CHAPTER 3
Patterns with Tiles and Cubes

Prerequisite chapters:

None

MATERIALS

For overhead projector:

Transparencies	graph paper, 2.5 cm squares	Worksheet 4
Clear acetate squares lettered *A* through *E*		Materials chapter, page 295
Tiles		Materials chapter, page 294
Cubes		Materials chapter, page 295

If no overhead projector is available:

Make charts in place of transparencies	Materials chapter, page 294
Square shapes lettered *A* through *E*	Materials chapter, page 295
Square shapes	Materials chapter, page 294

Student materials:

Dittos	Graph paper, 2.5 cm squares	
	Graph paper, 1.7 cm squares	Worksheet 5

Tiles
Cubes
Crayons
Unlined paper

The activities in this chapter familiarize students with the process of looking for patterns with tiles or cubes. The students look for patterns that can be extended by predicting additional steps in each sequence used. The ability to see patterns is an essential element in understanding mathematics and is closely connected to the ability to think logically.

LESSON 3-1

PATTERNS WITH TILES

PURPOSE:

To copy teacher-generated patterns and predict additional steps in a sequence

MATERIALS:

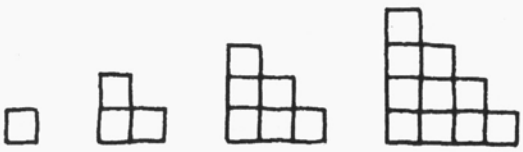
1. *If no overhead projector is available, square shapes*
2. *Tiles*

In the context of this book seeing a pattern means accurately predicting a step or steps in a sequence of events. In the previous chapter students used patterns to discover teacher created rules. In the lessons that follow students learn to construct their own patterns.

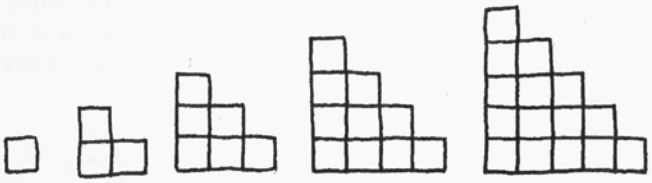
The teacher places tiles on the overhead projector.

Teacher: I have made three steps of a tile pattern. Please copy these steps with your tiles. If you think you can predict my next step, make it with tiles on your desk.

The teacher gives the students a few minutes to copy the pattern. Those who wish to, predict and build the next step. The teacher then places the fourth step to the pattern on the overhead.



Teacher: If you predicted my fourth step would look like this, you have probably figured out the pattern I am making. If your fourth step is different from mine, you are thinking of a different pattern. Look at my pattern again and try to predict what my *next* step will be. See if you can make it at your desk before I make it on the overhead.



After the students have the opportunity to make the step, the teacher builds it on the overhead. This process is repeated for two or three more predictions. The teacher then allows the students to extend the pattern on their own.

Teacher: Your predictions of the steps I add to my pattern are very good. See how many steps of my pattern you can make with your tiles without checking each new step with me. Keep adding steps to my pattern on your desk until you run out of tiles.

While the students are adding steps, the teacher walks around the room and observes. If any students are not extending the pattern begun on the overhead, the teacher returns to it and asks the class to describe how to add steps to the overhead pattern.

It is not important that every student predict from the pattern the teacher makes. It is important, however, that the teacher pick patterns simple enough to insure that at least some students know what comes next.

When the students have run out of tiles for making the first pattern, the teacher clears the tiles from the overhead and begins a new pattern, which the students copy.

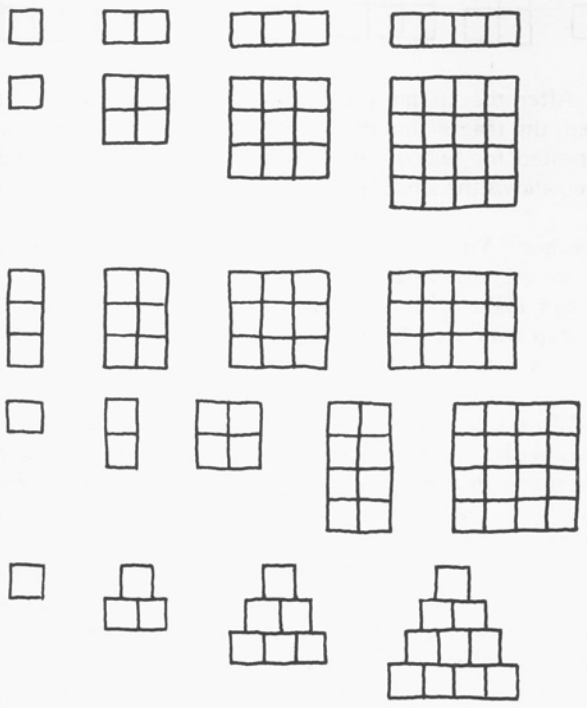
Each step of the first pattern was a separate unit of tiles. Patterns may also grow by the addition of a new row each step, thus creating a single attached design.

