CHAPTER 13
Sorting and Classifying

Prerequisite chapters:

None

MATERIALS

For overhead projector:

Transparencies Sorting tree _____ Worksheets 21–23
Buttons _____ Materials chapter, page 297

If no overhead projector is available:

Make charts in place of transparencies _____ Materials chapter, page 297
Button-shaped cutouts _____ Materials chapter, page 297

Student materials:

Dittos Sorting tree
Buttons in containers _____ Materials chapter, page 297
Variety of small objects in containers _____ Materials chapter, page 297
Individual blackboards _____ Materials chapter, page 294
Spelling notebooks _____ Materials chapter, page 296
Lined paper
Brown paper bags
Objects available inside and outside the classroom
String

The goal of sorting and classifying skills is to clarify thinking and improve students' facility with language.

Forming categories and dealing with their relationships is a part of logic; people utilize this form of logic constantly, often without being aware of it. The lessons in this chapter help students develop skills in thinking that enable them to employ logical relationships.

Sorting and classifying involve the recognition of *attributes*—properties of a given object or group of objects a person chooses to isolate and observe. Intellectual development consists in good part of learning to invent or discover attributes or categories relevent to a particular subject, then dealing with the relationships among various attributes. The child who learns to tell horses from cows has learned to classify and sort by attributes. The child who can tell one kind of horse from another has engaged in a more advanced form of sorting.

Through classifying and sorting, the student can make better sense out of a wide variety of impressions and handle increasingly complex situations that might otherwise prove overwhelming. The student restricts information by choosing from among attributes, thus limiting the scope of any one problem to something that is manageable.

LESSON 13-1

SORTING BY TWO ATTRIBUTES

PURPOSE:

To devise rules that adequately describe unannounced sorting systems used by the teacher

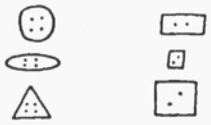
MATERIALS:

1. *Buttons, or button-shaped cutouts*
2. *Individual blackboards*

The activities in the first few lessons introduce students to sorting at the least complex level: sorting by attributes that place the objects in two distinct groups.

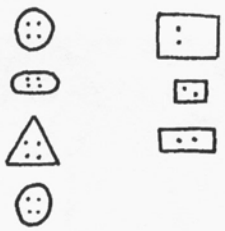
To insure students make maximum use of their logical thinking powers, they must decide the categories used for any of the sortings in this chapter, including those situations initiated by the teacher.

The teacher begins the lesson by creating two piles of buttons from an original pile. The teacher forms the two new piles according to an unannounced rule.



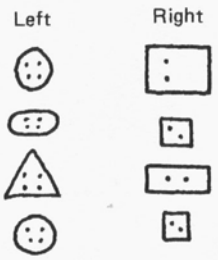
Teacher: I am dividing the buttons into two piles by a rule I have made up. Don't tell me my rule; see if you can predict where I will put this next button.

Student: That pile?
Teacher: Which pile?
Student: That one!
Teacher: I'll mark this the right pile, and this left pile.
Student: The left pile.

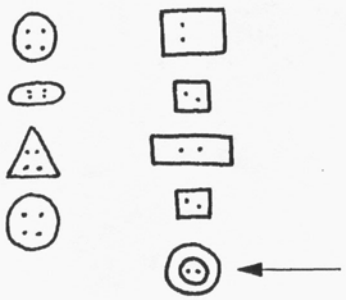


Teacher: Yes, by my rule, this button goes in the left pile. This button?

Student: The right.



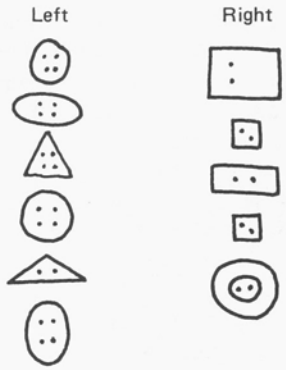
Teacher: Yes. This one?



