CHAPTER 6
Beginning Division

Lesson 6–1 page 51 Beans and Cups Students create and record division problems.	Lesson 6–2 page 52 Beans and Cups Students explore specific division problems for patterns.	Lesson 6–3 page 53 Beans and Cups Students continue looking for patterns while writing remainders in a new way.	Lesson 6–4 page 53 Beans and Cups Students explore specific division problems while answering teacher questions.
Lesson 6–5 page 54 Beans and Cups Students record answers to specific division problems on a matrix.	Lesson 6–6 page 55 Beans and Cups Students use bean and cup matrix to answer teacher questions.	Lesson 6–7 page 55 Tiles Students create and record division problems.	Lesson 6–8 page 57 Tiles Students create and record division problems while answering teacher questions.
Lesson 6–9 page 57 Tiles Students work teacher directed problems in three basic formats.	Lesson 6–10 page 58 Cross-line Division Students create and record division problems.	Lesson 6–11 page 59 Cross-line Division Students work teacher-directed problems in three basic formats.	

Prerequisite chapters:

None

MATERIALS

For overhead projector:

Transparencies	Ten by ten blank matrix	Worksheet 1
	Tile board	Worksheet 7
Acetate squares with circles drawn on them		Materials chapter, page 295
Beans		Materials chapter, page 295
Tiles		Materials chapter, page 294

If no overhead projector is available:

Make charts in place of transparencies	Materials chapter, page 294
Square shapes	Materials chapter, page 294
Circular cutouts	Materials chapter, page 295
Bean-shaped cutouts	Materials chapter, page 295

Student materials:

Dittos	Ten by ten blank matrix
	Tile board
Beans	
Tiles	
Cups	Materials chapter, page 295
Individual blackboards	Materials chapter, page 294
Lined or unlined paper	

The activities in this chapter provide students experience in constructing and checking their own division problems. To facilitate this process, the concept of *remainders* is introduced from the start.

LESSON 6-1

DIVISION WITH BEANS AND CUPS

PURPOSE:

To create and record division problems with beans and cups

MATERIALS:

1. *Circles on transparency squares, or circles cut from paper or made from string*
2. *Beans*
3. *Cups*
4. *Individual blackboards*
5. *Lined paper*

In this lesson, students learn to create and record division problems using beans and cups, which permits them to check the accuracy of their answers. To enable them to create a wide range of problems, the concept of a remainder is introduced.

Teacher: When we used beans and cups before, we discovered how many beans we had altogether with a particular number of cups and the same number of beans in each cup. This time we'll start backwards. I'll take a handful of beans, which will be how many I have altogether. Then I'll see how many beans I can put into each cup, making sure I end up with the same number in each cup.

Before I find out how many beans go in each cup, I'll count them all. I have 27 beans. Let's see how many I can put in each cup.

With six beans in each cup, I still have three left in my hand. Can I put one more in *each* cup?

Student: No.

Teacher: Why not?

Student: Because you have to have the same number of beans in each cup. You haven't got enough beans left over to put one more in each cup.

Teacher: How many beans in each cup?

Student: Six.

Teacher: How many left over?

Student: Three.

Teacher: Then the answer to this problem is six beans in each cup with three left over.

The teacher goes through one or two more examples at the overhead, starting with a new handful of beans each time. The students should then be ready to work an example for themselves.

Teacher: Everyone take out four cups. I'll take a handful of beans and count them. I want you to take out that many beans also.

I have 17 beans. Everyone count out 17 beans.

Now, see how many you get in each cup. Remember, you have to have the same number of beans in each cup. Keep any that are left over on your table or in your hand. When you've found your answer, write it on your blackboard.

When the class has completed the problem, the teacher works it out on the overhead as a check. The process is repeated for two or three more class examples. The students are almost ready to begin generating their own problems.