Teacher: This is row one of a pattern I will make. Watch as I add row two. Row three . . . and row four.
What will I add as my fifth row? Make this pattern with tiles, showing me what you think it will look like if I add another row.

Whether whole new steps are added, or only additional rows, the procedure is the same. The teacher starts the pattern. The students copy the first few rows or steps, then predict the next row or step. The teacher adds the next row or step of the pattern to the overhead so the students may check their predictions. When most of the students' predictions are accurate, they are asked to continue the pattern until they run out of tiles.

If there are students who are unable to foresee future steps of a pattern, the teacher asks those students who do see the pattern to explain why a certain number of tiles should be added to each step of the pattern.

The patterns are kept very simple at the beginning. Examples of the kinds of patterns used may be found in the figure below. As the students' skill increases, more complicated patterns may be attempted.



LESSON 3-2

PATTERNS WITH TILES

PURPOSE:

To copy teacher-generated patterns that change directions, and predict additional steps in each sequence

MATERIALS:

1. *If no overhead projector is available, square shapes*
2. *Tiles*

The patterns in Lesson 3-1 made each new step larger than the one before. It is not necessary, however, for a pattern to continually grow. It may change direction, it may grow to a certain height then decrease in size in an equally predictable manner. This figure represents steps in a pattern that changes direction.

Students can learn to predict next steps for change-of-direction patterns, too. The only difficulty comes when they guess a next step that is larger, while the next row placed on the overhead is smaller. The teacher then tells the students:

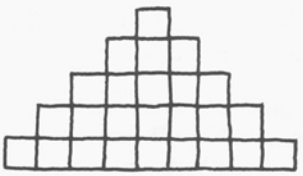
I see from your predictions you are thinking of a different pattern than I am using this time. Your predictions would have been correct if I were using *that* pattern. Now that you see what my pattern gives for my next row, what row am I going to add next?

There is more than one possibility for a next row that will fit into a predictable sequence. The only way students can know what comes next is to consider all that has gone before then assume that what is added follows from what they think they have already seen.

This may seem arbitrary, but most of science and, for that matter, most learning consists of observing closely what has already happened, and from that trying to project what *will* happen. The more reliable the patterns, the better the predictions. More difficult or less regular patterns lead to more tentative predictions.

Chemists mix two chemicals in known proportions, and feel they can predict with a high degree of accuracy what will happen next. Meteorologists, on the other hand, can give only an approximate weather forecast from their knowledge of weather patterns. *After* the front has passed, they can explain it. Although weather predictors know the logical consequence of any particular pattern they cannot guarantee which pattern will be in effect at any given moment.

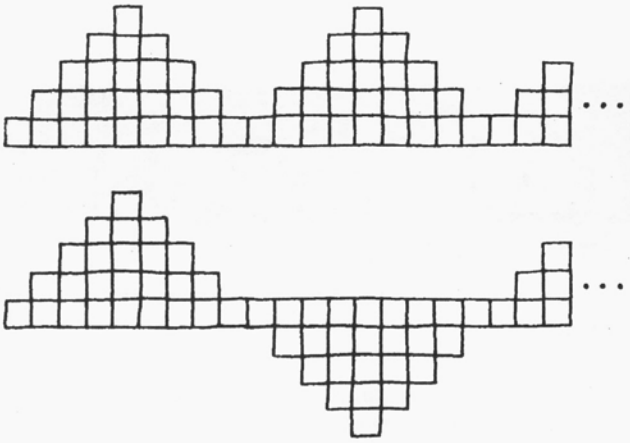
Change-of-direction patterns lead to an interesting question: when a pattern has gone up and back down again what will the next row look like?



