CHAPTER 12
Decimals

Lesson 12-1 page 178 Beans, Cups, Bowls, and Tin Cans Teacher assigns fractional values to beans, cups, and bowls.	Lesson 12-2 page 180 Beans, Cups, Bowls, and Tin Cans Teacher assigns decimal values to beans, cups, and bowls.	Lesson 12-3 page 181 Chips Teacher assigns fractional values to chips.	Lesson 12-4 page 182 Chips Teacher assigns decimal values to chips.
Lesson 12-5 page 182 Chips Students create addition and subtraction problems.	Lesson 12-6 page 183 Chips Students create addition and subtraction problems for dollars and cents.	Lesson 12-7 page 184 Chips Students discover decimal rules for multiplying a decimal by a whole number.	Lesson 12-8 page 185 10 x 10 Matrix Students discover decimal rules for multiplying 0.A times 0.C.
Lesson 12-9 page 186 100 x 100 Matrix Students discover decimal rules for multiplying 0.AB times 0.CD	Lesson 12-10 page 187 10 x 100 Matrix Students discover decimal rules for multiplying 0.A times 0.CD		

Prerequisite chapters:

Chapters 8, 9, and 11

MATERIALS

For overhead projector:

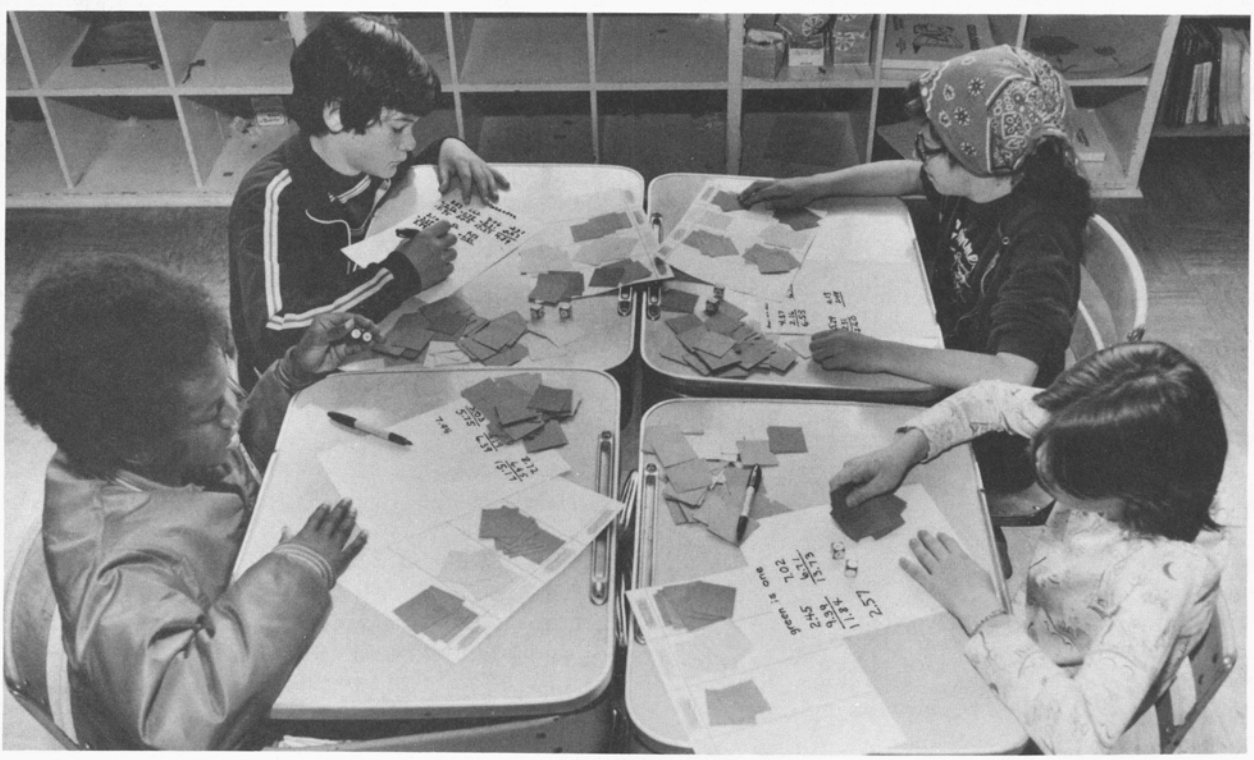
Transparencies	10 by 10 blank matrix	Worksheet 1
	100 by 100 blank matrix	Worksheet 19
	10 by 100 blank matrix	Worksheet 20
Acetate squares with circles of three different sizes drawn on them (one per square)		Materials chapter, page 295
Acetate squares in five different colors		Materials chapter, page 297
Washable color marking pens		Materials chapter, page 297
Overhead projector dice		Materials chapter, page 296
Beans		Materials chapter, page 295