Teacher: Before you begin making up your problems I want to show you how to record what you've done so I can see what problems you have worked out. First, I'll need to know how many beans you started with and how many cups you used. Then I'll need to know how many beans you put in each cup, and how many you had left over. I want you to call how many you had left over the *remainder.*

Beans	Cups	Beans in each cup	Remainder

At this point, the students begin making up and recording problems, deciding the number of cups and beans to be used. The teacher circulates among them, checking to see that everyone understands.

The students control the level of difficulty of their problems by using more or less beans and cups. Students who wish may try to predict the answer to a problem before they work it. This encourages them to look for relationships between each problem and its answer in a search for patterns that will permit them to make more meaningful

predictions. It also heightens their interest in the problem itself, since each new answer obtained becomes a check on their ability to predict.

LESSON 6-2

DIVISION WITH BEANS AND CUPS

PURPOSE:

To create and record division problems with beans and cups while searching for patterns

MATERIALS:

1. *Circles on transparency squares, or circles cut from paper or made from string*
2. *Beans*
3. *Cups*
4. *Individual blackboards*
5. *Lined paper*

The next five lessons call on students to organize the data from different series of division problems so the answers obtained may be examined for possible patterns. This search for patterns serves as both a motive for creating and solving division problems and a basis for acquiring an intuitive understanding of the division process.

Teacher: We will now begin looking for patterns that might help you work *some* problems more quickly. The best way I know to look for patterns is to do problems in a definite order.

First, write your record-keeping headings on your paper. We will begin looking for patterns when we use three cups, so for these first problems, everyone needs three cups. Instead of taking a handful of beans, begin by taking only one. What is the total number of beans you have?

Student: One.
Teacher: How many cups?
Student: Three.
Teacher: How many beans could you get in each cup?
Student: None.
Teacher: Then what is the remainder?
Student: One.
Teacher: Okay. That's the first problem. The second is for two beans. What is the total number of beans?
Student: Two.
Teacher: How many cups?
Student: Three.
Teacher: Beans in each cup?
Student: Zero.
Teacher: Remainder?
Student: Two.
Teacher: Now, take three beans. . . . Total beans?
Student: Three.
Teacher: Number of cups?
Student: Three.
Teacher: Number in each cup?
Student: One.
Teacher: Remainder?
Student: Zero.
Teacher: How many total beans will I ask you to take next?
Student: Four.
Teacher: How many cups?
Student: Three.
Teacher: How many beans in each cup?
Student: One.
Teacher: Remainder?
Student: One.
Teacher: Okay.

Beans	Cups	Beans in each cup	Remainder
1	3	0	1
2	3	0	2
3	3	1	0
4	3	1	1
5	3	1	2
6	3	2	0
7	3	2	1

The teacher asks the students to scan the columns for patterns to help them predict the numbers they expect for answers.

The above process is continued until either the numbers reach the bottom of the page or all the beans have been used. The teacher then helps the students start a new series of problems, using a constant of four cups and dividing using one, two, three, four, then five beans. The students then finish the remaining divisions on their own. When they have finished exploring with four cups, they start again with five. Beyond five, any number of cups is acceptable, including only one.

The students create as many or as few division problems for beans and cups as the time allowed for the lesson permits.

LESSON 6-3

DIVISION WITH BEANS AND CUPS

PURPOSE:

To continue searching for patterns while learning a new way to write remainders

MATERIALS:

1. *Circles on transparency squares, or circles cut from paper or made from string*
2. *Beans*
3. *Cups*
4. *Individual blackboards*
5. *Lined paper*

The activities in this lesson are an extension of those in the previous lesson. The only change is that the students are shown a new way to write remainders. They were not taught this new system earlier because the number of new ideas to be faced at any one time should be kept to a minimum. They already understand how to construct and record successive problems. Delaying the introduction of a new way of recording remainders until this point minimizes any potential confusion.

Teacher: Today I want you to write the numbers you get for remainders in a different way. You will begin writing the remainders the same way you will write them when you get to junior high or high school.

When we used four cups and added one more bean each time, this was what we recorded. What was the first remainder?