The teacher discusses various possibilities for the next row, then shows the class what it is according to the teacher's rule.

The teacher must emphasize that the pattern selected is only one of many possibilities. The object of this activity is to guess the teacher's pattern, although it is only one of many that might have been chosen.

There are many possible next steps for this pattern. Two of the possibilities are shown here.



LESSON 3-3

PATTERNS WITH TILES

PURPOSE:

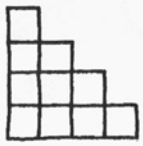
To provide experience creating and recording tile patterns

MATERIALS:

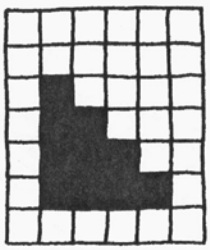
1. *Graph paper, 2.5 cm squares on a transparency, or on a large tagboard*
2. *If no overhead projector is available, square shapes*
3. *Tiles*
4. *Dittoed sheets of graph paper with 2.5 cm squares*

In the two lessons that follow, students have the opportunity to create and record their own tile patterns. As a test of the predictability of their patterns, students attempt to extend patterns created by their classmates. In addition to providing students practice in predicting next steps, these lessons reinforce the notion introduced in the preceding chapter that the creation of problems to be solved is not solely the province of the teacher.

Teacher: Copy my pattern on the overhead with tiles on your desk.



Now, watch as I copy this tile pattern square for square on my graph paper transparency. Copy the pattern on your sheet of graph paper square for square.

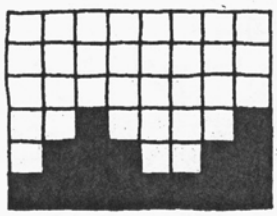


While the students copy the tile arrangement, the teacher circulates, observing how clearly they understood the instructions. When the students have completed copying one pattern from the overhead, the teacher arranges the tiles differently and the exercise is repeated.

When all students can successfully reproduce a tile pattern on graph paper, or are sitting next to a student who can, the teacher proceeds to the next step.

Teacher: I want each of you to make up a pattern using tiles. When you have a pattern you like, record it on a piece of graph paper. Make your pattern go all the way across the graph paper. You can make up as many patterns as you want, but put each on a separate sheet of paper.

Try to make up patterns for which someone can predict the next row or step. Each time you finish recording a pattern, please put it on my desk.



Anything the students make with their tiles and record on graph paper will result in a pattern; thus they conform to the assignment. Most students will not understand how to make a pattern from which someone may predict. The teacher will clarify this in the following lesson.

LESSON 3-4

PATTERNS WITH TILES

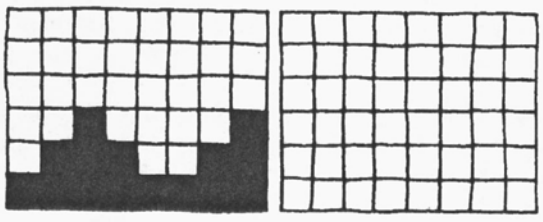
PURPOSE:

To provide experience in extending a variety of tile patterns

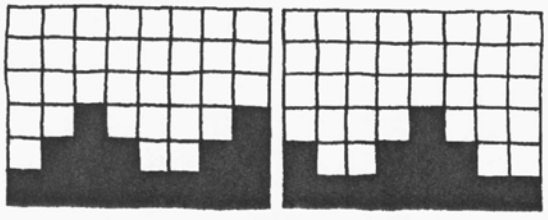
MATERIALS:

1. *Recordings on graph paper of the students' patterns*
2. *Graph paper, 2.5 cm squares on a transparency, or on a large tagboard*
3. *If no overhead projector is available, square shapes*
4. *Tiles*
5. *Dittoed sheets of graph paper with 2.5 cm squares*

Teacher: You each have a blank piece of graph paper and one of the pattern record sheets from yesterday. Please put the two papers together, like this.



Now, try to continue the pattern on the record sheet onto your graph paper. For the record sheet I held up, the pattern would be extended onto the graph paper like this.



When you have done this, pin the sheets next to each other on the bulletin board, and get another record sheet to predict from.

If you don't think you can extend the pattern for the record sheet I gave you, bring it to my desk and get another one.

