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**Tiles and People**

Students learn counting games for grouping by fours.

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### Lesson 8-2  page 82  
**Beans and Cups**

Students learn the *plus one* game for grouping by fours.

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### Lesson 8-3  page 83  
**Beans and Cups**

Students learn the *minus one* game for grouping by fours.

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### Lesson 8-4  page 85  
**Beans and Cups**

Students play *plus one* and *minus one* for groupings of five.

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### Lesson 8-5  page 85  
**Beans and Cups**

Students record numbers for *plus one* and *minus one* for groupings of four.

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Students record numbers for *plus one* and *minus one* for groupings of five.

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### Lesson 8-8  page 88  
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### Lesson 8-9  page 88  
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Students create and record 1-0-0-0 type subtraction problems for groupings of five.

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Students create and record 1-2-3-4 type subtraction problems for groupings of five.

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Prerequisite chapters: None

MATERIALS

For overhead projector:
- Acetate squares with circles of three different sizes drawn on them—one circle per square (Materials chapter, page 295)
- Acetate squares in five different colors (Materials chapter, page 297)
- Color marking pens—washable (Materials chapter, page 297)
- Overhead projector dice (Materials chapter, page 296)
- Tiles (Materials chapter, page 294)
- Beans (Materials chapter, page 295)

If no overhead projector is available:
- Circular cutouts of three different sizes (Materials chapter, page 295)
- Strips and squares of paper in five different colors (Materials chapter, page 297)
- Dice cards in bag (Materials chapter, page 296)
- Square shapes (Materials chapter, page 294)
- Bean-shaped cutouts (Materials chapter, page 295)

Student materials:
- Dittos (optional) Special paper for recording numbers in columns (Worksheets 8–10)
- Recording sheet for addition with beans (Worksheet 11)
- Recording sheet for subtraction with beans (Worksheet 12)
- Recording sheet for addition with chips (Worksheet 13)
- Recording sheet for subtraction with chips (Worksheet 14)
- Paper squares or chips in five different colors (Materials chapter, page 297)
- Dice (Materials chapter, page 296)
- Individual blackboards (Materials chapter, page 294)
- Tiles (Materials chapter, page 294)
- Beans, cups, bowls, tin cans (Materials chapter, page 295)
- Crayons
- Unlined paper
This chapter’s activities give students practice in adding and subtracting positive whole numbers. The activities utilize numbers in other bases to provide students with a framework for understanding addition and subtraction in base ten. All bases are presented through the use of materials that make clear to both teacher and student the working of each problem.

Those who were taught in base ten may think saving its presentation until the end of this chapter makes teaching addition and subtraction unnecessarily difficult. This is primarily because most adults were never allowed to see that patterns in other ways of grouping are present in base ten as well.

When students search materials for patterns in grouping of threes, fours, and fives, then see these same patterns repeat for groupings of ten, they achieve a far greater understanding of borrowing and carrying than is possible from studying base ten in isolation.

The true measure of the high level of proficiency and understanding achieved when base ten is presented only after the concept of grouping by any number is clear, is the achievement of the students themselves. Experience has shown that the sequence of activities in this chapter makes it possible for each student in class to master the operations of addition and subtraction.

**GROUPING ACTIVITIES WITH TILES AND PEOPLE**

**PURPOSE:**

*To learn a counting game for grouping by fours using people*

**MATERIALS:**

1. If no overhead projector is available, square shapes
2. Tiles

The concept of place value involves the notion of grouping numbers or amounts of objects. This is true for all bases. The activities in this lesson teach students a game in which they group by fours. In subsequent lessons this knowledge of grouping will be used to establish the concept of place value.

**Teacher:** Today we will play a counting game. No one can use the word “four” for the number four. I would like someone to give me a nonsense word we can use, in place of four.

**Student:** Zoobie.

**Teacher:** Okay. When we begin counting, instead of saying four, say zoobie.

I will place tiles on the overhead one at a time and I want you to tell me the number.

**Student:** One.

**Student:** Two.

**Student:** Three.

**Student:** Four.

**Teacher:** True, but by the rules of this game, we can’t say four. What word did we decide to use instead?

**Student:** Zoobie.

**Teacher:** Then how many do I have on the overhead?

**Student:** Zoobie.

**Teacher:** Very good! Let’s do it again.

Each time “zoobie” tiles is reached, the overhead is cleared and the counting started over. When the class can count, “one, two, three, zoobie, ... one, two, three, zoobie,” the students use zoobie to play the following counting game.

**Teacher:** In a moment I will ask you all to stand up ... but not yet! When you are all standing I want you to count off by zoobies. The zoobie-people will sit down. The people standing will keep counting. Let’s see what happens. Everyone stand up, please (see illustration on the following page).

The game continues until only one person is left standing. The purpose of this is to give students practice in counting by zoobies. When only one person is left standing, everyone stands up again and the count-off process is repeated.

To motivate the counting practice, the teacher asks the following questions:

Can you predict who will be the last person standing? Does it make any difference where we start counting? If we start with the same person twice, will the last person standing be the same twice? Why? Why not?
The illustration represents one playing of the game ... with repeated countings, until the last person is left standing.

**Key:**
- X = Seated student. No longer counts
- = Last person standing.

**GROUPING ACTIVITIES WITH BEANS AND CUPS**

**PURPOSE:**
To learn a counting game for grouping by fours using beans and cups

**MATERIALS:**
1. Clear acetate squares with circles drawn on them, or circular cutouts
2. If no overhead projector is available, bean-shaped cutouts
3. Beans
4. Cups
5. Unlined paper

In the next two lessons, students use their newly acquired technique of grouping by fours to count forward and backward by ones in base four.

**Teacher:** You need beans and cups and a piece of paper for this game. Draw a line down the middle of your paper like this. On the top right half, draw a bean-shaped dot, like this. On the top left half, draw a cup, like this. It doesn't have to be beautiful, just make it look more like a cup than a bean.

Now, when I say plus one, put one bean on the right half of your paper. Plus one. How many beans do you have on your paper?
Student: One.
Teacher: Plus one. How many do you have?

Student: Two.
Teacher: Plus one. How many?

Student: Three.
Teacher: Plus one. How many?
Student: Four.
Teacher: There aren't any fours in this game.
Student: Zoobie.
Teacher: How many zoobies?
Student: One zoobie.
Teacher: Okay. To show one zoobie on your paper, take the zoobie beans from the right half of your paper, put them all in a cup, a zoobie cup, and move the cup to the left half of your paper, like this. How many zoobie cups do you have on your paper?

Student: One.
Teacher: How many beans remain on the right half of the paper?
Student: None.
Teacher: What's the number in mathematics that means none?
Student: Zero.
Teacher: What you have, then, is one zoobie and zero. Plus one. How many do you have?

Student: One and one.
Teacher: One zoobie and one.
Student: One zoobie and one.
Teacher: Plus one. How many?

Student: One zoobie and two.
Teacher: Plus one. How many?

Student: One zoobie and three.
Teacher: Think about this next one. Remember there are no fours in this game. Plus one. How many?
Student: One zoobie and four.
Student: No! Two zoobies and zero.

The teacher explains why, by the rules of the game, two zoobies and zero is what goes on the papers. The teacher also illustrates the process on the overhead.

Student: Two zoobies and one.

The teacher continues saying "plus one" until the class reaches three zoobies and three. The students then clear their papers and the whole process is repeated. This time, the teacher does not demonstrate on the overhead, but walks around the room and observes what each student is doing as the directions are given.

**LESSON 8-3**

**GROUPING ACTIVITIES WITH BEANS AND CUPS**

**PURPOSE:**

To learn a counting game for grouping by fours with beans and cups

**MATERIALS:**

1. Clear acetate squares with circles drawn on them, or circular cutouts
2. If no overhead projector is available, bean-shaped cutouts
3. Beans
4. Cups
5. Unlined paper

Teacher: You need your beans and cups for this game. You also need a piece of paper with a line down the middle, like the one you used yesterday.
To start this game, you need three zoobie cups and three loose beans on your paper, like this. Now, let's count down instead of up.

![Image showing three zoobie cups and three beans on a paper]

Minus one... What do you think I mean when I say minus one?

Student: Take away.
Teacher: Yes, it means subtract. Every time I say minus one, subtract one bean from the right side of your paper. Let's try it.

Minus one. How many do you have left?

Student: Three zoobies and two.
Teacher: Minus one. How many?

Student: Three zoobies and one.
Teacher: Minus one. How many?

Student: Three zoobies and zero.

At this point, the teacher checks everyone's board and has them check with their neighbors, to see that each student has three zoobies and zero.

Teacher: Minus one.

The teacher does not demonstrate this step on the overhead yet, but allows the students to puzzle out the solution first.

