CHAPTER 7
People Problems and Word Problems

Lesson 7-1 page 63 People Teacher creates addition problems using people.	Lesson 7-2 page 63 People Teacher creates subtraction problems using people.	Lesson 7-3 page 64 People Students record teacher created addition and subtraction problems.	Lesson 7-4 page 64 People Students record teacher problems and illustrate with stick figures.
Lesson 7-5 page 65 People and Objects Students record teacher-created addition problems.	Lesson 7-6 page 65 People and Objects Students record teacher-created subtraction problems.	Lesson 7-7 page 66 People and Objects Students illustrate and record teacher-created addition and subtraction problems.	Lesson 7-8 page 66 Word Problems Students create "words" to accompany addition problems.
Lesson 7-9 page 67 Word Problems Students write words to accompany addition problems.	Lesson 7-10 page 67 Word Problems Students review written word problems in advance of writing new problems.	Lesson 7-11 page 68 Word Problems Students create "words" to accompany subtraction problems.	Lesson 7-12 page 68 Word Problems Students write words to accompany subtraction problems.
Lesson 7-13 page 68 Word Problems Students review written word problems in advance of writing new problems.	Lesson 7-14 page 69 People and Objects Multiplication Teacher creates and records repetitive addition problems.	Lesson 7-15 page 69 People and Objects Multiplication Teacher records problems using a new format.	Lesson 7-16 page 70 People and Objects Multiplication Teacher records problems on a matrix.
Lesson 7-17 page 70 People and Objects Multiplication Students use matrix to answer teacher questions.	Lesson 7-18 page 71 People and Objects Teacher creates division problems.	Lesson 7-19 page 71 People and Objects Students record teacher-created division problems.	Lesson 7-20 page 72 People and Objects Teacher records division problems on a matrix.
Lesson 7-21 page 72 People and Objects Students use division matrix to answer teacher questions.	Lesson 7-22 page 73 Word Problems Students create "words" to accompany multiplication problems.	Lesson 7-23 page 74 Word Problems Students write words to accompany multiplication problems.	Lesson 7-24 page 74 Word Problems Students review written-word problems in advance of writing new problems.
Lesson 7-25 page 74 Word Problems Students write addition, subtraction, and multiplication word problems.	Lesson 7-26 page 75 Word Problems Students create "words" to accompany division problems.	Lesson 7-27 page 75 Word Problems Students write words to accompany division problems.	Lesson 7-28 page 76 Word Problems Students review written-word problems in advance of writing new problems.
Lesson 7-29 page 76 Word Problems Students write addition, subtraction, multiplication, and division word problems.	Lesson 7-30 page 76 Word Problems Students review written word problems in advance of writing new problems.		

Prerequisite chapters:

The activities in this chapter are meant to parallel or follow the activities in Chapters 4, 5, and 6

MATERIALS

For overhead projector:

Transparencies Ten by ten blank matrix _____ Worksheet 1

If no overhead projector is available:

Make charts in place of transparencies _____ Materials chapter, page 294

Student materials:

Individual blackboards _____ Materials chapter, page 294
Spelling notebooks _____ Materials chapter, page 296
Objects available in the classroom, such as books
Lined paper

Students may be skilled at addition, yet not understand in what situation that skill might be applied. This failure to extrapolate is most apparent when students are asked to solve word problems. They may have the technical ability to solve problems when the numbers are provided, but be lost when asked to extract the same numbers from words.

The activities in this chapter provide students with the skills necessary to comprehend and solve word problems. By creating word problems of their own, they link what they are learning about numbers to problem situations from which these same numbers may have come.

Earlier chapters on numbers and operations presented students with concepts to explore. The lessons in this chapter parallel these explorations with specific examples of how the knowledge gained through exploration may be utilized to solve problems. Acquiring a concept through exploration requires time; the earlier lessons on numbers and operations provided that time. The process of applying skills, once learned, may be much more direct. Consequently, this chapter's lessons are meant to consume no more than ten minutes each day.

LESSON 7-1

ADDITION WITH PEOPLE

PURPOSE:

To observe the teacher create addition problems using people

MATERIALS:

1. No materials needed

In the first two lessons the students are introduced to a process of constructing addition and subtraction problems using people as the basis for the situations involved.

Teacher: Johnny, Sally, Susan, and José, will you please stand by the door. How many people are standing by the door?
Student: Four.
Teacher: Okay. I'll write four on the overhead. Edward, Brenda, and Sam, please come up and stand by me. How many people are standing by me?
Student: Three.
Teacher: Fine. I'll write three on the overhead, underneath the four. Now, I want both groups to stand together at the front of the room. How many people are there altogether?
Student: Seven.
Teacher: Then seven is the answer I'll write on the overhead.

$$\begin{array}{r} 4 \\ 3 \\ \hline 7 \end{array}$$

The teacher develops three additional problems using students, recording each problem on the overhead. After four examples the teacher checks to see if the class understands the process.

Teacher: What is the first thing I do to make up a problem?
Student: Have some people stand up.
Teacher: Okay. Russell, Carlos, and Debbie, please stand up. Now what?
Student: Have some more people stand up.
Student: No, those three have to go stand some place else, first.
Teacher: Where shall I have them stand?
Student: By the blackboard.
Teacher: Russell, Carlos, and Debbie, please go stand by the blackboard. Now what?
Student: Write a three on the overhead.
Teacher: Why?
Student: Because there are three people standing by the blackboard.
Teacher: Okay. . . . Now what?