As the students extend their classmates' patterns, the teacher asks them to think about the following questions:

Are there any patterns that can only be extended one way?

How many different next steps are there for the patterns on some task cards?

What kinds of patterns can we add to in the most different ways?

Are there any patterns on the task cards that no one can think of how to add on to?

What kinds of patterns could be drawn from which no one could predict?

LESSON 3-5

PATTERNS WITH CUBES

PURPOSE:

To copy teacher-generated patterns and predict additional steps in a sequence

MATERIALS:

1. *If no overhead projector is available, square shapes*
2. *Unifix cubes*

To insure students understand that the search for patterns is not confined to a specific material, unifix cubes are used to provide a second series of predicting opportunities. Although the element of color is introduced to the patterns that may be made from cubes, the basic purpose of the next four lessons is the same as that of the preceding four: to familiarize the students with the process of looking for patterns.

The first day the students use cubes for patterns, the sequence used in Lesson 3-1 is repeated. Cubes do not project color on the overhead, so the patterns seen by the students are essentially identical to those in Lesson 3-1.

LESSON 3-6

PATTERNS WITH CUBES

PURPOSE:

To add the element of color into the patterns to be created

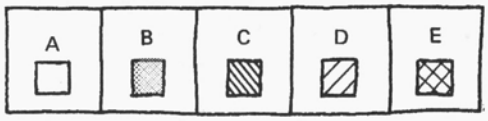
MATERIALS:

1. *Acetate squares lettered A through E, or square shapes lettered similarly*
2. *If no overhead projector is available, square shapes*
3. *Unifix cubes*
4. *Unlined paper*

When working with cubes, usually no two students have the same amounts of the same colors. Because not all students are using the same colors, color patterns to be copied by the students cannot be designated by assembling colored acetate squares on the overhead.

So the students are not confused about what pattern to copy, the teacher introduces a color reference framework before making any color patterns.

The students each draw five squares on a piece of paper. The squares are labeled from *A* to *E*. A different colored cube is placed in each square (no two squares may contain cubes of the same color). A student who has only three colors places them in squares *A*, *B*, and *C*, respectively (*D* and *E* are left empty). A student who has six or more colors will use only five.



The teacher draws five squares on the overhead and letters them in the same manner.

Teacher: Take any cube and put it in the square with the letter *A* on it. Look at the overhead then check with the person next to you to see if my instructions were clear.

Now, take a cube that is a different color from the cube you have in square *A* and put it in square *B*. Check with your neighbor.

Take a cube of still another color and put it in square *C*, again checking with your neighbor.

We won't put anything in the other two squares today. How many different colors do you have in the squares on your desk?

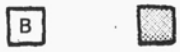
Student: Three.

Teacher: Okay. Leave *your* cubes on your paper, but I am going to take *my* cubes off the overhead and erase it.

Now, I'll tell you a pattern I want you to make by snapping some of your cubes together. Since you don't all have the same color cubes, I'll tell you what color cube I want you to use by telling you a letter. *A* means I want you to use the same color cube that is in your *A* square.

Take one *B* cube and hold it up so I can see it. Don't take the *B* cube from your *B* square. Take one that is the same color from your general pile instead.

The teacher puts one *B* acetate square on the overhead. The students have one *B* cube in their pattern.



Teacher: Look at the cube you are holding up. Is it the same color as the one on your *B* square?

Student: Yes.

Teacher: Is everybody holding up the same color?

Student: No.

Teacher: Why?

Student: Because we don't all have the same color cube in our *B* square.

Teacher: Okay. Take another *B* cube and snap it onto the one you already have. Add an *A* cube to your pattern, and hold it up.

The teacher continues to tell the students which color cube to add to their stack using letter names instead of color names. When the students can successfully transfer letters into colors, the next activity begins.

Teacher: Snap two of your *A* cubes together. Hold them up. Snap two *B* cubes onto what you have already. Hold it up. Add on two more *A* cubes. Hold it up. Two more *B* cubes. Hold it up.

What do you think I will have you add next?

Student: Two more *B* cubes?

Teacher: I could do that. It would make an interesting pattern, but that's not the pattern I am thinking of this time.

Student: Two more *A* cubes?

Teacher: Yes, for the pattern I am thinking of, please add two more *A* cubes.