Teacher: How many do I have left?
Student: You can't do it, there are no more beans left.
Student: No, two zoobies are left because you take away one zoobie cup.
Student: No, it's three zoobies, because you take one bean out of a zoobie cup and then you have three zoobie cups left.
Teacher: If, when we are subtracting one bean each time, we run out of beans, the rule for this game says we should dump the beans out of one zoobie cup back onto the bean side of our paper.

We can't borrow just one bean from a zoobie cup, and leave that cup a bean short, because the rule for this game says that all zoobie cups have to have exactly zoobie beans in them...no more and no less.

This is what I get when I "minus one" from three zoobies and zero.

![Image showing three zoobie cups and three beans on a paper]

Dump one of your zoobie cups on the bean side of your paper, take away a bean and see if you get the same thing. Okay, let's continue. Minus one—how many?

Student: Two zoobies and two.
Teacher: Minus one. How many?

Student: Two zoobies and one.
Teacher: Minus one. How many?

Student: Two zoobies and zero.
Teacher: Think carefully about this one before you do it. Minus one.

Instead of demonstrating the correct answer on the overhead, the teacher asks the students what the rules for the game direct when no more beans remain on the right side of the paper. When the rules are reviewed, the students dump the contents of a zoobie cup on the bean side of their papers, then remove one bean.

Teacher: Read me what you have on your papers.
Student: One zoobie and three.

Teacher: Okay, minus one. How many?

When the students have subtracted ones all the way down to zero, they place three zoobies and three beans back on their papers and begin the process again. The teacher walks around and observes each student while saying the directions.

ADVANCED ADDITION AND SUBTRACTION
GROUPING ACTIVITIES WITH BEANS AND CUPS

PURPOSE:

To learn a counting game for grouping by fives using beans and cups

MATERIALS:

1. Clear acetate squares with circles drawn on them, or circular cutouts
2. If no overhead projector is available, bean-shaped cutouts
3. Beans
4. Cups
5. Unlined paper

After the class has played the plus-one and minus-one games for fours, using zoobie in place of four, both processes are repeated for groups of five. A new name for fives comes from a student suggestion. For the following activities, the word *yuck* will be used to indicate the nonsense word the class chose for groups of fives.

All the activities in Lessons 8-3 and 8-4 for zoobies are repeated for yucks. The plus-one game is continued to four yucks and four. The minus-one game uses four yucks and four as a starting point.

Confusion might result from using yucks instead of zoobies: students might count “one, two, three, zoobie, yuck,” instead of “one, two, three, four, yuck.” However, for each new game, only one number is replaced with a special word.

GROUPING ACTIVITIES WITH BEANS AND CUPS

PURPOSE:

To record numbers from the counting games and examine them for patterns

MATERIALS:

1. Special paper for recording number patterns in columns
Plus one. Put a bean on your zoobie board. How many zoobies do you have?

Student: Zero zoobies.

Teacher: And how many beans?

Student: One.

Teacher: Okay. Let's write that on our recording strips of paper.

The plus-one game is continued up to three zoobies and three. At each step, the number of beans and cups on the zoobie board is written in numbers on the recording strip.

As the students work, the teacher poses the following questions. If a student feels he or she has an answer, it is shared with the others as they continue their work.

Look down the bean column on your recording strip. Can you see any number patterns that might help you predict the numbers?

Can you see any patterns in the zoobie column of numbers you can use to predict future amounts of zoobies?

Are the patterns you get on your recording strip for plus one the same as or different from the patterns you get for minus one? Why?

GROUPING ACTIVITIES WITH BEANS AND CUPS

PURPOSE:

To record numbers from the counting games and examine them for patterns

MATERIALS:

1. Special paper for recording number patterns in columns
2. Clear acetate squared with circles drawn on them, or circular cutouts
3. If no overhead projector is available, bean-shaped cutouts
4. Beans
5. Cups
6. Unlined paper

This lesson repeats the sequence of activities in the previous one using groups of five instead of groups of four.

The numbers for plus one and minus one are recorded as for groups of four except the teacher does not demonstrate the recording process on the overhead beyond one yuck and one.

The teacher says "plus one" and the students add the bean to their yuck boards. The students then state out loud how many yucks and how many beans they have, before writing the numbers on their recording strips.

The teacher continues saying "plus one" until the students reach four yucks and four. Then the class begins the minus one countdown.

As the students record the numbers, the teacher asks them to look for patterns and think about the same questions asked for zoobies in the previous lesson.
GROUPING ACTIVITIES WITH BEANS, CUPS, AND BOWLS

PURPOSE:

To extend the counting games for counting by fours using beans, cups, and bowls

MATERIALS:

1. Clear acetate squares with circles of two different sizes drawn on them (one circle per square), or circular cutouts in two sizes
2. If no overhead projector is available, bean-shaped cutouts
3. Beans
4. Cups
5. Bowls
6. Unlined paper

The following two lessons expand the students' ability to count in bases four and five from two-digit numbers to three.

Teacher: Today, we will play the plus-one game again. We won't write down any numbers, so all you need is your beans, cups, and zoobie boards. Plus one. How many?

Student: Zero zoobies and one.

Teacher: Plus one. How many?

Student: Zero zoobies and two.

Teacher: Plus one. How many?

The teacher continues to say "plus one," until the students have three zoobies and three on their boards.

Teacher: Plus one. How many?

Student: Zero zoobies and zero.

Teacher: Plus one. How many?

Student: Zero zoobies and two.

Teacher: Plus one. How many?

The teacher continues to say "plus one," until the students have three zoobies and three on their boards.

Teacher: Plus one. How many?

Student: Four zoobies and zero.

Teacher: That might be what it looks like, but the rules for this game say you can't have any fours. We'll have to call this something besides four zoobies.

Student: A grinch.

Teacher: That's a good idea, but I would prefer to have us call it something that has to do with zoobies, because zoobies is the word that tells us we're grouping by fours.

Student: Big zoobies.

Student: Giant zoobies.

Student: Zoolie zoobies.

Teacher: Let's vote... Okay, big zoobies is the winner. So, what will we call zoobie zoobie cups?

Student: One big zoobie.

Teacher: We'll need something to collect our little zoobies in. I have a lot of bowls we can use for our big zoobies.

Please dump all the beans from your zoobie cups into a big zoobie bowl. We'll have to make different zoobie boards, too. You'll need a new piece of paper.

This time I want you to divide your paper so it has three spaces. In the space on the right, draw a bean. In the middle space, draw a cup. In the space on the left, draw a bowl. The bowl doesn't have to be beautiful, but it should look like it is larger than the cup.

Okay. Take all the beans and cups you had on your other zoobie board and move them to your new zoobie board.

How far had we gone in our plus-one counting?

Student: Four zoobies and zero.

Teacher: Okay. On your new paper, put one bowl, zero cups, and zero beans. Plus one. How many?

The plus-one game is continued to three big zoobies, three zoobies and three. Then, the teacher takes the class back down to zero again with minus-one.

The minus-one process is essentially the same as the students used before big zoobies were introduced. When the students have three big zoobies, zero zoobies and zero beans a new problem is encountered.

Teacher: Minus one. How many?

Student: Two big zoobies, zero zoobies, and zero beans.

Teacher: Two big zoobies, three zoobies, and zero beans.

Student: Two big zoobies, three zoobies, and three beans.

Student: You still have three big zoobies because you haven't got any more beans you can take away.

The teacher and the students discuss the various possibilities suggested. The teacher then models the rules for this game, showing on the overhead how to take one bean from three big zoobies.

Teacher: If, when we are subtracting one bean at a time, we run out of beans, we already know we can get more beans by dumping all the beans out of a zoobie cup. If we run out of zoobie cups, the rule for this game says we can get more zoobie cups out of a big zoobie.

We can't borrow just one zoobie cup from a big zoobie, because the rule for this game says that all big zoobie bowls have to have exactly zoobie worth of zoobie cups in them... no more and no less.

How many zoobie cups in a big zoobie?

Student: Four.
Student: No! Zoobie!
Teacher: Okay, we can take zoobie worth of zoobie cups and put them here.

Now, can we borrow beans from any of these zoobie cups?
Student: Yes.
Teacher: How many zoobie cups do we need to get beans from?
Student: One.
Teacher: Okay. Now, can we take away one bean?
Student: Yes.

This is what I get when I take one bean from three big zoobies: zero zoobies and zero beans. Work this problem out on your zoobie boards and see if you get the same thing.

When the students have worked the minus-one problem, the teacher continues the minus-one subtractions down to zero, zero, zero.

**GROUPING ACTIVITIES WITH BEANS, CUPS, AND BOWLS**

**PURPOSE:**

To extend the counting games for grouping by fives using beans, cups and bowls

**MATERIALS:**

1. Clear acetate squares with circles of two different sizes drawn on them (one circle per square), or circular cutouts in two sizes
2. If no overhead projector is available, bean-shaped cutouts
3. Beans
4. Cups
5. Bowls
6. Unlined paper

In the next two lessons students once again record numbers on paper to search for patterns. The numbers they record permit them to examine patterns that extend much further than was possible in Lessons 8-5 and 8-6.

