CHAPTER 11
Fractions

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Lesson 11-5 page 156 Geoboards Students attempt to find areas for any triangle.	Lesson 11-6 page 157 Geoboards Students record triangles and areas on paper and check each other's work.	Lesson 11-7 page 158 Geoboards Students find and record areas for a variety of shapes.	Lesson 11-8 page 158 Geoboards Students find and record areas for a wider variety of shapes.
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Lesson 11-29 page 172 Geoboards Teacher guides students in determining fractional values for lengths, widths, and areas.	Lesson 11-30 page 173 Geoboards Students record fractional lengths, widths, and areas while searching for patterns.	Lesson 11-31 page 174 Geoboards Students record smaller fractional lengths, widths, and areas while searching for patterns.	

Prerequisite chapters: Chapter 7 for lessons 11-13, 11-14, 11-15

MATERIALS

For overhead projector:

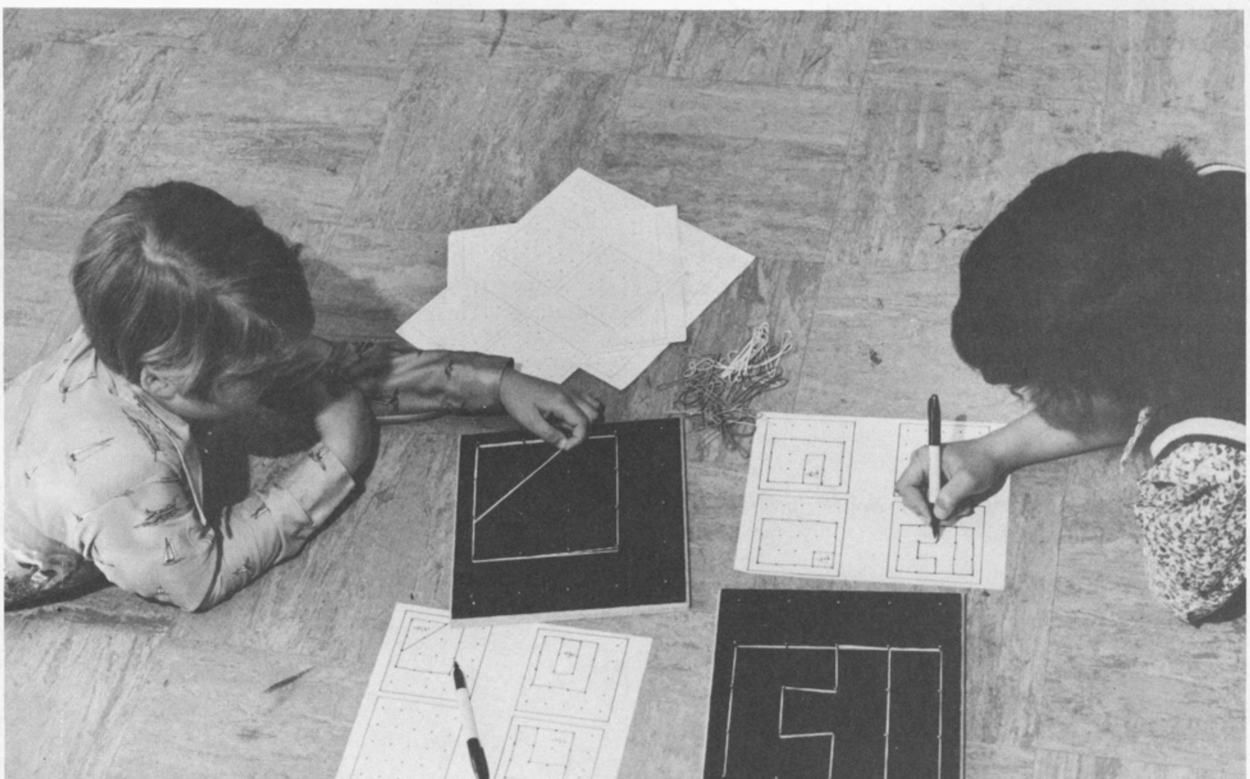
Making a Transparency (geoboard)	Page 317
Geoboard recording sheet	Worksheets 7-18

If no overhead projector is available:

Make charts in place of transparencies	Materials chapter, page 294

Student materials:

Dittos	Geoboard recording sheet
Geoboards	Materials chapter, page 297
Cubes	Materials chapter, page 295
Spelling notebooks	Materials chapter, page 296
Individual blackboards	Materials chapter, page 294
Lined and unlined paper	
String or yarn	



As the United States switches from U.S. Customary Units of Measure to the International System of Units (SI), more commonly known as metric units, the role of fractions in the mathematics curriculum shifts. Most calculations involving fractions will now be made with decimal fractions. This means it is no longer necessary to emphasize mastery of the basic operations involving nondecimal fractions. This does not mean, however, the study of fractions is no longer of value.

Three reasons exist for continuing to include nondecimal fractions in the curriculum. First, the study of fractions provides an excellent background for understanding decimal fractions. Second, not all situations involving fractions can be handled in decimal form. Third, the arithmetic operations associated with adding and subtracting simple fractions provide students who need it, practice in addition and subtraction that does not seem like remediation.

This chapter gives students experience in adding, subtracting, and multiplying fractions. The lessons are designed to establish a background for understanding decimal fractions, to familiarize students with a variety of fractional forms and equivalent forms, and to allow ample practice in simple addition and subtraction. Each lesson is structured around concrete examples of fractional numbers so students can visualize the concepts associated with these numbers. The lessons emphasize discovering rules for working with fractions through examining various fraction problems for patterns.

LESSON 11-1

UNDERSTANDING FRACTIONAL VALUES

PURPOSE:

To create shapes with whole number areas on the geoboard

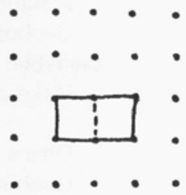
MATERIALS:

1. *Simulated geoboard on a transparency, or on a large tagboard*
2. *Geoboards*

Before the students learn to identify fractional areas on their geoboards, they must know which areas represent whole numbers. This lesson familiarizes them with whole number areas on their geoboards.

Teacher: I've made a shape on my geoboard, but do not copy it yet. This shape has an area of one square unit. Is it possible to make a shape on your geoboard that has an area of two square units? Hold up your board when you think you've made a shape with two square units.

Student: Here.



Teacher: Is it possible to make any other shape with an area of two square units?

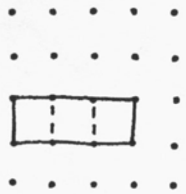
Student: This?

Teacher: Any more?

The teacher copies each new shape shown on the overhead.

Teacher: You've shown me ways to make shapes with an area of two square units. How about an area of three square units?

Student: This?



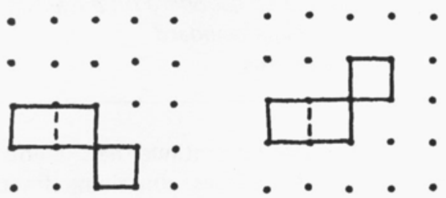
Teacher: Any more ways?

Student: Yes.

Teacher: Any more?

Student: Hey! That isn't a different way.

The students decide whether the two ways are the same or different. An interesting discussion is often generated over what is meant by "the same." If the shapes in the previous two figures are the same, are the shapes in the next figures the same? The students decide how they want to define "same" and "different" for this lesson.



When the students have found a variety of ways to make shapes with an area of three, they try four, five, six, and so on. It isn't necessary for them to exhaust the possibilities for each new area. What is important is that they discover there is more than one way to construct a shape with a particular area.

The teacher continues to assign new areas, illustrating on the overhead, throughout the time available for the lesson.

LESSON 11-2

UNDERSTANDING FRACTIONAL VALUES

PURPOSE:

To create shapes with fractional areas using paper, then geoboards

MATERIALS:

1. *Simulated geoboard on a transparency, or on a large tagboard*
2. *Geoboards*
3. *Unlined paper*

Teacher: How many pieces of paper do you have?
Student: One.

Teacher: Okay. Fold it like this, then tear it along the fold line. How many pieces do you have now?
Student: Two.
Teacher: How many pieces did you start with?
Student: One.
Teacher: And what part of the one whole piece that we started with is this?

Student: What do you mean?