If the students understand how to construct people problems for addition, they continue to direct the teacher in creating problems for the remaining few minutes of the lesson. If not, the teacher provides an additional example as a model, then lets the students try again to demonstrate their understanding of the process. The teacher alternates between providing examples and asking students to describe the process until they can direct the teacher in creating a problem.

LESSON 7-2

SUBTRACTION WITH PEOPLE

PURPOSE:

To observe the teacher create subtraction problems using people

MATERIALS:

1. No materials needed

Teacher: Stephanie, Richard, Ronald, and Rhonda, please stand by the door. How many people are standing by the door?

Student: Four.

Teacher: Okay. I'll write four on the overhead. Richard, Ronald, and Stephanie, please sit down. How many people did I have sit down?

Student: Three.

Teacher: Okay. I'll write three on the overhead, underneath the four. How many people are left standing?

Student: One.

Teacher: Then one is the answer I'll write on the overhead.

$$\begin{array}{r} 4 \\ 3 \\ \hline 1 \end{array}$$

After creating three additional problems, the teacher checks to see if the class understands the process.

Teacher: What is the first thing I do to make up a problem?
Where shall I have them stand?
What next?

If the students understand how to create people problems for subtraction, they continue to direct the teacher. If not, the teacher proceeds as in the previous lesson.

LESSON 7-3

ADDITION AND SUBTRACTION WITH PEOPLE

PURPOSE:

To record teacher-created addition or subtraction problems

MATERIALS:

1. Individual blackboards

In the next two lessons students learn to record addition and subtraction problems involving people, using both numbers and representative illustrations. This is an important link in the students' ability to attribute specific meanings to abstract mathematical symbols.

Teacher: This row of people stand up. Record how many are in this row on your blackboards. Everyone from Larry to Debbie in this row stand up. Record how many are in this group. How many people are standing? Record the answer on your blackboards.

The students who stand up as a part of any problem are also expected to record it on their blackboards.

For these problems, the teacher does not provide the numbers to be written—the students do their own counting.

The first problem is recorded on the blackboards, which are then held up for the teacher to see. The teacher writes the most frequently occuring problem and answer on the overhead as the majority opinion of the class. The teacher then presents a second kind of problem to record.

Teacher: All of the first row stand up. Record how many are in this row. Everyone from Russell to Brenda in the first row sit down. Record how many sat down. How many people are standing?

Often when confronted with a problem situation expressed in words rather than numbers, students ask their teacher, Do I add or subtract? The students must see for themselves which kinds of situations produce which kinds of problems.

The question How many people are standing? is used to elicit answers in both the addition and subtraction situations described above. This means the students cannot tell from the teacher's statements which operation is being enacted. They must decide when to add and when to subtract.

The teacher constructs people problems for both addition and subtraction for the entire lesson. The students record the appropriate numbers on their blackboards. Each problem ends with the question, How many people are standing?

LESSON 7-4

ADDITION AND SUBTRACTION WITH PEOPLE

PURPOSE:

To record teacher-created addition or subtraction problems, using stick figures to represent people

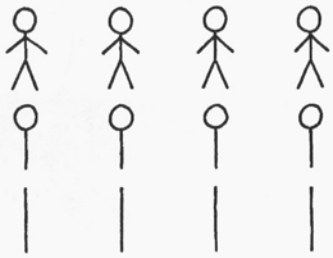
MATERIALS:

1. Individual blackboards

Teacher: I'll give you a people problem to record on your blackboards. This time I won't have anybody stand up. If you want to draw people to keep track of what the problem asks you to do, you may.

If you draw people, they don't have to be beautiful. When I draw people for problems, mine look like this. If I'm in a hurry, I leave off the arms and legs and they look like

this. If the people problems get too big and I want to draw people really fast, I leave off their heads too, and they look like this.

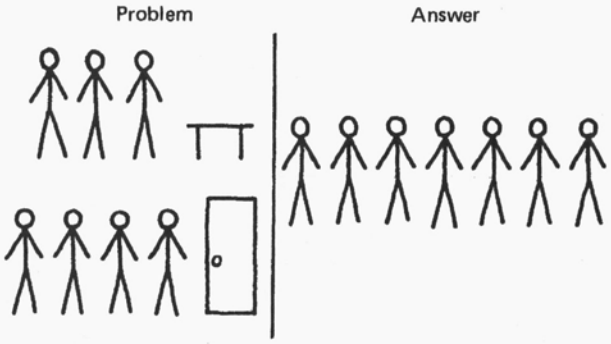


Here is the problem I want you to work: three people stood by the teacher's desk. There were four more by the door. When the teacher told them all to go to the front of the room, how many were at the front of the room?

Don't just write the answer. I would like to see the whole problem on your blackboard, written as we did when we recorded people problems before.

When the students finish recording, the teacher works the example on the overhead, drawing stick figures as illustrations. If the teacher uses the same computation aid recommended for student use, it lends credence to such use for solving problems. This may seem too simple a step for such an easy problem, but the problems become more difficult. An excellent aid in solving mathematics problems, in school or out, is drawing a picture. Pictures help students conceptualize the situation described in the problem.

The teacher continues creating addition and subtraction problems for the students to record, and illustrating what the words directed.