If no overhead projector is available:

Make charts in place of transparencies	Materials chapter, page 294
Circular cutouts of three different sizes	Materials chapter, page 295
Strips and squares of paper in five different colors	Materials chapter, page 297
Dice cards in bag	Materials chapter, page 295
Bean-shaped cutouts	Materials chapter, page 295

Student materials:

Dittos	10 by 10 blank matrix	Worksheet 1
	100 by 100 blank matrix	Worksheet 19
	10 by 100 blank matrix	Worksheet 20
	Special multiplication matrix	Worksheet 15
Beans, cups, bowls, tin cans		Materials chapter, page 295
Dice		Materials chapter, page 296
Individual blackboards		Materials chapter, page 294
Paper squares or chips in five different colors		Materials chapter, page 297
Crayons		
Unlined paper		
Restaurant menus, mail-order catalogs, newspaper advertisements		



The activities in this chapter provide students with the opportunity to use decimals in addition, subtraction, and multiplication. The students search problems and answers for patterns leading to rules for the correct placement of the decimal point in decimal problems. They are allowed to discover the rules for themselves so their learning will be a product of understanding, not memorization.

The students' work with decimals must be preceded by the activities in Chapters 8, 9, and 11. The background knowledge they bring to the lessons in this chapter is reflected in their ability to learn from the materials at an accelerated rate. The pacing of the following lessons reflects this growth.

LESSON 12-1

DECIMAL NOTATION

PURPOSE:

To assign fractional values to the beans, cups, and bowls on a bean trading board in preparation for using it to represent decimal fractions

MATERIALS:

1. *Clear acetate squares with circles drawn on them, or circular cutouts*
2. *If no overhead projector is available, bean-shaped cutouts*
3. *Beans*
4. *Cups*
5. *Bowls*
6. *Tin cans*
7. *Individual blackboards*
8. *Unlined paper*

Teacher: For this activity you will use your bean trading boards again. Today we will be working with groups of tens, so if you don't have enough beans yourself, you may want to team up with one or two other people.

Put one bean in the bean column and one cup in the cup column on your trading board. How many beans go in the cup?

Student: Ten.

Teacher: Put one bowl in the bowl column. How many cups in a bowl?

Student: Ten.

Teacher: How many beans in the bowl?

Student: One hundred.

Teacher: Put one tin can in the tin can column. How many bowls in a tin can? . . . How many cups in a tin can? . . . How many beans in a tin can?

With the aid of these materials, the students can answer each question. It is important that they discover the answers on their own, because they will need to know how to determine those relationships in the activities that follow.

Teacher: You or your group should each have a bean trade-ing board with one tin can, one bowl, one cup, and one bean.

When we were working with fractions, we made up different cube sticks and called them *one*. We can do the same thing now with the materials on your trading boards. If I say the cup on your trading board is one, what fraction of the cup is one bean?

It may help you to understand if you pretend the beans are people and the cups are classrooms. A person can be one person and at the same time a fraction of all the people in the classroom. What fraction is one person of the whole class if there are only ten people in class?

Student: One over ten.

Teacher: What fraction of the beans that make up a cup is one bean?

Student: One over ten.

Teacher: Why?

Student: Because there are ten beans in the cup. Each bean is one out of ten.

Teacher: How would you write that?

Student: 1/10.

Students have little difficulty accepting fractional values for the beans they've formerly called *one* if this new process is related to experiences with which they are already familiar.

Teacher: If I decide to call the bowl one, what fractional value would each cup have with respect to it.

It might help if you thought of the bowl as a whole school and each cup as a classroom. If a school has ten classrooms, what fraction of the whole school is one classroom?

Student: One over ten.

Teacher: What fraction of the cups contained in a bowl is one cup?

Student: One over ten.

Teacher: How would you write that?

Student: 1/10.

Teacher: If I still say the bowl is one, what fraction of the bowl would one bean be?

It may help you also to think what fraction of the whole school one student is if there are ten students in each class and ten classes in the whole school.

Student: One over 100.

Teacher: Why?

Student: Because there are 100 beans in a bowl, so one bean would be 1 out of 100.

Teacher: And how would you write it?

Student: 1/100.

Teacher: Okay, if I still say the bowl is one, then a cup would still be 1 over 10 and a bean would still be 1 over 100. What would two cups be?

Student: Two over ten.

Teacher: Three cups?

Student: Three over ten.
Teacher: What would two beans be?
Student: Two over 100.
Teacher: Three beans?
Student: Three over 100.
Teacher: If I decide to call the tin can one, what fraction of it would one bowl be?
Maybe it will help you to think of the tin can as a whole school district and each bowl as a school in the district. If a district has ten schools, what fraction of the whole district is one school?
Student: One over ten.
Teacher: What fraction of the bowls that make up a tin can is one bowl?
Student: One over ten.
Teacher: And how would you write it?
Student: 1/10.
Teacher: Now, if the tin can is still one, what fraction of the tin can would each cup be?