If no one knows half a piece of paper is called *one-half,* the teacher defines this for the students in the following way:

Teacher: How many pieces did we start with?
Student: One.
Teacher: This small part of the whole is called *one part.* Into how many equal parts did we tear the paper?
Student: Two.
Teacher: This part is said to be *one part of two.* What would the other part be called?
Student: The other part of two.
Teacher: It could be called that, but it is also called one part of two. It can be written as the number of parts we are holding, over the number of parts there were altogether, like this.

$$\frac{1}{2}$$

It can also be said *one over two,* meaning one part of two parts.

The discussion acquaints the students with the meaning of the fraction one-half. Tearing the paper may be supplemented by any other appropriate divisions, such as cutting cupcakes in half, or dividing apples.

Although the term "one-half" need not be avoided, it is easier for students to learn to read it as "one of two parts" or "one over two." This system of saying fractions permits students to read and think about what fractional numbers mean long before they have absorbed the more common terms such as halves, thirds, fourths, and so on.

Teacher: If this is one square unit, then what is this?
Student: I don't know.
Teacher: How many little triangles like this would it take to make the one square unit we started with?
Student: Two.

Teacher: Show me, please.
Student: Like this.

Teacher: As was true for folding and tearing the paper, each small part of the whole is said to be one part. How many parts did we divide the square into to get the small triangles?
Student: Two.
Teacher: Okay, then one triangle is said to be one part of two. We would write the area of this triangle like this,

$$\frac{1}{2}$$

and we would say this triangle has an area of one over two, meaning its area is one part of two equal parts.
If I say the small square has an area of one, can you make a triangle on your geoboards that has an area of one over two?

When the students have made the triangle on their geoboards, the teacher introduces a second name for this particular unit.

Teacher: If I say the little square is one unit, then I can either say the little triangle has an area of one over two or I can say it has an area of one-half. Both terms are correct; you may use either.
Now, can you show me a shape on your geoboard with an area of one and one-half square units? . . . Two and one-half square units? . . . Three and one-half square units?

The process continues as time permits.

LESSON 11-3

UNDERSTANDING FRACTIONAL VALUES

PURPOSE:

To create shapes with areas assigned fractional values using geoboards

MATERIALS:

1. *Simulated geoboard on a transparency, or on a large tagboard*
2. *Geoboards*

The activities in this lesson continue the students' exploration of ways to make shapes containing fractional or mixed-number areas.

Teacher: Show me a shape on your geoboard with an area of one half-square unit.
Student: This.

Teacher: Can you show me a shape with an area of two half-square units?
Student: Yes.

Teacher: Can you find another way?
Student: Yes.

Teacher: And another?

As students explore different ways to make shapes with an area of two half-square units, the teacher asks them to think about the following questions:

Why can two halves be made into the same kind of shape as one square unit?
Do *all* the two half shapes have the same area as one square unit?
Could we count the two half-square shapes as other ways to make shapes with an area of one square unit?

The students then explore ways to make shapes having areas of three half-square units, then four, five, six and so on. As the students create shapes for each new area the teacher asks:

Which of the shapes you are making now look like shapes you made with areas of one square unit, two, three, and so on?

Which look like shapes you made with areas of one and one-half square units, two and one-half, three and one-half, and so on?

It is not important at this point that students see two halves is the same as one whole, or three halves is equivalent to one and one-half. It is enough that they begin looking for the possible relationships or patterns that exist between fractions and whole numbers.

LESSON 11-4

AREA ON THE GEOBOARD

PURPOSE:

To compute areas for triangular shapes constructed on the geoboard

MATERIALS:

1. *Simulated geoboard on a transparency, or on a large tagboard*
2. *Geoboards*
3. *Individual blackboards*

In this lesson the students learn the difference between the fractional value one-half as applied to half of any shape and the value of one half-square unit, which applies only to a specific area. This knowledge in turn is used to help them calculate areas for triangle shapes constructed on their geoboards.

Teacher: What is the area of this triangle?
Student: One half-square unit.
Teacher: Okay. What is the area of *this* triangle?

Student: I don't know.
Teacher: How many units is this?

Student: One.
Teacher: How many units is this?

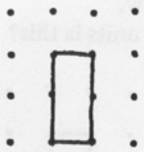
Student: One-half.
Teacher: How many units is this?

Student: Two.
Teacher: How do you know?
Student: Because we can count them.
Teacher: Okay. I'll put a line through the two-square unit area like this. How many pieces did I divide it into?

Student: Two.
Teacher: Then is one piece one-half of the whole shape?
Student: Yes.
Teacher: What was the area of the whole shape?
Student: Two.
Teacher: Then what is the area for half of the shape?
Student: I don't know.

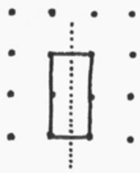
Some students may realize the area of the triangle is one square unit, because it is half the area of the two-square unit rectangle. Some students may also think the area for the shape is one-half. The triangle *is* one-half of the rectangle, but the teacher is asking for the area of the triangle.

A semantics problem is involved in the question which must be overcome. Half a cake is much bigger than half an apple, but both are referred to as one-half. An answer of one-half is correct, if the triangle is compared to the rectangle, but it is not appropriate if the area of the rectangle is asked for.



Teacher: What is the area of this shape?
Student: Two.

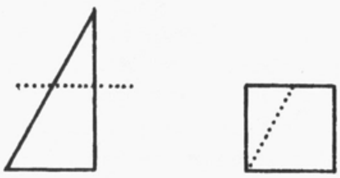
Teacher: If I divide it in half this way, what is the area of one part?
Student: One.



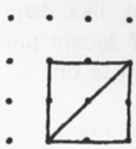
Teacher: If I divide it in half this way, what would the area of one part be?
Student: One.

Teacher: Then, if I divide it in half this way, what would the area of one part be?
Student: One.
Teacher: So the area of the triangle is . . .
Student: One.

If the students are unable to answer "one," the teacher physically demonstrates the area by cutting a triangle the same size as the triangle in question from a piece of paper. The top of the paper triangle can now be cut off and placed next to the bottom, as shown in this figure. The resulting shape clearly has an area of one square unit.



Teacher: What is the area of this triangle? If I put it inside a rectangle like this, can you use what you know about the area of the rectangle to find the area of the triangle? When you think you have figured out how many square units are in the triangle, write your answer on your blackboard.



The teacher scans the answers and writes the answer appearing most frequently as the class answer on the overhead. The students with the class answer explain how they arrived at that area.

The teacher must make sure the students understand that a triangle that is one-half a rectangle does not necessarily also have an area of one half-square unit. Beyond this point, the teacher does not tell them the area of any triangles.

For this lesson, the teacher places right triangles (formed by dividing a square or rectangle in half on the diagonal) on the overhead. This process continued as time permits.

LESSON 11-5

AREA ON THE GEOBOARD

PURPOSE:

To compute areas for triangular shapes constructed on the geoboard

MATERIALS:

1. *Simulated geoboard on a transparency, or on a large tagboard*
2. *Geoboards*
3. *Individual blackboards*

The triangle in this figure is not a right triangle—its area cannot be found by constructing a rectangle for which its area is one-half. The teacher will show the students one method to calculate the area of other kinds of triangles. Once this method is introduced, however, the students will again be responsible for finding their own areas for shapes on the geoboard.

First, the students are given the opportunity to discover the area for themsleves. The teacher's way is only one of many. The students may come up with methods they prefer over the teacher's eventual suggestion.

After the students have explored possible ways of determining the triangle's area, the teacher presents a particular method to the class, based on what they already know about finding areas of right triangles. The only difference is that for the triangle in the above figure, it may be necessary to use more than one rectangle to compute the area. Using two rectangles, the teacher divides the triangle into two parts, and calculates the area of each part separately. When the separate areas are known, they are added together to get the total area for the triangle.

The two-rectangle technique is shown to the students to expand their awareness of techniques that may be employed in determining areas. Knowledge of this or any other specific method is not a vital element in the fraction lessons that follow.

Different triangles may have their areas computed through the use of one, two, or three rectangles. Which technique applies to which triangle is left for the students to discover. Once the teacher has demonstrated the two-rectangle method, the class must create any new techniques necessary to find the area for each new triangle placed on the overhead—no other hints are provided.

The students copy the triangle on their geoboards, then determine its area, and write it on their blackboards. The teacher records the most common answer as the class answer. Students who had the class answer explain to their fellow classmates how they determined that area. A student who thinks the class answer is incorrect is free to explain his or her technique to the class. If there is a triangle for which no student can find an area, the problem remains unsolved.