Student: The left pile.
Teacher: You may be thinking of a very good rule, but by my rule, I cannot put this button in the left pile.
Student: The right pile, then.

In most situations when students are providing answers, the teacher determines the correct answer by surveying the answers written on the individual blackboards and selecting a class answer. However, because the placement of each button has already been determined by the teacher's unannounced rule, the student *guesses* have no effect on the answer. The teacher eliminates any stigma associated with providing a wrong answer by responding: "You may be thinking of a very good rule, but by the rule I am thinking of, I cannot put this button in that pile."

Next, the teacher has the students predict the placement of each button using their blackboards to indicate left or right. Once all the buttons in the original pile have been divided, they are asked what rule they think the teacher was using.



Teacher: All the buttons are in two piles. Can anyone tell me what rule I might have used to get these two piles?
Student: All the big ones on one side, all the little ones on the other.
Teacher: Let's see if all the buttons fit that rule. Are all the big buttons in this pile?
Student: Yes. That pile has all big buttons.
Teacher: Are all the little buttons in this pile?
Student: Yes. All the little buttons are in that pile.
Student: No! There are two big buttons in that pile, too.
Teacher: I thought you said all the big buttons were in this pile.
Student: That pile does have all big buttons!

Sorting and classifying lessons offer an excellent opportunity to observe much of the confusion in our language. The confusion in this instance comes from what the word "all" means. One student says one pile has all big buttons; another says not all the big buttons are in that pile. We cannot expect them to have clarified what they mean by their words at this stage. We can expect, however, that their use of such words will improve. This is one of the goals of the sorting and classification lessons.

Teacher: Are there any big buttons not in this pile?
Student: Yes, there are two in the little pile.
Teacher: Then by my rule, all the big buttons didn't go into just one pile.
Student: Move those two big buttons over.
Teacher: If my rule were big and little then I would have to. But by my rule they belong in the pile with all the little buttons.
Student: Round buttons and square buttons.
Teacher: Round buttons and square buttons what?
Student: Round buttons in one pile and square buttons in the other.
Teacher: What about these triangle buttons?
Student: Rounds and triangles in one pile.
Teacher: Could that be my rule?
Student: No . . . there is a round with the squares.
Teacher: If my rule were rounds and squares, I would have to move this round one, but by my rule it has to stay with the squares.
Student: Two holes and four holes. All the ones on the right have two holes. All the ones on the left have four holes.
Teacher: Are there any buttons in the right pile that don't fit that rule?
Student: No.
Teacher: Are there any in the other pile that don't fit?
Student: No.
Teacher: Then does the way I divided the buttons fit that rule?
Student: Yes.
Teacher: That could have been my rule. Is there another rule I might have used?

The class may have come up with the same rule the teacher used for dividing the buttons, or with a different rule that divides the buttons into the same groups. Regardless of the situation, the students are never told what rule the teacher used.

If the teacher stops asking the students to look for rules when *one* way has been found, they may reasonably assume they have found the only way—the teacher's way. The task is to discover rules that fit the sorting, not to guess the teacher's way. The search should only stop when they can find no other.

Once the class has decided on an appropriate rule(s), the teacher divides the same group of buttons again, using a different rule. The material's used for sorting should be selected carefully so they may be divided several different ways.

If the class is unable to identify a rule for a division, the teacher goes on to other sortings, and later redivides the buttons using the undiscovered rule. After the class has had the opportunity to figure out two or three other rules, their ability to come up with plausible explanations will have increased through practice. If they still cannot come up with the rule, it remains unexplained.

If the students believe the teacher will provide them with answers they do not discover for themselves, their own efforts at discovery fall off markedly.

The teacher continues dividing the buttons by unannounced rules throughout the time available for the lesson.