The teacher's questions continue until the students have determined what fractions of a tin can both cups and beans would be. The teacher then has them clear all the beans, cups, bowls, and tin cans from their trading boards.

Teacher: All the questions I ask you now are for the tin can as one. Put one bean on your trading board. On your blackboards, write the fraction one bean represents.
Student: 1/1000.
Teacher: Okay. Now put three beans on your trading boards–three beans altogether, not three more. Write the fraction three beans represent.
Student: 3/1000.
Teacher: Now put seven beans total on your board and write the fraction they represent.
Student: 7/1000.
Teacher: Now put ten total beans on your board and write the fraction they represent.
Student: 10/1000.
Teacher: Remember, the beans are on your trading board. What happens when you get ten beans on your trading board?
Student: We turn them into a cup.
Teacher: Okay, do that please . . . now write the fraction you get.
Student: 1/100.
Teacher: Before I reminded you to turn your beans into a cup, what fraction did you write?
Student: Ten over 1,000.
Teacher: Why?
Student: Because we had 10 beans out of 1,000 beans.
Teacher: And what fraction did you write for one cup?
Student: One over 100.
Teacher: Does the fraction 10 over 1,000 have the same value as 1 over 100?
Student: What do you mean?
Teacher: Did 10 over 1,000 represent 10 beans?
Student: Yes.
Teacher: Did 1 over 100 represent the same 10 beans you had put in a cup?
Student: Yes.
Teacher: Then maybe 1 over 100 is just another way of writing 10 over 1,000, like 4 over 8 and 2 over 4 are just other ways of writing 1 over 2. We'll have to gather some more information before we know for sure.
Put two cups total on your trading board and write the fraction for it.
Student: 2/100.
Teacher: Now, three cups total and write the fraction.
Student: 3/100.
Teacher: Eight cups and write the fraction.
Student: 8/100.
Teacher: Ten cups and write the fraction.
Student: 10/100.
Student: 1/10.
Teacher: I see 10 over 100 on some blackboards and 1 over 10 on others. Why?
Student: Because some people remembered to turn the cups into a bowl, and some people didn't.

When the students have placed various numbers of cups on their trading boards and written fractions for them, the process is repeated with bowls. When the tenth bowl has been placed on the trading board and the fraction to be written for it agreed on, the teacher presents the students with a different kind of problem.

Teacher: The tin can is still one. Put one cup and nine beans on your trading board. Write the fraction.
Student: How?
Teacher: What seems to be the difficulty?
Student: One cup is 1 over 100, and 9 beans is 9 over 1,000. How can we write them as one fraction? They don't have the same bottom number.
Teacher: Do you know how to find a number for both fractions that will give you the same bottom number?
Student: With start-with–go-by's, and cubes, but that would take an awful lot of cubes.
Teacher: Well, work out the start-with–go-by–both columns and if we need to, we can all work together on the cube stick.
What number was the first number in the both column?
Student: One thousand.
Teacher: You already have one fraction with 1,000 on the bottom, so you only need to find how many 1 over 100s is the same as something with 1,000s on the bottom.
Student: But we already know 1 over 100 is the same as 10 over 1,000.
Teacher: How do you know that?

If none of the students know 1 over 100 has the same value as 10 over 1,000, they may come to the same conclusion using a very long cube stick. If cubes are not available in the room, graph paper strips taped together will work.

Because the students have already formulated rules for determining equivalent fractions, it is highly likely they will have little difficulty realizing that 1 over 100 has the same value as 10 over 1,000. When they do, the teacher's questions continue.

Teacher: What's 10 over 1,000 plus 9 over 1,000?
Student: Nineteen over 1,000.
Teacher: Then you would write the fractional value for one cup and nine beans as 19 over 1,000. Can you see a pattern for the cup and beans and the numbers in the fraction that might help you to predict how to write fractional values for cups and beans?
If you think you see a pattern, test it by predicting the answer to the next problem I give you before we work it out.
The tin can is still one. Put two cups and eight beans on your trading board. Write the fraction.

The teacher continues directing the students to place a specified number of beans and cups, and then beans, cups, and bowls on their trading boards. For each new amount, the students write the fractional value on their blackboards. There is no required amount of problems; for each, the teacher allows them as much or as little time as they need to discover a solution.

