# CHAPTER 5
## Beginning Multiplication

**Prerequisite chapters:**

None

## MATERIALS

**For overhead projector:**

Transparencies — ten by ten blank matrix _____ Worksheet 1
Multiplication matrix _____ Worksheet 6
Acetate squares with circles drawn on them _____ Materials chapter, page 295
Tiles _____ Materials chapter, page 294
Beans _____ Materials chapter, page 295

**If no overhead projector is available:**

Make charts in place of transparencies _____ Materials chapter, page 294
Square shapes _____ Materials chapter, page 294
Circular cutouts _____ Materials chapter, page 295
Bean-shaped cutouts _____ Materials chapter, page 295

**Student materials:**

Dittos _____ ten by ten blank matrix
Multiplication matrix
Tiles
Beans
Cups _____ Materials chapter, page 295
Individual blackboards _____ Materials chapter, page 294
Cross-line flash cards _____ Materials chapter, page 296

The activities in this chapter provide experience in constructing multiplication tables through the use of a variety of concrete objects. The lessons emphasize understanding the derivation of the numbers. Students examine multiplication tables in search of countless patterns. They also learn a system for finding answers to individual multiplication problems whose products are less than 100.

# LESSON 5-1

## MULTIPLICATION WITH TILES

PURPOSE:

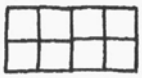
*To create and record repetitive addition patterns with tiles*

MATERIALS:

1. *If no overhead projector is available, square shapes*
2. *Tiles*
3. *Individual blackboards*
4. *Paper*

In the first four lessons students assemble rectangular patterns with tiles and record them in numbers. These patterns form the basis of the multiplication matrices the students will construct in Lessons 5-5 and 5-6.

**Teacher:** I'll build a pattern with tiles. Please build it along with me. As you work, keep a record of the tiles you use.

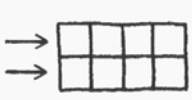


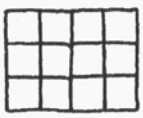
Make this pattern on your desk. How many rows does it have?

**Student:** Two.

**Student:** Four.

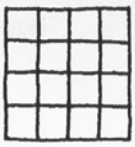
**Teacher:** I think I'd better change my question, since the way I asked it, you could both be right. When I say "rows," I mean lines across, like this. In *this* example, there are two rows. How many rows in the next one?





**Student:** Three.

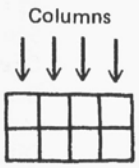
**Teacher:** How many rows in this one?



**Student:** Four.

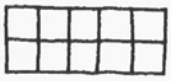
**Teacher:** Let's see. . . Yes, there are four. When I say rows I mean lines of tiles going across. I'll use the word "columns" for lines of tiles going down, like this.

How many columns in this one?




**Student:** Four.

**Teacher:** How about the next one?



**Student:** Five.

**Teacher:** Let's see. . . Yes, five. Okay. Let's go back to our first example. This time I'll keep track of our results on the overhead. How many rows?

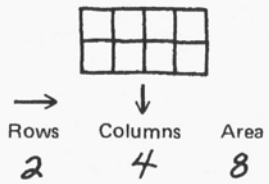
**Student:** Two.

**Teacher:** How many columns?

**Student:** Four.

**Teacher:** Now, tell me how many tiles we have altogether.

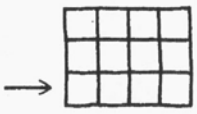
**Student:** Eight.




**Teacher:** When I use tiles to make rectangles, I call the number of tiles altogether the "area." Now, I want to add another row to the pattern. Which way should I put it?

**Student:** Another line across.

**Teacher:** Okay. How many rows?



**Student:** Three.

**Teacher:** How many columns?

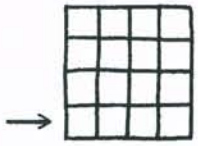
**Student:** Four.