The activities are essentially the same as those in Lesson 8-5. The students add or subtract one bean at a time from their zoobie boards while recording the appropriate numbers on long strips of paper.

The only difference is the columns of numbers generated for Lesson 8-5 are extended much further this time, to three big zoobies, three zoobies, and three.

**LESSON 8-9**

GROUPING ACTIVITIES WITH BEANS, CUPS, AND BOWLS

**PURPOSE:**

To record numbers from the counting games and examine them for patterns

**MATERIALS:**

1. Special paper for recording number patterns in columns
2. Clear acetate squares with circles of two different sizes drawn on them (one circle per square), or circular cutouts in two sizes
3. If no overhead projector is available, bean-shaped cutouts
4. Beans
5. Cups
6. Bowls
7. Unlined paper
The students will need to add a bowl-like figure to the top of their recording strips, so they relate to the revised zoobie boards.

The teacher assists the students in recording through one big zoobie, one zoobie, and one. Beyond this point, the students work on their own. The teacher does not say "plus one" each time. When they reach three big zoobies, three zoobies, and three, they then subtract one bean at a time until they reach zero, zero, zero.

As the students work, the teacher poses the questions from Lesson 8-5, with the addition of the following:

Are there patterns in the big zoobie column that tell you when you are going to add another big zoobie to your zoobie board?

---

**GROUPING ACTIVITIES WITH BEANS, CUPS, AND BOWLS**

**PURPOSE:**
To record numbers from the counting games and examine them for patterns

**MATERIALS:**
1. Special paper for recording number patterns in columns
2. Clear acetate squares with circles of two different sizes drawn on them (one circle per square), or circular cutouts in two sizes
3. If no overhead projector is available, bean-shaped cutouts
4. Beans
5. Cups
6. Bowls
7. Unlined paper

This lesson uses the previous one's activities. The only difference is the students record numbers up to four big yucks, four yucks, and four, before counting back down to zero.
Student: A four.
Teacher: Okay. Russell, ask Brenda for four beans and put them on your yuck board.

Russell's turn... What did Lonnie get?
Student: A two.
Teacher: Lonnie, get two beans from Brenda and put them on your yuck board.

Russell's turn again... Russell get a three. What should he do?
Student: Get three beans from Brenda.
Teacher: Right. How many beans does Russell have in his beans space?

Student: Seven.
Teacher: But this is a yuck board. The rule for the yuck board is, you can't have five beans or more in the bean space. What do you think I'll have Russell do?
Student: Put some of his beans in a yuck cup.
Teacher: How many?
Student: Five. Yuck!
Teacher: Okay Russell, show yuck beans to Brenda and she'll give you a cup to put them in. Brenda, you have to make sure Russell has enough beans to get a cup before you give him one.

Whose turn is it?
Student: Lonnie's...

Once the students have seen the game demonstrated at the overhead, they divide into groups of three and begin playing. If the class does not divide evenly into threes, then four students play together. Every time there is a winner, the loser trades places with the banker and the game begins again.

Students need to clarify the rules of a game before they start playing, so they don’t argue about them in the middle of a game. Learning to resolve conflicts peacefully is an important part of anyone’s education; games and their rules provide an excellent model for learning this.

Every time students question what to do in a given situation, they should be encouraged to make up a rule that will cover that and all similar situations. If they cannot decide on a rule among themselves, their problem should be submitted to the entire class for discussion and resolution by vote.
Our students have been taught all their lives that rules are "given," not something to be reflected on. Rules are no more than common agreements accepted by all, because they serve us well. If they do not, they should not be accepted without question. The greatest, most creative advances in the field of mathematics have come from persons who accepted the rules as a starting place rather than the end.

**LESSON 8-12**

GROUPING ACTIVITIES WITH BEANS, CUPS, BOWLS, AND DICE

**PURPOSE:**

To learn a grouping game that involves counting by the numbers zero through five

**MATERIALS:**

1. Overhead projector dice, or dice cards in bag
2. Dice, numbered zero to five
3. Clear acetate squares with circles of two different sizes drawn on them (one circle per square), or circular cutouts in two sizes
4. If no overhead projector is available, bean-shaped cutouts
5. Beans
6. Cups
7. Bowls
8. Unlined paper

The game presented to the students in the previous lesson involved racing to a big yuck. In this lesson, the students race to a big zoobie.

The students play in groups of threes; the loser becomes the banker. The rules decided on by the class for yucks are still in effect for zoobies.

**LESSON 8-13**

GROUPING ACTIVITIES WITH BEANS, CUPS, BOWLS, AND DICE

**PURPOSE:**

To learn a grouping game that involves counting by the numbers zero through five

**MATERIALS:**

1. Overhead projector dice, or dice cards in bag
2. Dice, numbered zero to five
3. Clear acetate squares with circles of two different sizes drawn on them (one circle per square), or circular cutouts in two sizes
4. If no overhead projector is available, bean-shaped cutouts
5. Beans
6. Cups
7. Bowls
8. Unlined paper

Teacher: Today we'll play the dice-rolling game a different way. You will start with a big yuck on your yuck board and roll your dice until you've subtracted all your beans. Before you start, I'll demonstrate what I mean on the overhead.

First, I place a big yuck on my yuck board. Then I roll my dice. What did I get?

Student: A three.

Teacher: The three means I take three beans away from the bean column. How can I do that? I don't have any beans in my bean column.

Student: You can get five yucks... I mean yuck yucks out of the big yuck. That gives you five yuck cups in the yuck column. Then you can dump the beans out of one yuck cup into the bean column and take one bean away. Just like we did for the minus-one game.

Teacher: Do you think you can start with a big yuck and take beans away each time you roll the dice till you get back to zero?

Student: Yes.

Teacher: Then you may begin playing the game.
The students know how to do the necessary regrouping to remove beans from their yuck boards. The structure of the game for racing to a big yuck is the same when racing back. Switching from an addition activity to subtraction generally gives students no difficulty.

If, for some reason, the switch from racing up to racing back is not easily made, the teacher selects three students to play at the overhead and guides them step by step through a game while the rest of the class watches.

The rules decided for the addition version of the game apply for subtraction. All new rule questions are decided as before.

LESSON 8-14

GROUPING ACTIVITIES WITH BEANS, CUPS, BOWLS, AND DICE

PURPOSE:

To play a grouping game that involves counting by the numbers zero through five

MATERIALS:

1. Dice numbered zero to five
2. Beans
3. Cups
4. Bowls
5. Unlined paper

The game presented in the previous lesson involved racing from a big yuck to zero. In this lesson, the students start with a big zooibie, and play without any formal instruction at the overhead. They begin with a big zooibie and race down to zero.

All rules established for earlier games remain in effect.

LESSON 8-15

PLACE VALUE ACTIVITIES WITH BEANS, CUPS, BOWLS, AND RICE

PURPOSE:

To look for place value patterns

MATERIALS:

1. Overhead projector dice, or dice cards in bag
2. Dice numbered zero to four
3. Clear acetate squares with circles of two different sizes drawn on them (one circle per square), or circular cutouts in two sizes
4. If no overhead projector is available, bean-shaped cutouts
5. Beans
6. Cups
7. Bowls
8. Individual blackboards
9. Unlined paper
10. Lined paper

*Note: For the activities that follow, the students’ dice have numbers from zero to four. This can be accomplished by placing a small gummed label over the five. Any number between zero and four can then be repeated on the label.

In the next two lessons students learn and practice a dice game which, when combined with the dice game played in the previous four lessons, provides them the necessary skill to create and solve addition and subtraction problems involving borrowing and carrying.

Teacher: Today we will play a new game. Each person will roll his or her dice three times. The first roll will be for beans, the second for yucks, and the third for big yucks. When you have rolled three times and your partner has rolled three times, the winner is the person with the most beans, counting the big yucks, yucks, and beans columns.

I'll demonstrate what I mean on the overhead. First, I roll my die... what did I get?

Student: Two.

Teacher: Okay. That means I put two beans in my bean column. Now, I roll again. What did I get?

Student: Three.

Teacher: That means I put three yuck cups in my yuck column. I roll again. What did I get?

Student: Four.

Teacher: That means four big yucks in the big yuck column.

I've shown you how to roll your dice and fill in amounts of big yucks, yucks, and beans on your paper. I haven't shown you how somebody wins, because I didn't have a partner. The winner is the person with the most beans. How many beans do I have in my bean column?