LESSON 13-2

SORTING BY TWO ATTRIBUTES

PURPOSE:

To devise a variety of rules for sorting piles of buttons into two groups

MATERIALS:

1. *Buttons in containers*

Teacher: Decide who you want to work with—you may work in groups of two or three. When you are in a group, send one person to the front counter to get a jar of buttons, then divide them into two groups, using a rule you invent. Then, tell me how you did it and I'll write your rule on the overhead.

Each time a group divides the buttons one way, the teacher asks them to think of a different rule.

Many students, initially, will use the same rules they think the teacher used earlier. To encourage a greater range of thinking, the teacher draws attention to ways of dividing the buttons that have not been discussed by the whole class.

Teacher: Eric and Mike have found a rule for dividing their piles we didn't use before. They have all the black buttons in one pile and all the other colors in another. Has anyone else thought of a *new* rule for sorting the buttons?

The list on the overhead encourages diverse thinking. The more obvious ways to divide the buttons are the first written; the only way the list may be added to is by inventing a new way to sort the buttons.

The class is also guided into new areas of thinking by the teacher's questions:

John and Marty sorted their buttons by rough and smooth. Are there other ways to divide the buttons by texture?
Are all the buttons made out of the same materials? Is it possible to divide the buttons by how they are made?
Do all the buttons rest on the table in the same way?
Are all the buttons the same on both sides?
Look at the list of all the ways you have found so far. Can you find any new ways?

The questions are not intended to lead in any one direction, only to increase the range of possibilities. In any lesson on sorting and classifying, the goal is the process, not the product.

LESSON 13-3

SORTING BY TWO ATTRIBUTES

PURPOSE:

To devise a variety of rules for sorting several different kinds of objects into two groups

MATERIALS:

1. *Containers holding a variety of small objects*
2. *Lined paper*
3. *Spelling notebooks*

Teacher: I have several plastic bags on the front counter, each containing something different. For example, one has seeds and pits, another small lids from jars, another a variety of nuts and bolts, and so on. By each bag I have placed a sheet of lined paper.
In a moment you are to divide into groups of two or three. Your group will send one person to get a bag and paper, then the group will sort the objects into two piles as many different ways as possible. Every time you find a new way to sort the objects, write it on the paper. When you can't think of any more ways, someone from your group takes your bag of objects and paper back to the front counter, then picks another, and you repeat the process.
If you need a word spelled, I'll write it in your spelling notebook. Are there any questions?

Very quickly, all the bags on the front counter will have been sorted by a group—this means the lined paper will already have several sorting possibilities written on it. The first group to sort a particular set of objects may contribute the greatest number of ways to divide the items, but it is unlikely any group has thought of *all* the ways. Each new group adds as many categories to the lists as possible in the time available. The object of the lesson is to generate lengthy lists of ways to sort objects.

The teacher never gives the students the impression that *all* ways to sort any one group of objects have been found.

LESSON 13-4

SORTING BY TWO ATTRIBUTES

PURPOSE:

To devise a vareity of rules for sorting the objects in each student's desk

MATERIALS:

1. Contents of each student's desk

Teacher: You have been sorting objects like buttons or keys into two piles by different rules. Today you will sort another group of objects—the contents of your desk. Make sure you have a reason why objects go in one pile or the other.

When you have everything from your desk sorted into two piles, tell me what rule you used and I'll make a list of them on the overhead.

Okay. Take everything out of your desk and begin sorting. . . . John, have you found a way?

Student: Things that have writing on them and things that don't.

Teacher: If there's anyone who hasn't thought of a way yet, try John's way. John, see if you can find another way. Sally, what's your way?

Student: Big things and little things.

Student: I was doing big and little too!

Teacher: It's alright if you both use the same way. I'll write big and little and put tally marks after it for each person using those two categories.

How many people are using big and little? No, I don't want to know who *isn't* using large and small. Raise your hand if you *are* using it. Has anyone found another way?

The students continue to sort the objects in their desks throughout the time available.