**Teacher:** What is the area now? Remember, area means how many tiles there are altogether.

**Student:** Twelve.

| → Rows | ↓ Columns | Area |
|---|---|---|
| 2 | 4 | 8 |
| 3 | 4 | 12 |

**Teacher:** Now, I'll add another row. How many rows? How many columns? What is the area?



When five or six rows have been added, the teacher asks the students if they can see any patterns for the rows, columns, or area. If they do, they are asked to use those patterns to predict the next numbers to be placed on the overhead. Then, the teacher adds the next row, the tiles are counted, and the results are compared with the predictions. The process is repeated as the teacher adds four or five more rows.

**Teacher:** We looked for patterns when I used two rows and four columns and added more rows. Now, let's see what patterns we might find if we start with two rows and five columns and keep adding rows. I will start the pattern on the overhead. As I make it, you copy the pattern on your desk and keep track of the numbers, because I want you to work this problem on your own.

The teacher sets the problem up on the overhead. After five or six rows have been added, the students are asked to finish assembling the tiles and recording the numbers independently, using the format established by the teacher. They continue adding rows to their tiles until their supply is exhausted.

As the students work, the teacher walks around the room checking to be sure the assignment is understood. When the supply of tiles is exhausted each student begins a new pattern of tiles with two rows and six columns, adding successive rows while keeping the number of columns fixed. When students have assembled and recorded the pattern starting with two rows and six columns, they may start with two rows and seven columns, then two rows and eight columns and so on, as time permits.

# LESSON 5-2

## MULTIPLICATION WITH TILES

PURPOSE:

*To create and record repetitive addition patterns with tiles*

MATERIALS:

1. *If no overhead projector is available, square shapes*
2. *Tiles*
3. *Individual blackboards*
4. *Paper*

The activities and procedure for this lesson are the same as for the previous lesson. The only change is that for this lesson, the teacher keeps the *rows* constant, while adding successive columns.

Using a constant-row, changing-column format, the first tile arrangement begins with two rows and three columns. The next series of problems would start with three rows and three columns, then four rows and three columns, five rows and three columns, and so on.

| → Rows | ↓ Columns | Area |
|---|---|---|
| 2 | 4 | 8 |
| 2 | 5 | 10 |
| 2 | 6 | 12 |
| 2 | 7 | 14 |

# LESSON 5-3

## MULTIPLICATION WITH TILES

PURPOSE:

*To create and record repetitive addition patterns using an abbreviated format*

MATERIALS:

1. *If no overhead projector is available, square shapes*
2. *Tiles*
3. *Individual blackboards*
4. *Paper*

The activities and procedure are the same as for Lesson 5-1. For this lesson, the column numbers are kept constant while more rows are added to the original arrangement of tiles.

The only change is in the way the number patterns are recorded.

**Teacher:** Start with a tile pattern that has one row and two columns. Record the number of rows, the number of columns, and the area on your blackboards.
Add another row . . . record what you have. Add another row . . . record it. Keep adding rows and recording the numbers until you have eight rows altogether.
Hold up your boards and show me what you have. What pattern do you see for the column numbers?

| Row | Column | Area |
|---|---|---|
| 1 | 2 | 2 |
| 2 | 2 | 4 |
| 3 | 2 | 6 |
| 4 | 2 | 8 |
| 5 | 2 | 10 |
| 6 | 2 | 12 |
| 7 | 2 | 14 |
| 8 | 2 | 16 |

**Student:** All the numbers are the same.
**Teacher:** Do you think the numbers will change if you add more tiles?
**Student:** No.
**Teacher:** Why not?
**Student:** Because that number says how many columns. You said not to change the number of columns.
**Teacher:** Okay. If the column number is not going to change for this pattern, then you don't need to write that number each time. If you want to, you can write it like this.

Column 2

| Row | Area |
|---|---|
| 1 | 2 |
| 2 | 4 |
| 3 | 6 |
| 4 | 8 |
| 5 | 10 |
| 6 | 12 |
| 7 | 14 |

Once the teacher has shown the students how to record the tile patterns in an abbreviated format, they repeat the pattern explorations in Lesson 5-1 using the shortened format for recording.

# LESSON 5-4

## MULTIPLICATION WITH TILES

PURPOSE:

*To create and record repetitive addition patterns using an abbreviated format*

MATERIALS:

1. *If no overhead projector is available, square shapes*
2. *Tiles*
3. *Individual blackboards*
4. *Paper*

The activities and procedure are the same as for the previous lesson. The only change is that, for this lesson, the *rows* are kept constant as successive columns are added.