When the students can predict what comes next, the teacher has them extend the pattern as far as their cubes allow. Students who run out of the colors representing *A* and *B* in their patterns may trade unused colors with fellow students to gain more *A*'s and *B*'s.

A cube pattern may extend beyond a student's desk. If this happens, students may break the pattern at the edge of the desk and form a second row across its width.

When the students have extended a pattern to the limit their cubes will allow, the teacher begins another pattern. The students extend each new pattern as far as their pile of cubes permits.

LESSON 3-7

PATTERNS WITH CUBES

PURPOSE:

To provide experience in creating color patterns with cubes

MATERIALS:

1. *Unifix cubes*

Teacher: I want each of you to choose two colors of cubes you will use. Take the two colors and make a pattern about 15 cubes long. Try to make a pattern that enables someone else to predict the cubes to be added to extend the pattern correctly. When you finish, switch cube patterns with your neighbor.

Can you make your neighbor's pattern longer? Have your neighbor try to make your pattern longer, too.

When the patterns have been extended, the originators check the extensions to see if they are consistent with the original pattern. When the checking is complete the stack of cubes is disconnected and the cycle is begun anew. Each group of students makes, trades, and extends as many cube patterns as time permits.

LESSON 3-8

PATTERNS WITH CUBES

PURPOSE:

To provide experience in recording color patterns with cubes

MATERIALS:

1. *Unifix cubes*
2. *Dittoed sheets of graph paper with 1.7 cm squares*
3. *Crayons*
4. *Unlined paper*

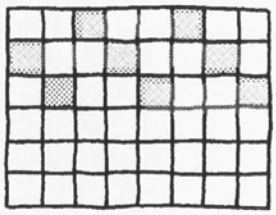
To begin this lesson students reconstruct the same five lettered squares used in Lesson 3-6 as a color reference. Different color cubes are placed in some or all of the squares.

Teacher: Snap two *A* cubes together . . . now a *B* cube . . . now two *A* cubes . . . now a *B* . . . now two *A*s. Do you know the pattern I am using? See how far you can extend the pattern before you run out of cubes.

Now, I'll show you how I want you to record your cube patterns on the graph paper. How many squares are there across the top of your graph paper?

Student: Eight.

Teacher: Okay, break eight cubes off the start of the pattern and lay them on the top row of your graph paper. Break off the next eight cubes and put them on the second row. And the next eight . . . and the next . . . and the next.



This process is continued until each student runs out of cubes or paper.

Teacher: Choose one crayon to match each color cube in your patterns. Remove the last row of cubes you put on your graph paper. Color the squares the same colors as the cubes that were on top of them.

Now, take off the next row of cubes and color in the squares underneath them. Continue until you have removed all the cubes.

The process is repeated until all the cubes in the linear pattern have been recorded or colored in on the graph paper. When one pattern has been recorded on graph paper, the students make up linear patterns of their own and record them in the same manner.

LESSON 3-9

PATTERNS WITH CUBES

PURPOSE:

To examine recorded cube patterns for the answers to teacher-generated questions

MATERIALS:

1. *Unifix cubes*
2. *Dittoed sheets of graph paper with 1.7 cm squares*
3. *Crayons*
4. *Unlined paper*

Although "seeing a pattern" means predicting a next step, the avenues of exploration associated with patterns are not limited to finding a next step. In this lesson students are asked a variety of questions, the purpose of which is to cause them to examine patterns they've seen for patterns they might *not* have seen.

Once the students learn the process of recording linear cube patterns with crayons on graph paper, as described in Lesson 3-8, the teacher encourages the creating and recording of many patterns like this.

As the students record their cube patterns on graph paper, the teacher asks them to think about the following questions:

Do all the patterns look the same?

How many different kinds of patterns could we make?

Can anyone make a pattern with the cubes having the same patterns going down *and* across?

What would happen to the cube patterns if we recorded them on graph paper with fewer squares in each row across?

If I give you a piece of graph paper colored in, can you build the cube pattern from which it was made?

How many rows do you need to color with the help of your cubes before you can color all the squares without needing any more cubes?

The goal of this and the preceding chapter has been to familiarize students with the process of searching for patterns, but it does not end here. The mathematical experiences that lie ahead are based on the belief that patterns form the core of all mathematics. The search has just begun.