 1/2 + 1/4 is 3/4 and not 2/6.

Lesson Seven Summary

Goals for our students:

Learn to see fractions as a part of measurement and not just as slices of a pie.
Learn to estimate fractions of a length.

How to reach the goals:

Connect the estimation learned in *Measurement* with estimating fractional lengths.

What is important to assess:

Students' ability to see fractions in length.

The lesson:

We ask our students to use strips of paper to estimate, then measure objects in the room.
The strips of paper we provide produce fractions in the measurements our students make.
We teach our students ways to find fractional values of the strips of paper we provide.
The measuring strips become an introduction to the reasons for the mark on rulers.

Measure to see if the goals have been reached:

We ask our students to show us fractions on a line that we call "one."

Lesson Eight Summary

Goals for our students:

Learn what a ratio is.
Learn to see practical applications of ratios and equivalencies in life.

How to reach the goals:

Teach students how to use calculators to convert numbers into ratios.
Give students practical problems to solve that use ratios.

What is important to assess:

Students' ability to find ratios.
Students' ability to use the ratios they have found to make predictions for numbers equivalent to the ratios.

The lesson:

We use shadows, bouncing balls, diagonals of rectangles and any other opportunities that we can find to give our students practical experience in finding ratios.
Our students use the ratios that they can find by measuring shadows to estimate heights of objects too high to reach.

Measure to see if the goals have been reached:

We observe to see if our students know which two numbers they should divide to find a ratio and if they know to keep the order of division the same for each equivalent ratio that they calculate.

Lesson Nine Summary

Goals for our students:

Learn what rates and rate tables are.
Learn to see practical applications for rates in life.

How to reach the goals:

Teach students what a rate is.
Give students practical problems to solve that use rates.

What is important to assess:

Students' ability to make a rate table.
Students' ability to use rates and rate tables to answer questions.

The lesson:

We teach our students a definition of rate.
We ask our students to search out examples of rates in the newspaper and in their lives.
We teach our students how to make rate tables for selected rates that we and they have found.
We ask our students to use the tables they create to answer questions about rates.

Measure to see if the goals have been reached:

We observe to see if our students can create rate tables.
We observe to see if our students can use their rate tables to answer our questions about rates.

Keep on teaching until everyone understands:

Each new kind of fraction lesson that we teach provides more practice with the fraction concepts that have gone before. Rates are ratios extended over time. Fraction lessons are extended over time, as well. There is no end to the opportunities we provide.

Lesson Ten Summary

Goal for our students:

Learn what kind of fractions decimals are.

How to reach the goal:

We use base-ten blocks and rectangles to refine the decimal knowledge that our students already possess.

What is important to assess:

Students' ability to place the decimal correctly.
Students' ability to read the decimal answer that they write.
Whether students know what their answers mean.

The lesson:

We use base-ten blocks to teach our students the kind of fraction that a decimal is.
Once we have presented the concept of the decimal fraction, the questions that we ask show us the assistance our students need.
We teach our students how to use their calculators to convert fractions to their decimal equivalents.
We use rectangles and "one is what we say it is" to allow our students to explore decimal multiplication for the patterns they find.

Measure to see if the goal has been reached:

We observe the decimal answers that our students write to see if our students know where the decimal goes.
We ask our students to read their decimal answers aloud.
We listen as our students explain why .5 is equivalent to 1/2 and 5/10.
We listen to our students explain why, when they multiply fractions or decimals together, the answer is smaller than either of the fractions or the decimals that they started with.

Lesson Eleven Summary

Goals for our students:

Learn the difference between decimals and percents.
Learn to understand the questions that percents represent.

How to reach the goals:

Teach students to understand the questions represented by percents.

What is important to assess:

Students' ability to use their knowledge and their reasoning to determine what the numbers mean.

The lesson:

We talk about the meaning of percent.
Every question that we ask is drawn from the answer that precedes it.
We give our students practice with percents using tipping, shopping, sports statistics and other examples from our students lives.

Measure to see if the goals have been reached:

We observe to see whether our students can calculate a percent.
We ask our students to explain the meaning of their answers.

Lesson Twelve Summary

Goal for our students:

Learn about money from using it at school and home.

How to reach the goal:

Capitalize on all the opportunities we can find to use real examples to teach money to the students in our class.

What is important to assess:

Students' ability to solve the money problems we present.