# LESSON 5-5

## MULTIPLICATION WITH TILES

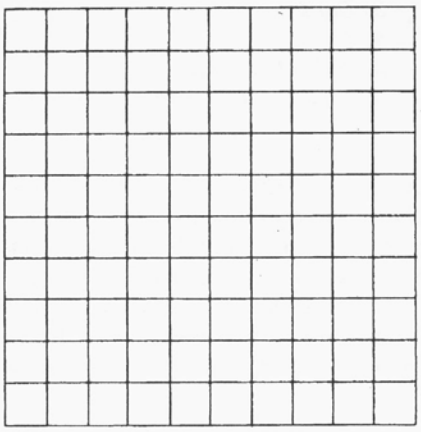
PURPOSE:

*To record repetitive addition patterns on a matrix*

MATERIALS:

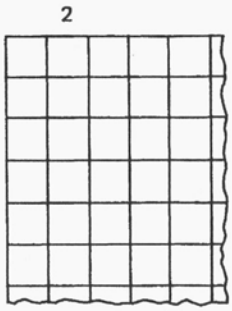
1. *Blank matrix, ten squares by ten squares on a transparency or on a large tagboard*
2. *Dittoed copies of blank ten by ten matrix*
3. *Tiles*
4. *Individual blackboards*

In the next two lessons students record the numbers from their earlier rectangular tile patterns on matrices. The matrices are in turn examined for patterns. This permits students to become familiar enough with each matrix that any recurrence of the same matrices in the recording of seemingly unrelated activities will be recognized.

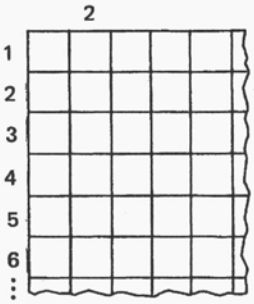


**Teacher:** Today I will show you how to record the tile patterns you've been working on in a different way.

Look at the matrix on the overhead projector. I'll count two squares on the top row and write the number *two*. Notice, I wrote it *outside* the square on the top line.




Now, I'll number from one to ten on the outside of the matrix from top to bottom like this. Please copy this on your paper too. Check with your neighbor to see if you both have what I have.

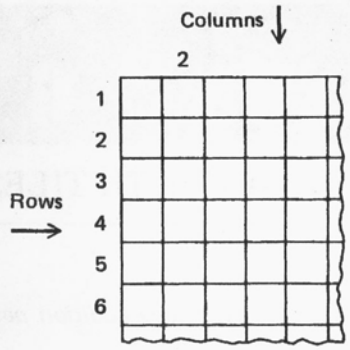



The number I wrote at the top is for columns, so I'll write *columns* at the top and put a column arrow (see illustration at top right). Copy this on your paper. All the numbers on the side are for the rows we'll add, so we'll label them *rows* and put in the row arrow. Check each other's papers to see if what I have said is clear.

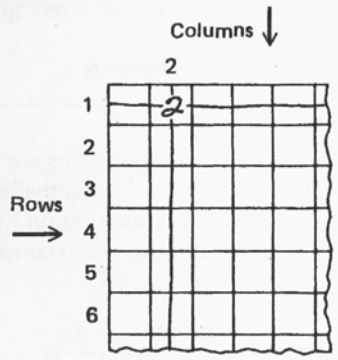
Okay. I want you to make a tile pattern on your desk with one row and two columns. What is the area?

**Student:** Two.

**Teacher:** Let's see. Here's two columns, and here's one row. Yes, the tiles have an area of two. On your paper, in the square down from the *two* columns and across from the *one* row, write two.




The teacher demonstrates exactly where to write the two by drawing a light line down the *two* column on the overhead, and another line across the *one* row. Where the two lines cross, the teacher writes in a two.