Beans	Cups	Beans in each cup	Remainder
1	4	0	1
2	4	0	2
3	4	0	3
4	4	1	0
5	4	1	1

Student: One.
Teacher: What did a remainder of one mean?
Student: That we had one bean left over.
Teacher: In high school, they write one bean left over using four cups, like this. The top number means how many were left over; the bottom number means how many cups altogether.

$$\frac{1}{4}$$

To read this remainder they say "one-fourth" or "one over four." How would they write the remainder for two beans left over with four cups?
Student: 2/4.
Teacher: For three beans and four cups?
Student: 3/4.
Teacher: For four beans and four cups?

Some students might write 4/4, others might suggest there is no remainder. In either event the teacher discusses the variety of possibilities for what four over four might mean in terms of beans and cups.

When the teacher has explained the system of writing the remainder over the number of cups, the students practice writing remainders on their blackboards, first for four cups, then for a variety of cups. The teacher then checks the blackboards.

This system is taught only as a method of writing remaining numbers, not as a lesson in fractions. Operations with fractions are taught in a later chapter.

LESSON 6-4

DIVISION WITH BEANS AND CUPS

PURPOSE:

To create and record division problems with beans and cups while exploring answers to teacher-directed questions

MATERIALS:

1. *Beans*
2. *Cups*
3. *Lined paper*

This lesson continues the activities in Lessons 6-2 and 6-3. As the students conduct their explorations, the teacher asks them to think about the following questions:

What patterns can you see in each column on your paper?
What patterns can you see for the number of beans in each cup?
What patterns can you see for the remainders?
Can you use the patterns you found for, say, four cups, to help you know what the patterns for five cups might look like? For six? Seven?
If I told you the number of cups, the number of beans in each cup, and the remainder, could you tell me the total number of beans you started with? How?
What if I told you the total number of beans, the number in each cup, and how many you had as a remainder. Could you tell me how many cups you would have? How?

Searching for answers to questions can be a highly effective motivating force for our students. We examine things more closely if a reason exists for looking. Questions provide that reason.

LESSON 6-5

DIVISION WITH BEANS AND CUPS

PURPOSE:

To record division patterns on a matrix to facilitate the search for patterns

MATERIALS:

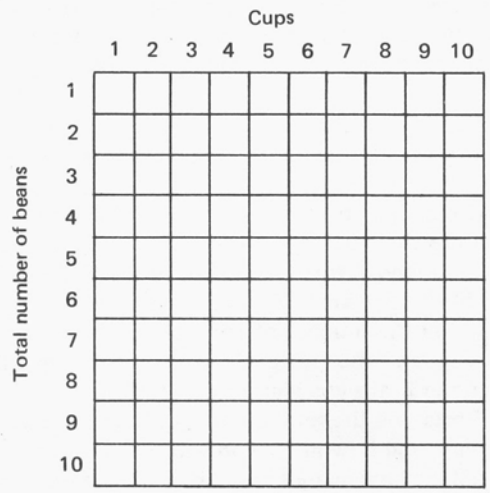
1. *Blank matrix, ten squares by ten squares on a transparency or on a large tagboard*
2. *Blank ten by ten matrix dittos*
3. *Beans*
4. *Cups*
5. *Individual blackboards*

Teacher: I have a blank ten by ten matrix on the overhead. You should each have the same blank matrix at your desks.

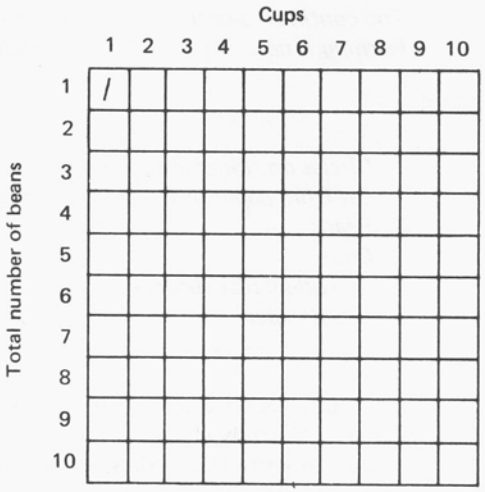
Fill in your matrix the same way I fill in mine. At the top, write "cups." Leave enough room to write some numbers underneath. Check each other's work so you can tell me if my instructions are clear.

