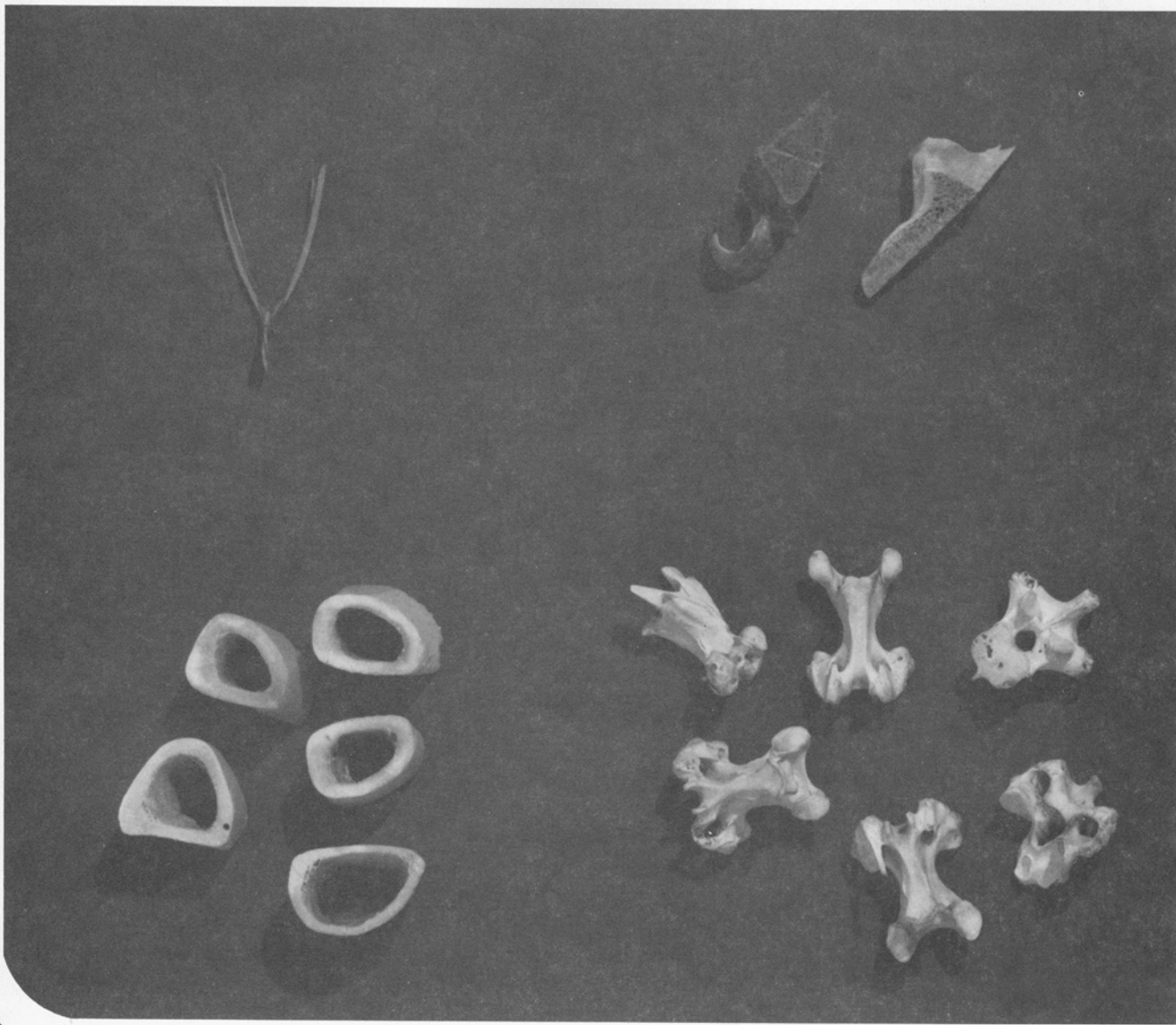
7

NUMBER AT THE CONCEPT LEVEL



7

SKILLS AND CONCEPTS

Exploring and describing the possible arrangements of a given number of objects
Exploring addition, subtraction, multiplication, and division with concrete materials
Invariance or conservation of number

SELF CONCEPT AND SOCIAL INTERACTION

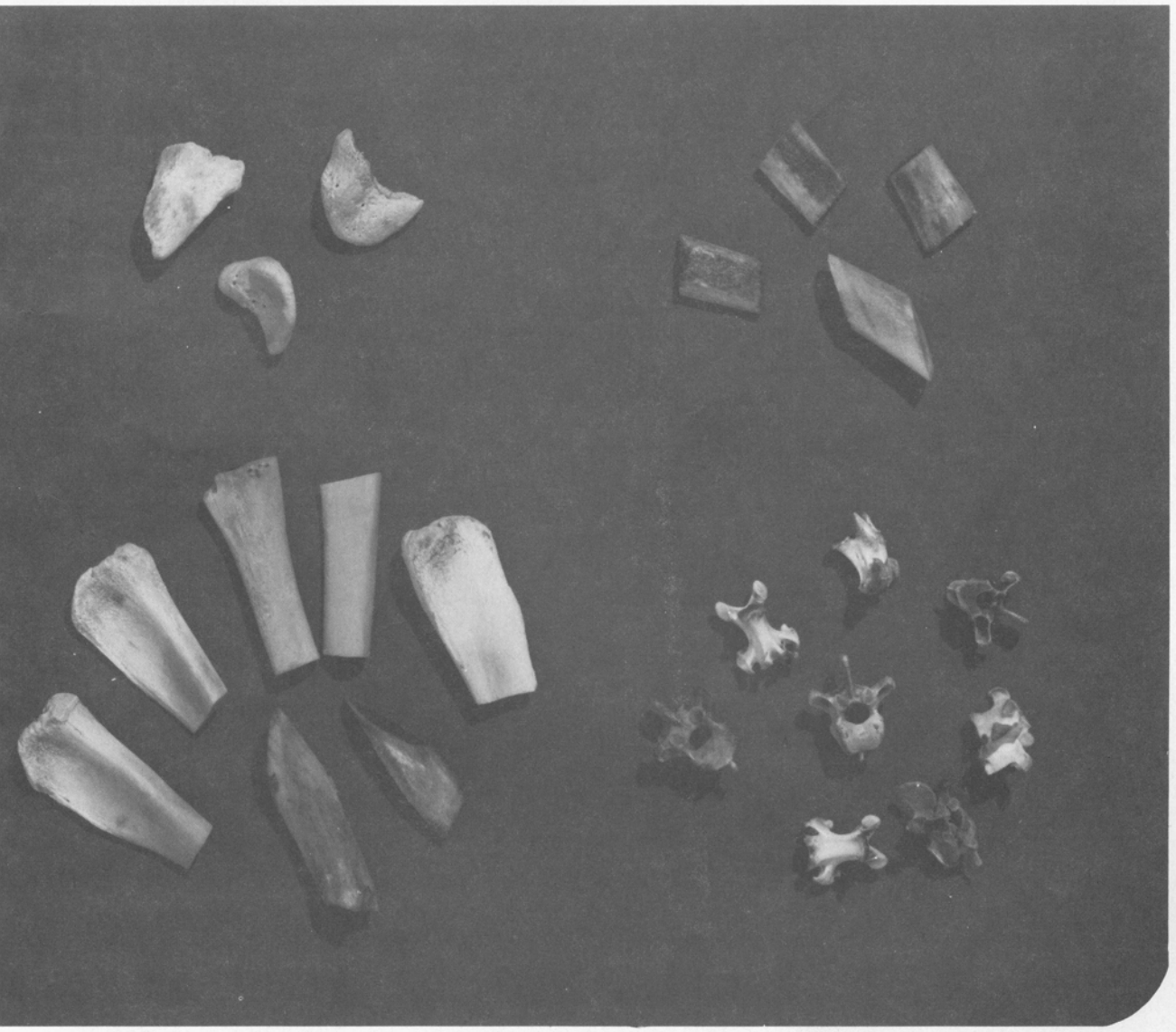
Gaining a feeling of self-worth through participating as a "teacher" as well as a learner
Enhancing one's image of self through seeing one's own work used by the whole class as an integral part of the classroom learning materials
Heightening awareness and visual imagery as a result of using all five senses in learning
Experiencing a variety of solutions to a problem, each value for its uniqueness

FUTURE APPLICATIONS

Recognizing designs and patterns in nature
Understanding the use of symbols to represent number concepts
Problem solving

PREREQUISITE CHAPTERS

Free Exploration; Counting; Comparing



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NUMBER AT THE CONCEPT LEVEL

INTRODUCTION

The activities in this chapter are used to surround children with the concept of number. Each child will explore the numbers from one to ten in a variety of ways using many different materials: toothpicks, tiles, geoboards, pattern blocks, jewels, Unifix cubes, wooden cubes, beans, and junk boxes.

The concept of number will be extended further as the children explore the relationships between the mathematical operations of addition, subtraction, multiplication, and division. The goal of these activities is the integration of mathematical relationships so the children will be able to deal flexibly with number and move from one operation to another with equal skill.

Before your class begins exploring the number stations, take half the class at a time and do the following: arrange four toothpicks in a design and show it to all the children. Then ask the children to make a different design with four other toothpicks. Talk together about the arrangements, pointing out all the different designs the children found. Allow the children to continue exploring "four" making as many designs as they can.