**Teacher:** Add another row to your tile pattern. How many rows do you have now?

**Student:** Two.

**Teacher:** How many columns?

**Student:** Two.

**Teacher:** Did the number of columns change?

**Student:** No.

**Teacher:** What is the new area?

**Teacher:** Four.

**Teacher:** Write the area for two rows and two columns straight across the *two* row line in the *two* column line.

The teacher demonstrates by drawing light lines down the *two* column and across the *two* row on the overhead. The teacher and the class together fill in the *two* column. The students then fill in the numbers for constant columns of three, four, five, and beyond on their own.

As the students work, the teacher asks them to think about the following questions:

What patterns can you see as you look down the columns of numbers on your papers?

Can you see any patterns across the rows of numbers, as well?

Are the patterns across the same as or different from the patterns down? In what way? Why?

Is it always necessary to work out every number in a row or column? Are there patterns you can use to predict what some of the numbers might be?

# LESSON 5-6

## MULTIPLICATION WITH TILES

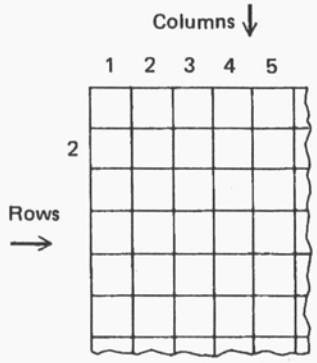
PURPOSE:

*To record repetitive addition patterns on a matrix*

MATERIALS:

1. *Blank matrix, ten squares by ten squares on a transparency or on a large tagboard*
2. *Dittoed copies of blank ten by ten matrix*
3. *Tiles*
4. *Individual blackboards*

The activities, procedures, and questions are the same as for the previous lesson. The only change in the lesson is that the teacher writes all the column numbers on the matrix in advance, and each row in succession is held constant.




# LESSON 5-7

## MULTIPLICATION WITH BEANS AND CUPS

PURPOSE:

*To create and record repetitive addition problems with beans and cups*

MATERIALS:

1. *If no overhead projector is available, bean-shaped cutouts*
2. *Acetate squares with circles drawn on them, or circular cutouts*
3. *Cups*
4. *Beans*
5. *Individual blackboards*
6. *Lined or unlined paper*

The students have assembled a multiplication matrix using rectangular patterns of tiles to produce the appropriate numbers. In the next four lessons students use beans and cups to generate number patterns, which they will record on a matrix. It will be examined for patterns that should enable students to recognize that the beans and cups matrix contains the same numbers as did the earlier tile matrices.

**Teacher:** You will need two cups and a pile of beans for the lesson. Each time I tell you to add a bean be sure to add one to *each* cup. Put a bean in each cup. How many cups do you have?

**Student:** Two.
**Teacher:** How many beans do you have in *each* cup?
**Student:** One.
**Teacher:** How many beans altogether?
**Student:** Two.
**Teacher:** Add another bean to each cup. How many cups?
**Student:** Two.
**Teacher:** How many beans in each cup?
**Student:** Two.
**Teacher:** How many beans altogether?
**Student:** Four.
**Teacher:** Okay. I'll record what we are doing on the overhead so we can keep track of the numbers and see if we can find any patterns to make our work go more easily.

| Beans in each cup | Number of cups | Total number of beans |
|---|---|---|
| 1 | 2 | 2 |
| 2 | 2 | 4 |

Add another bean to each cup. How many beans in each cup?
**Student:** Three.
**Teacher:** How many cups altogether?
**Student:** Two.
**Teacher:** What is the total number of beans in all the cups?
**Student:** Six.

| Beans in each cup | Number of cups | Total number of beans |
|---|---|---|
| 1 | 2 | 2 |
| 2 | 2 | 4 |
| 3 | 2 | 6 |

The students are able to answer the questions posed so far by simply counting the beans and cups in front of them. After they have added five beans to each cup, the teacher phrases the questions in a different way:

How many beans will I ask you to put in each cup next?
How many beans will that make in each cup?
How many cups will there be altogether?
What do you think the total number of beans will be?
Add in the beans and check all your predictions.

When the students can answer these less direct questions, they continue the pattern on their own, recording each new step on paper until they run out of beans or the numbers reach the bottom of the page. The beans are then dumped out and a third cup is added. The procedure is the same for three cups as for two. After the three-cup process is completed, four, five, six, etc., cups are added.