LESSON 7-5

ADDITION WITH PEOPLE AND OBJECTS

PURPOSE:

To observe and record teacher-created addition problems using people and objects

MATERIALS:

1. *Objects available in the classroom, such as books*
2. *Individual blackboards*

The next three lessons expand the students' ability to record addition and subtraction problems involving people to include problems involving people and objects together. As more elements are introduced into the problem, students begin to learn which elements deserve their attention and which information needs to be disregarded. The ability to make the distinction between necessary and unnecessary information is vital if students are to generate correct solutions to word problems.

Teacher: Eddie, take five books from the shelf and stand by the window. How many books does Eddie have?

Sandy, take six books and stand by the door. How many books does Sandy have? Now, Sandy and Eddie, bring your books over here and make a pile of them on my desk. How many books are there in this pile?

The teacher writes the student responses to the first problem on the overhead. Then the students record each new problem on their blackboards. When they understand the process of recording the teacher asks the questions in a different way.

Teacher: Sam, bring the row of books on the top shelf of our book case up here, please. Everyone record how many books Sam brought me.

Sharon, please bring me the books from the shelf next to the top. Everyone record how many books Sharon brought me. Now, how many books are in my pile?

The students must now count the books and decide which numbers to write on their blackboards. The teacher continues to create problems for the students to record throughout the remaining minutes.

LESSON 7-6

SUBTRACTION WITH PEOPLE AND OBJECTS

PURPOSE:

To observe and record teacher-created subtraction problems using people and objects

MATERIALS:

1. *Objects available in the classroom, such as books*
2. *Individual blackboards*

Teacher: Rosalind, take five books from the shelf and bring them here. How many books does Rosalind have?
Gregory, take three of Rosalind's books and put them back on the shelf. How many books did Gregory put back? How many books are in Rosalind's pile now?

The teacher writes the student responses on the overhead. The students then record each new problem on their blackboards. Next, the teacher changes the questions.

Teacher: Tyrone, bring me all the books in your desk. Everyone record how many books Tyrone brought me. Now, Tyrone, take all the books for reading and science. Everyone record how many books Tyrone took back. How many books are in my pile? Record the answer.

The students count the books and decide which numbers to write. The teacher continues to create problems as time permits.

LESSON 7-7

ADDITION AND SUBTRACTION WITH PEOPLE AND OBJECTS

PURPOSE:

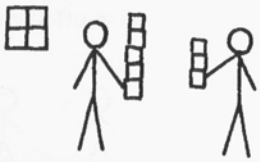
To record teacher-created addition or subtraction problems using drawings to represent people and objects

MATERIALS:

1. Individual blackboards

Teacher: I will give you a book problem to record on your blackboards. This time, I won't have anybody actually carry any books. If you want to draw books to keep track of what the problem asks, you may. When I want to draw a book, I just draw a little square or rectangle. If I'm in a hurry, I draw *X*s instead.
Here is the problem: one person took five books from the the shelf and stood by the window. Another person took three books and went over by the window. How many books did the people by the window have?

The students write the problem, then the teacher works the example on the overhead, accompanying the numerals with drawings of square-shaped books and stick-figure people to illustrate what happened. This process is continued for each problem.



LESSON 7-8

ADDITION WORD PROBLEMS

PURPOSE:

To create addition word problems

MATERIALS:

1. No materials needed

In the next six lessons students first create verbally, then describe in writing, problems involving addition and subtraction. Inventing a situation to accompany a predetermined abstract number problem is the reverse of the process calling for students to use numbers to record a preexisting situation. Both skill are important if students are to understand how abstract numbers and concrete experiences interrelate in the word problems they will later be asked to solve.

$$\begin{array}{r} 4 \\ +\,3 \\ \hline 7 \end{array}$$

Teacher: Here is an arithmetic problem. I want you to give me a word problem to go along with the numbers I have written. For example: Sam had four books and Eddie had three. When they put their books together they had seven. Can anyone give me another word problem for these numbers?
Student: There were four boys standing by the door and three boys standing by the window and the teacher told them all to go to the front of the class, and when they got to the front, there were seven in the front.
Teacher: Okay. Let's see if those words fit the numbers on the overhead. I'll draw a picture for what you said and we'll see what we get.
How many boys did you say were standing by the door?
Student: Four.

Teacher: Okay. And how many by the window?
Student: Three.
Teacher: And then what happened?
Student: They all went to the front of the class.
Teacher: Then how many were at the front of the class?
Student: Seven.

Teacher: Does this problem fit the numbers on the overhead?
Student: Yes.
Teacher: Fine, that's one word problem for these numbers. Can anyone give me another?

The teacher checks each student's words against the numbers on the overhead either by illustrating the words or by having students act them out. If students are to understand when their words relate to the numbers and when they don't, they must be able to *see* what they have said. Illustrations provide this essential visual clue.

The students describe different situations they think might fit the numbers on the overhead. This lesson will acquaint them with a variety of possibilities for a single set of numbers. After each new set of words has been checked against the numbers, the teacher asks: Can anyone tell me a different way?

LESSON 7-9

ADDITION WORD PROBLEMS

PURPOSE:

To write student-generated addition word problems

MATERIALS:

1. *Spelling notebooks*
2. *Lined paper*

Teacher: Yesterday, you made up word problems. Now I'll give each of you paper and ask you to *write* a word problem that could go with the same numbers.
Try to create a problem different from the ones everyone made up yesterday. If you can't think of a different one, just write the one you liked best from yesterday.
If you need a word spelled, bring me your spelling notebook . . . you may begin.