LESSON 11-6

AREA ON THE GEOBOARD

PURPOSE:

To compute areas for triangular shapes constructed on geoboards and recorded on paper

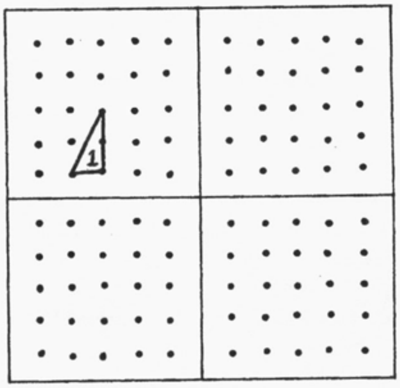
MATERIALS:

1. *Geoboard recording sheet on a transparency, or on a large tagboard*
2. *Dittoed copies of geoboard recording sheet*
3. *Simulated geoboard on a transparency, or on a large tagboard*
4. *Geoboards*
5. *Individual blackboards*

Teacher: Today, I want you to find the area for triangles you make up yourselves. Record your work on a geoboard recording sheet. I'll show you an example of what you are to do. First, I'll make a triangle on my geoboard. What is its area?

Student: One.

Teacher: Okay. Now I'll copy the triangle onto my recording sheet and write its area inside the shape, like this.



For each new triangle you make up, use a new space on your recording sheet. If you make a triangle on your geoboard for which you can't find the area, copy it on your paper and put a question mark inside.

As the students work, the teacher walks around checking to see that each person understands.

When about a third of the lesson time remains, the teacher has each student exchange papers with a classmate to check the areas of one another's triangles. If any disagreement arises, they show each other how they found the triangle's area. If they still cannot agree, they bring it to the teacher. Near the end of the lesson, the teacher presents any disputed triangles to the whole class for solution. All students find the triangle's area and write it on their blackboards—the class answer is accepted as the area.

Finding areas for triangles on a geoboard is a good opportunity for students to learn to rely on their own thinking. The two lessons that follow provide additional opportunities for self-reliance.

LESSON 11-7

AREA ON THE GEOBOARD

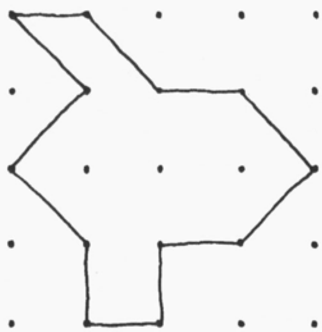
PURPOSE:

To compute areas for assorted shapes constructed on geoboards and recorded on paper

MATERIALS:

1. *Geoboard recording sheet on a transparency, or on a large tagboard*
2. *Dittoed copies of geoboard recording sheet*
3. *Simulated geoboard on a transparency, or on a large tagboard*
4. *Geoboards*
5. *Individual blackboards*

Teacher: Today, you will make shapes on your geoboards, find the areas of those shapes, and copy them onto your recording sheets. You may make any kind of shape you want, not just triangles. Here is a shape you might make. How can you find the area of this shape?



The students offer suggestions for possible ways of computing the area of this shape. When an area has been found, the teacher copies the shape onto the recording sheet transparency and writes the area inside. The students then are ready to construct their own shapes, compute areas for them, and record both the shape and area on their paper. The lesson continues as in the preceding one.

The students may create any shape they wish on their geoboards as long as the rubber band used to make it does not cross over itself at any point. Shapes made from crossed rubber bands are reserved for the following lesson.

LESSON 11-8

AREA ON THE GEOBOARD

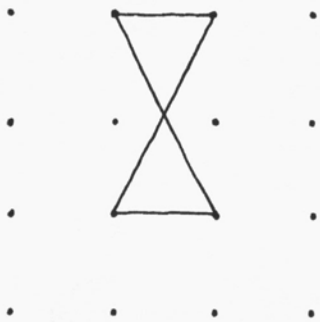
PURPOSE:

To compute areas for assorted shapes constructed on geoboards and recorded on paper

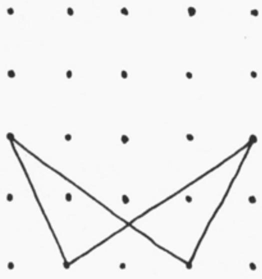
MATERIALS:

1. *Geoboard recording sheet on a transparency, or on a large tagboard*
2. *Dittoed copies of geoboard recording sheet*
3. *Simulated geoboard on a transparency, or on a large tagboard*
4. *Geoboards*
5. *Individual blackboards*

The activities for this lesson are essentially the same as those in the previous lesson, but now the rubber bands may cross. Any shape that can be made on the geoboard is permissible. (An example is this figure.)



The most difficult shapes on the geoboard for which to find areas are those constructed with crossed rubber bands. It is quite likely that some students may create shapes for which no one in class will be able to compute an area (see figure below). Such shapes should either be pinned to the bulletin board or placed in a special unsolved-problem box. Students who wish to, may continue searching for their areas during free time.



LESSON 11-9

UNDERSTANDING FRACTIONAL VALUES

PURPOSE:

To attribute fractional values to individual students

MATERIALS:

1. *Individual blackboards*

Teacher: There is one whole class of students in this room. How many students are in this room?
Student: Thirty-two.
Teacher: What fraction of the class is each student?
Student: What do you mean?
Teacher: Altogether you are one whole class. Each one of you separately is a part, or a fraction, of the class. Each person is one part of 32 parts or the fraction 1 over 32, written like this.

$$\frac{1}{32}$$

Write down the fraction of the class you are on your blackboard.

When the students demonstrate their understanding that each of them is 1/32 of the class, the teacher asks them to answer other questions based on this knowledge:

What fraction of the class are boys? Girls?
What fraction sits in row one? Wears glasses?
What fraction has on long pants? And so on.

The students respond to each question on their blackboards. The teacher scans the boards for a class answer then writes it on the overhead. The students who have the class answer share their methods.

The teacher asks them as many questions of this sort as time permits.

LESSON 11-10

UNDERSTANDING FRACTIONAL VALUES

PURPOSE:

To attribute fractional values to individual students

MATERIALS:

1. *Individual blackboards*

Teacher: Look at how many people are in your row and decide what fraction of your row you are. Write it on your blackboard.

Whole numbers usually represent only one concept at a time; fractional numbers represent two things at once. To understand what a fraction means, a student must know what both the things represented are.

When students correctly determine what fractional part of their row they represent, the teacher asks questions based on this knowledge:

What fraction of your row has short sleeves? Tennis shoes? Red? A dress or skirt?

The students answer on their blackboards. The teacher does not take a class answer, but asks the students in each row to look at one another's boards and decide on a row answer. Those with the row answer may explain how they arrived at their result to any student in their row with a different answer.

The teacher then writes all the row answers on the overhead and asks why there are so many different answers to the same question.

This question and the discussion of it focuses student attention on one of the special properties of fractional numbers. It is possible for the same person to be called 1/32, 1/6, 1/7, or a variety of other fractional numbers, and it is possible for each new fraction to offer a correct description. For fractions, knowing the whole is equally as important as knowing the parts.

LESSON 11-11

ADDITION OF FRACTIONS

PURPOSE:

To create addition problems with fractions using the students

MATERIALS:

1. *Individual blackboards*

Teacher: How many people in class?
Student: Thirty-two.
Teacher: What fraction of the whole class is the first row?
Student: Six over 32.

Teacher: What fraction is the second row?
Student: Seven over 32.
Teacher: If I added the people in the first and second rows together, what fraction of the class would I have then?
Student: Thirteen over 32.
Teacher: That problem is written like this. Can you see any patterns in the numbers that might help you predict the answer?

$$\frac{6}{32} + \frac{7}{32} = \frac{13}{32}$$

It is not expected that the students will see any connection between the numbers to be added and the numbers in the answer after only one example. The question is asked to start them looking for possible relationships.

The teacher continues to create addition problems using the whole class as the denominator, and varying the groups of students as the numerator. The students record the appropriate numbers on their blackboards. The teacher writes the class answer and the numbers for the problem on the overhead, and the students look for patterns.