For each series of recordings, the teacher asks the students if they see any patterns in the numbers that would help them predict future numbers. The beans and cups are then used to check the validity of the predictions.

# LESSON 5-8

## MULTIPLICATION WITH BEANS AND CUPS

PURPOSE:

*To create and record repetitive addition patterns using an abbreviated format*

MATERIALS:

1. *If no overhead projector is available, bean-shaped cutouts*
2. *Acetate squares with circles drawn on them, or circular cutouts*
3. *Cups*
4. *Beans*
5. *Individual blackboards*
6. *Lined or unlined paper*

The activities and procedure are the same as for the previous lesson. The only change is in the way the number patterns are recorded.

Since the number of cups remains the same, the students may use an abbreviated recording format such as is shown above right.

Cups 4

| Beans | Total |
|---|---|
| 1 | 4 |
| 2 | 8 |
| 3 | 12 |
| 4 | 16 |
| 5 | 20 |
| 6 | 24 |
| ⋮ | ⋮ |

The teacher presents the students the option of a shortened recording method as was done for tiles in Lesson 5-3.

Once the abbreviated format is introduced, some students will want to repeat the activities in the previous lesson starting with two cups, then three, then four. The teacher may encourage other students to go off in different directions by asking questions like the following:

What kinds of patterns would you get if you used 5 cups, then 10, then 15? Would the patterns help you know what would happen for 20 or 25?
What would happen if you added two beans each time, instead of one? Would you still get a pattern from which you could make predictions?
Would adding three beans at a time give you a pattern? How about four? Five?
What number of beans can you add each time that *won't* result in a pattern?

No one *has to* go off on a tangent. The basic structure of the lesson is always there—the questions are to spark exploration. If other questions occur, either to the teacher or the students, they can be asked of the whole class.

# LESSON 5-9

## MULTIPLICATION WITH BEANS AND CUPS

PURPOSE:

*To record repetitive addition patterns on a matrix*

MATERIALS:

1. *Blank matrix ten squares by ten squares on a transparency or on a large tagboard*
2. *Dittoed copies of blank ten by ten matrix*

3. *Cups*
4. *Beans*
5. *Individual blackboards*

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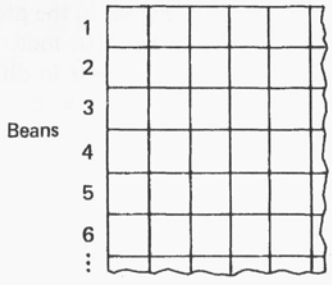
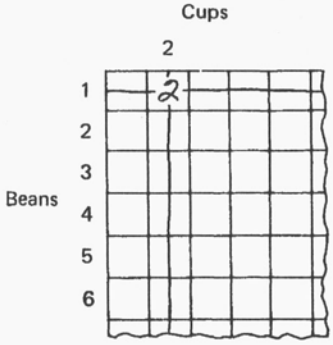
**Teacher:** Today we'll keep track of the number patterns we get with the beans and cups in a different way. Take out a handful of beans and two cups. Now, put one bean in each cup. I will record what you have on the ten by ten matrix on the overhead. As I write the numbers on my matrix, write the same numbers on your matrix.

The first thing I'll write are the numbers from one to ten. Please put them on the outside of your matrix. Check with your neighbor so I can see if what I have said so far is clear. These are the numbers that tell us how many beans we've put in each cup, so we'll write *beans* along the outside like this.



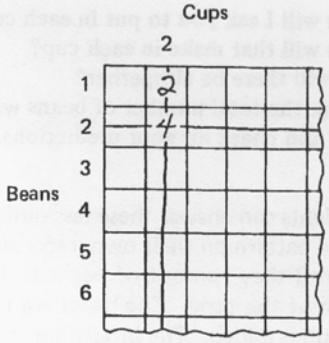

Next, I want you to write a two above the second column where I put my two. Notice, it's not inside the square. The two stands for the number of cups we will use in this pattern, so write *cups* above the two. Now, how many cups do you have on your desk?

**Student:** Two.
**Teacher:** How many beans in each cup?
**Student:** One.
**Teacher:** How many beans altogether?
**Student:** Two.
**Teacher:** The total number of beans in each cup goes in the space straight down from the number of cups, and straight across from the number of beans in each cup.