The lesson:

We use real money and real money situations to teach our students how to find answers to real-life problems.
We help the parents of our students to recognize the opportunities at home to teach money problems.

Measure to see if the goal has been reached:

We ask our students to tell us what they understand. We use their answers to determine what they understand and what is left to learn.

Chapter 12

Advanced Addition and Subtraction

Lesson One Summary

Goals for our students:

Learn that *base* means the number they are grouping by.
Learn to plus and minus one in selected bases three through ten.

Learn to see patterns in the counting numbers in selected bases three through ten.

How to reach the goals:

Show our students how to plus and minus ones in selected bases, base three through base ten.
Have students record the numbers they create and examine them for patterns.

What is important to assess:

Students' ability to plus and minus one in any base we ask.
Students' ability to record their plus and minus ones and see the patterns in the numbers.

The lesson:

We teach our students the technique of adding one over and over again in different bases.
We also teach our students the technique of taking one away.
Our students record their plus and minus numbers on recording strips and examine the numbers that they write for the patterns to be found.

Measure to see if the goals have been reached:

We observe each student's growing understanding as each student records, to know when every student understands how to plus and minus one.

Keep on teaching until everyone understands:

We teach until every one is taught.

Lesson Two Summary

Goal for our students:

Learn how to make exchanges for squares, cups and bowls when more than plus and minus one are involved.

How to reach the goal:

Teach our students racing-up and racing-back place-value games.

What is important to assess:

Students' knowledge of when to put the squares in a cup and a cup into a bowl.
Their knowledge of when to take the squares and the cups out again.

The lesson:

We teach our students how to race up to a bowl in different bases.
We teach our students how to start with one bowl, one cup and one square and race until their board is clear.

Measure to see if the goal has been reached:

Our measurements are the observations that we make as we watch our students play the games.

Lesson Three Summary

Goal for our students:

Learn that the value of a number changes depending on its place.

How to reach the goal:

Teach our students a place-value game to play.

What is important to assess:

Students' knowledge that 321 is a better arrangement then 231 or 132, when the assignment is to make the largest number they can.

The lesson:

We teach our students a three-dice place-value game.
Our students record the totals for each game they play.

Measure to see if the goal has been reached:

We look at the written records for the games our students play to see which students know the best arrangements for the numbers they have rolled.

Keep on teaching until everyone understands:

Our students play the game in base after base until we can see that every student in our room knows how to make the best use of a roll.

Lesson Four Summary

Goals for our students:

Learn to create addition and subtraction problems in any base three through ten.
Learn that they do not need the teacher to check their answers. The answers flow from the materials. The teacher is not the only one who knows what's right.

How to reach the goals:

Show our students how to create addition and subtraction problems on their trading boards.

What is important to assess:

Students' knowledge of what we are looking for on their papers as we walk around the room. Can they find it before we do?
Can our students tell if they have forgotten to use the rules for bases without our reminding them?

The lesson:

We teach our students how to create addition and subtraction problems on their trading boards.
We teach our students how to check the answers to the problems they create.

Measure to see if the goals have been reached:

The answers to the problems flow from the materials. We observe to see if students use the materials correctly.

Lesson Five Summary

Goals for our students:

Learn to see more patterns that bases have in common with other bases.
Learn to use materials to prove the answers that they find.

How to reach the goals:

Have students examine a matrix for place-value patterns in bases.
Have students calculate answers and then use materials to prove their answers.

What is important to assess:

Students' use of materials to check the answers to the problems they create with dice.

The lesson:

We ask our students to record on a matrix the number of cubes (or squares) it takes to make the first value of each column in each base.
Our students look for patterns in the matrix they create.
Our students create problems and use their skills from Lessons One through Four to predict the answers they expect to find.
Our students use materials to see how close predictions are to reality.

Measure to see if the goals have been reached:

Our observations of our students are all the measurement that we need.

Lesson Six Summary

Goal for our students:

Learn the names for numbers that they write.

How to reach the goal:

Teach students how to read large numbers.

What is important to assess:

Our students' ability to read large numbers.

The lesson:

We teach our students the names for the commas that we use to know the names for larger numbers.

Measure to see if the goal has been reached:

Our observations of our students.

Lesson Seven Summary

Goals for our students:

Learn that addition and subtraction are tools we use for finding out.
Learn to see the mathematics in nearly everything they do. Mathematics is already everywhere we look.
Learn to use mathematics to solve problems in their lives.