When five or six problems and answers are written on the overhead, the students are given fractions to add without any descriptive words.

$$\frac{2}{6} + \frac{3}{6}$$

Teacher: Add these two fractions together on your blackboards. Show me your answers. . . . Some of you have written this, and some have written this.

$$\frac{5}{6} \qquad \frac{5}{12}$$

Now, I'll tell you the words for the problem so you can see which answer it would give us. How many people in row two?
Student: Six.
Teacher: What fraction of the row is each person?
Student: One over six.
Teacher: What fraction of the row has brown hair?
Student: Two over six.
Teacher: What fraction has black hair?
Student: Three over six.
Teacher: What fraction would I get if I added all brown-haired and black-haired people together?
Student: Five over six.
Teacher: Then what is the answer to the problem on the overhead?
Student: Five over six.
Teacher: Did the people with that answer see a pattern in the numbers for other problems that helped them know the answer wouldn't be five over twelve?

The students who see 5/6 as the answer and not 5/12, have seen the pattern for adding fractions with like denominators.

The teacher alternates between the two formats in presenting problems to the students: with and without descriptive words. The students should find the answers and, eventually, the rules for predicting an answer without having to know what the numbers refer to.

LESSON 11-12

SUBTRACTION OF FRACTIONS

PURPOSE:

To create problems involving subtraction of fractions using the students in the classroom

MATERIALS:

1. Individual blackboards

The activities for this lesson are essentially the same as those in the previous lesson. However, the students are now given subtraction rather than addition problems.

Teacher: What fraction of this row are boys?
Student: Four over six.
Teacher: Eddie, please stand up. What fraction did I have stand up?
Student: One over six.
Teacher: What fraction of the row are the boys who are still sitting?
Student: Three over six.
Teacher: That problem is written like this.

$$\frac{4}{6} - \frac{1}{6} = \frac{3}{6}$$

Can you see any patterns in the numbers to help you predict the answer?

The teacher continues to create problems, alternating between two formats from the previous lesson.

LESSON 11-13

FRACTION WORD PROBLEMS

PURPOSE:

To create word descriptions for fractional numbers

MATERIALS:

1. *Spelling notebooks*
2. *Lined paper*

Teacher: Look at this fraction.

$$\frac{7}{18}$$

Can you give me words that would tell me where the numbers came from?

I'll give you an example of what I mean. There were 18 apples in a box. Seven of the apples had worms in them. What fraction of the box of apples had worms?

Can you give me another example of words that might describe these numbers?

Student: There were 18 boys in class. Seven had on tennis shoes. What fraction of the boys had on tennis shoes?

Student: There were 18 girls in class. Seven had on long pants. What fraction of the girls had on long pants?

When students can attribute words to a fraction with ease, the teacher has them describe several fractions in writing. The teacher writes five or six fractions on the overhead and the students write a situation to describe each fraction. Any words a student needs spelled are written in the spelling notebook. The students write as many descriptions as time permits. Students who cannot think of many different situations may use the numbers for each new fraction in one or two familiar situations. Others may describe one or two fractions in several different ways.

Before the end of the lesson, the teacher collects all the papers and selects two kinds of problems to read aloud. The first type, inappropriate descriptions, is paraphrased to avoid embarrassing the student who wrote it. An inapropriate description might be: There were eighteen boys and seven girls. The class discusses why the description is inappropriate and how it might be converted to an appropriate description.

The second kind of problem read aloud is the creative or unusual description of the fractions. Eighteen monsters escaped from the Black Lagoon. Seven monsters were friendly. What fraction of the monsters were friendly?

The teacher reads inappropriate responses to help the students having difficulty learn how to adapt their descriptions to a more appropriate format. The reading of creative, appropriate responses acquaints all students with a wider variety of correct formats. Writing the descriptions prepares them for writing addition and subtraction word problems for fractions, the subject of the two lessons that follow.

LESSON 11-14

FRACTION WORD PROBLEMS

PURPOSE:

To create word descriptions for problems involving addition of fractions

MATERIALS:

1. *Spelling notebooks*
2. *Lined paper*

Teacher: What is the answer to this problem?

$$\frac{2}{7} + \frac{3}{7}$$

Student: Five over seven.

Teacher: Can you tell me a word problem from which these numbers might have come?

Student: There are seven people in that row. Two have on red shirts, three have on white shirts. Altogether there are five people who have red or white shirts.

Teacher: Okay. Who can think of another way?

Once the students have heard several word problems for the first set of numbers, the teacher writes four or five new problems on the overhead. The students write as many or few words descriptions for each problem as time allows. Before the end of the lesson the teacher collects the students papers and selects examples of problems to be read aloud in the same manner as the previous lesson.

LESSON 11-15

FRACTION WORD PROBLEMS

PURPOSE:

To create word descriptions for problems involving subtraction of fractions

MATERIALS:

1. *Spelling notebooks*
2. *Lined paper*

Teacher: What is the answer to this problem?

$$\frac{5}{8} - \frac{4}{8}$$

Student: One over eight.

Teacher: Can you tell me a word problem from which these numbers might have come?

The activities for this lesson are essentially the same as those for the previous lesson. The only difference is the students write about subtraction of fractions as opposed to addition.

LESSON 11-16

EQUIVALENT FRACTIONS

PURPOSE:

To develop lists of equivalent fractions

MATERIALS:

1. *Unlined paper*
2. *Individual blackboards*

Through this lesson, the teacher guides the students in generating lists of equivalent fractions by folding paper. At the end of the lesson they are expected only to have the beginning notion that some fractions represent the same value as certain other fractions—they are not yet expected to know how to compute lowest common denominators.

Teacher: How many pieces of paper do you have?

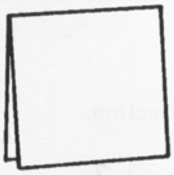
Student: One.

Teacher: Fold it in half like this. If we tore it along the fold line, how m any pieces would we have?

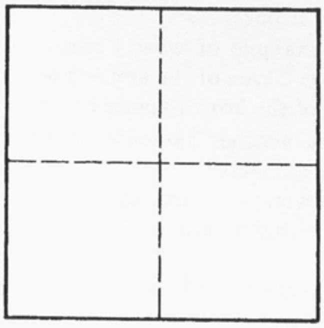
Student: Two.

Teacher: Don't tear it! I just said *if* we tore it!
What fraction of the whole piece of paper would one of the two pieces be?

Student: One over two.



Teacher: Okay. Fold it in half again. Now how many pieces would we get if we tore along each of the fold lines?

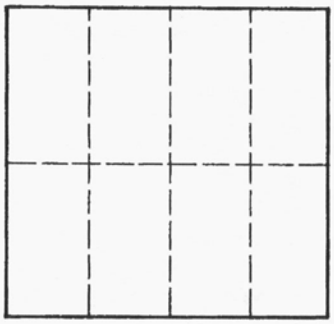


Student: Four.

Teacher: And what fraction would each piece be of the whole sheet?

Student: One over four.

Teacher: Fold it in half again, please. What fraction would each piece be now?



Student: One over eight.

Teacher: How many pieces do you think there will be altogether when we fold it in half again? Do it and see.

The paper is folded in half as many successive times as possible. The process is then repeated using a new piece of paper.

Teacher: How many pieces of paper are you starting with?

Student: One.

Teacher: Fold in in half. How many pieces if we tore along the fold line?

Student: Two.

Teacher: What fraction is one piece?

Student: One over two.
Teacher: What is one over two plus one over two?
Student: Two over two.
Teacher: If I put two halves together, what do I get?
Student: Two pieces over two pieces.
Teacher: For this paper, do the two half pieces equal one whole piece?
Student: Yes.
Teacher: So, could I write this?

$$1 = \frac{2}{2}$$

Student: Yes.
Teacher: Okay. Fold your paper in half again. What fraction is the smallest piece?
Student: One over four.
Teacher: How many of the one-over-four pieces are in the one half piece?
Student: Two.
Teacher: How would we write the answers to one over four plus one over four?
Student: 2/4.
Teacher: Then can I write this?

$$\frac{1}{2} = \frac{2}{4}$$

Student: Yes.
Teacher: How many one over fours are on the whole page?
Student: Four.
Teacher: How would I write that as a fraction?
Student: 4/4.
Teacher: Do all four parts equal the whole page?
Student: Yes.
Teacher: Then, can I write this?

$$1 = \frac{4}{4}$$

Student: Yes.
Teacher: Is four over four the same amount of paper as two over two?
Student: Yes.
Teacher: Then I could also write it like this.

$$1 = \frac{2}{2} = \frac{4}{4}$$

Fold your paper in half again. Now what is the fraction for the smallest piece of paper?