Check each other's papers to see if my instructions have been clear so far. Add another bean to each cup. How many beans in each cup?

**Student:** Two.
**Teacher:** How many cups?
**Student:** Two.
**Teacher:** How many beans total?
**Student:** Four.
**Teacher:** Where do you think I want you to write the total?




Since the class has already recorded similar information in a similar way for tiles, the process of teaching them to record the cups and bean data goes much more quickly. The teacher records the information for two cups on the overhead. Beyond two cups, students fill in the matrix on their own.

As the students work at filling the table, the teacher asks them to consider the following questions:

How is this table the same as the ones you made for tiles?
How is it different?
Why are they the same or different?
Are there any patterns in the columns or rows that you can use to help you fill in the table more quickly?
Are the patterns for the columns the same as or different from the patterns for the rows? Why?

# LESSON 5-10

## MULTIPLICATION WITH BEANS AND CUPS

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PURPOSE:

*For each student to use his or her multiplication matrix to explore answers to teacher-directed questions*

MATERIALS:

1. *Blank ten by ten square matrix transparency or a large tagboard, as filled in for Lesson 5-9*
2. *Dittoed copies of blank ten by ten matrix filled in by students for Lesson 5-9*
3. *Cups*
4. *Beans*
5. *Individual blackboards*

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**Teacher:** You recorded information from your beans and cups on this matrix. Now I want you to use the matrix to answer some questions.

Cups

| Beans | 1 | 2 | 3 | 4 | 5 . . . |
|---|---|---|---|---|---|
| 1 | 1 | 2 | 3 | 4 | 5 |
| 2 | 2 | 4 | 6 | 8 | 10 |
| 3 | 3 | 6 | 9 | 12 | 15 |
| 4 | 4 | 8 | 12 | 16 | 20 |
| 5 | 5 | 10 | 15 | 20 | 25 |
| 6 | 6 | 12 | 18 | 24 | 30 |
| ⋮ | | | | | |

I have five cups and six beans in each cup. Look at the matrix and write on your blackboards the number of beans you think I have altogether. . . . Most of you have written 30 on your blackboards. Tell me how you used your matrix to get 30.

The teacher has one or two students explain how they used the matrix to get their answer. This permits students who do not understand the connection between the questions asked and the numbers on the matrix to see and hear examples of how their classmates have attacked the problem.

When students can regularly find the total number of beans, given the number of cups and of beans per cup, they are asked a different kind of question:

**Teacher:** You are quite good at telling me how many beans there are altogether when I tell you how many cups you have and how many beans in each cup. Can you answer this question?

If you have 24 beans altogether, and you have the beans in four different cups, how many beans would you have in each cup? Can you find the answer by looking at your table?

If the students cannot find the answer using the matrix, they reconstruct the problem using beans and cups. When they've found the beans-and-cups answer, the teacher asks them to find that answer on the matrix, and if there was a way they could have used their matrix in the first place.

It is important that the total number of beans be a number that can be found on the matrix. For example, if the teacher asks students to figure out how many beans are in each cup for 5 cups and 41 beans, the answer is not found on the matrix, hence the question is inappropriate. Though the students will be solving problems like this at a later time, the present exercise is to provide practice in using the matrix to find answers.

When the students can calculate how many beans are in each cup, given the total number of beans and cups, the teacher asks them to explore a third possibility:

**Teacher:** If you have 30 beans altogether and I ask you to put 6 in each cup, how many cups would you have to use? Can you tell by looking at the numbers on the table?

If you can't find it by looking at the table, could you find it by using your beans and cups?

The method for solving this kind of problem is the same as that for discovering the number of beans in each cup, given the number of cups.

If students are unable to devise a solution to the problems, no solution should be given. It is more important that they have an opportunity to think about a solution than that they actually find one. If students are to rely on their own thinking, it must be this thinking, not the teacher's knowledge, that ultimately produces answers.