LESSON 12-2

DECIMAL NOTATION

PURPOSE:

To learn different ways to write fractions whose denominators are powers of ten

MATERIALS:

1. *Clear acetate squares with circles drawn on them, or circular cutouts*
2. *If no overhead projector is available, bean-shaped cutouts*
3. *Beans*
4. *Cups*
5. *Bowls*
6. *Tin cans*
7. *Individual blackboards*
8. *Unlined paper*

Teacher: Today I will show you a special way to write the beans, cups, and bowl fractions you worked with yesterday. This system was thought up by people who didn't want to write out the whole fraction each time they worked a problem. You've already used this system before, though you may not have known it. It's called the *decimal system.*

1
10
100
1000

Look at the numbers in this pattern. What number would come next?
Student: A one and four zeros.
Teacher: And after that?
Student: A one with five zeros.
Teacher: Okay. A decimal fraction is a fraction that has as its bottom number 10, 100, 1,000 or any other numbers in this pattern. The only difference between decimal and regular fractions is decimal fractions are written without any bottom numbers.
For the problems I will give you, the tin can is still one. Put a tin can and a bowl on your trading board. How many tin cans do you have?
Student: One.
Teacher: And what is the fractional value of the bowl?
Student: One over ten.
Teacher: That would be written like this.

$1\frac{1}{10}$

Put out two tin cans and two bowls and write the numbers for what you have.
Student: 2 2/10.
Teacher: To write that as a decimal number I take away the bottom number, like this.

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So I can tell this number is two and two over ten, and not twenty-two, you must add what is called a decimal point, like this.

2.2

The decimal point or dot tells me where the whole numbers stop and the fractional numbers begin.
Show me what this number would look like on your trading boards. The tin can is still one. . . .

2.3

Okay. Most of you have two tin cans and three bowls on your trading board. Why?

The teacher alternates between giving the students fractions to change into decimal fractions and decimals to convert back into fractions. When the students understand how to write various numbers of tin cans and bowls both ways, cups are added to the trading board as well.

Teacher: Put one tin can, one bowl, and one cup on your trading board. The tin can is still one. How would you write the fractional value of one bowl and one cup together?
Student: 11/100.

Teacher: Okay. As a decimal fraction, that is written like this

.11

If I include the tin can in the number, too, it is written like this.

1.11

Put two tin cans, two bowls, and two cups on your trading board and write the decimal fraction on your blackboards.

The alternating process is repeated; then beans are added.

Once the students understand how to write decimal fractions for all the materials, they are shown the following rule for writing decimal fractions.

Teacher: Please put one tin can and one bowl on your trading board. Write the whole number and the fraction for what you have altogether.
Student: 1 1/10.
Teacher: Now write it using a decimal fraction.
Student: 1.1
Teacher: Clear your trading board and put one tin can and one cup on it. Write the whole number and the fraction for how much you have altogether.
Student: 1 1/100.
Teacher: Now, write it using a decimal fraction.
Student: 1.1
Teacher: Most of you wrote one point one for one tin can and one bowl and for one tin can and one cup. I can see why you did that, because I said to write the top number. Sometimes writing the top number can make things a little confusing. When you write a top number, I need to know which fraction that number stands for. Right now, I can't tell if one point one means tin cans and bowls or tin cans and cups.
Slide the tin can and cup to the top of the board. Now, put another tin can and a bowl toward the bottom. Are the two rows the same?
Student: No.
Teacher: Why not?
Student: Because one row has a cup in it and the other has a bowl.
Teacher: If we were adding on our trading boards and I got an answer of one bowl and one bean, could I write this?

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Student: No.
Teacher: Why not?
Student: Because that means one cup and one bean.
Teacher: How would I write one bowl and one bean?
Student: 101.
Teacher: Why do I need the zero?
Student: To show you don't have any cups.
Teacher: The rule for decimal fractions is the same. If you have a column that doesn't have anything in it, then you have to write a zero for it. How would you write the decimal fraction for the row with one tin can and one bowl?
Student: 1.1
Teacher: For one tin can and one cup?
Student: 1.01
Teacher: Here are some more bowls, cups, and beans fractions to change into decimal fractions. Let's try them together.

As the students alternate between writing decimal fractions and fractions, the teacher asks them to think about the following questions:

Is it easier to figure out the decimal number or the fraction to write for each new amount of beans, cups and bowls? Why?
Can you see a relationship between the quantities in the columns on your trading boards and the decimal fractions you write that might help you predict the decimal fraction before you figure it out with the materials? Between columns and fractions?

The teacher provides numbers for beans, cups, bowls, and tin cans to convert to fractions and decimals throughout the time available for the lesson.