Above the top squares, write the numbers one through ten. Again check with your neighbor.

Now, I want you to write the numbers one through ten down the side, then write "total number of beans" along that same side. What do you think the top number one stands for?



Student: One cup.
Teacher: What does the side number one mean?
Student: One bean.
Teacher: Okay. If I take one cup and one bean, how many beans in each cup?
Student: One.
Teacher: What is the remainder?
Student: Zero.
Teacher: So my answer goes here.



What answer goes in the space beneath the one?

If the students know the answer, it is written in the space. If not, the teacher asks:

What does the number at the top of the column mean?
What does the number at the side of the row mean?
How many cups and beans should we use to work out the answer?

The students and teacher work out five examples for the one-cup column together. Then, without finishing that column, teacher and students begin again with two cups. For the two-cup column, the teacher points to an empty space and the students write the number they think should fill it on their blackboards. If they wish, they may use their beans and cups to help them find the answer.

When the students consistently have correct numbers on their blackboards, they begin compiling cup and bean data for their blank matrices. As they fill them in, the teacher asks the following questions:

Can you see any patterns in any of the columns that help you know what numbers to expect next?
Are there any patterns in the rows that might be useful for making predictions?
How about the diagonals?
Why do you think you get these patterns?

LESSON 6-6

DIVISION WITH BEANS AND CUPS

PURPOSE:

To utilize the division matrix to explore answers to teacher-directed questions

MATERIALS:

1. *Blank ten by ten matrix, transparency, or large tagboard, as filled in for Lesson 6-5*
2. *Dittoed copies of blank ten by ten matrix filled in by students for Lesson 6-5*
3. *Beans*
4. *Cups*
5. *Individual blackboards*

Cups

Total number of beans	1	2	3	4	5	6	7	8	9	10
1	1	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{6}$	$\frac{1}{7}$	$\frac{1}{8}$	$\frac{1}{9}$	$\frac{1}{10}$
2	2	1	$\frac{2}{3}$	$\frac{2}{4}$	$\frac{2}{5}$	$\frac{2}{6}$	$\frac{2}{7}$	$\frac{2}{8}$	$\frac{2}{9}$	$\frac{2}{10}$
3	3	$1\frac{1}{2}$	1	$\frac{3}{4}$	$\frac{3}{5}$	$\frac{3}{6}$	$\frac{3}{7}$	$\frac{3}{8}$	$\frac{3}{9}$	$\frac{3}{10}$
4	4	2	$1\frac{1}{3}$	1	$\frac{4}{5}$	$\frac{4}{6}$	$\frac{4}{7}$	$\frac{4}{8}$	$\frac{4}{9}$	$\frac{4}{10}$
5	5	$2\frac{1}{2}$	$1\frac{2}{3}$	$1\frac{1}{4}$	1	$\frac{5}{6}$	$\frac{5}{7}$	$\frac{5}{8}$	$\frac{5}{9}$	$\frac{5}{10}$
6	6	3	2	$1\frac{2}{4}$	$1\frac{1}{5}$	1	$\frac{6}{7}$	$\frac{6}{8}$	$\frac{6}{9}$	$\frac{6}{10}$
7	7	$3\frac{1}{2}$	$2\frac{1}{3}$	$1\frac{3}{4}$	$1\frac{2}{5}$	$1\frac{1}{6}$	1	$\frac{7}{8}$	$\frac{7}{9}$	$\frac{7}{10}$
8	8	4	$2\frac{2}{3}$	2	$1\frac{3}{5}$	$1\frac{2}{6}$	$1\frac{1}{7}$	1	$\frac{8}{9}$	$\frac{8}{10}$
9	9	$4\frac{1}{2}$	3	$2\frac{1}{4}$	$1\frac{4}{5}$	$1\frac{3}{6}$	$1\frac{2}{7}$	$1\frac{1}{8}$	1	$\frac{9}{10}$
10	10	5	$3\frac{1}{3}$	$2\frac{2}{4}$	2	$1\frac{4}{6}$	$1\frac{3}{7}$	$1\frac{2}{8}$	$1\frac{1}{9}$	1