# LESSON 5-11

## USING A MULTIPLICATION MATRIX

PURPOSE:

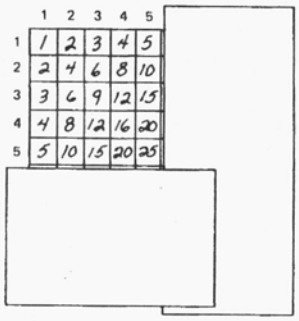
*To utilize a multiplication matrix to explore answers to teacher-directed questions*

MATERIALS:

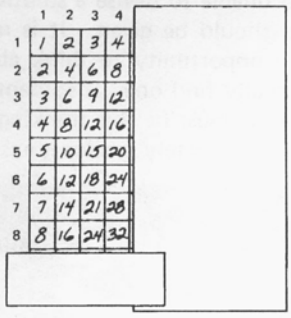
1. *Multiplication matrix on a transparency or on a large tagboard*
2. *Dittoed copies of multiplication matrix*
3. *Individual blackboards*
4. *Unlined paper*

In this lesson, students are given a matrix and are told from the start it is a multiplication matrix. Through the teacher's questions it is examined for patterns that might not have been discovered in the student's earlier explorations. The students' discoveries increase their understanding of the relationship of the numbers on the matrix to one another and to the matrix as a whole.

**Teacher:** I'll cover up part of this multiplication table with two pieces of paper. Look at the space that *isn't* covered. How many little squares make up this rectangle? Make this rectangle on your own table and count the squares to find out . . .



**Student:** Twenty-five.
**Teacher:** What number is in the lower right-hand corner of this rectangle?
**Student:** Twenty-five.
**Teacher:** Okay. I'll form a new rectangle with my two pieces of paper. How many squares are in this rectangle?




**Student:** Thirty-two.
**Teacher:** What is the number in the lower right-hand corner?
**Student:** Thirty-two.

The teacher continues covering parts of the multiplication matrix. The students count the squares left uncovered and write the amount on their blackboards. They then erase their boards and write the number that appears in the lower right-hand corner of the uncovered area. This process continues until the students see patterns for what the teacher is doing.

Students who see one pattern no longer erase the number between the two questions. Those who dispense with counting the squares and look only at the number in the lower right corner for both answers see another pattern.

As the teacher leaves different areas of the matrix uncovered the students are asked to think about the following questions:

Do you see a pattern for what I am doing?
If there is a pattern, why does it work?
Which rectangles on the matrix will the pattern work for? Which won't it work for? Why?
Look at the numbers on the outside of the matrix. Can you use any of them to help you discover how many little squares are in the rectangle? If you could . . . how?

# LESSON 5-12

## MULTIPLICATION WITH CROSSED LINES

PURPOSE:

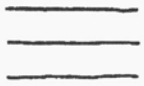
*To provide students with a quick method of determining specific multiplication facts*

MATERIALS:

*1. Unlined paper*

The teacher should post a large multiplication matrix where it is visible to all students; access to this is the best way for students to learn individual math facts. Each student should also keep one in his or her desk. The ready availability and frequent use of a matrix causes most students to effortlessly commit all or most of the multiplication facts to memory.

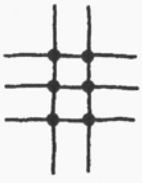
Times will occur, however, when a student who has not yet committed the facts to memory needs to know, say, eight times seven, and no matrix is available. For these special circumstances, the teacher can present students with a way of determining multiplication facts without having them rely on memory or reconstructing an entire table.



**Teacher:** How many lines have I drawn?
**Student:** Three.
**Teacher:** Across or down?
**Student:** Across.

**Teacher:** How many new lines did I draw?
**Student:** Two.
**Teacher:** Across or down?
**Student:** Down.
**Teacher:** Okay. I have three lines across and two lines down. At how many places do all these lines touch?

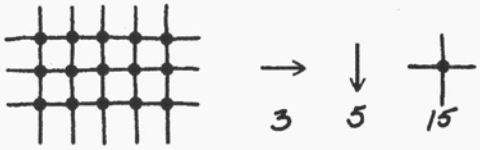
To illustrate, the teacher darkens in the intersections.



**Student:** Six.
**Teacher:** New problem. How many across?
**Student:** Two.
**Teacher:** How many down?
**Student:** Four.