The goal of this activity is to help the children make the transition from free exploration to directed exploration. Since the children's previous experience was in exploring freely, in the beginning you may see children at the number stations making patterns with numbers other than the designated number. You may need to draw some children's attention to the given quantity several times before you are successful. Try redirecting their attention by selecting several designs, one at a time, and helping the whole group count the toothpicks in each one. Push any designs which are not formed with the correct quantity back into the pile. Leave the ones built with the correct quantity intact.

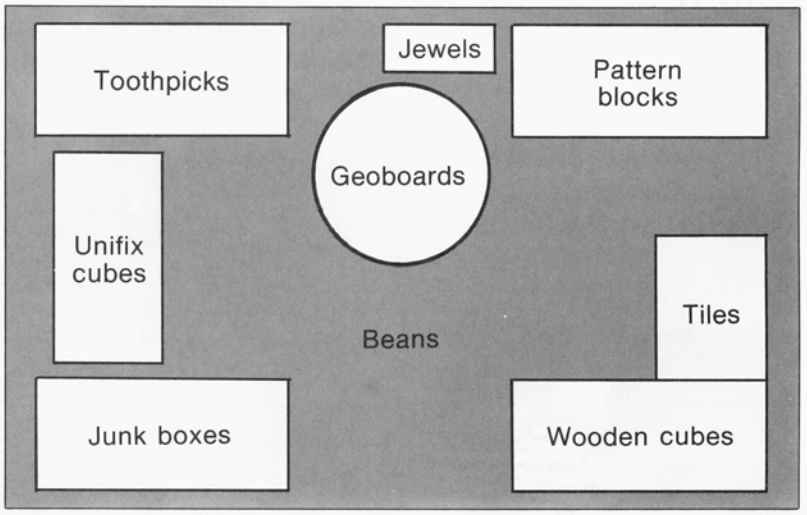
Designs have a way of evolving on their own as the children work. Often an incorrect total is the result of the child's becoming fascinated with the design and losing track of the number with which sheorhe should be working. You need to be sensitive to this, and as you redirect the child's attention, discarding the incorrectly formed designs, comment appreciatively on the designs, but remind the child that the class is to make patterns with only four objects.

All children, regardless of their grade level or apparent facility with abstract number, will benefit most from exploring the number stations by beginning with three, progressing to four, and then five, building gradually to ten. There is a great deal more happening than counting objects at these stations. Chil-

dren are discovering and internalizing the unique patterns and combinations each number forms naturally. The understanding children gain from exploring the numbers below six have a profound impact on their later explorations, for understanding all the numbers greater than five is based on concepts discovered for five and below. If a child sees a group of seven objects pushed together sheorhe would not immediately think seven. The child separates them in hisorher mind, seeing perhaps three and four, and then thinking, seven.

These experiences are the building blocks for the later stages of the child's number concept development in mathematics; if this foundation is firmly laid, dealing with abstract number will be effortless.

To initiate the following activities, place the toothpicks, tiles, and other materials at the different stations. Begin by having the children explore "three" with each material. Encourage them to move from station to station when they feel ready to explore a new material.



This gives your students the responsibility for making their own decisions. Observe how each student handles this freedom. Expect to see some of the children spending only a few minutes at each station and two or three children spending the whole period intensely absorbed in exploring one material. The majority of the class will move to a new station every ten or twelve minutes.

It is not important for every child to have equal contact with each material. What is important is that the children have the freedom to explore a given number with whatever materials are especially appealing to each one of them at a given moment. You will observe certain materials fading in and out of popularity with individual children as well as with the entire class. This evolution is natural and will increase your personal enjoyment and knowledge as you observe it over the year.

The Number Stations

Toothpicks

SKILLS

Counting
Discovering relationships among different quantities
Creative expression
Learning invariance of number
Exploring the possible arrangements for a given quantity of objects

MATERIALS

Several boxes of flat toothpicks

ACTIVITY

A group of children work together exploring a given quantity of toothpicks, making as many arrangements as possible. The teacher encourages the children to describe the toothpick patterns in a variety of ways.

Tiles

SKILLS

Counting
Discovering relationships between different quantities
Creative expression
Learning invariance of number
Exploring the possible arrangements for a given quantity of objects

MATERIALS

Ceramic tiles*

ACTIVITY

A group of children work together exploring the possible arrangements of a given quantity of tiles. Every tile must touch a corner or at least part of the side of another tile. Encourage the children to describe the patterns in a variety of ways.

Pattern Blocks

SKILLS

Counting
Discovering relationships among different quantities
Creative expression
Learning invariance of number
Exploring the possible arrangements for a given quantity of objects

MATERIALS

Pattern blocks* in two different shapes

ACTIVITY

A group of children arrange a given quantity of pattern blocks, making several different designs. The teacher encourages the children to describe the patterns in a variety of ways.

Jewels

SKILLS

Counting
Pattern
Learning invariance of number
Exploring the possible arrangements for a given quantity of objects

MATERIALS

Jewels* and counting cups*

ACTIVITY

Each child takes a handful of jewels and divides the jewels into counting cups so that each cup holds the quantity being explored.

Expect to see the children creating a wide variety of combinations. Anything that totals five, in this case, is correct. The children will discover on their own that some jewels can never be used for particular numbers. For example, when exploring three, all the four and five jewel lengths will be sorted out naturally.

When all the jewels are divided, the children sort the cups according to the combinations formed.

Unifix Cubes

SKILLS
Counting
Thinking logically
Making patterns
Learning invariance of number
Exploring the possible arrangements for a given quantity of objects

MATERIALS
Unifix cubes* in two colors

ACTIVITY
Each child in the group uses two colors to make different patterns with the number being explored. The teacher asks the children to describe the stacks of cubes by determining the total number of each color or the quantity of each color that is grouped together as they read from left to right.

Wooden Cubes

SKILLS

Counting
Discovering relationships among different quantities
Creative expression
Learning invariance of number
Exploring the possible arrangements for a given quantity of objects

MATERIALS

Wooden cubes*

ACTIVITY

The children take a given quantity of cubes and arrange them in many different designs. In this activity children have an opportunity to build multi-layered patterns. The teacher should encourage the children to describe the arrangements in different ways as they build.

Beans

SKILLS

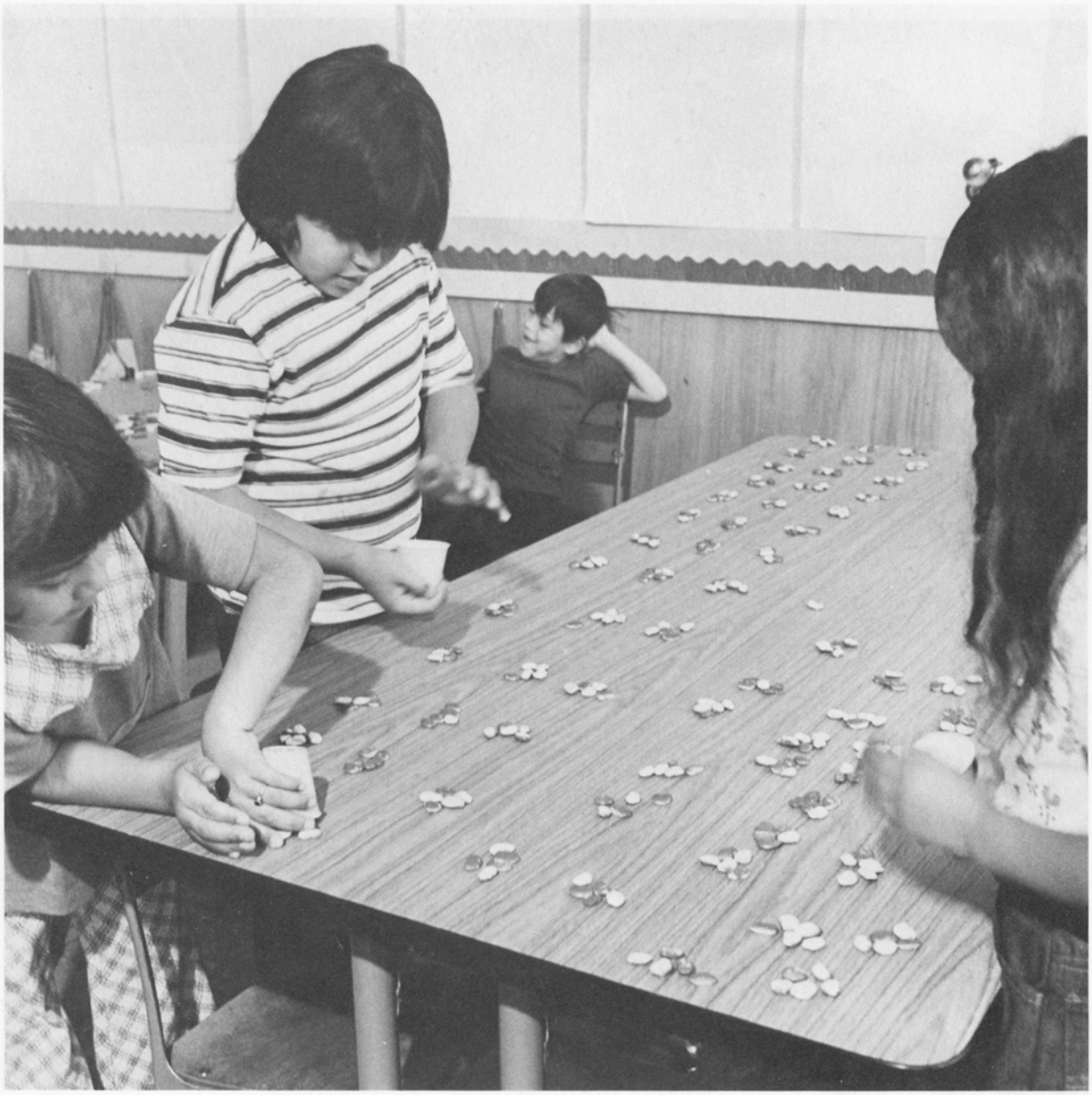
Counting
Probability
Making and checking predictions
Learning invariance of number
Exploring the possible arrangements for a given quantity of objects

MATERIALS

Two-sided beans* and paper cups

ACTIVITY

Each child takes a given number of beans and shakes them in a paper cup. The beans are spilled out onto the table and the children read the combinations. To establish a consistent working pattern, have the children read the painted beans first.