When the students finish their word problems, the teacher collects them. The problems form the basis for the next lesson.

LESSON 7-10

ADDITION WORD PROBLEMS

PURPOSE:

To review written word problems before writing new ones

MATERIALS:

1. *Written word problems from Lesson 7-9*
2. *Spelling notebooks*
3. *Lined paper*

The teacher reads the students' word problems from Lesson 7-9 and selects two different kinds of problems to be read aloud: those not reflecting the numbers from which the problem was supposed to originate and those that are appropriate and unusual or creative.

The inappropriate examples are used only as models; they are paraphrased, not read aloud word for word, and are not identified with a student. No papers, good or bad, are returned to the students so that the identity of the authors of the inappropriate problems cannot be known.

By discussing examples that do not reflect the numbers, students have an opportunity to analyze the difference between effective and ineffective descriptions. This discussion loses its value, however, if any student feels that his or her work is unworthy.

An example of a student problem representing a misunderstanding of the assignment is:

There were four boys and three boys and seven boys.

Paraphrasing this problem, the teacher would read aloud:

I had four tiles and three tiles and seven tiles.

Once an inadequate example is read, the class must change the written statement to make it match the original numbers on the overhead. Students who have missed the point of the assignment can, anonymously, see how they might correct their problem.

The teacher helps the class analyze the inappropriate descriptions by illustrating, or having students act out, what the words mean.

Once students can write numbers describing what objects or people are doing, they can learn to write words that describe these same events. However, this learning does not take place automatically; it is best accomplished when students can repeatedly evaluate the success of their transfer of number knowledge to words. This evaluation should continue until they can make the transition with ease.

The second kind of problem to be read aloud involves unusual or creative thinking. By hearing these examples

read, students experience the teacher's acceptance of a wide range of ideas. This encourages divergent thinking in writing word problems.

"Johnny was walking to the store to shop for his mother. On the way he saw four baby kittens who were lost. He picked them up and carried them from house to house asking everyone where they had come from. Finally he found their owner and he gave the cats to him. The owner said there were three more just like them inside, so he went in and looked and he saw the three other ones, so he knew there must be seven kittens altogether."

"King Kong stepped on four tanks and three jeeps and squashed them, so altogether King Kong squashed seven things."

"Pinocchio's nose grew one centimeter every time he told a lie. On the way to school he told four lies. On the way home he told three more. When he got home he looked in the mirror and saw his nose had grown seven centimeters."

Examples of written problems that go beyond simple patterns or descriptions discussed aloud by the class in earlier lessons are not paraphrased. The teacher may, however, unobtrusively alter the students' grammar. When a teacher takes dictation from a student for an experience story, the student's speech patterns should be recorded intact. But, the teacher reading the student's grammar does not sound as natural as when reading his or her own grammar. If the grammar is switched for the reading, what is read aloud will sound right. This is important to the student whose words are being shared with the class.

Reading unusual problems encourages students to abandon the "four books plus three books is seven books" format, while reinforcing those who wrote creative problems.

Once several examples have been read, the teacher writes a new addition problem on the overhead, and the students write words for it. The teacher collects their papers—these problems will be read and discussed at the beginning of the next lesson.

LESSON 7-11

SUBTRACTION WORD PROBLEMS

PURPOSE:

To review written addition word problems; to create subtraction word problems

MATERIALS:

1. *Written word problems from Lesson 7-10*

The first few minutes are devoted to a discussion of the student word problems from Lesson 7-10. When a brief selection has been read and discussed, the teacher writes a new problem on the overhead.

$$\begin{array}{r} 4 \\ -\,3 \\ \hline 1 \end{array}$$

Teacher: Look at this problem. Give me a word problem to go along with these numbers. For example: Sam had four books. After he gave three to Eddie, he had only one left.

Can anyone give me another word problem for these numbers?

The students provide an assortment of word problems that they think match the numbers on the overhead. The teacher checks each example by illustration or having students act it out. The students are encouraged to devise as many different examples of subtraction word problems as time permits.

LESSON 7-12

SUBTRACTION WORD PROBLEMS

PURPOSE:

To write subtraction word problems

MATERIALS:

1. *Spelling notebooks*
2. *Lined paper*

The process used in Lesson 7-9 for writing words for a single problem is used in this lesson, for subtraction problems.

When the students finish writing their word problems, the teacher collects them, to be read and discussed at the beginning of the next lesson.

LESSON 7-13

ADDITION AND SUBTRACTION WORD PROBLEMS

PURPOSE:

To review written word problems before writing word problems for both addition and subtraction

MATERIALS:

1. *Written word problems from Lesson 7-12*
2. *Spelling notebooks*
3. *Lined paper*

The first few minutes are spent discussing the previous lesson's word problems.

Once examples of both inappropriate and creatively appropriate problems have been read, the teacher writes one addition and one subtraction problem on the overhead. The students are then asked to write two word problems; one for addition and one for subtraction. These are saved to be read at the beginning of Lesson 7-25.

LESSON 7-14

MULTIPLICATION WITH PEOPLE AND OBJECTS

PURPOSE:

To observe the teacher create repetitive addition problems using people and objects

MATERIALS:

1. *Objects available in the classroom, such as books*
2. *Individual blackboards*

The next four lessons provide students with a basis for relating repetitive addition problems involving people and objects to their experiences with multiplication in Chapter 5. These lessons give them the necessary background for creating and writing multiplication word problems. The actual creation of such problems by the students, however, will not begin until Lesson 7-22.