LESSON 13-5

SORTING BY TWO ATTRIBUTES

PURPOSE:

To devise a variety of rules for sorting selected objects obtained outside the classroom

MATERIALS:

1. Paper bags
2. Objects from outside the classroom

For this lesson, the students take a paper bag outside at any convenient time and collect ten objects in it.

The students sort the objects from outside just as they sorted the items in their desks. The number of ways objects may be sorted into two groups is nearly infinite. The more knowledge students gain, both about sorting and the materials to be sorted, the longer the lists will become.

LESSON 13-6

SORTING BY TWO ATTRIBUTES

PURPOSE:

To devise rules that adequately describe unannounced sorting systems used by the teacher

MATERIALS:

1. No materials needed

Teacher: Today I will sort *you*. I will make up a rule and ask you to stand at the front or back or the room according to my rule.

John, I want you to go to the back. Bill, you go to the back. Diana, you come up front. Susie, up front. David, in back. Daniel, in back. Janet, up front.

I haven't told you what my rule is. See if you can guess what is the same about all the students at the front that makes them different from those at the back. Don't say the rule out loud, but if you think you know it, tell me where I will put Brenda.

Student: Up front.

Teacher: Yes, by my rule Brenda comes up front. Where will Larry go?

Student: In back.

Teacher: Yes, by my rule, Larry goes in back. Where will Gregory go?

The process continues until the class is divided into two groups, then the students are asked to state what they think the rule is.

Even if a description is adequate, the class is asked to find another that might also work. When the class can think of no more ways, everyone sits down and the teacher redivides them using another unannounced rule; this continues throughout the lesson.

LESSON 13-7

SORTING BY TWO ATTRIBUTES

PURPOSE:

To devise a variety of rules for sorting people into two groups

MATERIALS:

1. No materials needed

Teacher: Brenda, John, and Janet, please step outside the door when I say to.

The rest of the class will make up a sorting rule to divide everyone into two groups. When you come back inside, you will try to guess how the class sorted itself. Now, please step outside. . .

Okay. What rule do you want to use to divide yourselves.

When the class is ready, the outside people return. As "guessers," they may ask as many questions as they want, but they may make only one guess as to what the rule is, agreed on by all three. Ideally, the students from outside would not be told if their guess was correct. In a class of thirty students, however, there will always be someone who cannot resist telling his or her friend what the rule was. Therefore, the game rule lets them tell the guessers if they guessed correctly.

Since the students may have difficulty separating a question from a guess, the teacher introduces a special rule defining which questions may be asked. The students may only ask whether they or someone else fit into a group, or what group they would go in if they had some specific trait:

Would I go in this group? Would Sally go in that group? If I had on red would I go in this group?

After one group makes its guess, another team of students is sent outside and the class divides up using a different rule. As many different teams have the opportunity to guess class rules as time permits.

LESSON 13-8

SORTING BY THREE ATTRIBUTES

PURPOSE:

To devise rules to adequately describe unannounced sorting systems used by the teacher and systems created by the students

MATERIALS:

1. *If no overhead projector is available, button-shaped cutouts*
2. *Buttons*
3. *Individual blackboards*

The activities in the next few lessons introduce students to sorting by attributes that divide objects or people into three distinct groups. The previous two-group sorting activities form the basis for more advanced forms of sorting in later lessons. Three-group sortings are introduced now to expand the students' thinking about ways objects may be classified, and to show they can be sorted into more than two groups.

This lesson's activities are essentially the same as those in Lessons 13-1 and 13-2, but the buttons are now sorted into three groups.

The two-group sortings placed an emphasis on "either/or" categories; grouping involving three categories is not as easily established.