Student: Two.
Teacher: How many beans do I have in my yuck cup column?
Student: Three yuck cups.
Teacher: But how many beans?
Student: What?
Teacher: How many beans are there in one yuck cup?
Student: Five.
Teacher: Then how many beans in two yuck cups?
Student: Ten.
Teacher: Okay. How many beans in three yuck cups?
Student: Fifteen.
Teacher: Then how many beans do I have in my yuck cup column, if I have three yuck cups in it?
Student: Fifteen.
Teacher: How many beans do I have in both my yuck cup column and my bean column?
Student: Seventeen.
Teacher: How many beans do I have in my big yuck column?
Student: I don’t know.
Teacher: How many beans in one big yuck?
Student: I don’t know.
Teacher: How many yucks in a big yuck?
Student: Five.
Teacher: How many beans in each yuck?
Student: Five.
Teacher: How many beans in five yucks?
Student: Twenty-five.
Teacher: Then how many beans in one big yuck?
Student: Twenty-five.
Teacher: Two big yucks?
Student: . . . Fifty?
Teacher: Three big yucks?
Student: Seventy-five.
Teacher: Four big yucks?
Student: One hundred.
Teacher: How many beans in all the big yucks, yuck cups, and beans columns?
Student: . . . One hundred and seventeen.
Teacher: Okay. That means my total number of beans is 117. If I were playing someone, then I would win if that person had less than 117, and that person would win if he or she had more than 117.

The easy part for students to learn is to roll the dice and put big yucks, yucks, and beans in the appropriate columns—the hard part is figuring out who has the most total beans. An amount like 3 big yucks, 4 yucks, and 4 beans might mean counting out 99 beans. The only way for students to avoid the lengthy counting is to search for patterns within the numbers of big yucks, yucks, and beans, which can help them know who has more without always having to count every bean.

Patterns do exist that, when seen, eliminate the need for counting. To increase the students’ potential for seeing these patterns, the teacher has them record their games in numbers.

Teacher: When you were racing to or back from big yucks, the games took long enough so I could come around and see what you were doing. The games you are to play now will be over too quickly for me to see very many of them. For this reason, I want you to record your scores for each game on paper, so when I come around I can see a record of all the games you have played.

You will need a record sheet on which to keep score. Here is one example of a record sheet you might use. You need a piece of lined paper. Draw a line down the middle of it, write one person’s name on one half, and the other person’s name on the other half. On each half, make a separate column for big yucks, yucks, and beans. Then, on each line write how many big yucks, yucks, and beans each person collected for that game.

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The teacher draws a record sheet on the overhead for the students to copy, then selects two students to play a sample game at the overhead. The students each roll the dice three times and set out the appropriate number of big yucks, yucks, and beans on their yuck boards.

John Marsha

John

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Marsha

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Teacher: You can see how I wrote the numbers of yucks on my recording sheet. This is how I want you to keep track of your scores.

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I need you to add one extra thing to your recording sheet. Who won this game?
Student: Marsha.
Teacher: Okay. You need to indicate who won. I know you could think of several good ways to indicate who the winner was, but I will show you a special symbol mathematicians use to indicate which number they think is bigger or smaller than another. The symbol is written like this,
Whenever you want to show which number you think is bigger, you point the big end toward the bigger number. The little, or pointed, end goes toward the little number. Write two and seven on your blackboards and show me how you think you would indicate which one is bigger.

Student: Like this?

Teacher: Yes. How would we show that Marsha had won? Show me on your blackboards, please.

Student: This?

Teacher: Okay. Whenever you figure out who has more beans for any of your games, write the symbol in the correct direction on your recording sheet.

Once the students have learned how to keep score, they begin rolling against each other to see who gets the larger number. When they have had an opportunity to roll, compute the larger number, and examine their findings for patterns to make their work simpler, the teacher changes the game slightly. The students roll the dice three times just as before, but this time they may select the column into which they wish to put each successive number. For example, under the old rules students who rolled a 4, then a 1, then a 3, ended up with the number 314 on their record sheets. In this new version of the game, they might elect to place the 4 in the big yuck column, the 1 in the beans column, and the 3 in the yucks column. In this case, instead of 314, the number would be 431.

This new form of the game may be played in one of two ways. The students choose the column in which they want to record each roll of the dice before the roll and may not change their minds once they see what the roll produces; or they place their numbers in any order they wish after they have rolled the dice three times.

Either version of the game produces the same result: any numeral to the left is worth more beans than any numeral to its right. This pattern is called place value. When students think they see patterns in the numbers with which they are working, the teacher asks them what the pattern is, how they think it works, if they think it will always work, and if they can find a time when it doesn’t.

Is there a quicker way to find how many beans you have in all your big yucks, yucks, and beans columns than counting out the beans each time? Could you make a list of how many beans you have in one big yuck, then two big yucks, then three, and so on, to find out how many beans you have without having to recount each time? Are there patterns you can see that would permit you to look just at the numbers on your recording sheet and know who has more beans? Remember, all you have to know is who has more, not how many more.

The teacher can ask students to share their discoveries with one another. Students who are using a shortcut to counting are asked to explain their discovery to their classmates. Different ways exist to shorten the counting process—each new one found is shared with the whole class.

Some students understand numbers and place value so well that a glance at the numbers on their recording sheet is sufficient to tell them who has more. Others, despite repeated explanations from their classmates of quicker ways, count the beans one by one each time. Both methods should receive an equal amount of acceptance from the teacher.

**LESSON 8-16**

**PLACE VALUE ACTIVITIES WITH BEANS, CUPS, BOWLS, AND DICE**

**PURPOSE:**

To look for place value patterns

**MATERIALS:**

1. Overhead projector dice, or dice cards in bag
2. Dice numbered zero to three*
3. Clear acetate squares with circles of two different sizes drawn on them (one circle per square), or circular cutouts in two sizes
4. If no overhead projector is available, bean-shaped cutouts
5. Beans
6. Cups
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LESSON 8-17
ADDITION WITH BEANS, CUPS, BOWLS, AND TIN CANS

PURPOSE:

To create addition problems with beans, cups, bowls, and tin cans

MATERIALS:

1. Overhead projector dice, or dice cards in bag
2. Dice numbered zero to four
3. Clear acetate squares with circles of three different sizes drawn on them (one circle per square), or circular cutouts in three sizes
4. If no overhead projector is available, bean-shaped cutouts
5. Beans
6. Cups
7. Bowls
8. Tin cans
9. Unlined paper

Over the next lessons, students apply the knowledge of place value they have gained so far to creating, recording, and checking addition and subtraction problems involving borrowing and carrying.

Teacher: Today you need to make a new paper on which to put your beans, cups, and bowls—this will be used for addition problems.

First, make four columns on your paper, like this. Starting with the right-hand column, draw a bean, then a cup, then a bowl, as you did on your earlier papers. In the fourth column, the one on the far left, draw a tin can. Remember, it doesn’t have to be perfect. This time your papers will have rows in addition to columns on them. Make rows across your paper like this. Please notice I have made the last row at the bottom by drawing double lines.

We will now use the new papers to work some addition problems. For my convenience, I will call the paper on which you’ve put all your rows and columns a trading board.

I’ll give you an example of how to make up an addition problem before you begin doing your own.

The teacher fills in the top row of the trading board by rolling a die three times, as in Lessons 8-15 and 8-16. The teacher rolls first for beans, then for yucks, then for big yucks. The fourth column remains empty. When there are beans, yucks, and big yucks in the first row, the second row is filled in using the same procedure.

An example of how the trading board would be filled in for successive rolls of two, one, three... four, zero, and three, can be seen in this figure.

Teacher: I have filled in two rows on my trading board by rolling my die. I am now ready to add. To do this, I slide all my beans, cups, and bowls below the double line on my trading board.
That's all there is to adding yucks, except, by the rules, I can't have any fives in any column, so I have to make some exchanges, or trades. That's why I call this a trading board.

How many beans are in the bean column?
Student: Six.
Teacher: I can't have six, so I have to get rid of some beans. What do I do to them?
Student: Put five in a cup and put the cup in the yuck column.

Teacher: Okay. How many yucks do I have now?
Student: Two.
Teacher: Do I need to make any trades?
Student: No.
Teacher: How many big yucks do I have?
Student: Six.
Teacher: Do I need to make any trades?
Student: Yes. Take five big yucks and change them into a tin can.

Teacher: What shall we call the tin can column?
Student: A giant yuck.
Teacher: Can I make any more trades?

Student: No.
Teacher: If I can't make any more trades, this is the answer to my yuck addition problem. How would you say the answer?
Student: One giant yuck, one big yuck, two yucks, and one.

When the first example is finished, the teacher works a second one, in which the students tell the teacher what to do each step. When they can successfully direct the teacher, they begin constructing their own problems on their trading boards. As the students work, the teacher walks around the room checking their efforts.