Teacher: Place two books on top of your desk. When I call your name, bring your books up to the front of the room . . . David. How many students are standing in the front of the room?
Student: One.
Teacher: How many books does he have?
Student: Two.
Teacher: How many books altogether?
Student: Two.
Teacher: Edna, stand next to David. How many students in the front of the room?
Student: Two.
Teacher: How many books does each student have?
Student: Two.
Teacher: How many books do they have altogether;
Student: Four.
Teacher: Okay. Before I have anybody else come up, I'll record what we have so far. This will help us look at the numbers for patterns.

Students	Books for each student	Total number of books
1	2	2
2	2	4

Francis, bring your two books up now, and let's see what happens to our pattern . . .

As the numbers are recorded, the teacher encourages students to predict future numbers.

Teacher: How far will the pattern go before we run out of people? Could we continue the pattern if we had more people carrying books? How far?
Are there any students who won't be able to be included in this pattern? Why?

When the supply of students or the space for writing is exhausted, the students return to their desks. The teacher then asks each student to take out three books and the process is repeated, as time permits.

LESSON 7-15

MULTIPLICATION WITH PEOPLE AND OBJECTS

PURPOSE:

To observe the teacher create repetitive addition problems using a short-form recording system

MATERIALS:

1. *Objects available in the classroom, such as books*
2. *Individual blackboards*

The activities for this lesson are the same as those in the previous one. The only change is in the way the teacher records the number patterns.

Teacher: Since the number of books each student brings up for any one group of problems is the same, it would work just as well if I simply wrote the number once at

the top. Okay, everybody take out four books, and we'll see what kind of pattern we get.

Books for each student

4

Students Total books

The students continue carrying books to the front of the room while examining the teacher's columns of numbers for patterns.

LESSON 7-16

ADDITION WITH PEOPLE AND OBJECTS

PURPOSE:

To observe the teacher record the repetitive addition problems on a matrix

MATERIALS:

1. *Blank matrix ten squares by ten squares on a transparency, or blank matrix on a large tagboard*
2. *Objects available in the classroom, such as books*
3. *Individual blackboards*

The activities are the same as in Lesson 7-15, with a change in the way the teacher records the number patterns.

Teacher: Today, I'll record the numbers we get on a matrix. Across the top, I'll write "books for each person," and number from one to ten. Down the side, I'll write "people," and also number from one to ten. Let's start with each person having two books and record what happens on the matrix.

Books for each person

People	1	2	3	4	5	6	7	8	9	10
1		2								
2		4								
3		6								
4		8								
5		10								
6		12								
7		14								
8		16								
9		18								
10		20								

The teacher constructs the matrix as the students again carry books to the front of the room. As the matrix is filled in, the teacher asks the students the following questions:

Do you see any patterns in the columns or rows of this matrix we could use to help us fill in the table more quickly?

Are the patterns for columns the same as or different from the patterns for rows? Why?

Have you seen patterns like this on a matrix before? When? Why are they the same?

LESSON 7-17

MULTIPLICATION WITH PEOPLE AND OBJECTS

PURPOSE:

To use the teacher's matrix to explore an-answers to teacher-directed questions

MATERIALS:

1. *Filled-in matrix from Lesson 7-16*
2. *Objects available in the classroom, such as books*
3. *Individual blackboards*

Books for each person

People	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

Teacher: These numbers record what we found out about people and piles of books. Now, I want you to use this matrix to answer some questions.

Three students are at the front of the room, each holding a pile of four books. Look at the matrix and write on your blackboards how many books you think the students have altogether.

Most of you have written twelve. Can you tell me how you found twelve by using the matrix?

The teacher allows one or two students to explain how they used the matrix to get an answer. This enables those who do not understand the connection between the question and the numbers on the matrix to see examples of how their classmates handled the problem.

If the students cannot answer the question using the matrix, they work it out using books and students. They then figure out how the matrix might have been used to produce the same answer.

When the students can, with regularity, find the total number of books, given the number of students and the number of books for each student, they are asked a different kind of question:

Teacher: If you have 40 books altogether and you have 8 students each holding an equal number of them, can you tell me how many books each student is holding?
Can you tell me by looking at the matrix? If you can't could you work it out using books and people?

When the students can work this problem, the teacher asks them to explore a third possibility:

Teacher: If there were 35 books altogether and each student was holding 5 of them, could you use the matrix to tell me how many students were holding the books?
If you can't find it on the matrix, could you figure it out using students and books?

Knowledge of how to use the people-and-books matrix is not vitally important to students' learning. Any answer not found with the matrix can always be found by acting out the situation with people and books.

To discover for themselves the workings of the matrix, students have to think; this thinking is the point of all our questions and, in the end, is the point of all mathematics.

LESSON 7-18

DIVISION WITH PEOPLE AND OBJECTS

PURPOSE:

To observe the teacher create division problems using people and objects

MATERIALS:

1. *Objects available in the classroom, such as books*
2. *Individual blackboards*

The next four lessons provide students with a basis for relating their experiences with division in Chapter 6 to division problems involving people and objects, giving them the necessary background to create division word problems.