LESSON 12-3

DECIMAL NOTATION

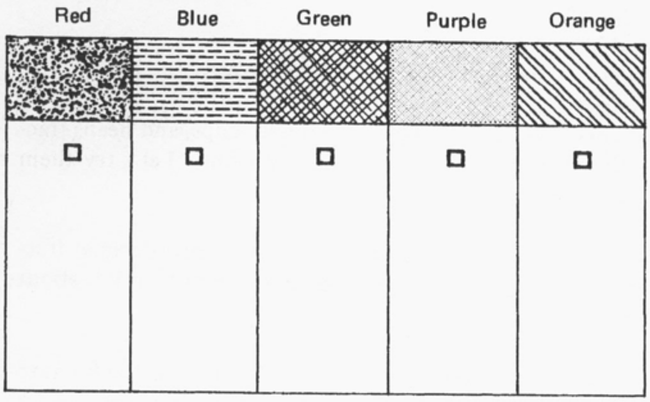
PURPOSE:

To assign fractional values to chips on a chip trading board

MATERIALS:

1. *Acetate squares or squares of paper in five separate colors*
2. *Colored marking pens or strips of paper in five separate colors*
3. *Paper squares or chips*
4. *Crayons*
5. *Individual blackboards*
6. *Unlined paper*

Beans, cups, bowls, and tin cans are useful in helping students visualize concepts. Once the concept is understood, this knowledge is transferred to chips. The beans provide the necessary background for understanding the relative value of each chip; chips provide a more efficient method of working problems.



Teacher: Place one red, blue, green, purple, and orange chip on your chip trading board. If orange is one, and it takes ten oranges to make a purple, how many oranges make two purples?
Student: Twenty.
Teacher: How many purples to make one green?
Student: Ten.
Teacher: How many greens to make a blue?
Student: Ten.
Teacher: Blues to make a red?
Student: Ten.
Teacher: How many oranges to make a green?
Student: One hundred.
Teacher: We called a cup, bowl, or tin can *one* on the bean trading board, to determine the fractional values for anything on the board smaller than one. We can do the same thing on the chip trading board.
I'll say a purple chip is one. What fraction of a purple is one orange?
Student: One over ten.
Teacher: Two oranges?
Student: Two over ten.
Teacher: Three oranges?
Student: Three over ten.
Teacher: Now a green chip is one. What fraction of a green is the purple?
Student: One over ten.
Teacher: What fraction of a green is an orange?
Student: . . . One over 100.

This process is essentially the same as that used in Lesson 12-1 for beans, cups, bowls, and tin cans. The same questions are asked. The two differences are: (1) chips are used instead of beans, cups, and bowls, and (2) the fractional values have the potential of reaching one ten-thousandth, if the red chip is said to be one.

It is not necessary that the teacher have the students work with fractions as small as ten-thousandths. Usually, working with decimals to three places is sufficient for elementary school use. If so, the red chip as one need not be included.

LESSON 12-4

DECIMAL NOTATION

PURPOSE:

To assign decimal values to chips on a chip trading board

MATERIALS:

1. *Acetate squares or squares of paper in five separate colors*
2. *Colored marking pens or strips of paper in five separate colors*
3. *Paper squares or chips*
4. *Crayons*
5. *Individual blackboards*
6. *Unlined paper*

The activities for this lesson are essentially the same as those activities of Lesson 12-2, but the fractional numbers the students convert to decimal values come from chips on the chip trading boards.

LESSON 12-5

ADDITION AND SUBTRACTION WITH DECIMALS

PURPOSE:

To create decimal problems for addition and subtraction using chip trading boards

MATERIALS:

1. *Acetate squares or squares of paper in five separate colors*
2. *Colored marking pens or strips of paper in five separate colors*
3. *Paper squares or chips*
4. *Crayons*
5. *Individual blackboards*
6. *Unlined paper*
7. *Overhead projector dice, or dice cards in bag*
8. *One pair of dice per student: one numbered from zero to four the other from zero to five*

Teacher: You already know how to use your chip trading boards to make up problems for groupings by tens. (See Lessons 8–43 through 8–46.) Today you will use them to make up addition and subtraction problems for decimals.

I will roll my dice to give you numbers for the top row of each addition problem. . . . What did I get?

Student: Seven blues, five greens, four purples, and three oranges.

Teacher: For all the problems we make up today, blue will be one. How would you say the decimal number I have?

Student: Seven point five, four three.

Teacher: And how would you write it?

Student: 7.543

Teacher: Now, I'll roll for the next row. . . . What did I get?

Student: Three blues, five greens, seven purples, and two oranges.

Teacher: What is the decimal number?

Student: Three point five, seven, two.

Teacher: How would you write it?

Student: 3.572

Teacher: Now I'll add the two rows together and make the trades. What's my answer in decimal numbers?

Student: One, one, point one, one, five.

Teacher: And how would you write it?

Student: 11.115

The example for addition is followed by an example for subtraction, then the students create as many or few addition and subtraction problems as time permits.

Since the students already know how to create addition and subtraction problems, the only difficulty they might encounter is where to place the decimals. Some students may pay more attention to keeping the numbers themselves in columns than to lining up the decimals one under another. Since the decimal fractions represent particular columns of numbers on a chip trading board, there is ample evidence available for the students to decide which method of recording numbers is more appropriate.