How to reach the goals:

We search our students' lives and our own for examples of problems that need solving.

What is important to assess:

Students' ability to see and solve problems on their own.

The lesson:

We work with our students to create real problems that use the skills we have taught.
We give our students reasons to use their growing skills in math.

Measure to see if the goals have been reached:

We measure our own ability to see the mathematics in nearly everything we do.

Lesson Eight Summary

Goals for our students:

Learn to look for patterns in the numbers they see everywhere.
Learn that they can make predictions from the numbers that they see.

How to reach the goals:

Ask students questions for the numbers that they see:
Why do you think so?
What would you predict?

What is important to assess:

Students' use of the patterns that they see to anticipate numbers coming next.

The lesson:

We ask our students to look for patterns in the numbers that they find.
We have our students look for patterns in the bases.
We have our students look for patterns in palindromes.
We have our students look for patterns in ordinary events.

Measure to see if the goals have been reached:

We observe our students anticipating what the next numbers will be.

Lesson Nine Summary

Goals for our students:

Learn that mathematics is flexible and free.
Learn that finding answers is inventive.
Learn that any way they find an answer is still called math.
Learn to get answers right enough when only thinking is allowed.

How to reach the goals:

Talk about thinking processes and strategies when finding answers in our heads.

What is important to assess:

We do not assess. We simply wonder at and encourage the imagination and inventiveness our students use in solving problems in their heads.

The lesson:

We teach our students to solve problems in their heads.
Our students use whatever they know to solve problems.
They share their techniques with one another and we share ours.

Measure to see if the goals have been reached:

We observe each student's ability to know when an answer to a problem is right enough.

Chapter 13

Advanced Multiplication and Division

Lesson One Summary

Goals for our students:

Learn what it means to multiply in any base.
Learn that there are patterns within and between matrices.
Learn a different way to think about the multiplication patterns in base ten.

How to reach the goals:

Teach students how to make multiplication matrices for every base from three to ten.
Ask students to look for and describe the patterns they can find in every matrix they create.

What is important to assess:

Students' ability to fill in as much of a blank matrix as they can from the patterns they see in their matrices.
The patterns seen are not as important as learning to look.

The lesson:

We show our students how to make a multiplication matrix in base three.
We ask them to make a matrix for base four.
We use the matrices for bases three and four to predict as much as possible what a matrix will look like for base five.
For each succeeding base, we ask our students to use the previous matrices to see what they can learn from the patterns alone.
We use squares to verify that the predictions reflect the reality of the base.

Measure to see if the goals have been reached:

Students collectively share the patterns they see; we make no individual student measurement.

Lesson Two Summary

Goals for our students:

Learn how to represent a multiplication problem larger than what a matrix shows.
Learn how to multiply in any base.

How to reach the goals:

Give students a visual image of what it means to multiply.

What is important to assess:

Students' ability to represent the abstract numbers of a multiplication problem with squares, cups and bowls.

The lesson:

We ask our students to solve larger multiplication problems than can be found on their matrices.
We provide guidance in how to use our knowledge to find the answers with squares, cups and bowls.

When our students understand the process, we ask them to use dice to create their own problems in different bases.
If we have them, we use multibase blocks.

Measure to see if the goals have been reached:
We observe our students as they create, represent and solve problems.

Lesson Three Summary

Goal for our students:
Learn practical, nonthinking ways to find answers to problems meant only for school.

How to reach the goal:
Teach students ways to find answers that are required only to pass an end-of-year test.

What is important to assess:
Students' ability to produce the kind of nonthinking answers that may still be required of them on tests meant only for school.

The lesson:
We teach our students how to calculate answers to multiplication problems when the rules of the school say that no materials, calculators, or other reasonable ways may be used.

Measure to see if the goal has been reached:
We give our students a test.

Lesson Four Summary

Goals for our students:
Learn that not every problem is a different problem to be solved.
Learn to look for patterns in the zeros of the numbers that they multiply.
Learn how to find a reasonable answer to problems that are large.

How to reach the goals:
Ask students to look for patterns in the answers for multiplying numbers by 1, 10, 100, 1,000, 10,000; 2, 20, 200, 2,000, 20,000; and so on.

What is important to assess:
Students' ability to see the effect that zeros have when added to a number by which our students multiply.

The lesson:
We give our students a worksheet for keeping track of series that they multiply.
Our students do the multiplication and look for patterns in their answers.
We ask our students to apply their growing knowledge of zeros to make reasonable approximations of answers not yet found.