Teacher: I have 23 books on this table. Shortly I will ask 4 of you to come up here and each take the same number of books. Before I do, however, write on your blackboard how many you think each person will have in his or her pile, and if there is a remainder, what you think it will be.

The students write their predictions and the teacher scans the blackboards and selects the most frequently predicted number to write on the overhead.

Four students then divide up the books and check the prediction.

The teacher creates similar problems, changing either the number of students who come up or the number of books in the pile. For each new problem, students predict the answer.

LESSON 7-19

DIVISION WITH PEOPLE AND OBJECTS

PURPOSE:

To record teacher-created division problems

MATERIALS:

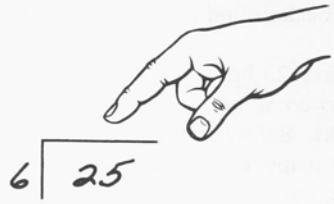
1. *Objects available in the classroom, such as books*
2. *Individual blackboards*

The activities are the same as in the previous lesson. The only change is the information that the students record on their blackboards.

Teacher: This time when you write your predictions, you are to record the *whole* problem as well. This is how I want you to write the problem:
First, write a sideways L. I'm not sure why the sideways L is used for division, but it is, so, we'll use it. Inside, write how many books there are altogether—in this case, 25. I'll have 6 people come up and take piles. The 6 goes here.

$\overline{}$ $\quad \overline{)25} \quad 6\overline{)25}$

Now, write your prediction up here, because for *these* problems that's where the answer goes.



This format is an arbitrary system of writing a particular kind of problem. When we teach arbitrary things, there is no point in striving to understand why we write them that way. As much as possible, we should keep arbitrary rules out of teaching, but we cannot eradicate them completely.

If no clear logic exists for what we ask students to do, it is simpler to say "the rule for this game is. . . , you have to write it like this. . . ," or "I want you to write your work in this way because it makes it easier for me to see what's happening. . . ," or "please record your work like this so your teachers in junior high and high school will be able to more quickly recognize what you are doing."

The students continue to predict answers for new problems. The teacher records the predictions and the full problem on the overhead. Students then divide the books and check the predictions.

LESSON 7-20

DIVISION WITH PEOPLE AND OBJECTS

PURPOSE:

To observe the teacher record division problems on a matrix

MATERIALS:

1. *Blank matrix, ten squares by ten squares on a transparency, or blank matrix on a large tagboard*
2. *Individual blackboards*

Teacher: Today, I will give you more division problems for students and books, only in a more systematic way. I'll write the answers on this matrix.

Across the top, I'll write "total books," and number from one to ten. Down the side I'll write "students dividing books," and also number from one to ten.

Let's start with a pile of ten books altogether and see what numbers we get for the matrix.

Total Books

Students dividing Books	1	2	3	4	5	6	7	8	9	10
1										10
2										5
3										$3\frac{1}{3}$
4										$2\frac{1}{4}$
5										2
6										$1\frac{4}{6}$
7										$1\frac{3}{7}$
8										$1\frac{2}{8}$
9										$1\frac{1}{9}$
10										1

The teacher constructs the matrix as the students predict answers on their blackboards then divide the books to check their predictions. As the matrix is filled in, the teacher poses these questions:

Are there any patterns in the columns or rows of this matrix we could use to help us fill in the table without having to divide each new pile of books?

Can you use the patterns to help you make more accurate predictions?

Are the patterns for columns the same as or different from those for rows? Why?

LESSON 7-21

DIVISION WITH PEOPLE AND OBJECTS

PURPOSE:

To use the teacher's matrix to explore answers to teacher-directed questions

MATERIALS:

1. *Filled-in matrix from Lesson 7–20*
2. *Objects available in the classroom, such as books*
3. *Individual blackboards*

Total Books

Students dividing books	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	$\frac{1}{2}$	1	$1\frac{1}{2}$	2	$2\frac{1}{2}$	3	$3\frac{1}{2}$	4	$4\frac{1}{2}$	5
3	$\frac{1}{3}$	$\frac{2}{3}$	1	$1\frac{1}{3}$	$1\frac{2}{3}$	2	$2\frac{1}{3}$	$2\frac{2}{3}$	3	$3\frac{1}{3}$
4	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{3}{4}$	1	$1\frac{1}{4}$	$1\frac{2}{4}$	$1\frac{3}{4}$	2	$2\frac{1}{4}$	$2\frac{2}{4}$
5	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{3}{5}$	$\frac{4}{5}$	1	$1\frac{1}{5}$	$1\frac{2}{5}$	$1\frac{3}{5}$	$1\frac{4}{5}$	2
6	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{3}{6}$	$\frac{4}{6}$	$\frac{5}{6}$	1	$1\frac{1}{6}$	$1\frac{2}{6}$	$1\frac{3}{6}$	$1\frac{4}{6}$
7	$\frac{1}{7}$	$\frac{2}{7}$	$\frac{3}{7}$	$\frac{4}{7}$	$\frac{5}{7}$	$\frac{6}{7}$	1	$1\frac{1}{7}$	$1\frac{2}{7}$	$1\frac{3}{7}$
8	$\frac{1}{8}$	$\frac{2}{8}$	$\frac{3}{8}$	$\frac{4}{8}$	$\frac{5}{8}$	$\frac{6}{8}$	$\frac{7}{8}$	1	$1\frac{1}{8}$	$1\frac{2}{8}$
9	$\frac{1}{9}$	$\frac{2}{9}$	$\frac{3}{9}$	$\frac{4}{9}$	$\frac{5}{9}$	$\frac{6}{9}$	$\frac{7}{9}$	$\frac{8}{9}$	1	$1\frac{1}{9}$
10	$\frac{1}{10}$	$\frac{2}{10}$	$\frac{3}{10}$	$\frac{4}{10}$	$\frac{5}{10}$	$\frac{6}{10}$	$\frac{7}{10}$	$\frac{8}{10}$	$\frac{9}{10}$	1

The teacher asks three basic types of questions, which the students answer with the help of Lesson 7-20's matrix.