12.301	12.301
4.37	4.37
1.5	1.5
7.334	7.334
20.0.8.7	25.505

The teacher does not teach them any particular rules for placing the decimal point.

In the case of addition and subtraction, the students may find placing the decimal point correctly is so obvious that they need not state a formal rule for it.

If a more formal statement of rules is desired, the teacher may ask the following:

Can you tell where the decimal point will go in your answer before you slide all the chips together and make all the exchanges?

Could you write the decimal point down before you find the answer?

Does the place where the decimal point goes change if you have to make many chip trades before finding your answer?

How can you tell where the decimal point goes? Could you make up a rule to predict where it goes for addition and subtraction problems?

LESSON 12-6

ADDITION AND SUBTRACTION WITH DECIMALS

PURPOSE:

To create problems involving adding and subtracting dollars and cents

MATERIALS:

1. *Acetate squares or squares of paper in five separate colors*
2. *Colored marking pens or strips of paper in five separate colors*
3. *Paper squares or chips*
4. *Crayons*
5. *Individual blackboards*
6. *Unlined paper*
7. *Menus, catalogs, and newspaper advertisements*

Teacher: I want you, without getting too excited, to pretend the green chip is one dollar. What, then, would a blue be worth?

Student: Ten dollars.

Teacher: And red?

Student: . . . One hundred dollars.

Teacher: What fraction of one dollar is a purple?

Student: One over ten.

Teacher: How much money would a purple be worth?

This question might lead to considerable discussion, because many students do not have a clear idea of the relationship of various coins to a dollar. Students who can calculate dollars-and-cents problems abstractly still may not know a dime is one-tenth of a dollar.

Teacher: If green is one, how would you write one green and one purple as a decimal fraction?

Student: 1.1

Teacher: Okay. When I say I want green to stand for a dollar, it's the same as when I say green is one. How would you write one dollar and one dime in decimals?

Student: 1.1

Teacher: What fraction of a dollar is an orange chip?
Student: One over 100.
Teacher: And how much money would an orange chip be worth?

This question might also lead to considerable discussion, although it is more likely the facts necessary to determine the value of one-hundredth of a dollar have been brought out in the discussion of the monetary value of a tenth.

Teacher: If green is one, how would you write one green, one purple, and one orange as a decimal number?
Student: 1.11
Teacher: And how would you write a dollar, a dime and a penny as a decimal number?
Student: 1.11
Teacher: You know how to add and subtract decimal fractions. The rules for the other decimals work for dollars and cents as well. There are only two differences. First, you write a dollar sign in front of the whole number so people can tell the decimal numbers stand for dollars and cents. Second, you have to write the decimals using two places, even if the orange column is empty.
How do you write a green and a purple in decimal numbers if green is one?
Student: 1.1
Teacher: That's right. How would you write it if green is one dollar and purple is a dime? Remember, you have to write a number for the orange column, even if it's empty.
Student: 1.10
Teacher: Don't forget the dollar sign.
Student: $1.10
Teacher: If green is one dollar, write the decimal number for one green, two purples, and five oranges. Don't forget the dollar sign.
Student: $1.25
Teacher: What does the one mean? . . . The two? . . . The five?

When the students can state the monetary values of the various chips, the teacher changes the questions.

Teacher: Write the decimal number for $2.36 on your blackboards. Now, show me on your trading board the chips that represent it.

When the students can convert abstract numbers for dollars and cents into chips and reverse the process, they are ready to create their own addition and subtraction problems for dollars and cents.

Such problems may be generated by asking students to keep a record of mythical purchases. Addition problems sum up lists of students' desires; subtraction problems give students an imaginary amount of money to spend and require them to keep track of the amount remaining after each expenditure. The children obtain the numbers from the menus, mail-order catalogs, and newspaper advertisements.

Without specific lessons equating the decimal experiences on the chip trading board with the decimals used in dollars-and-cents problems many students fail to see the connection between decimal values and monetary values. This lack of transfer is also a difficulty if they are not shown a direct relationship between their chip trading boards and various metric units.

To use the board as an aid in adding and subtracting metric units, the teacher assigns one chip a specific metric value; all other chips receive their values accordingly. For example, in linear measure, if the green chip represented one meter, this would make purple a *deci*meter, orange a *centi*meter, blue a *deka*meter and red a *hecto*meter. The color on the left of red, if needed, would be a *kilo*meter.

This use is shown to the students as soon as they begin to learn such measuring units. Metric measure is discussed in Chapter 16.