Junk Boxes

SKILLS

Counting
Learning invariance of number
Exploring the possible arrangements for a given quantity of objects

MATERIALS

Junk boxes*

ACTIVITY

A group of children work together with a given quantity, making as many different arrangements of junk as possible. The teacher encourages the children to describe the various arrangements.

Geoboards

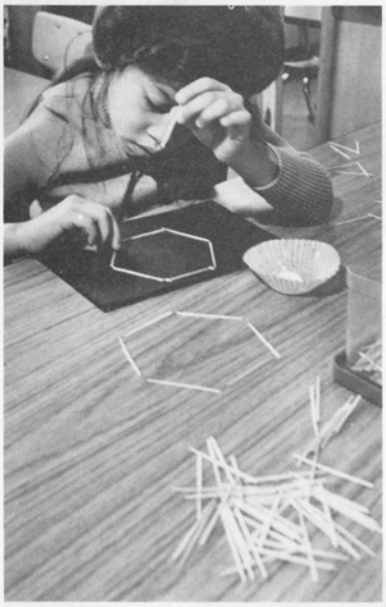
SKILLS
Counting
Exploring the possible arrangements for a given quantity of objects

MATERIALS
Geoboards, geobands, squares of paper which fit between the nails on the geoboards

ACTIVITY
Several children work together exploring a given quantity. They attempt to find many different arrangements. Each square must be placed adjacent to another square.

Recording at the Number Stations

When the children have explored a given number using many different materials and arrangements, they are ready to record their explorations of that number.

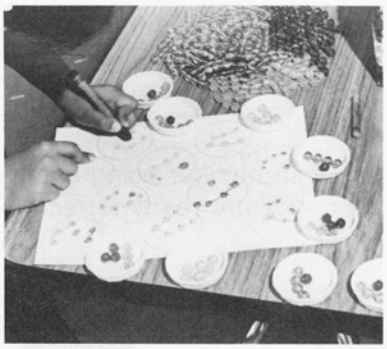


Toothpick patterns: Glue toothpicks onto 15 cm × 23 cm or 6″ by 9″ pieces of black construction paper.

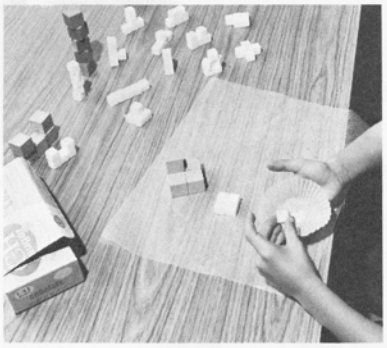
Tile patterns: Glue pre-cut squares of construction paper onto 15 cm × 23 cm or 6″ by 9″ pieces of drawing paper or trace the outlines with the square template* and color in the designs.

Geoboard patterns: Glue construction paper squares onto geoboard dot paper or trace the outline of the rubber band onto the paper. (See Worksheet 17.)

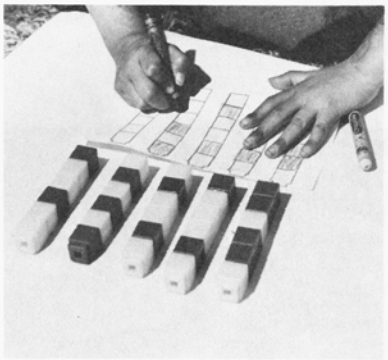
Pattern block designs: Use the pattern block template* or glue colored construction paper shapes onto 15 cm × 23 cm or 6″ by 9″ drawing paper (cut out by the children from Worksheets 2–6).



Jewels: Color in the appropriate jewel color on the worksheet (see Worksheet 27).

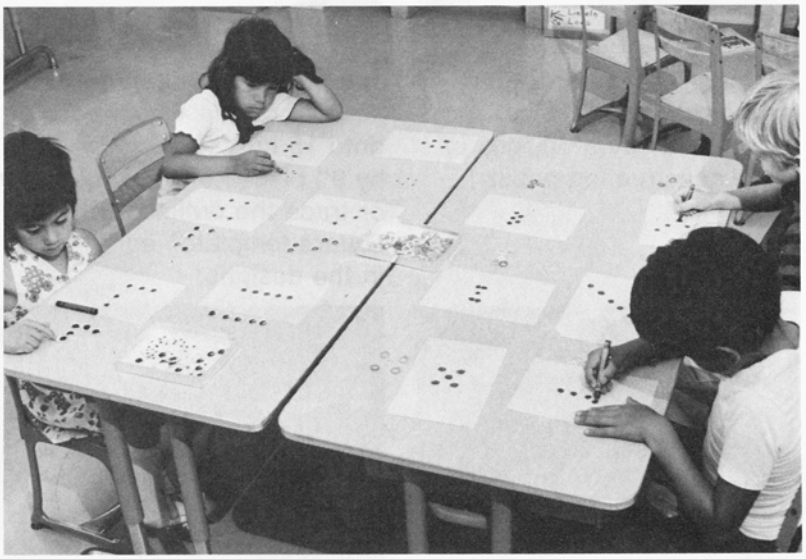


Cube patterns: Glue sugar, Styrofoam, or balsa wood cubes together with white glue.



Unifix patterns: Color in the appropriate squares on the Unifix worksheet. (See Worksheets 28–32.)

Beans: Color the bean shapes on the bean worksheet (see Worksheets 33–39).



Junk: Color black dots on 15 cm × 23 cm or 6″ by 9″ drawing paper to show the arrangement of the junk.

The children's records should not be made for the sake of recording. The records made at this time will be used for additional learning experiences at a later time. These records should be kept at school for use at the symbolic stage of number concepts, when they will be made into number books.

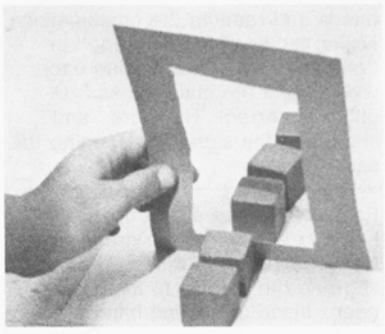
APPLICATION AND EXTENSION OF NUMBER AT THE CONCEPT LEVEL

When the children know the routine and are confidently exploring and recording their results at the number stations, take half the class at a time and teach them the following three games: The Hand Game, Lift the Bowl, and Peek Through the Wall.

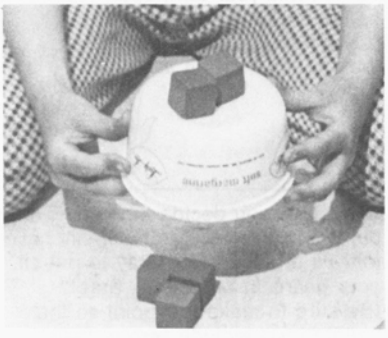
You will know that the children are ready for the games when they work independently and you begin to feel you are dispensable. This is usually after the children have explored and recorded "three," have moved on to exploring "four," and are ready to record patterns of "four."

The above three games deal with the combinations of addition and subtraction in an informal and concrete way. Through these games the children will develop the mental images which they will need at the abstract level of number concepts.

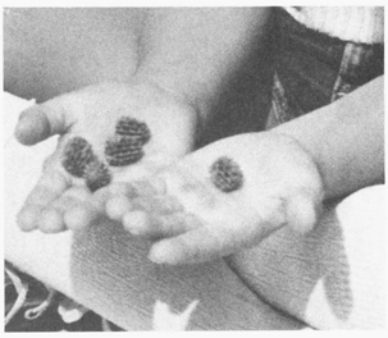
After introducing the three games one at a time, the children should play the games one after another in the same work period. This allows the children to see the same concept presented with different materials and will result in their internalizing this concept more firmly.



Peek Through the Wall



Lift the Bowl



The Hand Game

The Guide to Facilitate Classroom Planning in the appendix suggests a model for implementing the Three Games into your math period (see pp. 367–384). In addition, you should read pp. 179–187, 221–222, and 236–238 to understand the entire sequence.

The Three Games

The Hand Game

SKILLS

Subtraction
Addition
Commutativity
Visual imagery

MATERIALS

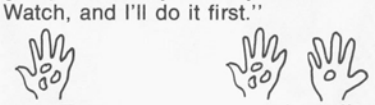
Beans or junk box* objects

ACTIVITY

The children work together in small groups separating the given quantity of beans in different way and verbalizing the combinations that result.

Teach the game initially to half the class at a time in the following way.

SAMPLE TEACHING STRATEGY

TEACHER	ACTIVITY
"Take four beans and put three of them in one hand and one in the other" (Remove the container of beans so that each child has only four beans.)	The children follow the directions.
"When it is your turn, open your hands one at a time and tell the group how many beans you see. Watch, and I'll do it first."  "Three and one."	The children practice opening their hands and reading the combinations, saying either "three and one" or "one and three," depending upon which hand the child opens first—either is correct. The word "and" should be the signal for opening the second hand.
"This time *you* decide how many to put in each hand. You must still have four all together. Is it okay to put all your beans in one hand? Sure!" (Be sure to make this point so the children will include zero in their combinations.)	Each child takes a turn opening hisorher hands and the group reads the combinations formed. The child showing the objects to the group opens hisorher second hand at the word "and": Two and two. Three and one. Four and zero. One and three.