LESSON 8-18

ADDITION WITH BEANS, CUPS, BOWLS, AND TIN CANS

PURPOSE:

To create and record addition problems

MATERIALS:

1. Overhead projector dice, or dice cards in bag
2. Dice numbered zero to four
3. Clear acetate squares with circles of three different sizes drawn on them (one circle per square), or circular cutouts in three sizes
4. If no overhead projector is available, bean-shaped cutouts
5. Beans
6. Cups
7. Bowls
8. Tin cans
9. Unlined paper
10. Individual blackboards
11. Dittoed copies of recording sheet for addition with beans (optional)

Teacher: Yesterday, as I watched you make up addition problems on your yuck trading boards, I could tell you were doing an excellent job. My problem is, I can't get around fast enough to see all of the work you do. I want you to keep track of the problems you make up—write them down on paper, so I can have a record of what you have done. To keep track of your work, write the numbers from the top row of your problem like this, the second row underneath, like this, then draw a line and put your answer below, like this.
The teacher follows this explanation by creating beans, cups, and bowls problems on the overhead, then works the problems together with the students. They then record the problems on their blackboards.

If the students understand the recording process after two or three problems, they begin creating addition problems for beans, yucks, and big yucks on their own. If some students have difficulty recording numbers for their materials, the teacher may introduce a recording sheet that contains copies in miniature of the students trading boards (see black-line master 11). As they put their beans, cups, and bowls on their trading boards, the students write the numbers for these amounts in the corresponding spaces on their recording sheets.

The greatest difficulty students have in learning to record problems, with or without the recording sheet, is in forgetting to write the numbers that composed the problem before the numbers are added. Students often take the two rows of beans, cups, and bowls, slide them below the double line, make all the necessary changes, then try to write numbers for the whole problem. This is a difficult task, because only the answer is available, and the recorded problem will consequently be incomplete. The teacher cannot tell from an answer if the student added correctly.

Students construct and record as many addition problems as time permits. As the students work, the teacher walks around the room and assists them.

**LESSON 8-19**

**ADDITION WITH BEANS, CUPS, BOWLS, AND TIN CANS**

**PURPOSE:**

To create and record addition problems

**MATERIALS:**

1. Overhead projector dice, or dice cards in bag
2. Dice numbered zero to three
3. Clear acetate squares with circles of three different sizes drawn on them (one circle per square), or circular cutouts in three sizes
4. If no overhead projector is available, bean-shaped cutouts
5. Beans
6. Cups
7. Bowls
8. Tin cans
9. Unlined paper
10. Individual blackboards
11. Dittoed copies of recording sheet for addition with beans (optional)

This lesson combines the activities from Lessons 8-17 and 8-18, but now, the students add groups of four instead of groups of five.

The teacher shows them how to create addition problems on their trading boards using dice. When they have had about ten minutes to work problems with materials, the teacher reintroduces the recording process. They then create problems, and record them.

**LESSON 8-20**

**SUBTRACTION WITH BEANS, CUPS, BOWLS, AND TIN CANS**

**PURPOSE:**

To create subtraction problems with beans, cups, bowls, and tin cans

**MATERIALS:**

1. Overhead projector dice, or dice cards in bag
2. Dice numbered zero to four, three dice per student
3. Clear acetate squares with circles of three different sizes drawn on them (one circle per square), or circular cutouts in three sizes
4. If no overhead projector is available, bean-shaped cutouts
5. Beans
6. Cups
7. Bowls
8. Tin cans
9. Unlined paper
10. Individual blackboards
11. Dittoed copies of recording sheet for subtraction with beans (optional)

Teacher: Today you will make a new paper for subtraction problems for yucks. The paper is almost the same as the one we made for addition. You need four columns: they have a bean, a cup, a bowl, and a tin can in them just like we drew for addition. You don't need any other lines on your paper for subtraction.
A difficulty is encountered in setting up subtraction problems not found for addition. For subtraction, the top row of numbers has to represent a larger total than the bottom row. This means random rolls of dice, by themselves, will not consistently produce workable subtraction problems. The following method is designed to eliminate this difficulty.

Teacher: I'll make up some subtraction problems on the overhead. After I am sure the instructions are clear, you can make up subtraction problems on your own. These are created differently than the addition problems were so you must watch closely.

First you start each problem with one giant yuck, zero big yucks, zero yucks, and zero beans in the top row. Then you roll three dice at one time. Here's my roll... what did I get?

Student: Four, one, and two.

Teacher: Okay. I'll put one die under the beans column, one under the yucks column, and one under the big yucks column. Notice, I didn't put the dice on my trading board. The dice are off the board.

Now, I'll subtract the number on the die from the column it's under. That means I'll subtract two beans from the bean column, one yuck from the yuck column, and four big yucks from the big yuck column.

Let's start with the beans column. How can I take two beans from the beans column if there aren't any beans there?

Student: You do it like we did it for the minus-one game. Take five big yucks out of the giant yuck. Then take five yucks out of a big yuck. Then take five beans out of a yuck.

Teacher: Like this?

Student: Yes.

Teacher: Let's do the subtracting and see what we get. How would you read the answer?

| 4 | 1 | 2 |

When the first example is completed, the teacher works a second one, in which the students direct the teacher. When they can successfully do that, they begin constructing problems on their trading boards. As the students work, the teacher walks around the room and assists them.

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**LESSON 8-21**

**SUBTRACTION WITH BEANS, CUPS, BOWLS, AND TIN CANS**

**PURPOSE:**

To create and record subtraction problems

**MATERIALS:**

1. Overhead projector dice, or dice cards in bag
2. Dice numbered zero to four
3. Clear acetate squares with circles of three different sizes drawn on them (one circle per square), or circular cutouts in three sizes
4. If no overhead projector is available, bean-shaped cutouts
5. Beans
6. Cups
7. Bowls
8. Tin cans
9. Unlined paper
10. Individual blackboards
11. Dittoed copies of recording sheet for subtraction with beans (optional)

Teacher: You've been doing a really great job making up subtraction problems on your trading boards. I can't get around fast enough to see all your work, though, so this time I want you to write your problems on paper so I'll have a record of them.
I’ll put a problem on the overhead then show you how to record it. Here’s the first part of the problem. How many giant yucks?

Student: One.
Teacher: Big yucks?
Student: Zero.
Teacher: Yucks?
Student: Zero.
Teacher: Beans?
Student: Zero.
Teacher: Okay. I write that 1000, which means one giant yuck, zero big yucks, zero yucks, and zero beans. That’s the first row.

Now I’ll roll my dice for the second row.

Student: Four.
Teacher: How many yucks?
Student: Three.
Teacher: How many beans?
Student: One.
Teacher: I write that as 431, which means four big yucks, three yucks, and one bean.

Now, before we record the answer, we have to work the problem.

How many giant yucks in the answer?
Student: Zero.
Teacher: How many big yucks?
Student: Zero.
Teacher: How many yucks?
Student: One.
Teacher: How many beans?
Student: Four.
Teacher: Okay. I write that 0014 and say it: zero giant yucks, zero big yucks, one yuck, and four.

If the students understand the recording process after two or three problems have been worked on the overhead and recorded on their blackboards, they begin constructing and recording subtraction problems on their own. If some students have difficulty, the teacher introduces a recording sheet that is an exact miniature of the subtraction trading boards (see black-line master 12).

When working a problem with materials, the students use the trading board for the starting row of the problem and the answer row. This is reflected on the recording sheet by the inclusion of two miniature trading boards for each problem to be recorded. The starting row is recorded in the upper miniature trading board, the dice numbers in the appropriate dice squares, and the answer in the lower miniature trading board. The problem written above would be recorded as in this figure.

Students create and record as many subtraction problems as time permits.

**LESSON 8-22**

**SUBTRACTION WITH BEANS, CUPS, BOWLS, AND TIN CANS**

**PURPOSE:**

To create and record subtraction problems

**MATERIALS:**

1. Overhead projector dice, or dice cards in bag
This lesson combines the activities for Lessons 8-20 and 8-21, but here the students begin each subtraction problem with a giant zoobie instead of a giant yuck.

The students create subtraction problems on their trading boards. After about ten minutes the recording process is introduced and the students keep track of each new problem they create.

**LESSON 8-23**

**SUBTRACTION WITH BEANS, CUPS, BOWLS, AND TIN CANS**

**PURPOSE:**

To create and record subtraction problems

**MATERIALS:**

1. Overhead projector dice, or dice cards in bag
2. Dice numbered zero to four
3. Clear acetate squares with circles of three different sizes drawn on them (one circle per square), or circular cutouts in three sizes
4. If no overhead projector is available, bean-shaped cutouts
5. Beans
6. Cups
7. Bowls
8. Tin cans
9. Unlined paper
10. Individual blackboards
11. Dittoed copies of recording sheet for subtraction with beans (optional)

This lesson repeats the activities from Lessons 8-20 and 8-21. The difference is in how the students determine the numbers with which to begin the problem. In Lesson 8-20, each new subtraction problem was begun by placing a giant yuck on the trading board. The students then rolled three dice to establish the numbers to be subtracted from 1000. In this lesson they use their dice to determine beginning numbers as well.