LESSON 12-7

MULTIPLICATION OF DECIMALS

PURPOSE:

To examine the answers to specific decimal multiplication problems for patterns leading to the discovery of rules for multiplication of decimals

MATERIALS:

1. *Paper squares or chips*
2. *Crayons*
3. *Unlined paper*
4. *Individual blackboards*
5. *Dittoed copies of special multiplication matrix*

Teacher: Find the answer to this problem using your chip trading board.
Four people each had $1.25. How much money did they have altogether? Please write your answers on your blackboards.
Student: $5.00.
Teacher: I'll record on the overhead the numbers for all the problems. Although you could also get the answer to the first problem by adding, I'll record it as a multiplication problem.

$1.25 Money
x 4 People
$5.00 Money Altogether

Here's the next problem. Three girls had $2.50 each with which to buy toy soldiers. How much money could they spend altogether? Write your answers on your blackboards.

Student: $7.50.

$2.50 Money
X 3 Girls
$7.50 Money Altogether

Teacher: Okay. Here's the next problem . . .

When the answers to four or five problems on the overhead have been calculated on the chip trading boards, the teacher gives the students a more difficult problem.

Teacher: For this problem, you may use your special multiplication matrix and the boxes.
There are 32 people in class, each with $2.25. How much money does everyone have altogether?
I see most of you have the number seven, two, zero, zero for the answer. I also see you don't all have the decimal point in the same place. Look at the problems I have already written on the overhead. Can you find a pattern for where the decimal goes that will help you know where it goes for this problem?

The students state where they think the decimal point goes. The teacher then draws a chip trading board on the overhead and each student brings up $2.25 worth of chips. The teacher makes all the exchanges and the chip trading board dictates where the decimal should go. If students have accurately predicted the decimal place, their rule is probably a good one. If not, they reexamine all the earlier problems, including the one just completed, and come up with a new way of forecasting the decimal place for the next problem.

The teacher continues to assign problems involving multiplication of a decimal number by a whole number. Each new problem is used as a check on possible rules for placement of the decimal. The students first work the problem abstractly with the boxes, then predict where the decimal goes using the rule or pattern. The appropriate number of students carry chips to the teacher, thereby checking the decimal placement.

The rules for multiplying a decimal by a whole number are relatively straightforward. The difficulty comes in knowing where to put the point when one decimal is multiplied by another. The numerical answer to .5 times .7 is 35. For this problem, however, the correct placement of the decimal point cannot be found by adding .5, .7 times. How it can be found is the subject of the next lesson.

LESSON 12-8

MULTIPLICATION OF DECIMALS

PURPOSE:

To examine the answers to specific decimal multiplication problems for patterns leading to rules for multiplication of decimals

MATERIALS:

1. *Blank ten by ten matrix on a transparency or a large tagboard*
2. *Dittoed copies of blank ten by ten matrix*
3. *Individual blackboards*

Teacher: Look at your matrix. How many spaces on each side?

Student: Ten.

Teacher: If I say the length of one whole side is one, then what fraction of the side is each small space?

Student: One over ten.

Teacher: Since the fraction has ten as a bottom number, you can write it as a decimal. How would you write one over ten as a decimal?

Student: .1

Teacher: I'll write that here. What fraction would two spaces together be?

Student: Two over ten.

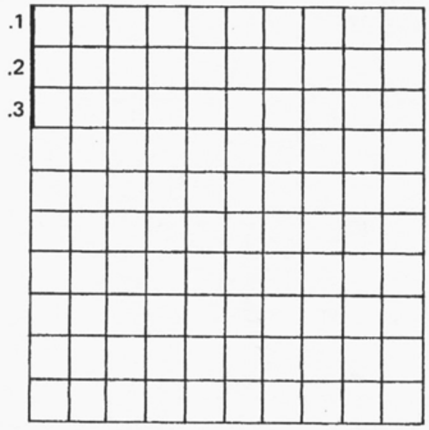
Teacher: How would you write that as a decimal?

Student: .2

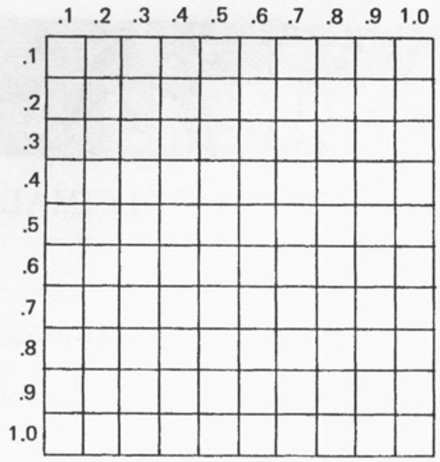
Teacher: That goes here. What decimal would you write for three spaces together?

Student: .3

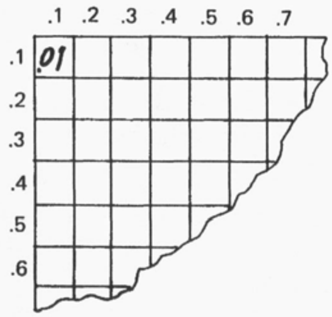
Teacher: That goes here. Four spaces together?