Measure to see if the goals have been reached:
We ask our students to tell us the effect that certain numbers of zeros in a multiplication problem will have on the answer. We listen to what our students say.

Lesson Five Summary

Goals for our students:
Learn to apply the skills they now have.
Learn that problem solving and thinking go hand in hand.

How to reach the goals:
Connect the process of multiplying with the uses of multiplication.

What is important to assess:
Students' knowledge of when it is appropriate to use multiplication to find an answer to a problem.

The lesson:
We look around our world for real problems we can give our students.
We teach our students to find problems of their own to solve.
Most of the problems we give require multiplication, but most does not mean all.

Measure to see if the goals have been reached:
Our goals are reached when our students come to us with problems that they have found to solve.

Lesson Six Summary

Goal for our students:
Tie all the learning that has gone before to an understanding of what it means to divide numbers in any base and any size.

How to reach the goal:
Show students how to find answers to division problems in different bases using squares, cups and bowls.
Have students practice their dividing skills with problems they create with dice.

What is important to assess:

Students' ability to translate the abstract numbers of a division problem into a problem they can solve with squares, cups and bowls.

The lesson:

We give our students a base-five division problem to solve and the materials with which to solve it.
We ask our students to find the answer any way they can.
We discuss the ways they have used and, if needed, add ways of our own.
We show our students how to create base-five division problems using dice.
Our students keep creating problems in different bases until we feel confident they understand the process.
We save base ten and calculators until our students can already divide.

Measure to see if the goal has been reached:

We make our measurement as we walk around our room observing the understanding our students demonstrate in their work.

Lesson Seven Summary

Goal for our students:

Learn again that mathematics is a tool we use for finding out.

How to reach the goal:

We use division to analyze and take averages.

What is important to assess:

Students' application of the division skills they have learned in school.

The lesson:

We look around for problems that use division and all the other kinds of math we teach.

Measure to see if the goal has been reached:

The most important measurement we make is of our own ability to see problems we can use to show our students the usefulness of math.

Chapter 14

Algebra

Lesson One Summary

Goal for our students:

Learn to write symbolic equations or formulas for familiar patterns and experiences..

How to reach the goal:

Students look at geoboard, Power Block, and wooden cube patterns and learn to use letters to record the patterns that they see.

What is important to assess:

Students' ability to translate numerical patterns into letter form.

The lesson:

We ask our students to look at patterns they have seen before in S-1 squares, geoboards, wooden cubes and Power Blocks.
We teach our students to record numeric patterns algebraically in letters we call formulas.

Measure to see if the goal has been reached:

The difference between arithmetic and algebra is not the concepts taught, it is the recording used. We observe to se if our students can use letters to record patterns seen.

Lesson Two Summary

Goal for our students:

Learn to plot coordinate points on a graph.

How to reach the goal:

Have students play games like coordinate tic-tac-toe and battleships that use coordinates to mark their moves..

What is important to assess:

Students' ability to describe a single point on a grid or graph using ordered pairs of numbers.

The lesson:

We play games of coordinate tic-tac-toe with our students, adding the negative quadrants once the positive is understood.
We have our students play the game of battleships.

Measure to see if the goal has been reached:

We observe to see if our students can use pairs of numbers to describe points. Do our students know that (2, 3) describes a different point than (3, 2)?.

Lesson Three Summary

Goal for our students:
Learn to add, subtract, and multiply signed numbers.
How to reach the goal:
Teach letter carrier stories to our students that provide a framework for understanding arithmetic operations with positive and negative numbers.
What is important to assess:
Students' knowledge of the rules for operations with signed numbers.
The lesson:
We tell letter carrier stories as our student write the signs and numbers that the stories represent.
We write signed number problems and ask our students to tell us stories for the numbers and the signs..
Measure to see if the goal has been reached:
We listen to the stories our students tell to accompany the problems that we write to see if they know the rules for working with positive and negative numbers.

Lesson Four Summary

Goals for our students:
Learn to write tables for paired numbers that are functions of each other.
Learn to predict unseen numbers from the numbers seen.
How to reach the goals:
Teach our students the use of a function machine.
Have students construct their own number relationships for use in the machine.
Have students write formulas or equations for the patterns seen.
What is important to assess:
Students' ability to predict the number out from the number in.
The lesson:
We introduce the function machine "Magic Box" with pre-made cards.
Students create their own cards for use in the machine.
Numbers in and out are recorded in table format.
Students write formulas or equations for the cards' rules.
Measure to see if the goals have been reached:
We observe our students' ability to make tables for numbers that are ordered pairs.