**Teacher:** How many intersections? How many meeting points?
**Student:** Eight.
**Teacher:** Okay. This time I'll keep track of what we find out. Here are the column headings I will use.

How many across?
**Student:** Three.
**Teacher:** How many down?
**Student:** Five.
**Teacher:** How many intersections?
**Student:** Fifteen.




The teacher records three or four examples on the overhead, then the students practice recording examples on their blackboards. When they have demonstrated an understanding of the recording process, the students construct their own cross-line problems and record the numbers on paper.

The students may make up problems at random. They may also choose to explore what happens to the number of intersections as more down lines are added to constant cross lines, or more cross lines are added to constant downs.

# LESSON 5-13

## MULTIPLICATION WITH CROSSED LINES

PURPOSE:

*To record cross-line number patterns on a matrix*

MATERIALS:

1. *Blank matrix, ten squares by ten squares on a transparency or on a large tagboard*
2. *Dittoed copies of blank ten by ten matrix*
3. *Individual blackboards*
4. *Unlined paper*

When students have had the opportunity to create their own cross-line problems, the teacher presents the problems in a more systematic way.

The activities and procedures for this lesson are the same as for Lesson 5-5. The only change is that the teacher writes all the cross-line numbers on the matrix in advance, while holding each successive down-line number constant.

As the students work, the teacher asks them to think about answers to the following questions:

What patterns can you see as you look down the columns of numbers on your paper?
Have you seen these patterns before? Where? Why?
Can any patterns be seen across the rows of numbers?
Have you seen these patterns before? Where? Why?
Is it necessary to work out all the numbers that go on the matrix? Or, are there patterns you can use to predict what some of the numbers might be?

# LESSON 5-14

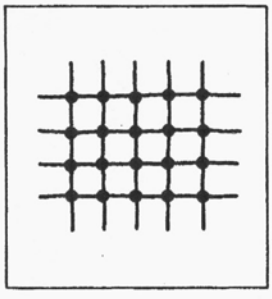
## MULTIPLICATION WITH CROSS LINES

PURPOSE:

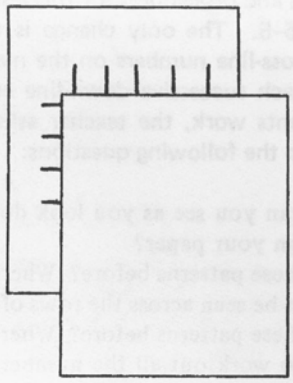
To use teacher-made flash cards for multiplication facts practice

MATERIALS:

1. *Teacher-made flash cards*
2. *Individual blackboards*



**Teacher:** How many lines down on this card?
**Student:** Five.
**Teacher:** How many lines across?
**Student:** Four.
**Teacher:** How many intersections?
**Student:** Twenty.
**Teacher:** Okay . . . now look at this card (see illustration, page 48). I've covered up the intersections. How many lines down?



**Student:** Five.
**Teacher:** How many lines across?
**Student:** Four.
**Teacher:** And how many intersections?

To answer questions like these, the students must either visualize four lines crossing five lines then mentally count the intersections, or know four times five is twenty. Either way is acceptable. It is also acceptable if a student doesn't know the answer using either method. The lines may always be drawn, and the intersections counted out.

The teacher presents the cross-line flash cards to the whole class. After the initial example, each card is shown first with the intersections covered. As a check on the students' responses, the covering paper is removed and the intersections counted. When the class as a whole has gone through the entire pile, the cards are left available for individuals to use at their leisure. The teacher may also periodically have the whole class go through the cards again. This keeps the cross-line technique fresh in the students minds.

The cross-line flash cards are not intended to cause students to memorize multiplication facts, but to help them learn that the lines seen on top and the lines on the side have a definite bearing on the number of intersections seen in the middle.

This skill allows a student to know what 8 times 7 is, if no multiplication matrix is handy, because the 56 intersections that are produced by 8 lines down and 7 lines across is the answer. The cross-lines provide the students with a method of determining all the multiplication "facts" they will ever need to know.

The students now have a framework for giving meaning to the numbers appearing in a multiplication matrix. This understanding of the multiplication matrix provides the basis for the later activities in the chapter Advanced Multiplication.