The process continues for each fold. In each case successive amounts of the smallest section of paper formed by the fold lines are related to earlier, larger folds. The teacher records the resulting fractional relationships on the overhead.

$$1 = \frac{2}{2} = \frac{4}{4} = \frac{8}{8} = \frac{16}{16} = \frac{32}{32}$$

$$\frac{1}{2} = \frac{2}{4} = \frac{4}{8} = \frac{8}{16} = \frac{16}{32}$$

$$\frac{1}{4} = \frac{2}{8} = \frac{4}{16} = \frac{8}{32}$$

$$\frac{1}{8} = \frac{2}{16} = \frac{4}{32}$$

When no more folds can be made, the students are asked to look at the top and bottom numbers of the fractions to see what patterns they can find. Are there patterns in the numbers that could have been used to predict the fractions present in each row before the paper had been folded?

After the students have had an opportunity to look for patterns, the teacher copies the list onto a sheet of paper and posts it on the bulletin board so students can reexamine it as the results of new paper folding exercises become known.

LESSON 11-17

EQUIVALENT FRACTIONS

PURPOSE:

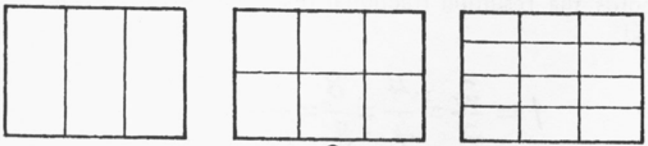
To develop lists of equivalent fractions

MATERIALS:

1. *Unlined paper*
2. *Individual blackboards*

This lesson continues the activities of the previous lesson. However, the papers no longer need to be folded into successive halves. The only restriction placed on how the paper may be folded is that the folds made at any one time must yield spaces of equal size. For example, a sheet of paper may be folded in thirds, then in half, then in thirds again. A fold that divides half the paper into fourths or sixths while leaving the other half undivided would be inappropriate.

An example of an equivalency chart produced by folding a paper into thirds, in half, then in thirds again can be seen in this figure.



$1 = \frac{3}{3}$

$1 = \frac{3}{3} = \frac{6}{6}$ $\quad \frac{1}{3} = \frac{2}{6}$

$1 = \frac{3}{3} = \frac{6}{6} = \frac{12}{12}$ $\quad \frac{1}{3} = \frac{2}{6} = \frac{4}{12}$ $\quad \frac{1}{6} = \frac{2}{12}$

Regardless of the foldings, the teacher presents the same sequence of activities from the previous lesson. The folding process to be used is gone through once, so the fractions become familiar. The process is then repeated with a new piece of paper as the teacher directs the students in examining which fractional amounts relate to others. The second time, the teacher asks the students which are equivalent. Each new equivalent fraction is written on the overhead.

The students examine the numbers for patterns to help them anticipate which fractions will have the same value as others. After the students have described all the patterns they see, the teacher copies the fractions onto a sheet of paper and posts it on the bulletin board. The students can refer to the equivalent fractions they have already discovered when they begin to add unlike fractions in the next lesson.

LESSON 11-18

ADDITION OF UNLIKE FRACTIONS

PURPOSE:

To use paper folding and lists of equivalent fractions as aids in formulating rules for addition of fractions with unlike denominators

MATERIALS:

1. *Posted lists of equivalent fractions from Lessons 11-16 and 11-17*
2. *Individual blackboards*
3. *Unlined paper*

Teacher: Look at this problem and tell me the answer.

$$\frac{1}{2} + \frac{1}{4} =$$

Student: Two over six.

Student: No, we can't do that one, because the bottom numbers aren't the same.

Teacher: Well, let's assume two over six is the answer and then fold some paper to see if it's right.

Each person take a sheet of paper and fold it in half. What fraction of the whole sheet is one part?

Student: One over two.

Teacher: Okay. Fold it in half again. What fraction is the smallest piece now?

Student: One over four.

Teacher: Point to the part of the paper representing one over two. Now, find another part of the paper that stands for one over four.

What fraction of the whole paper do you get when you add the piece that stands for one over two to the piece that stands for one over four?

Student: Three over four.

Teacher: Then what is one-half plus one-fourth?

Student: Three over four.

Teacher: Look at the numbers in the problem. Can you see any pattern that might have helped you predict the answer?

If you can't see a pattern yet, can you find a way you might have used the numbers we found before for the paper folding to help you predict the answer without having to refold the paper?

Not many students will see any relationship between the numbers to be added and the numbers in the answer after only one example. The purpose of these questions is to cause them to look for possible patterns and to reexamine the numbers copied onto charts from their earlier paper-folding activities.

The teacher continues to present addition problems for fractions with unlike denominators, from those that have denominators represented on the posted equivalency charts. For each problem the students first write their predictions. The teacher and students then work the answer together using paper folding. Each problem and its answer are then examined for patterns that might permit the students to correctly forecast the answer to the next problem.

Whether or not the students uncover any relationships they will learn at least one thing: answers for the addition of unlike fractions are not obtained by adding the bottom numbers together.

LESSON 11-19

SUBTRACTION OF UNLIKE FRACTIONS

PURPOSE:

To use paper folding and lists of equivalent fractions as an aid in formulating rules for subtraction of fractions with unlike denominators

MATERIALS:

1. *Posted lists of equivalent fractions from Lessons 11-16 and 11-17*
2. *Individual blackboards*
3. *Unlined paper*

The activities for this lesson are essentially the same as those presented in the previous lesson, but now the teacher presents the students with subtraction problems for fractions with unlike denominators.

LESSON 11-20

EQUIVALENT FRACTIONS

PURPOSE:

To increase the students' capabilities of finding equivalent values for fractions

MATERIALS:

1. *Unifix cubes*
2. *Individual blackboards*

In this lesson the students use sticks of Unifix cubes to generate a greater variety of equivalent fractions than was possible through paper folding. These cube sticks will also help students develop rules for finding lowest common denominators in a later lesson.

Teacher: You should each have a stick of eight cubes—all your cubes hooked together. How many sticks do you have?
Student: One.
Teacher: What fraction of the whole stick is one cube?
Student: One over eight.
Teacher: Can you break your stick into four equal parts?
Student: Yes.
Teacher: Do it please. If the stick you started with was one whole stick, what fraction of it would one of your parts be?
Student: One over four.
Teacher: How many one over eights are in each one over four?
Student: Two.
Teacher: How would you say that in a fraction?
Student: Two over eight.
Teacher: Is two over eight the same as one over four?
Student: Yes.
Teacher: Then I can write it like this.

$$\frac{1}{4} = \frac{2}{8}$$

Put your stick back together, then break it into two equal piles. If the whole stick was one, what fraction of the stick is one of your new piles?
Student: One over two.
Teacher: How many one over eights are in the one over two pile?
Student: Four.
Teacher: How would you say that in a fraction?
Student: Four over eight.
Teacher: Then I can write it like this.

$$\frac{1}{2} = \frac{4}{8}$$

How many cubes were in a one-over-four piece of the stick?
Student: Two.
Teacher: How many one-over-four pieces are in the one-over-two piece of the stick?
Student: Two.
Teacher: How would you say that as a fraction?
Student: Two over four.
Teacher: Then is two over four the same as one over two?
Student: Yes.
Teacher: So I can write all that like this.

$$\frac{1}{2} = \frac{2}{4} = \frac{4}{8}$$

The process of finding equivalent fractions for cubes is essentially the one used in the paper folding activities in Lessons 11-16 and 11-17. The teacher asks the leading questions and the students provide the answers with their cube sticks. The teacher makes a list on the overhead of all equivalent fractions found.

The one rule for finding fractional values with cubes is all piles into which a cube stick is divided must be of equal size.

Teacher: Now, I want you to make a new cube stick, with nine cubes in it. How many different ways can you break your cube stick into piles that are exactly equal? Each time you find a way, tell me what fraction of the whole stick each equal pile would be. I'll keep track of all the different ways on the overhead.
Student: We can make piles that have one cube each in them.
Teacher: Are all the piles of equal size?
Student: Yes.
Teacher: What fraction of the whole stick is each equal pile?

Student: One over nine.

Teacher: Okay. Any more ways?

Student: We can make piles that have two cubes each in them.

Teacher: Are all the piles of equal size?

Student: Four of them are.

Teacher: The rule for this activity is that *all* the piles must be the same size.