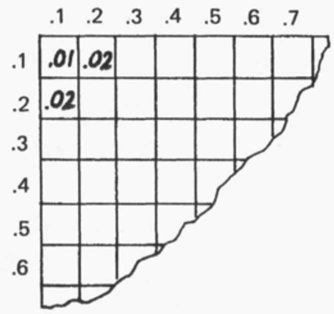
The questioning process is continued until the entire side is filled in. Decimal fractions for the top spaces of the matrix are obtained the same way.



Teacher: How many little squares are there altogether on the matrix?
Student: One hundred.
Teacher: If I say the whole matrix is one big square, what fraction of the big square would a little square be?
Student: One over 100.
Teacher: How would you write that as a decimal?
Student: .01



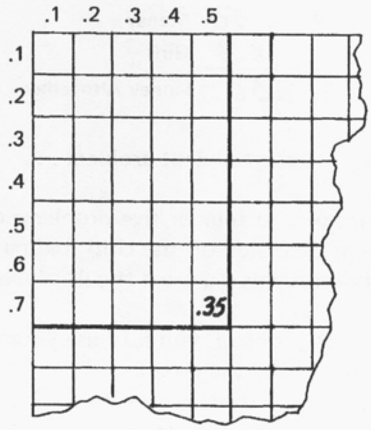
Teacher: What fraction of the big square would two little squares together be?
Student: Two over 100.
Teacher: Write that as a decimal, please.
Student: .02
Teacher: I'll write that here, and here.



Now, let's see if you can tell me which decimal fractions to write in some of these spaces. Write on your blackboards what you think would go here.

The teacher points to spaces that are next to those for which decimal values have already been assigned. As a whole, the class should have little difficulty assigning decimal values to all spaces on the matrix.

Teacher: What is .5 times .7? Write the answer on your blackboards. . . . I see most of you have the number three, five, with the decimal point in a variety of places. Let's see if we can find where it goes by using our decimal multiplication matrix.



Students usually can place the decimal point correctly after only a few problems without having to refer to the matrix. In verbalizing the rule they are using, however, they may be surprised to learn not all those who have been making accurate predictions are doing so for the same reasons.

Because the matrix produces answers that have two-place decimals, as did the problems in the previous lesson, some students may think *all* problems involving multiplication of decimals have answers with two-place decimals. Others may confine their predictions to the situation at hand.

The test of any rule is whether it is capable of accurately predicting answers. The teacher's role is to provide new situations so the rules receive an adequate test.

LESSON 12-9

MULTIPLICATION OF DECIMALS

PURPOSE:

To examine the answers to specific decimal multiplication problems for patterns leading to rules for multiplication of decimals

MATERIALS:

1. *Blank 100 by 100 matrix on a transparency or a large tagboard*
2. *Dittoed copies of blank 100 by 100 matrix*
3. *Individual blackboards*

The activities for this lesson are essentially the same as those for the preceding lesson, although a 100 by 100 matrix is used, and not every number on the sides or in the little squares need be filled in before the students are given problems.

Once the students know how to find the decimal values for the spaces along the outside of the matrix as well as for the small squares inside, they need only calculate the decimal answers to each problem as needed.

When each side is called one, and the matrix itself is also called one, the kinds of decimal answers a 100 by 100 matrix produces can be seen in this figure.

$$.01 \times .01 = .0001$$

$$.12 \times .12 = .0144$$

This matrix produces answers with the decimal point four spaces over from the right.

Students who, from earlier experiences, formulated the rule that the decimal point in a multiplication answer always goes two places over from the right will need to rethink their rule. The greater the variety of decimal problems students face in multiplication, the harder it becomes to find a single rule to predict the location of the decimal point for *any* problem. The teacher's responsibility is to provide a variety of problems from which the students may gather evidence.

LESSON 12-10

MULTIPLICATION OF DECIMALS

PURPOSE:

To examine the answers to specific decimal multiplication problems for patterns leading to rules for multiplication of decimals

MATERIALS:

1. *Blank 10 by 100 matrix on a transparency or a large tagboard*
2. *Dittoed copies of blank 10 by 100 matrix*
3. *Individual blackboards*

The activities in this lesson are essentially the same as those in the previous lesson, but the matrix used is now 10 by 100. Examples of the decimal multiplication problems produced by a 10 by 100 matrix may be seen in this figure.

$$.1 \times .01 = .001$$

$$.5 \times .25 = .125$$

The rules for decimals are more easily discovered if one is allowed to look for them—thirty minds looking at once makes it even easier. Students can learn how to add, subtract, and multiply decimals, not because they were told how, but because they were directed where to look and saw for themselves what needed to be done.