Teacher: Look at the matrix you filled out yesterday for cups and beans. See if you can use it to answer this question: you have seven beans and five cups; how many beans in each cup and what's the remainder?

If the students can use the matrix to find the answer, they verify it by working out the solution with beans and cups. If they cannot, they solve the problem first with beans and cups, then find their answer on the table and attempt to discover how the table might have been used to produce the answer.

The teacher continues to ask questions using the format:

You have _____ beans altogether and _____ cups. How many beans in each cup and what's the remainder?

When the students can answer questions in the first format, questions stated in two additional formats are used.

The second questioning pattern is:

There are _____ beans in each cup and _____ beans left over. If there are _____ cups, can you use your matrix to tell me how many beans you started with?

The third questioning pattern is:

There are _____ beans in each cup and _____ beans left over. If there are _____ beans altogether, can you use your matrix to tell me how many cups you started with?

The same pattern of verifying answers with beans and cups used for the first questioning format is used with the second two.

What students see objects do can be recorded in numbers. These same numbers can also be used to help predict what objects will do. Our students should have the opportunity to use numbers to record their experiences with materials, and to use these numbers to forecast what new materials will do. Numbers record; recorded numbers can also be used to predict.

LESSON 6-7

DIVISION WITH TILES

PURPOSE:

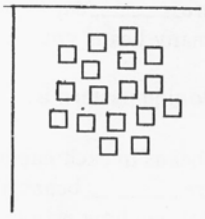
To create and record division problems with tiles

MATERIALS:

1. *Tile board transparency, or L-shaped cut-out and cardboard squares*
2. *Tile board dittos*
3. *Tiles*
4. *Individual blackboards*
5. *Unlined paper*

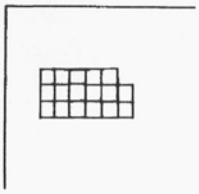
The next two lessons allow students to discover that although the materials used may be different, the numbers recorded for dividing with tiles produce the same answers as do beans and cups.

Teacher: I call this a *tile board.* I will take a handful of tiles and put them where it says "tiles" on my tile board. How many tiles do I have on my board?



Student: Seventeen.

Teacher: Okay. For each of the problems we do today, we will make rectangles having only 3 rows across. I'll see what rectangle I can make with these 17 tiles that has only 3 rows across. Is this a rectangle?



Student: No.

Teacher: Why not?

Student: It has to be even on all sides with no bites out.

Teacher: How can I make it into a rectangle?

Student: Add one more tile.

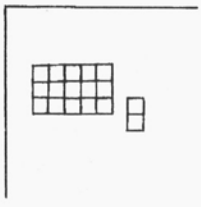
Teacher: Okay. That's one way I could make it into a rectangle. Any other ways?

Student: Take two off the end.

Teacher: Okay. Any others?

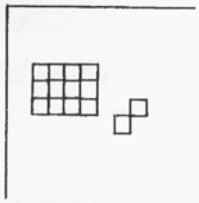
Student: Try making one with a different number of rows.

Teacher: That might work. There are probably many ways to make these tiles into a rectangle. So that we all do it the same way I'll give you a rule. Make the biggest possible rectangle, using the number of rows I tell you and the number of tiles in your handful. When you've used all the tiles you can, just slide the rest over to one side of your tile board—that will be the remainder.