Examples of three-group sorting categories for buttons are:

Small	Medium	Large
Round	Square	Everything else
Two holes	Four holes	No holes
Big, with two holes	Small, with two holes	All others
Smooth on front	Bumpy on front	Bumpy and smooth on front

LESSON 13-9

SORTING BY THREE ATTRIBUTES

PURPOSE:

To devise several rules for sorting different kinds of objects into three groups

MATERIALS:

1. *A variety of small objects*
2. *Lined paper*
3. *Spelling notebooks*

This lesson is essentially the same as Lesson 13-3, but the objects are sorted into three groups rather than two.

LESSON 13-10

SORTING BY THREE ATTRIBUTES

PURPOSE:

To devise a variety of rules for sorting the objects in each student's desk

MATERIALS:

1. *Contents of each student's desk*

The activities are essentially the same as those in Lesson 13-4—now the contents of the desks are sorted into three groups rather than two.

LESSON 13-11

SORTING BY THREE ATTRIBUTES

PURPOSE:

To devise a variety of rules for sorting selected objects obtained outside the classroom

MATERIALS:

1. *Paper bags*
2. *Objects from outside the classroom*

This lesson is essentially the same as Lesson 13-5, only now the objects collected from outside are sorted into three groups.

LESSON 13-12

SORTING INTO PILES OF ONE

PURPOSE:

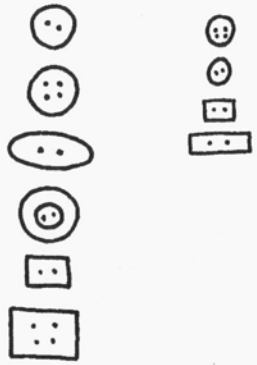
To distinguish one object from another through the use of attributes

MATERIALS:

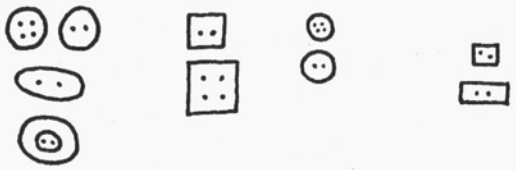
1. *Buttons, or button-shaped cutouts*
2. *Buttons in containers*

The activities in the next few lessons capitalize on the students' knowledge of two-group sortings to help them learn a process of identifying individual objects by their attributes. Objects are classified by sorting and resorting them into successively smaller groups, until each sorted group consists of a single object.

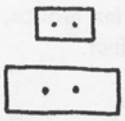
Teacher: What was one of the ways we found to sort these buttons into two separate piles?
Student: Big and little.
Teacher: Okay. I'll sort them into big and little piles. How many piles do I have?



Student: Two.
Teacher: Earlier, when we finished dividing the buttons into two groups, we pushed the piles back together and sorted them again in a new way. This time I want you to think of a rule for each pile so it is divided in two again.
Student: Sort the big buttons by holes and the little buttons by shape.
Teacher: That's a good idea, but I want you to use the *same* rule for both piles.
Student: Sort both piles by round and straight sides.
Teacher: Can I do that? Will it work for both?
Student: Yes.



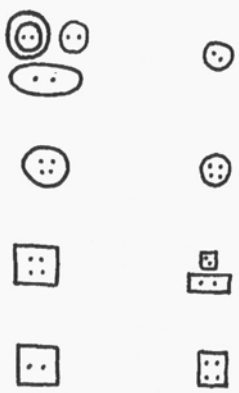
Teacher: How many piles do I have now?
Student: Four.
Teacher: Now is there a way to divide each of the four piles in half again, using only one rule?
Student: Two holes and four holes.
Student: That won't work, because one pile only has two-hole buttons in it . . . so you can't divide that one in half.
Teacher: You're right, so I'll add a new rule. Could there be any four-hole buttons in this pile?