Teacher: I will make up a different kind of subtraction problem on the overhead. Once I am sure my instructions are clear, you will make up subtraction problems on your own. I start each problem with one giant yuck, as we've already been doing. Then I roll three dice and place them on my trading board.

Now, I replace each die with the appropriate number of beans, cups, or bowls.

Now I've got the top row of my problem. What do you think I will do next?

Student: Roll the three dice to see how much to take away.

Teacher: Okay. Now what?

Student: Now do the subtracting.
Teacher: Okay. What's the answer?
Student: One giant yuck, zero big yucks, three yucks, and three.

When the students understand the process they begin creating and recording their own subtraction problems.

LESSON 8-24

SUBTRACTION WITH BEANS, CUPS, BOWLS, AND TIN CANS

PURPOSE:
To create and record subtraction problems

MATERIALS:
1. Overhead projector dice, or dice cards in bag
2. Dice numbered zero to three
3. Clear acetate squares with circles of three different sizes drawn on them (one circle per square), or circular cutouts in three sizes
4. If no overhead projector is available, bean-shaped cutouts
5. Beans
6. Cups
7. Bowls
8. Tin cans
9. Unlined paper
10. Individual blackboards
11. Dittoed copies of recording sheet for subtraction with beans (optional)

The activities in this lesson repeat those for the previous one, however, the students now use zoobies for each problem instead of yucks.

The activities in Lessons 8-1 through 8-24 involved the students in adding and subtracting with zoobies and yucks. Using two different groupings is an effective way to practice without having the repetition seem like remediation.

The repetition of yuck lessons with zoobies or zoobie lessons with yucks is meant to be a flexible model, not a rigid guide. In classrooms where students catch on quickly, yucks followed by zoobies is usually sufficient to make the point of the lessons clear. If they have difficulty mastering the concept, yucks and zoobies can be followed by groupings of sevens or sixes. These new groupings, for which students create names, allow them to examine in greater detail the patterns and processes that lead them to a clear understanding of addition and subtraction in base ten.

LESSON 8-25

GROUPING ACTIVITIES WITH PAPER CHIPS

PURPOSE:
To learn a counting game for grouping by threes, using a chip trading board; to record numbers for the counting game and examine them for patterns

MATERIALS:
1. Acetate squares or squares of colored paper in five different colors
2. Marking pens with washable ink, or colored strips of paper in five different colors
3. Paper squares or chips
4. Crayons
5. Special paper for recording number patterns in columns
6. Unlined paper

In the previous 24 lessons, students used beans and cups to learn how to count, add, and subtract in bases four and five. In the next 16 lessons, many of these earlier experiences are repeated using cutout squares of paper called chips. Beans and cups provide students a framework for visualizing the relative value of numerals as they are used to represent increasingly larger quantities in a place value system of numeration. If students are to be able to deal with increasingly larger numbers, representing increasingly abstract notions of quantity, they need to internalize or abstract what they have learned about grouping and regrouping. The chips to which they will now be introduced provide the necessary link between the concrete representations of beans and cups and the more abstract concept of numbers representing unseen quantities.

Teacher: We will learn a new game today, but first you must make a new trading board. You need a blank piece
of paper and your red, blue, green, purple, and orange crayons.

The colors are arbitrary. What is important is that each student uses the same colors on his or her board in the same order.

Teacher: Divide your paper so you have five columns on it. Now, use your crayons to color a strip across the top of each column. Make sure you have your colors starting from the same side and going in the same order as the ones on the overhead. Check with your neighbor to see if my instructions are clear.

Red  Blue  Green  Purple  Orange

The game I will show you is called chip trading. The chips are your little squares of colored paper.

First, you will play on your chip trading boards the plus-one game. For this game, the rule is that you can't have three or more in any one column.

Take out some orange chips. Each time I say "plus one," put an orange chip in the orange column.

Plus one . . . Plus one . . . Plus one. How many orange chips do you have in your orange column?

Student: Three.

Teacher: By the rules of this game, you can't have three of anything in a column. The rule for the chip trading board is when you get too many in one column, you turn them in for one chip in the next column. By the chip trading rules, then, we have to turn three oranges in for one purple.

Check with your neighbor to see if I explained clearly what should happen to the orange chips.

Plus one . . . add another orange chip to your board. Plus one . . . Plus one. Think about what the rules say to do.

Some students will remember to make the exchange of three oranges for a purple, others won't. The teacher restates the rule and demonstrates the exchange process on the overhead. The students then check each other's chip trading boards to see if everyone has two purples and no oranges.

Teacher: Plus one orange . . . Plus one . . . Plus one. Think about what the rules say to do.

It is likely that a few students will forget to exchange the three oranges for a purple. It is more likely that they will remember the rule says they can't have three oranges, but will fail to see they now have three purples. Three of any color in a column is not allowed. Some students may actually exchange the three purples for a green without direc-
As they work, the teacher poses the following questions:

Look down each column on your recording strip. Do you see any number patterns to help you predict the numbers you expect to get in that column?
Are the patterns for one column the same as or different from the patterns in the column next to it?
Have you seen any of those patterns before? Where?

The teacher continues guiding the students through the plus-one game until one red chip has been reached. The students then clear their trading boards and the process is begun again. This time, however, the teacher does not demonstrate on the overhead what is to be done, but walks around and observes while saying plus one.

The third time the students play the plus-one game, they record the numbers of chips on long strips of paper, using the same procedures learned earlier for beans and cups. The difference now is that the top of the recording strip is marked with the same colors as the ones on the chip trading boards.

The banking game taught to the students for this lesson is essentially the same as the game taught in Lesson 8-11. The two differences are that the students play with chips instead of beans and cups, and the grouping used is three not five. The rules decided for earlier versions of this game are still in effect.
COUNTING AND GROUPING ACTIVITIES WITH PAPER CHIPS

PURPOSE:

To play counting and grouping games for grouping by fours

MATERIALS:

1. Overhead projector dice, or dice cards in bag
2. Dice, numbered zero to three
3. Acetate squares or squares of colored paper in five different colors
4. Marking pens with washable ink, or colored strips of paper in five different colors
5. Paper squares or chips
6. Crayons
7. Special paper for recording number patterns in columns
8. Unlined paper

The activities in this lesson combine those in Lessons 8-25 and 8-26, the only difference being the amount of chips allowed to remain in a column before an exchange is required. For Lessons 8-25 and 8-26, the limit was two per column. At three, an exchange had to be made. For this lesson, each column may have three chips; when there are four chips, they must be exchanged for a chip in the column to the left.

The teacher explains the revised chip trading rule to the students who then practice it as the teacher demonstrates a plus-one cycle on the overhead up to one red chip. The second time the students repeat the plus-one game, they write numbers for their chips on their recording strips. The teacher poses the same questions as in Lesson 8-25 for base three.

When most students have finished recording the plus-one activity, the banking game is reintroduced. After a quick review lesson on the overhead, the students divide into groups of three and race each other to one red chip, playing as many rounds as time permits.

The student-generated terms zoobie and yuck have unobtrusively faded from use with the introduction of the chip trading board. The words were to focus the child's attention on the grouping process. If the student has mastered the activities for beans and cups, the intended focusing has been accomplished and special words are no longer necessary.

The teacher notifies the students of the grouping to be used on the chip trading board by announcing the grouping rules for that day. Teachers who wish to, however, may introduce them to the more formal names for the groupings: grouping by threes is called base three, grouping by fours, base four, and so on.

GROUPING ACTIVITIES WITH PAPER CHIPS

PURPOSE:

To learn a counting game for grouping by threes, using a chip trading board; to record numbers for the counting game and examine them for patterns

MATERIALS:

1. Acetate squares or squares of colored paper in five different colors
2. Marking pens with washable ink, or colored strips of paper in five different colors
3. Paper squares or chips
4. Crayons
5. Special paper for recording number patterns in columns
6. Unlined paper

Teacher: We'll play the minus-one game with chips on your chip trading board. I'll start the game by placing one red chip on my board. Please put one red chip on your board, too.

Student: Orange.
Teacher: Orange is also the color chip I want you to take off your board each time I say "minus one" in this game. Are you ready? Minus one.

Some students may know they can turn one red into three blues, one blue into three greens, one green into three purples, one purple into three oranges and then take away the one orange meant by minus one. It is more likely, however, that the teacher will need to explain the process of reversing the exchanges they made earlier for plus one.

The students possess sufficient knowledge to master the minus one game for chips relatively quickly. They have already played the game for beans and cups and learned through the plus-one game the various trading relationships that lead to a red chip. This does not mean, however, that this knowledge will automatically transfer to the present situation.

To insure ample opportunity for the students to transfer their prior knowledge of minus one to chips, the teacher demonstrates each new exchange on the overhead after they make the exchange on their chip trading boards. The teacher continues guiding the students through the minus-one game until all the chips have been subtracted from the boards, one orange chip at a time. They then replace the red chip on their boards and the process is begun again. This time, however, the teacher does not illustrate what is to be done, but walks around observing each student while saying "minus one."