It is important that the child showing the objects not talk. This will encourage the group to focus on the objects. When the child talks, the group tends to look at the child's mouth instead of looking at the objects to determine the quantity.

When the children understand how to play the game, divide them into small groups. This enables them to see one another's hands easily. Now that they understand how to play the game, drop out as an active participant and supervise several groups playing at the same time. As the children go around the circle many combinations will be repeated. As a result the children hear and see the addition combinations over and over again. The combinations are also reversed (three and one, one and three), giving the children an opportunity to experience commutativity as a natural and incidental by-product of the game.

Children should play this game until they are very comfortable with it, changing their beans to form new combinations when each child in the rotation has verbalized the numbers in hisorher hands. (Within any one activity period the total number of beans used should be kept constant.)

Lift the Bowl

SKILLS

Subtraction
Addition
Commutativity
Visual imagery

MATERIALS

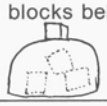
A bowl and wooden blocks for each child

ACTIVITY

The children work together in small groups arranging a given quantity of blocks and verbalizing the different combinations formed.

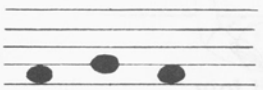
Teach the game initially to half the class at a time in the following way.

SAMPLE TEACHING STRATEGY

TEACHER	ACTIVITY
"Take four blocks and put them under your bowl." (Remove the extra blocks so that each child has *only* four blocks.)	Each child counts out four blocks and puts them under hisorher bowl.
"Take one block from underneath your bowl and put it on top." 	Each child removes one block from under hisorher bowl and puts it on top leaving, three blocks behind. 
"I am going to tell you how many blocks I have on top of my bowl, then I'll lift my bowl and tell how many blocks I have underneath. Listen to what I say and then repeat it."  "One  and three."	The children watch the teacher reading the blocks and lifting the bowl, and the group echos the combination: "One and three."
"Now, you try it. We'll say 'one and three' around the circle giving everyone a turn to lift hisorher bowl.	The children take turns practicing the procedure. Each child lifts hisorher bowl at the word "and".
"This time *you* decide how many of your four blocks to put on top of your bowl. You can use any number you want. Is it okay to use all your blocks on top? Sure!" ◆◆◆◆◆◆◆◆◆◆◆◆◆◆◆◆◆◆◆◆◆◆◆◆ *Make this point again, repeating it as often as necessary so that the children will retain the idea and include zero in their combinations.* ◆◆◆◆◆◆◆◆◆◆◆◆◆◆◆◆◆◆◆◆◆◆◆◆	Each child takes a turn to lift hisorher bowl for the group. As the combinations are shown the group reads them: "One and three; two and two; zero and four; three and one; one and three; two and two . . ."

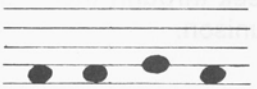
When the children understand the game, divide them in small groups of about five. This enables them to see one another's combinations easily. Drop out of the game entirely and supervise several small groups playing the game at one time. The children take turns until eveyone has had a chance to verbalize hisorher combinations. Then the whole group changes blocks and repeats the activity.

An enjoyable variation is the number song. Instead of *saying* the combination the child sings it. The children can use the simple tune (shown below) or make up their own for the group to echo.



("One and three.")
("Two and two.")

Because seven and zero are two syllable words, the children should use a slight variation when they occur. For example:



("Seven and two.")
("Zero and nine.")

Peek Through the Wall

SKILLS

Subtraction
Addition
Visual imagery

MATERIALS

Blocks, a piece of acetate with tape around the edges, one work space* per child

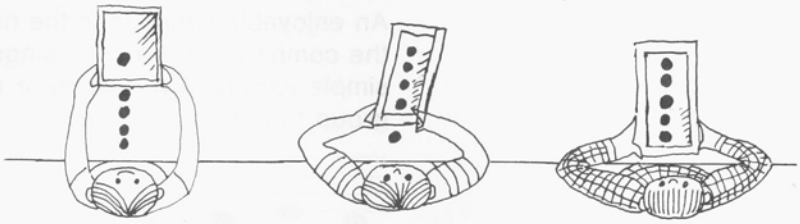
ACTIVITY

The children work together in a small group walling off their blocks and describing the combinations formed.

Ask the children to arrange four blocks on their work space in a vertical line with about an inch between each block.

Young children can easily reproduce a vertical or horizontal line if it is presented in relation to their bodies. For vertical, suggest they point the line at their stomachs. For horizontal, ask them to make a line like their arms when they are stretched out to each side. Always use the correct vocabulary saying, for example, "Make it look like your arms. That's a horizontal line."

Have the children use their hands as "walls" for the first few tries. This establishes the meaning of "wall off." For example, "wall off three" means place your wall so you see the three blocks closest to your body. This leaves one block *behind* the wall. Check every child each time you give a new direction until you are sure the language is clear to the children and they are responding correctly and with confidence. Ask the children to use the acetates as walls and repeat.



"Wall off four." "Wall off one." "Wall off zero."

Now the game begins. The teacher gives a direction, telling the children how many blocks to "wall off." When the children have done this correctly, the teacher says, "peek," and the children peek through their walls and call out the number of blocks in unison.

SAMPLE TEACHING STRATEGY

TEACHER	CHILDREN
"Wall off three."	Each child walls off the three blocks closest to hisorher body.
"Peek."	Each child peeks through the acetate. Then the group chants, "One and three."
"Wall off four."	Each child walls off four blocks.
"Peek."	Each child peeks through the acetate and the group chants, "Zero and four."

As soon as you are sure the children know how to play the game, ask a child to be the teacher and give the directions to the group. This frees you to observe and enables the children to play the game independently. Do not be concerned if the child who is the teacher tells the group to do something impossible, such as walling off six, when there are only four blocks. This is a valuable learning experience for the children. Trust that they will quickly learn *from their own experience* that six is not contained in four. When children have the opportunity to make such mistakes, they learn, in a way they do not forget, the possibilities and limitations of a given number.

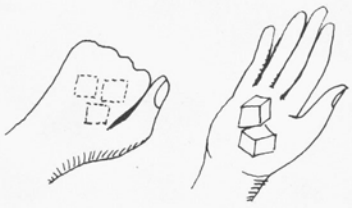
Individualizing the Three Games

When the children understand the *form* of the Three Games, they are ready to begin working at different levels, depending upon what is appropriate for them as individuals. The initial teaching of the games is done at "four" on the assumption that every child being taught the games can count to four and is ready to work on the combinations of four. This number may be "too easy" for some children, but it is used because it is a level at which *every child* can learn to play the games successfully.

To assess the level at which each child has a *real understanding* of the quantity of a given number, use the following procedure with each student individually.

Do not show the hidden blocks to the child after sheorhe has guessed the number that is hidden. This will minimize the assessment time. Try to be very careful not to indicate to the child by any nonverbal cues if sheorhe is unsuccessful. Remember, this is a diagnostic, not a teaching, tool and its purpose is to relay information quickly and easily.

SAMPLE ASSESSMENT STRATEGY

TEACHER	CHILD	INTERPRETATION
"Count five blocks into my hand." 	Child is able to count five blocks into the teacher's hand.	Appropriate level. Child demonstrates skills and understanding necessary to function at this level. (Ask the next question.)
	Child is unable to count out five blocks.	Inappropriate level. Child is being asked to function at too high a level. Attempt to make an assessment for "three." (If this is *not* successful, work on counting activities. Do not go on.)
"How many blocks do I have in my hand?"	Child says "five" without counting.	Appropriate level. Child shows an understanding of invariance for five. (Ask the next question.)
	Child must count to tell you "five."	Inappropriate level. Child needs more experience with the activities in the chapter on invariance and counting totals at the number stations. Attempt to make this assessment for "three." If this is not successful, work on counting activities.
The teacher hides a number of blocks in one hand and shows the others in an outstretched hand, "How many am I hiding?" 	Child says "three," instantly and confidently.	Appropriate level. Child appears to show a real understanding of five. (Check several other combinations for five before making a final judgement; then make this assessment for "six.")
	Child guesses incorrectly, cannot make a guess or does not know instantly with real confidence.	Child appears not to have a real understanding for "five." Continue checking other combinations for "five" until you feel confident to make a judgment. If the child does not understand, make this assessment for "three."

The Whale Game

SKILLS

Addition
Subtraction
Counting
Discovering relationships among different quantities
Visual imagery

MATERIALS

A paper cup with ten to fifteen Pepperidge Farm goldfish crackers for each child

ACTIVITY

The children act out the whale story as it is told by the teacher. Dramatic embellishments are invited from both teacher and students!