Student: The stick divides into piles of three.

Teacher: Are all the piles of equal size?

Student: Yes.

Teacher: What fraction of the whole stick is each equal pile?

Student: One over three.

Teacher: Okay. Any more ways?

Student: No.

Teacher: How many one over nines are in one over three?

Student: Three.

Teacher: How would I write one over nine plus one over nine plus one over nine as a single fraction?

Student: 3/9.

Teacher: Then can I write this?

$$\frac{3}{9} = \frac{1}{3}$$

Student: Yes.

Teacher: How many one over nines in the fraction two over three?

The teacher guides the students through developing equivalent fractions for a cube stick of nine. The students then add another cube and repeat the process for a stick of ten.

For the stick of ten cubes, the teacher provides none of the equivalencies; instead, the students find all the ways one fraction equals another. The teacher copies each new way on the overhead.

A stick of ten cubes presents the students with an interesting situation. Two-tenths is equivalent to one-fifth, and five-tenths is equivalent to one-half, but no amount of fifths is equivalent to a half. When the students were producing equivalent fractions through paper folding, all the fractions on a given page were related in some way to each of the other fractions on that same page. The ten stick is the students' first experience with a situation where all the fractions relating to a single whole do not necessarily relate to one another.

When the students have found as many equivalencies for a stick of ten as they can, they begin the process for eleven. This presents another situation they have not faced before: other than one-eleventh, eleven breaks into no fractions at all.

The students continue adding cubes and examining each new stick length for equivalent fractions as time permits.

While the students explore, the teacher asks them to examine each new set of numbers on the overhead for patterns that might enable them to predict which fractions are equivalent to which others. Patterns found that allow students to make accurate predictions are shared with the whole class.

As long as materials are available for students to use, all are equally capable of finding answers to the problems. Some will see patterns that permit them to abandon the materials quickly; others will alternate between concrete objects and abstract reasoning, or rely solely on physical aids. Each method is equally valid and each should be equally valued.

LESSON 11-21

EQUIVALENT FRACTIONS

PURPOSE:

To increase the students' capabilities of finding equivalent values for fractions

MATERIALS:

1. Unifix cubes

Teacher: Yesterday you used Unifix cubes to find which fractions had the same values as other fractions. Today, you'll do the same thing, only I'll let you choose what stick length you want to call one.

As you can see, I have marked off and written a number above 30 spaces on the blackboard. The number stands for the length of a cube stick. Select a number and build your cube stick with that many cubes, then see how many different fractions with the same values as other fractions can be made from it. For each fraction, go up to the blackboard and write what you've found underneath the number for that stick length.

Make sure you check the blackboard before you write what you've found—you only need to write what is discovered one time. Are there any questions? Then you may begin.

5	6	7	8	9	10
$1 = \frac{5}{5}$	$\frac{1}{3} = \frac{2}{6}$		$\frac{1}{4} = \frac{2}{8}$	$\frac{1}{3} = \frac{3}{9}$	$\frac{1}{5} = \frac{2}{10}$
	$1 = \frac{6}{6}$		$\frac{1}{2} = \frac{2}{4} = \frac{4}{8}$		$\frac{1}{2} = \frac{5}{10}$
	$\frac{1}{2} = \frac{3}{6}$				

This exercise generates many equivalent fractions. As the students write fractions and equivalents in the spaces on the blackboard, the teacher asks them to think about the following questions:

Can you see a pattern for fractions having the same value as other fractions?
Can you see any patterns to help you to know when two fractions that seem different really represent the same number of cubes in a cube stick?
Can you use any of the patterns to help tell you which fractions are going to be the same as others before you break up your cube stick?

Students may share their patterns with their classmates. The pattern may then be used to check the accuracy of all equivalent fractions on the blackboard. If a fraction doesn't fit the pattern, it is rechecked with cubes. If the cubes indicate the fraction is correct, then the pattern may not be valid.

LESSON 11-22

FINDING LOWEST COMMON DENOMINATORS

PURPOSE:

To learn a game called start-with–go-by

MATERIALS:

1. *Individual blackboards*
2. *Lined paper*

Once students can find equivalent fractions using cube sticks, they are ready to use this skill in determining lowest common denominators. First, however, they need to know which length cube stick to select as the basis for finding appropriate denominators. This selection is made through an activity called *start-with–go-by–both.* The start-with–go-by half of the activity is presented in the following lesson.

Teacher: Please give me a number between one and ten.
Student: Five.
Teacher: Five is the number we will start with. Now, I need a number between one and ten to go by.
Student: Three.

3
5

Teacher: Okay. That means we'll start with five and go by three. I'll show you what the next three numbers in the column are. What comes next?

3
5
8
11
14

Student: Seventeen.
Teacher: Okay. Next?
Student: Twenty.
Teacher: Next?

3
5
8
11
14
17
20

The teacher records the first start-with–go-by column on the overhead while the students watch. The students record the second column on their blackboards.

Teacher: Please give me a number between one and ten to start with.
Student: Four.
Teacher: Okay. I'll write four on the overhead; you write a four near the top of your blackboards, leaving a little room for the go-by number. Now I need a number to go by.
Student: Two.
Teacher: Okay, I'll write two in a circle above the four. Please copy this. Now, write the number you think would go underneath the four, for when we start with four and go by two. . . . Hold up your boards so I can see what you wrote.
Most of you wrote six, so I'll write that on the overhead. What comes after six? Write the next number on your blackboards.

The students record a third start-with–go-by column on their blackboards, but this time the teacher writes nothing on the overhead. One student suggests a number to start with, another, a number to go by. The students then complete the column on their own.

The students add the numbers for the fourth start-with–go-by column on lined paper. One student suggests a "start with" number. A second student suggests a "go by" number. Each student then completes the column on his or her own paper. A column is complete when there is a number on every line from top to bottom. The process is continued for more columns, as time permits.

While the students work, the teacher poses the following questions:

Can you see any patterns for the digits going down the right-hand side of your columns? The left?
Could you use any of the patterns you see to predict the numbers you will get further down the column?
Is it possible to pick a number to start with and a number to go by that won't produce a pattern in the numbers you write down?

LESSON 11-23

FINDING LOWEST COMMON DENOMINATORS

PURPOSE:

To learn the start-with–go-by–both game

MATERIALS:

1. *Individual blackboards*
2. *Lined paper*

Teacher: Please give me a number to start with and one to go by.
Student: Start with six and go by three.
Teacher: Okay. And what numbers should I write in the column?
Student: After the six . . . nine, twelve, fifteen . . .
Teacher: I'll leave that column on the overhead and put another start-with–go-by column next to it. Please give me two more numbers.
Student: Start with five and go by two.
Teacher: What numbers should I write in this column?
Student: After the five . . . seven, nine, eleven, thirteen . . .
Teacher: Now I'll start a third column which I'll call the *both column.* In the both column I will make a list of all the numbers that appear in both of the start-with–go-by columns. The first number I'll write is nine, because there are nines in both the start-with–go-by columns. What would the second number be?
Student: Fifteen.
Teacher: And the third?
Student: Twenty-one.
Teacher: The fourth?

The teacher completes one example on the overhead, then the students work the second on their blackboards while the teacher records each step on the overhead. The students write the third example on their blackboards with no assistance, and the fourth on lined paper. For the fifth example, the students create their own problems.

3	2	Both
6	5	9
~~9~~	7	15
12	~~9~~	21
~~15~~	11	⋮
18	13	
~~21~~	~~15~~	
24	17	
27	19	
30	~~21~~	
⋮	⋮	

As the students work, the teacher asks them these questions:

The go-by number tells us how far apart all the numbers in each start-with–go-by column are. Is there a number that is the go-by number for the both column as well?
Could you predict the go-by number for a both column by looking at the go-by number for the start-with–go-by columns? Why? Why not?
Does the both column always go by the same number?
Will there always be a both column?
Why does the both column do what it does?
Will it always do it?

LESSON 11-24

FINDING LOWEST COMMON DENOMINATORS

PURPOSE:

To learn a modified version of start-with–go-by–both

MATERIALS:

1. *Lined paper*

The activities for this lesson are essentially the same as those in the previous lesson. Instead of picking one number to start with and another to go by, the students use the same number to start with and go by. For example, if four is picked, then four is the number with which that column starts and goes by as well.