The third time the students play the game, they also write numbers for the chips on their long strips of recording paper, as in Lesson 8-25. Here the first number recorded is 10000, and each subsequent number is one less than the previous one.

The teacher assists the students until they reach zero reds, two blues, one green, one purple, and one orange (02111). Beyond this point, the students subtract their own orange chips, one at a time, and record their numbers. As they work the teacher poses the following questions:

As you look down the different color columns on your recording strip, can you see any number patterns to help you predict what you expect to get next in that column? Are the patterns for one column the same as or different from the patterns in the column next to it? Are the patterns for minus one the same as or different from the patterns you found for plus one? Why?

**LESSON 8-29**

GROUPING ACTIVITIES WITH PAPER CHIPS

**PURPOSE:**

To learn a grouping game that involves counting by the numbers zero, one, and two

**MATERIALS:**

1. Overhead projector dice
2. Dice, numbered zero, one, two
3. Acetate squares or squares of colored paper in five different colors
4. Marking pens with washable ink, or colored strips of paper in five different colors
5. Paper squares or chips
6. Crayons
7. Unlined paper

The students once again play the banking game first introduced in Lesson 8-11. It is essentially the same as the chip-trading version in Lesson 8-26, except the students start with a red chip on their trading boards and race down to zero. The rules decided on for earlier versions are still in effect.

**GROUPING ACTIVITIES WITH PAPER CHIPS**

**PURPOSE:**

To play counting and grouping games for grouping by fours

**MATERIALS:**

1. Overhead projector dice, or dice cards in bag
2. Dice, numbered zero to three
3. Acetate squares or squares of colored paper in five different colors
4. Marking pens with washable ink, or colored strips of paper in five different colors
5. Paper squares or chips
6. Crayons
7. Special paper for recording number patterns in columns
8. Unlined paper

This lesson combines the activities in Lessons 8-28 and 8-29. Now the number of chips that remain in any one column before an exchange is required is three; four must be exchanged.

The teacher begins by explaining the revised chip trading rule to the class and they practice through one minus-one cycle. The second time the students repeat the minus-one game, they also write numbers for their chips on their recording strips. The teacher asks the same questions as in Lesson 8-28 for base three.

When most students have finished recording the minus-one activity, the banking game is reintroduced. The students divide into groups of three and, starting with a red chip, race each other down to zero.

**LESSON 8-31**

**ADDITION WITH PAPER CHIPS**

**PURPOSE:**

To create addition problems using a chip trading board for groupings of threes

**MATERIALS:**

1. Acetate squares or squares of colored paper in five different colors
2. Marking pens with washable ink, or colored strips of paper in five different colors
3. Paper squares or chips
4. Crayons
5. Overhead projector dice, or dice cards in bag
6. Dice, numbered zero, one, two
7. Unlined paper

The activities in this lesson are essentially the same as those in Lesson 8-17. However, the addition problems are now created for paper chips instead of beans and cups, and the dice are rolled four times instead of three to create the numbers for each row.

To create a chip trading addition problem, the students roll their dice first for orange, then purple, then green, and then blue. After each roll, the indicated amount of chips is placed in the appropriate column on the chip trading board. The example in the figure below illustrates chips placed on a board for consecutive dice rolls of zero, one, two, and one. The dice are then rolled again to determine numbers for the bottom row. Chips for the second series of dice rolls are placed on the lower half of the trading board. The figure illustrates the second row of chips placed on the board for consecutive rolls of two, zero, one, and two.

When the students have rolled the dice to produce two rows of chips, the two separate rows are moved together. All the appropriate trades that can be made are made; when no more exchanges remain, the chips left on the board are the answer to the problem.

The sequence of events for chip trading addition is the same as for beans and cups in Lesson 8-17.
LESSON 8-32

ADDITION WITH PAPER CHIPS

PURPOSE:

To create and record addition problems using grouping of threes

MATERIALS:

1. Acetate squares or squares of colored paper in five different colors
2. Marking pens with washable ink, or colored strips of paper in five different colors
3. Paper squares or chips
4. Crayons
5. Overhead projector dice, or dice cards in bag
6. Dice, numbered zero to two
7. Unlined paper
8. Individual blackboards
9. Dittoed copies of the recording sheet for addition with chips (optional)

The activities are essentially the same as for Lesson 8-18, but the problems to be recorded now are for paper chips instead of beans and cups, and the answers usually extend to five places instead of four.

The teacher explains the need for recording the problems on paper. Then, the teacher and the students work problems on the overhead together. The students record the problems on their blackboards.

Students who understand the recording process begin creating and recording their own addition problems. Students who have difficulty in understanding the process are asked to modify their chip trading board as shown in the figure below. The first row is for the chips produced from the first series of dice rolls. The second row is for the second series of rolls. The third row, below the double line, represents the spaces into which the first two rows of chips are combined before the exchanges are made that produce the answer.

<table>
<thead>
<tr>
<th>Red</th>
<th>Blue</th>
<th>Green</th>
<th>Purple</th>
<th>Orange</th>
</tr>
</thead>
</table>

LESSON 8-33

ADDITION WITH PAPER CHIPS

PURPOSE:

To create and record addition problems for groups of fours

MATERIALS:

1. Acetate squares or squares of colored paper in five different colors
2. Marking pens with washable ink, or colored strips of paper in five different colors
3. Paper squares or chips
4. Crayons
5. Overhead projector dice, or dice cards in bag
6. Dice, numbered zero to three
7. Unlined paper
8. Individual blackboards
9. Dittoed copies of the recording sheet for addition with chips (optional)

This lesson combines the activities of Lessons 8-31 and 8-32, however, the students now add groupings of fours instead of threes.

LESSON 8-34

SUBTRACTION WITH PAPER CHIPS

PURPOSE:

To create subtraction problems on the chip trading board for groupings of threes
The activities are essentially the same as those in Lesson 8-20. The subtraction problems are now created for paper chips instead of beans and cups, and four dice instead of three are rolled to create the numbers to be subtracted.

To create a chip trading subtraction problem, the students start with a top row of one red chip and nothing else. Four dice are rolled together to produce the second row. The four dice are placed off the chip trading board, with one die underneath each column for blue, green, purple, and orange. The students with modified chip trading boards for addition will need to make new ones for subtraction.

The sequence of events for teaching chip trading subtraction is the same as for beans and cups in Lesson 8-20.

LESSON 8-36
SUBTRACTION WITH PAPER CHIPS

PURPOSE:
To create and record subtraction problems for groupings of fours

MATERIALS:
1. Acetate squares or squares of colored paper in five different colors
2. Marking pens with washable ink, or colored strips of paper in five different colors
3. Paper squares or chips
4. Crayons
5. Overhead projector dice, or dice cards in bag
6. Dice, numbered zero to three
7. Unlined paper
8. Individual blackboards
9. Dittoed copies of the recording sheet for subtraction with chips (optional)

This lesson is essentially the same as Lesson 8-21. The problems to be recorded now are for chips instead of beans and cups, and the top row of the problem is 10000 instead of 1000.

Students who have difficulty understanding the recording process are given a recording sheet, but it is not an exact miniature of their own subtraction trading boards. It is, instead, similar to the subtraction recording sheets first used in Lesson 8-21 (see black-line master 14).
The activities in this lesson combine those of Lessons 8-34 and 8-35. The only difference is for this lesson, the students subtract groupings of fours instead of threes.

The teacher repeats the process of showing the students how to create subtraction problems on their trading boards. When the students have had about ten minutes to construct problems, the teacher reintroduces the recording process.

### LESSON 8-37

**SUBTRACTION WITH PAPER CHIPS**

**PURPOSE:**

*To create and record subtraction problems for groupings of threes*

**MATERIALS:**

1. Acetate squares or squares of colored paper in five different colors
2. Marking pens with washable ink, or colored strips of paper in five different colors
3. Paper squares or chips
4. Crayons
5. Overhead projector dice, or dice cards in bag
6. Dice, numbered zero, one, two
7. Unlined paper
8. Individual blackboards
9. Ditoed copies of the recording sheet for subtraction with chips (optional)

This lesson repeats the activities for Lessons 8-34 and 8-35. The difference is in how the students determine the numbers for the top row in each problem. In Lesson 8-34, each new subtraction problem was begun by placing a red chip in the appropriate column on the trading board. The students then rolled their four dice to establish what numbers were to be subtracted from 10000.

For this lesson, the four dice are used to determine the numbers for the top row as well. The students place one red chip on their trading boards. They then roll their four dice and place them on their trading boards in the empty columns. Each die is then replaced with the appropriate number of chips.

Next, the students roll the dice to be placed off the board. The subtraction is then carried out.

The teacher demonstrates this process on the overhead. When the first example is completed, the teacher works a second one, in which the students direct each step. When
they can successfully do this, they begin creating their own problems on the trading boards. In about ten minutes, the teacher introduces the recording process. They then record on paper each problem they create.