SAMPLE TEACHING STRATEGY

TEACHER	CHILDREN
"This is a story about a whale. He was a very special whale and he loved to eat little fish. In this story *you* are going to play the part of the whale! Who's the whale?"	"We are." "Me!" "You said we get to be."
"And *this* is your ocean." (Teacher extends hisorher hands, palm up.) "Show me your ocean."	The children extend their hands, palm up.
"One bright day the whale was swimming around. All at once he saw two fish swimming near him. Show me what happened!"	Each child takes two fish out of hisorher cup and places them in the "ocean." 
"'Boy, am I hungry', said the whale, and he gobbled up one of the fish." (Teacher looks expectantly at the children.)	Each child eats one of the fish from the "ocean."
"And that left how many fish?"	"One."
"Pretty soon two other fish came swimming along. How many fish are in the ocean now?"	Each child places two more fish in the ocean and says, "Three!"
"It didn't take the whale long to spy those three fish. He swam by and in a flash gobbled up one of the fish.	Each child eats one fish and says,
That left . . ."	"Two!"
"The whale was *still* hungry so he ate one more fish. That left. . . ."	Each child eats one fish and says, "One!"
In a little while three more fish came by to join the one that was left. That made. . . ."	Each child adds three additional fish and says, "Four."

Continue to add and subtract fish until all the crackers are eaten. This game should be played over and over again to reinforce number concepts at different levels. Let the children improvise on the theme and take turns telling the story to the group.

You can vary this activity by using Fruit Loops or Cheerios, making up a similar donut story.

Listen and Count

SKILLS

Counting
Addition
Subtraction
Auditory perception

MATERIALS

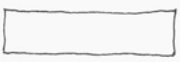
Bell, junk boxes,* one work space* per child, cards with + and − written on them.

ACTIVITY

The "teacher" rings the bell an arbitrary number of times and turns over a card. If the card is a + the children add the number of objects indicated by the bell to their paper. If the card is a − they remove the number of objects indicated by the bell.

Do not tell the children in advance that they will encounter impossible situations. Let them struggle with this when it occurs naturally. In this way children discover on their own that a larger number is not contained in a smaller one.

SAMPLE TEACHING STRATEGY

TEACHER	CHILDREN
Bell: ding, ding, ding Card: +	 "Three."
Bell: ding, ding, ding, ding Card: −	 "Impossible."
Bell: Ding, Ding, Ding Card: −	 "Zero."
Bell: ding, ding, ding, ding, ding Card: +	 "Five."
Bell: ding, ding Card: +	 "Seven."

The children continue playing the game adding and subtracting objects.

This same activity can be done without the teacher to ring the bell by having the children roll dice.

Concentration

SKILLS

Matching
Visual memory
Counting
Making and checking predictions

MATERIALS

Unifix cubes,* milk carton graphing boxes*

ACTIVITY

This game can be played with a team of two to six children who prepare the following stacks of Unifix cubes: two stacks of one, two stacks of two, and two stacks of three. Each stack is placed under a milk carton and the cartons mixed up and then arranged in some way. Each child takes a turn lifting two milk cartons trying to locate identical stacks. If the child is successful he keeps the stacks. The milk cartons are replaced before the next child takes a turn. The game continues until all the stacks have been discovered.

The number of stacks used in the game should increase as the children's skill in number increases.

The Cave

SKILLS

Visual memory
Addition
Subtraction

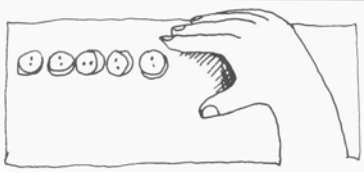
MATERIALS

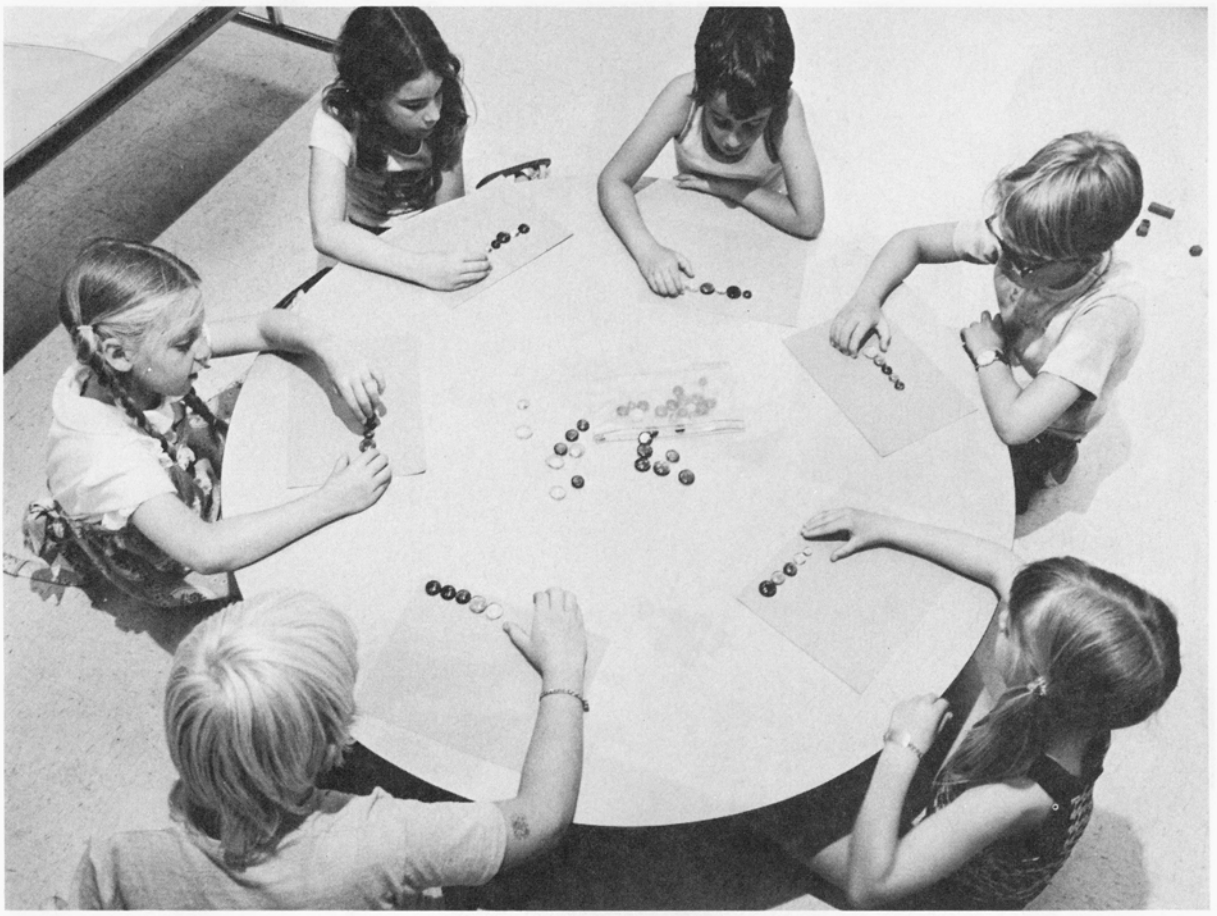
Junk boxes,* work space*

ACTIVITY

Have each child count out a given number of objects (whatever number is appropriate) and place them on a work space. The teacher should demonstrate how to make a "cave" by cupping the right hand on the right side of the workspace. The teacher tells how many objects are to be hidden and the children hide the objects and read the problem out loud. The children lift the "cave," exposing the previously hidden items and reread the problem.

SAMPLE TEACHING STRATEGY

TEACHER	CHILDREN
"Put five objects on your workspace and make the "cave."	
"Hide two."	
The teacher points to the uncovered objects, and says "three," and to the cave and says, "two."	The children imitate the teacher saying, "Three and two."
Teacher lifts the cave and repeats the combination: "Three and two."	The children lift the cave and repeat the combination: "Three and two." 
The teacher gives three or four more examples, leaving the total number of objects constant but changing the number of hidden objects. Then without demonstrating, the teacher tells the children how many to hide. From then on the teacher only says the number to be hidden. The children should be able to repeat the combinations independently at this point.	"One and four." (Lift cave) "One and four." 
"Hide three."	"Two and three." (Lift cave) "Two and three."
"Hide five."	"Zero and five." (Lift cave.) "Zero and five."



As the children become confident they should take turns giving the directions. If you hear a child giving an impossible direction such as, "Hide eight," when there are only five items on the work space, resist the urge to step in. Let the child learn through experience that this is impossible. Children need opportunities to solve problems and to learn from their mistakes on their own. The *materials* themselves will correct the child and teach himorher truth about numbers.

Subtraction Cards

SKILLS
Subtraction
Following directions

MATERIALS
Subtraction cards,* junk boxes,* work space*

ACTIVITY
This activity works best in pairs or in small groups. A child plays the role of the teacher. The group follows the teacher's directions using their materials as the subtraction cards are read one at a time.

SAMPLE TEACHING STRATEGY

TEACHER	ACTIVITY
The teacher reads the total on the card, "seven."	Each child counts out seven objects onto a work space.
The teacher checks to be sure the children are ready and then lifts the right hand flip, saying, "Take away three."	Each child takes three objects off hisorher work space, counts the remaining objects and says, "Four!"
The teacher shows the group the four remaining dots.	

The children should use the cards to review the number concepts appropriate for their level. Each total is a different color.

Always use language that is familiar and meaningful to the children while teaching a new concept. By using "take away" for minus and "and" for plus you can begin where the child is and link a new concept to this familiar language. Once the child understands the concept of subtraction, it is a very simple matter to substitute the more mathematical vocabulary. Premature introduction of technical vocabulary only complicates matters for the child.