Lesson Five Summary

Goal for our students:
Learn to graph functions and the equations the functions represent.
How to reach the goal:
Students graph the data from the tables form Lesson Four.
Students graph multiplication facts, toothpick patterns, and formulas like $lw = a$ or $c = \pi d$.
What is important to assess:
Students' ability to plot ordered pairs of numbers on a graph and connect the plotted points. Can our students see the connection between the numbers and the line?
The lesson:
We show our students how to graph equations like $x + y = 10$.
Students graph the data from the tables in Lesson Four.
Students graph areas for squares and rectangles and use their graphs to find lengths and widths for rectangles they cannot make with S-1 squares.
Students graph diameters and circumferences to discover π.
Students graph Number facts for multiplication.
Students graph toothpick designs.
Measure to see if the goal has been reached:
We observe our students graphs to see if they can record data coordinately.

Lesson Six Summary

Goal for our students:
Learn to look at $y = mx + b$ equations and anticipate each equation's graph.
How to reach the goal:
Students graph equations in $y = mx$ and $y = mx + b$ formats and look for slope and intercept patterns in the lines.
What is important to assess:
Students' knowledge of the relationship between slope, intercept, m and b in the $y = mx + b$ equation.
The lesson:
Students graph $y = x$, $y = 2x$, $y = 3x$ and so on, and predict the changes in their graphs the changing numbers produce.

Students graph y = x +2, y = x + 3, y = x + 4, and so on, and predict the changes in their graphs the changing numbers produce.

Measure to see if the goals have been reached:

We observe to see if our students can predict the effect of m and b on graphs of equations in the y = mx + b format.

Lesson Seven Summary

Goals for our students:

Learn to look at $y = mx^2 + b$ and $y = mx^3 + b$ equations and predict the equation's graph.
Learn the meaning of square root.

How to reach the goals:

Students graph equations in the $y = mx^2 + b$ and $y = mx^3 + b$ format and look for patterns in the lines.

What is important to assess:

Students' knowledge of the relationship between exponents and a graph's line.

The lesson:

Students graph equations in the $y = mx^2 + b$ and $y = mx^3 + b$ format and observe the effect that exponents have.

Measure to see if the goals have been reached:

We observe to see if our students can anticipate the effect exponents have on a graph.

Lesson Eight Summary

Goals for our students:

Learn to multiply (x + y)(x + y) equations.
Learn to connect algebra to arithmetic.

How to reach the goals:

Students use drawings to solve (x + y)(x + y) equations.
Students connect their algebraic solutions to arithmetic.

What is important to assess:

Students' knowledge of the connection between the letters and the arithmetic the letters represent.

The lesson:

We illustrate 12 x 12 in various forms on graph paper.
We show our students how to solve (x + y)(x + y) using drawings.
Students use drawings to solve similar equations.
Students substitute numbers for the letters in the equations to connect algebra to arithmetic.

Measure to see if the goals have been reached:

Can our students explain why $(x + y)(x + y) = x^2 + 2xy + y^2$.
Can our students connect the letters to the arithmetic the letters represent.

Lesson Nine Summary

Goal for our students:

Learn that the rates, ratios, proportions and equivalencies that were part of fractions are a part of algebra as well.

How to reach the goal:

Students link the rates, ratios, proportions and equivalencies of fractions to algebra by graphing the data and writing the algebraic formula for the graph.

What is important to assess:

The connection students make between past lessons and algebra. Can our students write formulas for past patterns they have seen.

The lesson:

Students graph relationships from earlier fraction lessons and look for equations in their graphs.

Measure to see if the goal has been reached:

Can our students graph data from earlier arithmetic experiences and convert the graph to an algebraic formula?

Lesson Ten Summary

Goal for our students:

Learn the answer to the question. "What do you use algebra for?"

How to reach the goal:

Create an environment in our classroom in which algebra is a natural part.
Connect algebra to life.

What is important to assess:

Students' knowledge that algebra has a use.

The lesson:

We create an environment in our classroom that uses algebra as a tool for finding out.

Measure to see if the goal has been reached:

Can our students find uses for algebra. Can we.

Chapter 15

Science

No Lesson Summaries