Student: What do you mean?
Teacher: Could I find another button to add to this pile so we would be able to divide it in two?
Student: I think so.
Teacher: What would it look like?
Student: It would have four holes.
Teacher: True, but what else would it need? Could it be big?
Student: No! That's the little pile.
Teacher: The button would need to have four holes and be little. What else?
Student: Straight sides.
Teacher: Okay. If I had a little button with four holes and straight sides, could I sort this pile by two holes and four holes?
Student: I think so.
Teacher: Do you think anyone has ever made any buttons like that?
Student: Yes, there are some in one of the button containers.
Teacher: I agree. I have seen buttons like that, but my set doesn't have any. So the rule I will add is, you can say you have sorted a pile into two groups even if you only get one group, *if* something really exists that you don't have in your set that would go into the other group.

The concept of having a group with nothing in it is easier for students to envision if they describe the object that could fill the space. At the start, it also helps if they actually add the missing item to the pile, if it is available in the room.

Teacher: Johnny, will you please get me a small button with four holes and straight sides from the button box. Now, can I sort these buttons by two holes and four holes?
Student: Yes.
Teacher: How many piles do I have?



Student: Eight.
Teacher: Can we sort the piles again?

After the teacher demonstrates the process of continually dividing the piles of buttons into smaller groups, the students divide their own sets of buttons in as many successive ways as they are able until they run out of ways to divide or they achieve a division leaving one button in each pile.

When students have practiced sorting the complete sequence of materials, in the lessons that follow they will repeat these same sortings while recording their groupings.

LESSON 13-13

SORTING INTO PILES OF ONE

PURPOSE:

To distinguish similar objects from one another through the use of attributes

MATERIALS:

1. A variety of small objects

The activities for this lesson are essentially the same as those in the previous lesson although the objects to be sorted are different.

The containers of objects to be sorted are placed on the front counter. The students divide into groups of two or three. Each group sends one person to the front of the room to select a container of objects. The group as a whole divides and redivides its set of objects into smaller and smaller groups, either until the students in that group run out of ways to divide, or until they achieve a division which leaves exactly one object in each pile.

When the students in a group feel they have exhausted the possibilities for their original set of objects, one student from that group returns the set of objects to the front counter and selects another for which the group may make successive divisions.

LESSON 13-14

SORTING INTO PILES OF ONE

PURPOSE:

To distinguish similar objects from one another through the use of attributes

MATERIALS:

1. Contents of each student's desk

This lesson continues the activities from the previous lesson. Now, the students attempt to sort the contents of their desks into continually smaller groupings; the goal of the successive divisions is piles of one.

If the students wish, two or three of them may sort the objects in one desk, then switch to the desk of another student within the team.

LESSON 13-15

SORTING INTO PILES OF ONE

PURPOSE:

To distinguish similar objects from one another through the use of attributes

MATERIALS:

1. Paper bags
2. Objects from outside the classroom

This lesson continues the activities of the previous two. For this lesson, the students sort the contents of their paper bags. Again, they may work in teams of two or three, combining all their objects into one large group, or sorting each bag separately.

LESSON 13-16

SORTING INTO GROUPS OF ONE

PURPOSE:

To distinguish students in the classroom from one another by various attributes

MATERIALS:

1. No materials needed

Teacher: What is one way you can sort yourselves into two groups?
Student: Boys and girls.
Teacher: All the boys go to the back of the room and all the girls come up front. Is there a way you can divide each of these groups into two more groups?
Student: Long hair and short hair.
Teacher: How many groups will there be altogether if you divide that way?
Student: Four.
Teacher: Where shall the four new groups stand?

The goal for the students is to sort and resort themselves until each student stands alone. If they accomplish this goal, they start the process over again, without using the attributes used the first time.