Capture

SKILLS

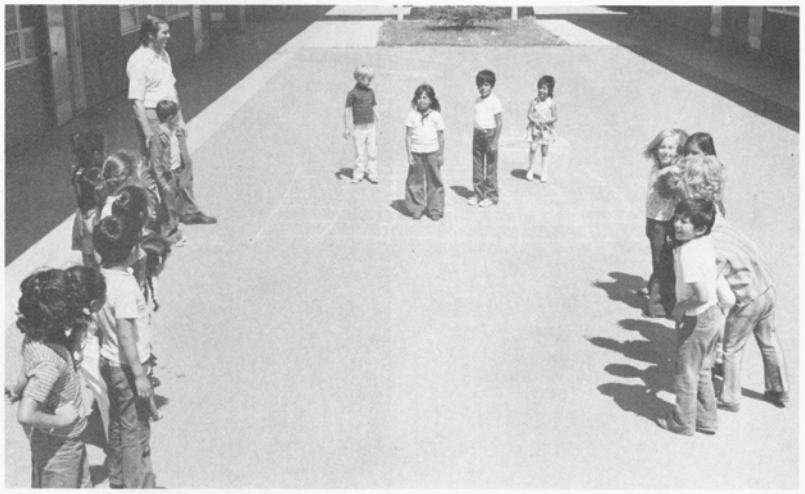
Counting
Comparing
Making and checking predictions
Learning coordinate graphing

MATERIALS

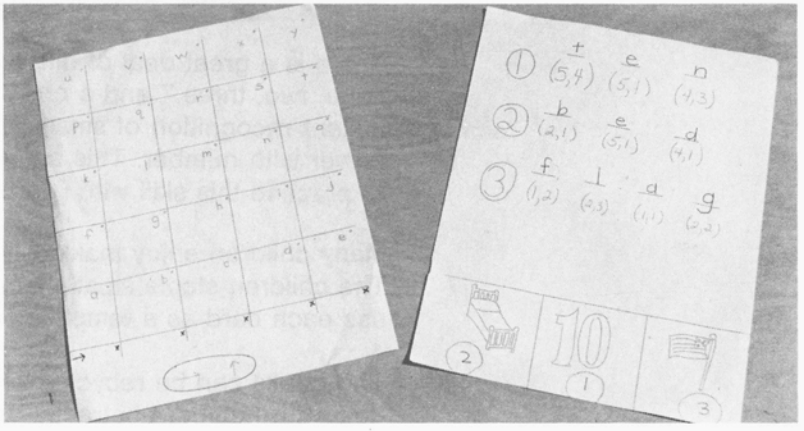
Chalk

ACTIVITY

Line off a gameboard with chalk. Have half the class stand on the left of the gameboard and half on the right. Select several children to stand on the gameboard. The goal of the game is for each team to capture as many children as possible from the gameboard. To capture a child, a team must say the child's name and the numbers needed to get the child off the board. The first number indicates how many X's must be paced off starting from the arrow. The second number indicates the number of dots up from the last X. The teams alternate giving numbers and pacing them off until all the children are captured. If a child gives a number which does not capture the intended child, the latter remains in that position on the gameboard.



This same game can be played on a reduced gameboard using letters in the squares to enable the children to spell out words.



"Say It Fast"

SKILLS

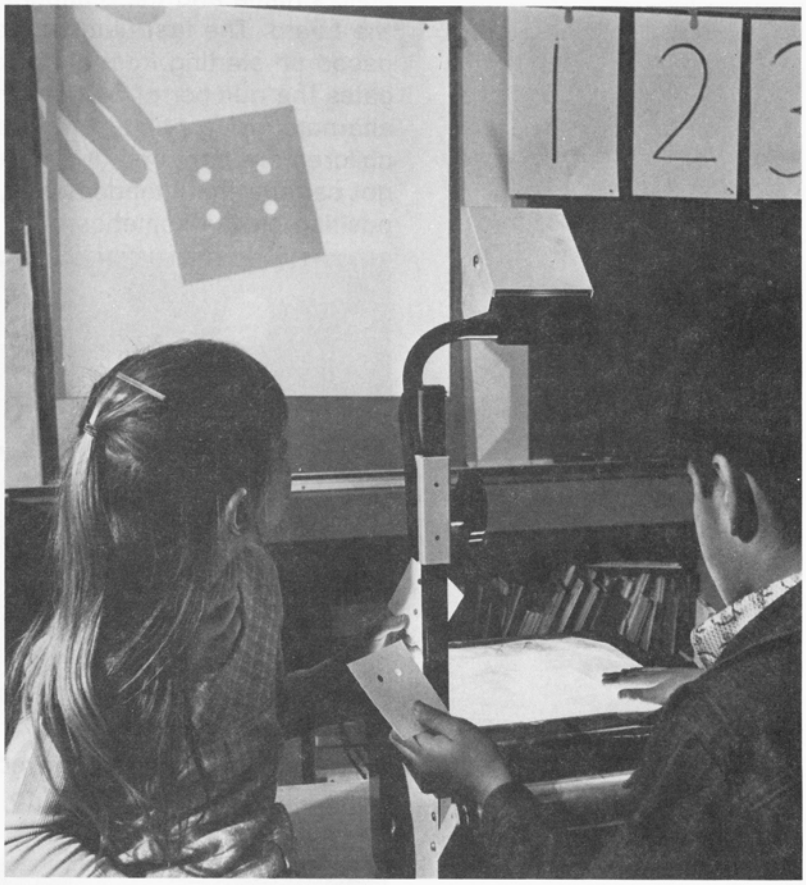
Recognizing a quantity without counting
Visual memory
Addition
Invariance of number

MATERIALS

Overhead projector, cards with holes punched to show combinations

ACTIVITY

Two children work together at the overhead projector. One child puts the "say it fast" cards down and the other takes them off. The other children watch the screen and tell how many holes they see as fast as they can.



There is a great deal of difference between a child counting, "one, two, three," and a child saying "three" instantly. Instant recognition of small groups can give children real power with number. This activity gives children an opportunity to practice this skill with groups of six or less.

Many children enjoy making little books from these cards. The children staple small pieces of paper together and use each card as a template to color groups of dots.

The books can be recycled when the children are ready to deal with number abstractly.

Presto-Change-O

SKILLS

Addition
Subtraction
Multiplication

MATERIALS

Wooden cubes,* junk boxes,* tiles,* one workspace* per child

ACTIVITY

The children work together in small groups following the teacher's directions.

SAMPLE TEACHING STRATEGY

TEACHER	CHILDREN
Put a star on the board to signal a silent activity. From now on the only talking that is allowed is the teacher's directions.	
"Put two stacks of three on your workspace."	
"Now I want three stacks of two on your workspace. I *only* want three stacks of two."	
"One stack of five."	
"Five stacks of one."	
"One stack of three."	
"Three stacks of one."	

Let the children struggle with these problems. Do not tell them what to do. Observe how each child solves the problem. You will notice some children clearing their workspace every time, treating each problem as a distinct one, unrelated to any other. Other children will occasionally reassemble the cubes that they have on their workspace. They treat the results of their occasional efforts at reassembling the cubes already on their workspace as accidental good luck, discovering with great delight that they have the exact number of cubes they need to make the second set of cubes. These children are just as likely to clear their paper for the next example as to try to reassemble the cubes that they have on their workspace. Still other children quickly abstract the pattern, sit back smugly, knowing they have cut their work in half and puzzle over their fellow student's tedious method.

Observe the growth of this understanding in each child individually as it develops from within. Do not try to teach it. This activity is an opportunity for a child to make a discovery. By keeping the game silent, you allow each child to abstract this discovery for himorherself.

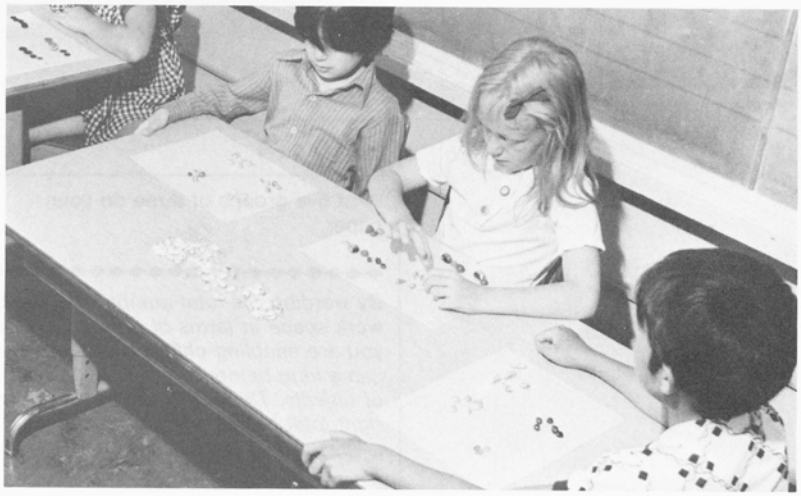
By observing your children during this activity, you will gain insight into their cognitive development. Each child will make the discovery when sheorhe is ready.

Continue giving directions, being careful not to use any numbers that are larger than the children are ready to work with. In each case, ask the reverse of the problem as the second step.

Repeat the entire activity with a new material.

TEACHER	CHILDREN
To indicate the direction of a row, ask the children to put their arms out to the side and to make their row in the same way.	
"Five rows of two."	
"Three rows of one."	
"One row of three."	
"Five rows of four."	
"Four rows of five."	

Continue giving directions for several more presto-change-o's and then switch materials and repeat the activity with junk. In this way, the child observes the concept in a variety of materials; sheorhe is surrounded with the idea and has several different experiences from which to abstract the concept. This variety helps prepare the child to deal with an abstract concept. The experience teaches that the idea is not tied to a particular material.



"Two groups of three."	
"Three groups of two."	
"One group of five."	
"Five groups of one."	