The teacher works two more examples for the class. Each time, the handful of tiles is formed into a rectangle with only three rows. The tiles that do not fit evenly into the three-row rectangle are moved to the right side of the tile board. When the students have seen the teacher work three examples, they begin working on their own tile boards.

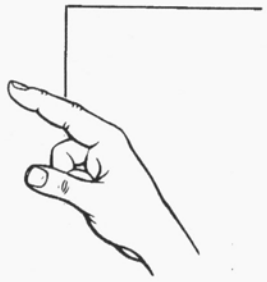
Teacher: Take 14 tiles and put them on your tile board. Using only 3 rows, make the biggest rectangle you can. If you have any left over, slide them to the right.



The teacher continues giving the students amounts of tiles to form into rectangles with remainders. As the students work, the teacher circulates among them, checking.

When the students have successfully made three-row rectangles with and without remainders, they are then taught a system for recording their work.

Teacher: You must keep track of your work on paper so I will have a way of checking your problems without seeing each one as you do it. The way I want you to record your work looks a little like your tile boards. I'll draw a miniature tile board on the overhead. To show you what numbers I want you to write, we'll use as an example the tile problem with 14 tiles in 3 rows, leaving a remainder. Look at your tile board. What do you think I would put here?

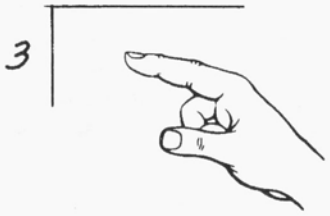


Student: Rows?

Teacher: That seems reasonable. How many rows are there in this problem?

Student: Three.

Teacher: Okay. Three goes here. What do you think I would record in here?

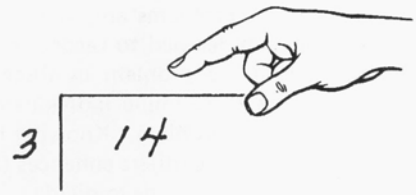


Student: The number of tiles altogether?

Teacher: How many is that?

Student: Fourteen.

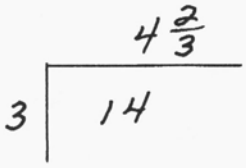
Teacher: Okay, 14 goes here. How about here?



Student: The number of columns?
Teacher: Okay. And what else?
Student: The remainder?

At this point, the students need to be shown how they are to record the columns and the remainder.

Teacher: In this space write the number of columns in the rectangle and how many tiles you had left over. For this problem, there were four columns and two tiles left over. Write the remainder (as you wrote the leftovers for beans and cups) over the number needed altogether to make a whole column. In this case, that means I would write two over three. How many columns in this problem?
Student: Four.
Teacher: That goes here. What is the remainder?
Student: Two.
Teacher: How do you write the remainder?
Student: Two over three.
Teacher: And that goes here.



Students record the numbers for their first few tile problems on blackboards. As they demonstrate an understanding of the recording process, the teacher asks them to begin devising their own tile problems and recording them on paper, continuing as time permits. Students whose blackboards indicate difficulty in understanding the process are provided individual assistance by the teacher.

LESSON 6-8

DIVISION WITH TILES

PURPOSE:

To create and record division problems with tiles while exploring answers to teacher-directed questions

MATERIALS:

1. *Tiles*
2. *Tile board dittos*
3. *Unlined paper*

This lesson continues the activities of the previous lesson. Now, however, the students are not confined to making rectangles with three rows—they may change the number of rows as they wish.

As the students work, the teacher poses the following questions:

What happens to the answers you get if you take the same number of tiles each time, but divide it into more and more rows?
What happens to the answers if you add one more tile to your handful each time while keeping the number of rows in the rectangle the same?
When you take a handful of tiles and make a rectangle out of it, you get an answer at the top of your tile board. Is it possible to get the same answer using a different number of tiles?