How many books for each student if there are three students and seven books altogether?

How many students if there are eight books, each student has six, and two are left over?

How many books altogether if there are three students who each have two books and two books are left over?

If the students cannot use the matrix, they work out the problem with actual people and books. The same basic questions may be asked repeatedly by changing the numbers the students work with.

The process is repeated until students understand that objects stand for numbers and numbers stand for objects—neither is independent of the other.

LESSON 7-22

MULTIPLICATION WORD PROBLEMS

PURPOSE:

To create multiplication word problems

MATERIALS:

No materials are needed.

In the next three lessons students first create verbally, then describe in writing, problems involving multiplication.

$$\begin{array}{r} 2 \\ \times\ 3 \\ \hline \end{array}$$

Teacher: What is the answer to this problem?

Student: Five.

Teacher: It would be five if I had written this,

$$\begin{array}{r} 2 \\ +\ 3 \\ \hline \end{array}$$

but I've written a multiplication problem instead. When I write a plus sign, it means add. When I write an x it means multiply. What is the answer to two times three?

Student: Six.

Teacher: How do you know?

Student: Because two times three is six.

Teacher: How do you know? Could you prove it?

Student: Look on a times table and you can see two times three is six.

Student: When we worked with tiles, two rows and three columns was six.

Student: When we worked with beans and cups, two cups and three beans in each cup was six.

Student: When we had books, two people who each had three books had six books altogether.

Teacher: Okay, you have given me pretty good proof that two times three equals six. What I would like you to do now is give me a word problem for that. I'll give you an example first, then you can make up one on your own.

There were two boys, and each had three pieces of candy in his hand, so altogether the boys had six pieces of candy.

Can anyone else think of a problem?

Student: I had two cups and I had three beans in each cup, so I had six beans altogether.

Teacher: Let me draw a picture of that and see what I get.

Student: I have three dolls and two girls.

Teacher: I'll draw that and see what I get.

Student: There are supposed to be six dolls!

Teacher: You said to draw three dolls and two girls, so that's what I did.

Student: And the two girls have three dolls.

Teacher: Well, there are three dolls here, so I guess that's what I've drawn.

Student: No! *Each* girl has three dolls!

Teacher: Oh, like this?

Student: Yes!
Teacher: Then what's the answer to your problem?
Student: Six.
Teacher: One, two, three, four, five, six, seven, eight. I see eight.
Student: Six *dolls*!
Teacher: Oh! Okay. I see six dolls. You mean that there were two girls and each had three dolls, so there were six dolls altogether?
Student: Yes!
Teacher: Okay. That problem works out. Can anyone think of another one?

The students describe as many situations as they can to accompany the numbers on the overhead. The teacher checks each new description by illustrating the words.

LESSON 7-23

MULTIPLICATION WORD PROBLEMS

PURPOSE:

To write multiplication word problems

MATERIALS:

1. *Spelling notebooks*
2. *Lined paper*

The teacher asks the students to write on paper word problems for multiplication. They are encouraged to create problems different from the previous day's problems, or if they can't, to write down the one they liked best from that group.

When the students finish, the teacher collects the problems to be used in the next lesson.

LESSON 7-24

MULTIPLICATION WORD PROBLEMS

PURPOSE:

To review written word problems before writing new multiplication word problems

MATERIALS:

1. *Written word problems from Lesson 7-23*
2. *Spelling notebooks*
3. *Lined paper*

This lesson is conducted as was Lesson 7-10, except that the problems to be read aloud are for multiplication.

Once the examples have been read, the teacher writes a new multiplication problem on the overhead and the students generate word problems to go with them. These will be read and discussed at the beginning of the next lesson.

LESSON 7-25

ADDITION, SUBTRACTION AND MULTIPLICATION WORD PROBLEMS

PURPOSE:

To review written word problems before writing word problems for addition, subtraction, and multiplication

MATERIALS:

1. *Written problems from Lesson 7-13*
2. *Written problems from Lesson 7-24*
3. *Spelling notebooks*
4. *Lined paper*

In this lesson students combine their ability to describe situations that correspond with numbers representing multiplication problems with their similar ability to associate situations with abstract addition and subtraction problems. This means students must separate three different kinds of problem formats from one another and create an appropriate description to go along with each.

The procedure is the same as for the previous lesson (and Lesson 7-10). The only change is that the teacher reads a greater variety of problems.

The teacher reads paraphrased problems that represent a misunderstanding of multiplication word problems. Examples of creative or unusual problems, however, are drawn from the problems written for both Lesson 7-13 and 7-24. This means the students hear examples of creative addition, subtraction, and multiplication word problems.

Once the examples have been read, the teacher writes one addition, one subtraction, and one multiplication problem

on the overhead; the students write words for each. The teacher collects them for use at the beginning of Lesson 7-29.