4	3	Both
8	6	12
~~12~~	9	24
16	~~12~~	⋮
20	15	
~~24~~	18	
28	21	
32	~~24~~	
⋮	⋮	

These questions are asked in addition to those in the previous lesson:

Can you find two start-with–go-by columns for which there is no both column; Why? Why not?

Have you seen the patterns the numbers make in each start-with–go-by column before? Where? Why?

LESSON 11-25

FINDING LOWEST COMMON DENOMINATORS

PURPOSE:

To learn a technique for finding lowest common denominators

MATERIALS:

1. *Unifix cubes*
2. *Individual blackboards*
3. *Lined paper*

The start-with–go-by–both activity can be combined with the use of cube sticks to determine the lowest common denominator for any two unlike fractions.

$$\frac{1}{5} + \frac{1}{4}$$

Teacher: Add these two fractions together, please.
Student: We can't.
Teacher: Why not?
Student: Because they have to have the same bottom number.
Teacher: Yes, but you've added other fractions that didn't have the same bottom numbers.
Student: But we changed the bottom numbers to be the same before we added.
Teacher: Well, can't you do that now?
Student: No, because we don't have both of those fractions on one of the charts we made for paper folding.
Teacher: Didn't you find fractions that were the same as one over four and one over five when you were working with cube sticks?
Student: Yes, but you erased them all off the blackboard!
Teacher: If you knew what size cube stick you had used, could you change the one over four and the one over five into fractions with the same bottom numbers?
Student: If there was a cube stick that had one over fours and one over fives in it at the same time. They'd both have to be from the same cube stick.
Teacher: Okay. I'll show you a way to decide how long to make a cube stick to find one with both one over fours and one over fives in it.
Take the bottom number from one over four and make a start-with–go-by column for it. Now, take the bottom number from one over five and make a start-with–go-by column for it. Make a both column. What is the first number in the both column?

4	5
8	10
12	15
16	~~20~~
~~20~~	25
24	30
28	35
32	~~40~~
36	45
~~40~~	50
⋮	⋮

Both
20
40
⋮

Student: Twenty.
Teacher: Make a cube stick twenty cubes long and see what fractions it breaks into. Can you break the cube stick into one over fours, then one over fives?
Student: Yes.
Teacher: Okay, put a pile of cubes the same as one over four on your desk, then one over five, too. Have you got one over four and one over five on your desk?
Student: Yes.
Teacher: Add them together and tell me what you get.
Student: Nine over twenty.
Teacher: Then, one over four plus one over five is nine over twenty.

The teacher gives the students one more example, then writes several problems on the overhead for which they must find answers on their own.

As the students work, the teacher asks:

Why do the start-with–go-by–both columns give you a cube stick length that helps you solve the problem.

Would the second both number also make a cube stick that would let you get an answer? The third number? The fourth?

Are there any cube sticks of different lengths than the number in the both column that would also give you the fractions you need to solve the problems?

Can you find any patterns in the cube sticks to help you predict what the answer will be?

Can you find any patterns in the start-with–go-by columns that tell you what the first both number will be before you write down the numbers to check?

Can you find any patterns in the columns to help you predict the answer?

LESSON 11-26

ADDITION AND SUBTRACTION OF FRACTIONS

PURPOSE:

To create addition and subtraction problems for fractions

MATERIALS:

1. *Unifix cubes*
2. *Lined and unlined paper*

Up to this point, the teacher has been in control of all the fraction problems the students have solved because they did not have the skill to deal with all fractions. Once the students have been shown a way to find lowest common denominators, they are potentially capable of solving any fraction problems involving either addition or subtraction.

For this lesson, the students may create any addition or subtraction fraction problems they wish, using folded paper, cube sticks, start-with–go-by columns, or their own imaginations. They may create problems having the same or different denominators for each new fraction. The problems may be difficult or easy.

Students create as many or few problems as the time available permits. Students who have difficulty creating their own problems may work problems their neighbors construct for them, or work along with a classmate who is not having difficulty.

The object of this lesson is to free students from the teacher-created problems. Students benefit from creating problems that follow their curiosity; teachers benefit from observing the kinds of problems their students create when they take full responsibility for both problems and answers.

LESSON 11-27

MULTIPLICATION OF FRACTIONS

PURPOSE:

To learn how to multiply whole numbers on a geoboard in preparation for multiplying fractions on the geoboard

MATERIALS:

1. *Simulated geoboard on a transparency, or on a large tagboard*
2. *Geoboards*
3. *Individual blackboards*
4. *Lined paper*

Length	Width	Area
3	2	6

Teacher: Please copy this rectangle onto your geoboard. What is its area?

Student: Six.

Teacher: You will make several different rectangles on your geoboards, then record and examine their lengths, widths, and areas for possible patterns. The first information I want you to record for each rectangle is the length–the longest side of the rectangle. The number for length is how many spaces are between the nails on that side. What is the length of this rectangle?

Student: Three spaces.

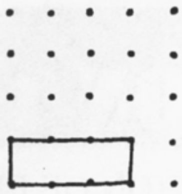
Teacher: Okay. The width is the shortest side, measured by counting the spaces between the nails. What is the width of this rectangle?

Student: Two spaces.

Teacher: Okay. And what is the area?

Student: Six.

Teacher: Now make this rectangle on your geoboards and record its length, width, and area on your blackboards.



Once the students demonstrate on their blackboards that they understand recording lengths, widths, and areas, they create their own rectangles and record the numbers on lined paper.

Length	Width	Area
4	3	12
2	2	4
4	1	4
3	3	9

The teacher next guides them through a more systematic examination of all the possible geoboard rectangles.

Teacher: Now I want you to do something a little different. I'll write some lengths and widths on the overhead and you make the rectangles that have those sides and find what the areas are. Keep track of each length, width, and area on your paper.

Here are the lengths and widths I want you to use. Are there any questions? . . . Then you may begin.

Length	Width	Area
4	1	
4	2	
4	3	
4	4	
3	1	
3	2	
3	3	
2	1	
2	2	
1	1	

As the students work, the teacher asks the following questions:

Can you see a pattern for length, width, and area that might help you predict the area for a rectangle before you make it on your geoboard?

Can you find a rectangle for which the pattern doesn't work?

Have you seen this pattern before? Where? Why?

The pattern produced from lengths, widths, and areas is one the students should have little difficulty recognizing. They will use it when they explore multiplication of fractions in the lessons that follow.

LESSON 11-28

MULTIPLICATION OF FRACTIONS

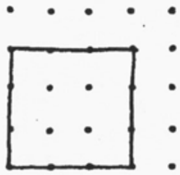
PURPOSE:

To learn values for the area of smaller squares or rectangles on the geoboard when a larger square or rectangle is assigned the value of one

MATERIALS:

1. *Simulated geoboard on a transparency, or on a large tagboard*
2. *Geoboards*
3. *Individual blackboards*
4. *Dittoed copies of geoboard recording sheets*

The next four lessons provide students with a method of finding answers to problems involving multiplication of fractions. Once the answers can be found the students are asked to discover any rules that might permit them to generalize the patterns they observe beyond their geoboards experiences.



Teacher: Copy this square onto your geoboard and tell me how many spaces it has on each side.

Student: Three.

Teacher: What is the area of the square?

Student: Nine.

Teacher: Remember when we made cube sticks? We could call any stick *one*, then use it to figure out what fractions the individual cubes were. We can do the same thing with squares and rectangles on the geoboard.

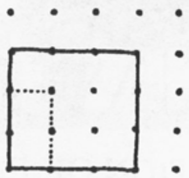
For now, the square on your geoboards is one big square. If the big square is one, what fraction of it is the littlest square?

Student: One over nine.

Teacher: Why?

Student: Because there are nine little squares in the big square, so each little square is one over nine.

Teacher: Okay. If one little square is one over nine, then what fraction of the big square is this?

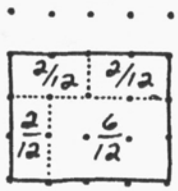


Student: Two over nine.

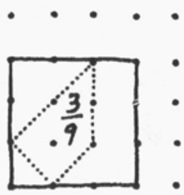
Teacher: Why?

The process is repeated for four or five more squares or rectangles. The teacher asks the students to make a rectangle on their geoboards with a value of one, and then decide what the fractional value of the littlest square would be. Then the teacher makes different-size squares or rectangles inside the first rectangle and the students attribute fractional values to each new shape.