LESSON 6-9

DIVISION WITH TILES

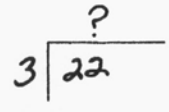
PURPOSE:

To expand awareness of the component parts of tile-board division through teacher-selected problems in three basic formats

MATERIALS:

1. *Tile board transparency, or L-shaped cut-out and cardboard squares*
2. *Tile board dittos*
3. *Tiles*
4. *Individual blackboards*

Multiplication and division are closely interrelated skills. This lesson allows students to look at division with tiles in a way that makes this relationship clear.



Teacher: I'll write a tile problem on the overhead. What does the three stand for?
Student: Rows.

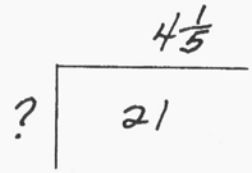
Teacher: How about the 22?
Student: How many tiles in the handful.
Teacher: What goes where the question mark is?
Student: How many columns and how many left over.
Teacher: Okay. Take your tiles and work out the number I should put where the question mark is.

The students work out the answer to the first problem and write it on their blackboards, then the teacher writes a similar problem on the overhead. The numbers used are changed; the questions asked remain the same.

The format for the first type of problem is basically the same as that used by students to record their tile problems. Both the number of rows and tiles altogether are known—the unknown is the number of columns and the remainder.

The second problem format presents a different unknown.

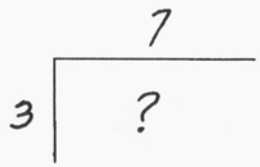
Teacher: Now I'll write a different kind of tile problem on the overhead. What does the 21 stand for?



Student: How many tiles altogether.
Teacher: What does the four, with a remainder of one over five stand for?
Student: How many columns and how many left over.
Teacher: What goes where the question mark is?
Student: How many rows.
Teacher: Okay. Use your tiles to find out how many rows there would be in this problem. When you think you know, write your answer on your blackboard.

Although the students have the technical knowledge to find the number of rows, some may have difficulty putting it to use. Even so, they should not be told the answer—the tiles will show them how many rows there should be.

The students work the problem and write the answer on their blackboards. The teacher then writes similar problems on the overhead. The students continue to work examples until they have demonstrated that, given the columns of a rectangle and the number of tiles left over, they can find the number of rows.



The third-problem format can be seen in the figure above. Once again, the teacher asks students to explain what each number means, and what number replaces the question mark. The teacher continues until they know how to find how many tiles altogether, when they've been told the number of rows and columns.

These three kinds of problems are all part of the one problem students already learned to record from their tile boards. By presenting the problem in three forms, the teacher forces students to examine individually the three components of any single problem. Knowing how each of the components relates to the others enhances the student's ability to understand what it means to divide.

LESSON 6-10

DIVISION WITH CROSSED LINES

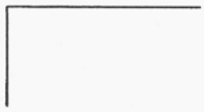
PURPOSE:

To create and record division problems with crossed lines

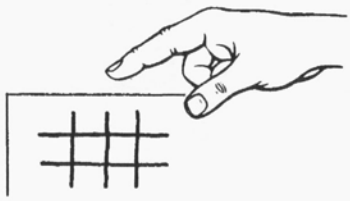
MATERIALS:

1. *Individual blackboards*
2. *Unlined paper*

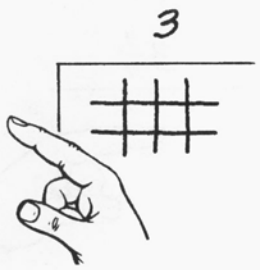
Students who learn a skill using only one material are apt to associate the skill with the material, failing to extrapolate a broader application from what has been learned. The use of a second or third material to present the same series of experiences helps the students extend and review the knowledge they have acquired.



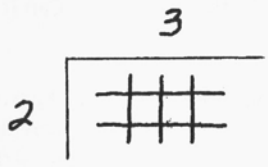
Teacher: I've drawn a cross-line board on the overhead. We will use this board much the same way we did our tile boards. Now, I'll put a cross-line problem on the board. What do you think I'll write here?



Student: How many lines down.
Teacher: Yes. How many lines down are there?
Student: Three.
Teacher: Then, three goes here. What will I write here?