"Line Them Up"

SKILLS

Problem solving
Division
Multiplication
Counting

MATERIALS

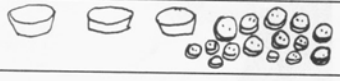
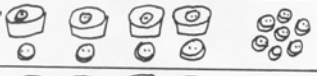
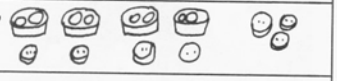
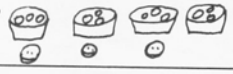
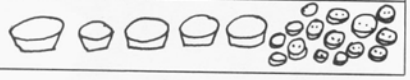
Counting cups,* junk boxes,* one work space* per child, bell

ACTIVITY

The children follow the teacher's directions, dividing a given number of objects in a variety of ways.

SAMPLE TEACHING STRATEGY

TEACHER	ACTIVITY
"Put five groups of three on your paper."	
By wording the total number of objects you want the children to have on their work space in terms of groups (five groups of three rather than saying fifteen) you are enabling children who can only deal with five objects to play this game long before they would be able to correctly count out the total number of objects. This also enables you to tell at a glance that every child has the right total before beginning the experiment.	
"Now push them all together."	
"How many cups shall we use for our experiment? One, two, three, four or five?"	A child suggests three.
"Put three cups in a line at the top of your workspace."	
"When I say, 'Line them up' put one object from your pile in front of each cup. Watch me try it. Line them up."	The children watch the teacher demonstrate and then imitate what they have seen.

"When I ring the bell, you answer *yes* or *no*. Yes means that there is an object in front of each cup. If there is, put each object in its cup when you tell me *yes*. If there is not an object for each cup, tell me *no* and do not put them in the cups. Ready?" Ding. "Yes or no?"	"Yes." Children put the object in front of each cup into the cup. 
"Ready, line them up." Ding. "Yes or no?"	"Yes." 
"Line them up." Ding.	"Yes." 
"Line them up." Ding.	"Yes." 
"Line them up." Ding.	"Yes." 
	
"Did it come out evenly? Are there any left over? Let's count the objects in each cup together."	"Yes." "No." "One, two, three, four, five; one, two, three, four, five; one, two, three, four, five."
"How many are in each cup?"	"Five."
"Empty the cups onto your workspace."	
"How many cups shall we use this time, one, two, three, four, or five?"	(Someone suggests four.) 
"Line them up." Ding.	"Yes." 
"Line them up." Ding.	"Yes." 
"Line them up." Ding.	"Yes." 
"Line them up." Ding.	"No." 
"Did our experiment come out evenly?"	"No."
"Are there any left over?"	"Yes. Three left over." 
"Let's count the objects in each cup."	"One, two, three; one, two, three; one, two, three."
"How many are there in each cup?"	"Three." 
"And how many are left over?"	"Three." 
"How many cups shall we use this time?"	

Continue experimenting with the children's suggestions. Repeat this lesson many times, changing the total number used each time. This activity gives children a problem solving tool which they will use again at the abstract level to make records of the results of their experimentation.

Unifix Trains

SKILLS Problem solving
Division
Multiplication
Counting

MATERIALS Unifix cubes,* work space*

ACTIVITY The children follow the teacher's directions and attempt to break a train of cubes into smaller equal groups.

SAMPLE TEACHING STRATEGY

TEACHER	ACTIVITY
"Put out four groups of three."	

This enables a child to get a total that otherwise might be difficult to count out correctly. It also enables the teacher to tell at a glance whether each child has the correct total.

"Snap your cubes together in a long train and match your train with a friend's."

This is a good problem solving tool which enables the children to double check their total.

"Shall we break our long train into ones, twos, threes, fours, or fives?"	A child suggests twos.
"Did it break evenly in twos?"	"Yes."
"Are there any left over?"	"No."
"How many little trains of two do you have?"	"Six."
"Snap your train back together and match with a friend."	
"The last time we made trains of two. How shall we break our long train up this time?"	Someone suggests five.
"Did our long train break up evenly in fives?"	"No."
"Are there any left over? How many?"	"Yes. Two."
"How many little trains of five did we get?"	"Two."
"How shall we break our long train up this time? . . ."	

Continue exploring the process of division informally. Expect the children to suggest impossible numbers such as dividing their train into fifteens. It is important to *try* these impossible experiments and allow the children to discover by trial and error which numbers are possible and which are impossible.

Word Problems

SKILLS

Problem solving
Connecting an abstract idea to the real world
Logical thinking
Counting
Addition
Subtraction
Multiplication
Division

MATERIALS

Whatever props are necessary from the classroom

ACTIVITY

The teacher reads a word problem for the children to act out. The problem is reread and the children check the action.

The following are models for other problems you and your students can make up. Substitute your childrens' names, the streets in the neighborhood, and familiar stores whenever possible. This makes the problems more meaningful and thereby more effective and enjoyable. The following sections represent different mathematical concepts. Make up at least twenty additional problems for each section you use in order to develop each concept fully. Vary the numbers used depending upon what is appropriate for the children you are working with.

Word Problems Using Children and Props

COUNTING

Sue is six years old today. What does her birthday cake look like?
John bounced a ball six times.
Sue put six boxes of crayons on the table.
Carolyn walked across the room carrying six books.
Leretha clapped her hands six times.
Jemell closed his eyes and counted to six.
Thomas' mother checked the car battery. She unscrewed all six plugs. What does it look like?

ORDINAL NUMBER

(Ask six children to get in a line.)
Give a book to the fourth person in line.
Shake hands with the last person in line.
Change places with the second person in the line.
Name the fifth person in line.
Stand behind the first person in line.
Ask the third person in line to sit down.

ADDITION

John put six chairs at the table. Some were on one side and some were on the other.
There were six children. Some were standing and some were sitting down.
Mark had six pencils. He gave three to Paul and three to Susan.
Donna gave four books to Ruthie. She got two more books and gave them to Ruthie, too.

SUBTRACTION

There were six Cheerios on the table. Mark ate one of the Cheerios.
There were six balloons on the table. Melissa knocked two of them off the table.
There were six children standing up. Four of them sat down.
There were six chairs in a row. Along came Batman and knocked over two of the chairs.

MULTIPLICATION

Frances put three hula hoops on the floor. She asked two children to stand in each hula hoop.
Catherine got six chairs. She told two children to sit on each chair.
There were six desks. Mark asked three children to get under each desk.
Donna chose four children. She gave each child five Cheerios.

DIVISION

Mary had six books. She gave Peter and Carole the same number of books.
Marty had six Cheerios. He gave them to Sue and Paul and Toni. Each child received the same number of Cheerios.
Edward had six pencils. He put them on six chairs. Each chair had the same number of pencils on it.
Tim got six coats. He put them on two tables. Each table had the same number of coats on it.

Word Problems Using Blocks And Paper

These problems are to be acted out symbolically with blocks representing cups of coffee, meatballs, chairs and people, and pieces of paper representing rugs, beds, tents, tables, etc., so that each child participating in the group will be able to act out the word problem simultaneously. These experiences draw on the images and understandings attained at the earlier, more concrete, stage. The present level of activities links these images with a symbolic representation in order to prepare the children to deal with word problems at the abstract level.

COUNTING

John put six cups of coffee on the table.
Carla's family went camping. They had six chairs in their tent.
Mom checked the water in the car's battery before starting on a trip. The battery had six places to be checked.
Paul had six puppies. He put out a dish of dogfood for each puppy.

ORDINAL NUMBER _____

Carolyn lined up six boxes of junk for the garbage man to collect. The men picked up the fourth box of junk first.
There were six houses on one side of Carter Street. A fire burned down the second and third house.
In a hospital ward there were six beds lined up. The patient in the sixth bed was wheeled into surgery.
Merlin the magician put out six blocks in a row. Abracadabra; he made the first and last blocks disappear.

ADDITION _____

Karl put two chairs on one side of the table and four chairs on the other side.
Four children were playing football on the field. Two of their friends were off the field watching them.
Joli had six boxes. She put some boxes on the floor and some on the rug.
Paula and five friends were playing in her room. Some were jumping on the bed and some were waiting their turn.

SUBTRACTION

There were six cars in the parking lot. Five of the cars drove out of the lot.
Six men got on an empty bus at Polk Street. At Washington Street four of them got off.
There were six TV's in Frank's TV store. On Saturday two people bought TV's.
There were two houses side by side. Each house was three stories high. A terrible tornado carried away the top stories of each house.

MULTIPLICATION

Michael baked six cakes. Each cake had two layers.
John fixed three plates of biscuits. There were four biscuits on each plate.
Jack built five buildings. Each of the buildings had three stories.
John had two plates of spaghetti. Each plate had six meatballs on it.

DIVISION

Ricky had six chairs. He put them at three tables. He put the same number of chairs at each table.
Susan had six boxes. She put them on the rug in two stacks. She put the same number of boxes in each stack.
Carla had six cans. She put them in two different cupboards. She put the same number in each cupboard.
John had six books. He put them into three piles. He put the same number of books in each pile.

These problems, and others like them, should be used over and over again, substituting different numbers each time. If it is appropriate at a later time, the children can verbalize the combinations formed and the total quantity which results, but this is a later step and should not be included until the children are successful at acting out the process.

The "multiplication" and "division" categories are no more difficult than addition and subtraction by this method and should be tried by children of every age.

7

NUMBER AT THE CONCEPT LEVEL

Word Problems

QUESTIONS FROM TEACHERS

WHY DON'T YOU USE *TOTALS* WHEN VERBALIZING AT THE NUMBER STATIONS OR THE THREE GAMES? WOULDN'T IT BE BETTER IF THE KIDS SAID "THREE PLUS (OR AND) TWO *EQUALS* FIVE?"