As soon as the students are able to correctly identify the fractional value of the smallest square, given a variety of large squares or rectangles as one, they are ready to decide for themselves which squares or rectangles to call one. They make each first on the geoboard, then copy it onto a geoboard recording sheet and draw smaller squares or rectangles inside it. Next, they determine the fractional value of each smaller shape and write it inside.



The students may draw different or the same sized one squares or rectangles in each new space on their recording sheets. They may fill each with smaller shapes and find fractional values or put only one or two shapes in each rectangle and attempt to find its fractional equivalent, as in this figure.



LESSON 11-29

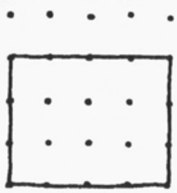
MULTIPLICATION OF FRACTIONS

PURPOSE:

To learn values for length, width, and area of smaller squares or rectangles on the geoboard when a larger square or rectangle is assigned the value of one

MATERIALS:

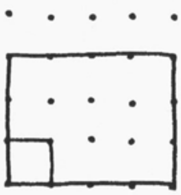
1. *Simulated geoboard on a transparency, or on a large tagboard*
2. *Geoboards*
3. *Individual blackboards*



Teacher: Please copy this rectangle, representing one, onto your geoboard. What fraction does each of the smallest squares represent?

Student: One over 12.

Teacher: I want you to make one of the small squares over in the lower left-hand corner, like this.

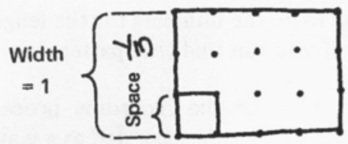


How many spaces are there along the width of the big rectangle?

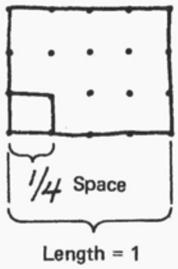
Student: Three.

Teacher: When we were working with cube sticks we said that any number of cubes hooked together could be one. As soon as we decided how long a stick we were going to use, we figured out what fraction of the stick each cube was. We can do that for the sides of this rectangle, too. If this whole side is one, what fraction of the whole side would one space be?

Student: One over three.

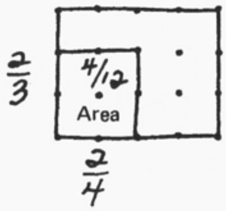


Teacher: Why?
Student: Because there are three spaces on that side.
Teacher: That means along this side, the littlest square has a side with the value of one over three. How many spaces are along the whole length of the big rectangle?
Student: Four.
Teacher: If this whole length is one, what fraction would one space be?
Student: One over four.



Teacher: So along the length the smallest square has a side with the value of one over four.
What was the value of the side of the little square along the the width of the large rectangle?
Student: One over three.
Teacher: And what was the area of the little square compared to the whole rectangle?
Student: One over 12.
Teacher: Now, can anybody explain to me how one little square can have all those different fractional values?

The process is repeated with a new shape inside the same large rectangle.



The teacher uses the same large rectangle until all the possibilities for squares or rectangles that can be made within its bounds have been exhausted, then selects another. For simplicity, the inner squares or rectangles are always constructed with one corner at the lower left-hand corner of the large rectangle. When the students begin devising problems on their own, however, they may wish to see what difference it makes if the small squares or rectangles are located in another position.

LESSON 11-30

MULTIPLICATION OF FRACTIONS

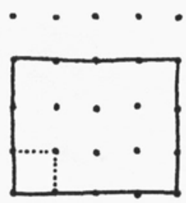
PURPOSE:

To create and record fractional numbers for lengths, widths, and areas; to examine the numbers recorded for patterns leading to rules for multiplication of fractions

MATERIALS:

1. *Simulated geoboard on a transparency, or on a large tagboard*
2. *Geoboards*
3. *Individual blackboards*
4. *Lined paper*

Teacher: Today I want you to make up your own one rectangles and find the fractional values for the sides and areas of the small squares or rectangles. Since you also need to examine the numbers you get for patterns, keep track of the fractional values for each small shape on a piece of paper.
Here's how I want you to record your information. First, make this one rectangle on your geoboards. Now, place a small square in the lower left corner, like this.



What is the fractional value for the side of the small square along the length of the rectangle?
Student: One over four.
Teacher: What is the fractional value for the side of the small square along the width of the rectangle?
Student: One over three.
Teacher: What is the area of the small square with respect to the area of the rectangle?
Student: One over 12.
Teacher: That is recorded like this.

Length	Width	Area
$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{12}$

The students record a second example on their blackboards. After this, the students create their own problems and record each new fractional number on lined paper.

When the students have worked for about ten minutes, the teacher stops them and says:

Teacher: When you were recording the lengths, widths, and areas you found on your geoboards for whole numbers, did you find a pattern for the lengths and widths that helped you predict each new area?
Student: Yes.
Teacher: What was it?
Student: Multiplication.
Teacher: Multiplication of what?
Student: Length times width equalled area.
Teacher: Was that true for all the numbers?
Student: Yes.
Teacher: Is it true for fractions, too?
Student: What do you mean?
Teacher: If you multiply the length of a rectangle times its width will you get the area?
Student: Yes.
Teacher: Even if the length and width are written as fractions?
Student: I don't know.
Teacher: Well, if you multiplied two fractions together would you get another fraction for the area if the first two stood for sides?
Student: I don't know.
Teacher: What is the side of this little square?
Student: One over three.
Teacher: And what is the other side?
Student: One over four.
Teacher: Now, what is the area of this square, expressed as a fraction of the area of the whole rectangle?
Student: One over 12.
Teacher: Do you know both sides?
Student: Yes.
Teacher: Are they both fractions?
Student: Yes.
Teacher: Do you know the area?
Student: Yes.
Teacher: Is it a fraction?
Student: Yes.
Teacher: Is the area equal to one side times the other?
Student: How can we know that?
Teacher: Does the area equal length times width?
Student: But how can we know that! We don't know how to multiply fractions!
Teacher: True, but you figured out a pattern for length times width when you were using whole numbers; did I tell you what the pattern was?
Student: No.
Teacher: Then how did you figure it out?
Student: We looked at the numbers and saw a pattern.
Teacher: Okay. Even though you may not know how to multiply fractions, you can still look at the numbers and see if you can find a pattern. Maybe it will also tell you how to multiply fractions. Do you think there may be a pattern between lengths and widths you could use to predict what the area might be . . . even for fractions?
Student: Maybe.
Teacher: As you write the numbers for the lengths, widths, and areas, see if you can find that pattern.

The students continue the recording process and the search for patterns. Any rule suggested as a way to predict areas from lengths and widths is tested against the data already gathered.

LESSON 11-31

MULTIPLICATION OF FRACTIONS

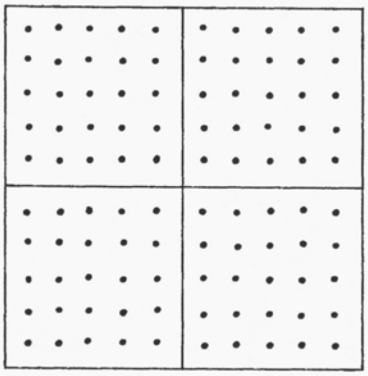
PURPOSE:

To create and record fractional numbers for lengths, widths, and areas; to examine the numbers for patterns leading to rules for multiplication of fractions

MATERIALS:

1. *Geoboards*
2. *Lined paper*
3. *String or yarn*

The activities for this lesson are essentially the same as those in the previous lesson. The added element is that the students may form the large rectangles on several geoboards placed together. Geoboards are constructed so the distance from the outside nails to the edge of the board is one-half the distance between the nails on the board itself. This means when two boards are placed together, the spaces between the adjacent nails on both boards remains the same as the spaces on a single board.



Four boards laid in a square array increase the fractional units that may be examined for patterns to a maximum of

1/9 on each side and 1/81 for each small square. There is no limit to the number of boards that may be joined.

As the boards are placed next to one another, the rubber bands used on the individual boards to form shapes are no longer adequate. In place of them, string or yarn may be used.

As the students increase the number of boards, they are asked the following:

Is there a one rectangle you can make that gives you fractional values that do not fit the same pattern for length, width, and area you found when you could only use a single geoboard?

This chapter on fractions has emphasized thinking more than knowledge. Nevertheless, each student now has the level of understanding necessary to prepare him or her for the lessons on decimals in the following chapter.