Student: How many lines across.
Teacher: How many are there?
Student: Two.
Teacher: Okay.



The teacher creates one more problem on the overhead and the students fill in the missing numbers. They then construct their own problems on paper, as time permits. Students who wish to may construct the cross-line part of the problem then exchange papers with a neighbor for filling in the down and across numbers.

LESSON 6-11

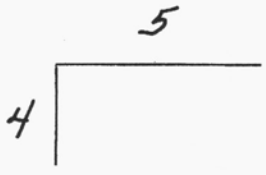
DIVISION WITH CROSSED LINES

PURPOSE:

To expand awareness of the component parts of cross-line division through teacher-selected problems in three basic formats

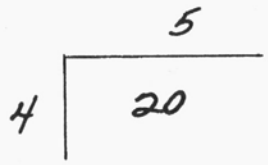
MATERIALS:

1. Individual blackboards



Teacher: I've written a cross-line problem on the overhead. What does the four stand for?
Student: Lines across.
Teacher: How about the five?
Student: Lines down.
Teacher: Then what number would go inside?

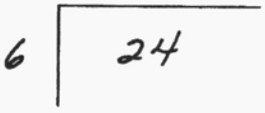
The students may construct the cross-line figure with four lines across and five lines down to assist them in finding the answer, however, that isn't the answer by itself. The teacher asked for the *number* that would go inside, not the figure. Students who include the figure rather than the number are told they have worked the problem correctly, but this time the teacher only wants to know the number of intersections. An example of how the answer is to be recorded may be seen in this figure.



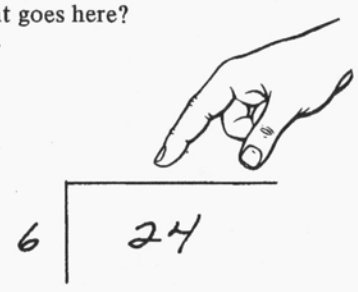
The students work out the answer to the first problem and write it on their blackboards. The teacher then writes a similar problem on the overhead, changing the numbers but asking the same questions. The teacher continues with the first format until the students can provide correct numerical answers.

The second format presents a different unknown.

Teacher: Now I'll write a different kind of cross-line problem on the overhead. What does the six stand for?



Student: How many lines across?
Teacher: What does the 24 stand for?
Student: The places where all the lines cross.
Teacher: What goes here?

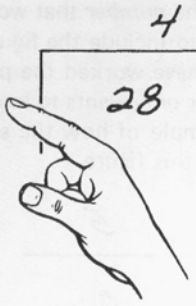


Student: How many lines down.
Teacher: Okay. How many lines down would you have to draw to get 24 crossing points altogether? When you think you know, write your answer on your blackboard.

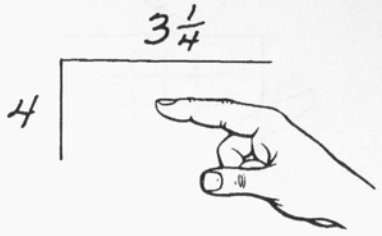
The students have not been presented with a cross-line problem expressed in this manner before. As a group, they will be able to figure out how many down lines it takes, helping each other and discussing possible answers. The teacher may also lead the class in a discussion of possible solutions to the problem but does not tell them how to solve it.

The teacher provides several examples that are essentially the same.

If the students can demonstrate on their blackboards an understanding of how to get correct answers for the second problem type, the teacher gives them a third form of the problem to think about, presenting it as before.



If the students can find answers for all three forms of the problem, the teacher may present them with a more difficult version of the third question:



Teacher: Look at this problem. Do you think you could tell me a way to write *down* and cross lines for it. Can it be done?

The more practice students receive doing their own thinking, the more likely the answer to "Can it be done?" whenever it is asked, is apt to be yes.

The students have made a good beginning in learning the fundamentals of division. This will provide the basis for the later activities contained in the chapter Advanced Division.