LESSON 7-26

DIVISION WORD PROBLEMS

PURPOSE:

To create division word problems

MATERIALS:

No materials are needed

In the next three lessons students first create verbally, then describe in writing, problems involving division.

$$3\overline{)\,6}$$

Division problems are harder to describe in words than are multiplication problems. A problem written as above may leave students at a loss to explain which number means what. The answer for three times two is the same as for two times three, but the answer for three divided into six is not the same as for six divided into three.

Teacher: What kind of a problem is this?
Student: Division.
Teacher: What is the answer?
Student: Two.
Teacher: When I asked you to prove your answer in multiplication you were able to give me lots of ways. Can you think of any ways to prove that *this* answer is two?
Student: I had three cups and six beans and so I had two beans in each cup.
Student: We took six cubes and made them into three piles, so there were two in each pile.
Student: Six tiles were a rectangle with three rows and two columns.
Student: I had six piles and three cups, so I had two beans.

When students are first asked to string words together to match a preestablished set of numbers, some, initially simply run together as many familiar words as they can associate with the operation, dropping numbers in the problem wherever they think one might fit. This is one reason why illustrating each problem is so important.

There may be a few students who have great difficulty writing word problems, and who find drawing pictures insufficient help. In these cases, the teacher has them construct problems with materials. The students record the numbers for the materials then write a description of the actual work they performed.

For example, a student takes a tile board (see page 56 of the chapter on beginning division) and places a rectangle on it that consists of twelve tiles in four columns and three rows. The student then records the numbers

$$\begin{array}{r} 4 \\ 3\overline{)\,12} \end{array}$$

and writes in words "I put twelve tiles on my tile board and then I made a rectangle. The rectangle had three rows and it had four columns."

This is a word problem. Word problems describe something that happened or could have happened. If students cannot look at numbers and imagine a situation from which they might have come, they need to look again at situations they have already seen and observe the numbers that are generated. Students must also be given the opportunity to discover the numbers they see tell a story, and the stories they tell also generate numbers.

The students describe as many situations as they can think of that could generate the numbers on the overhead. The teacher checks the validity of each new description by illustration.

LESSON 7-27

DIVISION WORD PROBLEMS

PURPOSE:

To write division word problems

MATERIALS:

1. *Spelling notebooks*
2. *Lined paper*

The process of directing students to write division word problems is the same one used for multiplication in Lesson 7-23.

When the students finish writing their word problems, the teacher collects them. The problems form the basis for the lesson that follows.

LESSON 7-28

DIVISION WORD PROBLEMS

PURPOSE:

To review written word problems before writing new division word problems.

MATERIALS:

1. *Written word problems from Lesson 7-27*
2. *Spelling notebooks*
3. *Lined paper*

The teacher has selected word problems from the previous lesson to be read and discussed. The procedures from Lesson 7-10 are used.

Once the various examples have been read, the teacher writes a new division problem on the overhead, the students then write words for it, and the problems are saved for use at the beginning of the next lesson.

LESSON 7-29

ADDITION, SUBTRACTION, MULTIPLICATION, AND DIVISION WORD PROBLEMS

PURPOSE:

To review written word problems before writing word problems for addition, subtraction, multiplication, and division

MATERIALS:

1. *Written problems from Lessons 7-25 and 7-28*
2. *Spelling notebooks*
3. *Lined paper*

The process for reviewing and discussing written word problems remains the same, but an increased variety of problems are read. Examples are drawn from the problems written for Lessons 7-25 and 7-28, so students hear examples of creative problems for addition, subtraction, multiplication, and division.

Once the examples have been read, the teacher writes one problem of each on the overhead and the students write words for them. These problems form the basis of the following lesson.

LESSON 7-30

ADDITION, SUBTRACTION, MULTIPLICATION, AND DIVISION WORD PROBLEMS

PURPOSE:

To review written word problems before writing word problems for addition, subtraction, multiplication and division

MATERIALS:

1. *Problems written during Lesson 7-29*
2. *For each repetition of Lesson 7-30, written word problems from earlier lessons 7-30*
3. *Spelling notebooks*
4. *Lined paper*

The lessons in this chapter have provided students with experience in creating word problems for addition, subtraction, multiplication and division. When students have written problems for all four of the basic arithmetic operations, they should continue to practice this skill.

$$\begin{array}{r} 5 \\ +\,7 \\ \hline \end{array} \qquad \begin{array}{r} 6 \\ -\,3 \\ \hline \end{array} \qquad \begin{array}{r} 4 \\ \times\,9 \\ \hline \end{array} \qquad 3\overline{)5}$$

At least once each week, the teacher should write four simple arithmetic problems on the overhead as above. The students then write problems for each operation; the teacher collects them for later use.

Each time this lesson is repeated, examples of problems from earlier repetitions are read to the students. The teacher selects examples of problems representing both misunderstanding and creative or unusual descriptions, drawn from all four arithmetic operations.

When the examples have been read, the teacher writes four new problems on the overhead and the students write

a story to illustrate each. Again, these are saved for use at the beginning of the next repetition of this lesson.

The students can now create and solve word problems for the four basic arithmetic operations. They also know numbers in a problem may come from a wide variety of situations. Numbers describe objects or situations, and vice-versa. Knowledge of this interrelationship permits the students to apply the number skills they gain in following chapters to real-life situations.