**LESSON 8-38**

**SUBTRACTION WITH PAPER CHIPS**

**PURPOSE:**

To create and record subtraction problems for groupings of fours

**MATERIALS:**

1. Acetate squares or squares of colored paper in five different colors
2. Marking pens with washable ink, or colored strips of paper in five different colors
3. Paper squares or chips
4. Crayons
5. Overhead projector dice, or dice cards in bag
6. Dice, numbered zero to three
7. Unlined paper
8. Individual blackboards
9. Dittoed copies of the recording sheet for subtraction with chips (optional)

This lesson repeats the activities of the previous one, but the students use groupings of fours for each problem instead of threes. The pattern of presentation for each new lesson should be as much like the patterns for old lessons as possible. In this way, the amount of new material presented to students is kept to a minimum.

**LESSON 8-39**

**ADDITION AND SUBTRACTION WITH PAPER CHIPS**

**PURPOSE:**

To create and record addition and subtraction problems for groupings of threes

**MATERIALS:**

1. Paper squares or chips
2. Dice, numbered zero, one, two
3. Chip trading boards made earlier on unlined paper
4. Dittoed copies of recording sheets for addition and subtraction with chips (optional)

The activities in this lesson repeat those for the previous one, but the students use groupings of fours for each problem instead of threes.

Lessons 8-25 through 8-40 involved the students in adding and subtracting on their chip trading boards using groupings of threes and fours. Alternating the two groupings is a continuation of the method of repeating experiences without having the repetition seem like remediation.

The groupings of threes and fours are not meant to be rigid guides for what must be done in a classroom. As was true for beans and cups, in classrooms where students catch on more quickly, two different groupings may be enough. If students need more experience to internalize a concept,
additional ones may be used: threes and fours can be followed by fives, then sixes.

**LESSON 8-41**

**GROUPING ACTIVITIES WITH PAPER CHIPS**

**PURPOSE:**

To learn a counting game for grouping by tens, using a chip trading board; to record numbers for the counting game

**MATERIALS:**

1. Acetate squares or squares of colored paper in five different colors
2. Marking pens with washable ink, or colored strips of paper in five different colors
3. Paper squares or chips
4. Crayons
5. Special paper for recording number patterns in columns
6. Unlined paper

The activities in the next six lessons allow students to apply all they have learned about place value, addition and subtraction in bases three, four, and five to the base in which most computation is performed—base ten. The effectiveness of preceding addition and subtraction in base ten with study in other bases is best shown by the students themselves. When addition and subtraction in base ten is learned in the context of a pattern for addition and subtraction regardless of the base, students exhibit none of the confusion traditionally associated with the operations of borrowing and carrying.

The activities in this lesson combine those in Lessons 8-25 and 8-28. The differences are: the students use groupings of tens, and only play the plus-one game up to one green chip before starting back down to zero with the minus-one game.

As they work at recording the numbers, the teacher asks them to think about the same questions asked in Lessons 8-25 and 8-28.

Students who wish to, may go beyond one green chip in the plus-one game. It is not important that each student have time to play the minus-one game. It is important that each student have the opportunity to examine long lists of numbers for patterns.

**LESSON 8-42**

**GROUPING ACTIVITIES WITH PAPER CHIPS**

**PURPOSE:**

To learn a grouping game that involves counting by the numbers zero through nine

**MATERIALS:**

1. Overhead projector dice, or dice cards in bag
2. One die per student, numbered zero through four
3. One die per student, numbered zero through five
4. Acetate squares or squares of colored paper in five different colors
5. Marking pens with washable ink, or colored strips of paper in five different colors
6. Paper squares or chips
7. Crayons
8. Unlined paper

This lesson combines the activities for Lessons 8-26 and 8-29. The only differences are (1) the students use groupings of tens, (2) the banking game is played first up to one green, then down from green, then, up again, then, down, and so on, and (3) two dice are rolled and added together to determine the orange chips to be added or taken away on any given turn.

**LESSON 8-43**

**ADDITION WITH PAPER CHIPS**

**PURPOSE:**

To create and record addition problems for groupings of tens using a chip trading board

**MATERIALS:**

1. Acetate squares or squares of colored paper in five different colors
2. Marking pens with washable ink, or colored strips of paper in five different colors
3. Paper squares or chips
4. Crayons
5. Overhead projector dice, or dice cards in bag
6. One die per student, numbered zero to four
7. One die per student, numbered zero to five
8. Unlined paper
9. Individual blackboards
10. Dittoed copies of the recording sheet for addition with chips (optional)

This lesson combines the activities for Lessons 8-31 and 8-32, but the students use groupings of tens, and two dice are rolled and their totals added together to find the number of chips to put in each row of each column.

**LESSON 8-44**

SUBTRACTION WITH PAPER CHIPS

**PURPOSE:**

To create and record subtraction problems for groupings of tens using a chip trading board

**MATERIALS:**

1. Acetate squares or squares of colored paper in five different colors
2. Marking pens with washable ink, or colored strips of paper in five different colors
3. Paper squares or chips
4. Crayons
5. Overhead projector dice, or dice cards in bag
6. One die per student, numbered zero to four
7. One die per student, numbered zero to five
8. Unlined paper
9. Individual blackboards
10. Dittoed copies of the recording sheet for subtraction with chips (optional)

The activities for this lesson are essentially the same as those for Lesson 8-37. The students use groupings of tens, and employ the same technique of the previous lesson, that produced numbers for the second row of the subtraction problems, to produce numbers for the orange, purple, green, and blue columns of the first row of the problems.

**LESSON 8-45**

SUBTRACTION WITH PAPER CHIPS

**PURPOSE:**

To create and record subtraction problems for groupings of tens using a chip trading board

**MATERIALS:**

1. Acetate squares or squares of colored paper in five different colors
2. Marking pens with washable ink, or colored strips of paper in five different colors
3. Paper squares or chips
4. Crayons
5. Overhead projector dice, or dice cards in bag
6. One die per student, numbered zero to four
7. One die per student, numbered zero to five
8. Unlined paper
9. Individual blackboards
10. Dittoed copies of the recording sheet for subtraction with chips (optional)

The activities for this lesson combine those for Lessons 8-34 and 8-35. However, the students use groupings of tens and do not roll four dice at once to determine what numbers to subtract from 10000. Instead, they roll their two dice to find each number to be subtracted, one number at a time, as they have already done for their various chip trading problems in addition. The first two-dice roll determines the number of orange chips to be subtracted, the second is for purple, the third for green, and the fourth for blue.

**LESSON 8-46**

ADDITION AND SUBTRACTION WITH PAPER CHIPS

**PURPOSE:**

To create and record addition and subtraction problems for groupings of tens using a chip trading board

**MATERIALS:**

The activities for this lesson combine those for Lessons 8-34 and 8-35.
MATERIALS:

1. Acetate squares or squares of colored paper in five different colors
2. Marking pens with washable ink, or colored strips of paper in five different colors
3. Paper squares or chips
4. Crayons
5. Overhead projector dice, or dice cards in bag
6. One die per student, numbered zero to four
7. One die per student, numbered zero to five
8. Unlined paper
9. Individual blackboards
10. Dittoed copies of the recording sheet for addition and subtraction with chips (optional)

The activities for this lesson combine what the students have learned about adding and subtracting with chips for groupings of tens. The students are asked to alternate between creating addition and subtraction problems.

When students can create problems in base ten, the lessons in the techniques of adding and subtracting are at an end. Our next responsibility is to provide them opportunities to use and perfect these skills. Subsequent chapters are based on the assumption that all students now know how to add and subtract, and can verify their answers. If some students have not yet mastered their chip trading boards, they can get answers using beans and cups.

If students are carried no further in learning to add and subtract, when are they weaned from their chip trading boards? The answer is not clear-cut. Some students internalize the board's use almost before they finish watching the teacher demonstrate it. Others hesitantly abandon it for some problems but revert to it as a security check when work becomes more difficult. Still others continue using it throughout the entire year. The internalization process is a direct function of how often the students use their boards and of how well they understand that use. Students must decide when, if ever, they wish to abandon aids to computation.

The measure of a person's practical ability in mathematics is whether he or she can produce accurate answers to problems. We must think carefully about our responsibilities as teachers. Are we to teach our students only those ways of producing answers that have been pre-judged as "acceptable" for use on a standardized test? Or is our goal, instead, to help each student find answers to problems in the best way he or she is able? In Japan, all students are allowed to work calculations on their sorobons. Japanese students register the highest level of arithmetic understanding of any of the nations tested by UNESCO, including the United States. The chip trading board is the equivalent of a multi-base sorobon or abacus.

Each student can now produce answers to addition or subtraction problems using positive whole numbers in base ten. This is the foundation for the multiplication and division work in the following two chapters.