What I learned from having the children include totals is that it encourages them not to pay attention to the combinations as much.

Let me explain. During the concept development stage (which this chapter on number represents) the "number" the children use at the number stations and at the three games remains constant. The children may be exploring four, for example, finding all the ways to make four (the possible combinations). If they verbalize the totals it is the *same* (four) for each combination and it gets monotonous very quickly. The children *intuit* the four even though they are not saying it each time. The focus here is on the combination—the way of grouping the total rather than the total itself. When the child progresses to the symbolic stage, sheorhe will make up problems with different totals and at this point the idea of "equals" will be introduced and included.

WITH THE WHALE GAME AND LISTEN AND COUNT YOU MIX UP ADDITION AND SUBTRACTION AT THE SAME TIME. WON'T THIS CONFUSE THE CHILDREN?

One of my goals in these activities is for the children to see the relationship between the process of addition and the process of subtraction. This can only be accomplished when both processes are included in the same lesson. By experiencing both operations simultaneously, the children are prevented from getting locked into one concept. This often occurs when addition is presented first and then subtraction. When the operations are presented simultaneously, although it is a bit slower in the beginning, the children grow to see the interrelation between the two operations; they are able to make use of their insight by using their knowledge of addition to help them in subtracting and their knowledge of subtraction to help them in adding. Mathematics stops being many separate concepts and begins to reveal itself as an interrelated pattern which forms a whole. For this reason I have found it very worthwhile to give children experience with both addition and subtraction at once.

WHY DO YOU HAVE ACTIVITIES LIKE UNIFIX TRAINS AND PRESTO CHANGE-O (MULTIPLICATION AND

It has taken me ten years to unravel the way children really learn mathematics best. I assumed that the sequence used in children's textbooks represented the most successful model. Years of research with children have convinced me that this model is a successful and necessary tool for *writing a*

DIVISION) MIXED IN WITH ADDITION AND SUBTRACTION ACTIVITIES? WON'T CHILDREN BE OVERWHELMED? DON'T THEY NEED TO REALLY UNDERSTAND *SETS* BEFORE TACKLING ADDITION AND MASTER ADDITION BEFORE SUBTRACTION, AND SO ON?

textbook but not for learning. An author faces an enormous amount of information that must be organized and then put down in a linear fashion. A book, by its very nature, has one page before another and one sentence before another. Because of this need the sequence of sets, addition, subtraction, multiplication and division evolved. Experience with children in *their* world does not show this sequence at all. Children first deal with *groups* (a pile of books, a collection of rocks, a bowl of Cheerios) and with division and fractions (a quarter of an apple, half a glass of milk, candies divided into equal groups to share with four friends).

Our children's world is not linear (one idea isolated and presented repeatedly until mastery); rather, it is geometric (many, many ideas presented simultaneously, half understood by the child, with each idea contributing to the growth of the other ideas, until the "light dawns" and makes clear several ideas at once).

The activities in this chapter are based on this real experience of childhood. You will find that it is no more difficult for your children (even for five-year-olds) to deal *at the concept level* with the *process* of all the operations at once. After all, they are all related, and the actual operation of addition or multiplication or division or subtraction depends only upon the focus, not on a totally different experience.

Children develop more understanding of the *whole* of mathematics and more flexibility in dealing with number if they have the opportunity to deal with all the operations at once.

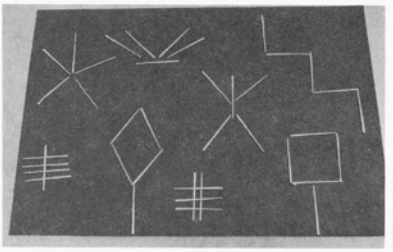
HAVEN'T YOU GOT SOMETHING LIKE THE THREE GAMES FOR SUBTRACTION?

Actually, the three games deal just as much with subtraction as they do with addition. It depends on what *process* you zero in on. For example, with the hand game, if you're exploring six and a child puts four in one hand and two in another the child has solved for $6 - 4 = 2$ as well as $4 + 2 = 6$. With Lift the Bowl, if the child is exploring eight, sheorhe starts out with eight blocks under the bowl. If the child takes out three blocks to put on top of the bowl, the subtraction process $8 - 3 = 5$ takes place. Peek Through the Wall is even more obviously subtraction. The child who starts with four objects and "walls off" three has subtracted $4 - 3 = 1$. The reason I have not encouraged you to focus on subtraction in the three games is that it is difficult to do subtraction at the connecting and symbolic stages. Addition works perfectly at all three stages. My children found it more helpful to use the subtraction cards* for subtraction. They work well at all three stages: concept, connecting, and symbolic.

I HAVE DIFFICULTY GETTING THE CHILDREN TO DESCRIBE THEIR DESIGNS AT THE NUMBER STATIONS. I'M NOT REALLY SURE WHAT THE CHILDREN ARE *SUPPOSED* TO SAY ABOUT THE DESIGNS OR HOW I, AS THE TEACHER, CAN GET THEM TO SAY WHATEVER IT IS THEY SHOULD BE SAYING.

When I go from group to group while the children are working at the number stations, I engage one group of children at a time in a discussion of their designs. First I quickly check to be sure all the designs on the table are the correct quantity. If I see two or three designs that are not correct, I do not point them out but rather, I get the children to count several designs—some of which have the correct quantity and some of which do not, allowing the *children* to discover the errors and correct them. Then I ask the children to look at the designs on the table and think of *other* things which the designs remind them of. This prompts the children to make verbal descriptions which is the thing that seems uppermost in their minds at first.

"I see a man with no head."
"I see two different ones."
"There's a kite."
"Stairs."
"A TV antenna."
"Do you see the sun?"



There is a point where the designs no longer spark ideas of "other things" and you can actually *feel* the energy in the group change. This is your signal to ask the next question: "If we wanted to describe this design (/\‾/\) with numbers, how might we do it?" "Two and one and two." The teacher immediately asks, "Is there a *different* way we could describe this design?" "Five." "Is there another way?" "How about one and four." Or, "I see one and three and one."

Through this technique the teacher is able to help the children become flexible in looking at and describing the designs. Each successive idea focuses on the design in a slightly different way. The children should look at the design and try to understand how a person could think of the design in the way mentioned. After first being allowed to think on their own, then the child who verbalized the numbers points to the parts of the design.

Do not be concerned if the children do not always look left to right to generate the pattern. Whatever is natural to the children is acceptable. A child may say "One and two and two" for "\\ //" and the other children will, through mental trial and error, find the way that number sentence can fit.

Another departure point for discussion is to describe a design with a number sentence ( "four and one") and then find other designs on the table that fit the same number pattern ().

WHEN DOING THE THREE GAMES FOR SIX AND BELOW THERE DIDN'T SEEM TO BE ANY PROBLEM, BUT WITH SEVEN AND ABOVE THE KIDS CAN'T TELL AT A GLANCE HOW MANY OBJECTS ANOTHER CHILD HAS IN HISORHER HAND WITHOUT COUNTING. THIS MEANS THE GAME LOSES ITS RHYTHMIC QUALITY. IS THIS OKAY?

I have also experienced this. My class enjoys changing the hand game slightly with numbers above six. Instead of opening both hands at once, the children open one hand and leave the second hand closed. The rest of the group tells *not* how many objects are in the open hand, but how many are in the *closed* hand. It's sort of a real-life missing addend problem. After the guessing is completed, the child opens hisorher hand as a check. We do the same thing with Lift the Bowl—the children arrange their cubes and then the group guesses the number of hidden cubes. Lifting the bowl checks the guess. For Peek Through the Wall the children use an index card (solid) as their wall and guess how many objects are hidden *behind* the wall. A peek *over* the wall checks the guess. Some children prefer the more tedious counting in the earlier version of the game even with the larger numbers. But for those children who enjoy this version more, it is a pleasant alternative.

I CAN'T KEEP TRACK OF WHO'S BEEN WHERE AT THE NUMBER STATIONS UNLESS I ROTATE EVERYONE FROM STATION TO STATION. HOW CAN I BE SURE THAT EACH CHILD IS GETTING THE EXPERIENCE SHEORHE NEEDS UNLESS I ROTATE?

I used to have my children make a tally after their name (on a duplicated class list placed at each station) so I could keep track of who had been where. What I learned was that it *doesn't matter* if every child uses every material. With the freedom to change stations at will, some children rotate themselves systematically through each station over two or three days. Other children seem to have two or three favorite materials where they repeatedly go and only occasionally seek out a different material. Since the children are "exploring four" or "exploring seven" rather than "exploring four *with pattern blocks*" or "exploring seven *with toothpicks,*" they are accomplishing the goal no matter how many or how few stations they use.

By allowing children this free choice, we have an opportunity to observe each child's differing need for variety or change, the time period that different materials interest children, and individual children's level of involvement with each material. Using the observation sheets or a tally system will help you gather information which you will find of interest. Only if we allow our children this free choice will they have the opportunity to learn in *their* way.

DO YOU LIMIT THE NUMBER OF CHILDREN ALLOWED AT A STATION AT ONE TIME?

Yes, I have the children set up enough stations (about seven) so that no more than five children are at a station at any one time. Sometimes I have a class where I feel six at a station works well, but five guarantees success. This allows plenty of extra places for children as they want to change stations. If I set up exactly enough places for the children, there are no empty places for children to change to and rotating is the only way to change.