LESSON 13-17

SORTING INTO PILES OF ONE

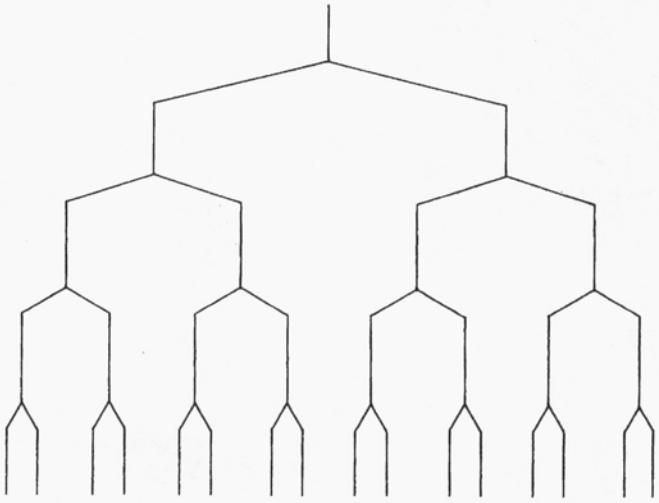
PURPOSE:

To learn and use a system for recording successive sorting by attributes leading to groups of one

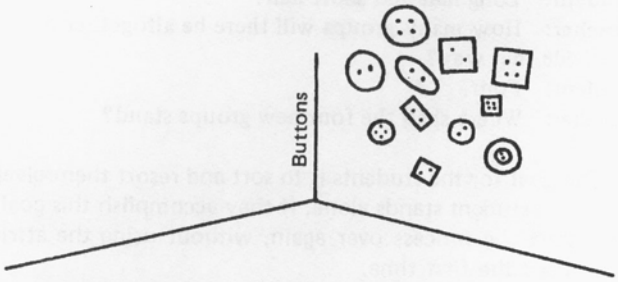
MATERIALS:

1. Sorting tree on transparency, or on a large tagboard
2. Dittoed copies of sorting tree
3. Buttons, or button-shaped cutouts
4. Buttons in containers
5. Spelling notebooks

The necessary link between the sorting skill the students learned in the preceding lessons and the process of identifying individual objects by their attributes is the system of recording presented in the following activities.

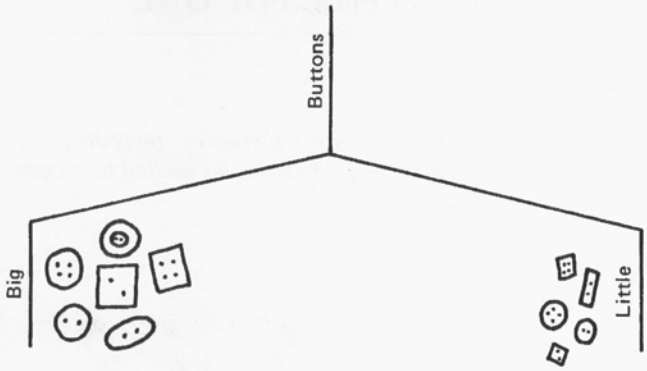


Teacher: This is called a *sorting tree.* I'll demonstrate how to use it. I put all my buttons next to this top line and write "buttons." What is a way we divided the buttons into two groups before?



Student: Big and little.

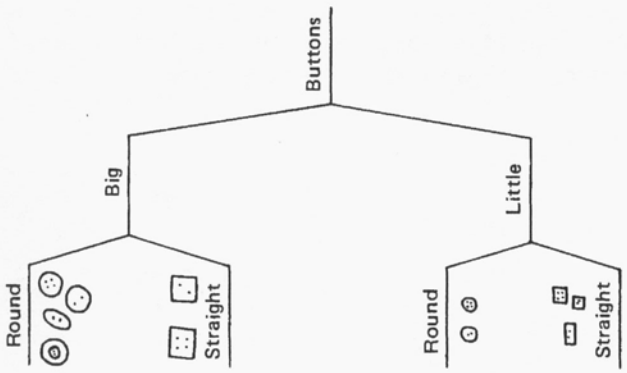
Teacher: Okay. I'll divide the buttons into big and little and move the two new piles to the two places where I have written "big" and "little" on the sorting tree.



What is another way we divided the buttons?

Student: Round and straight.

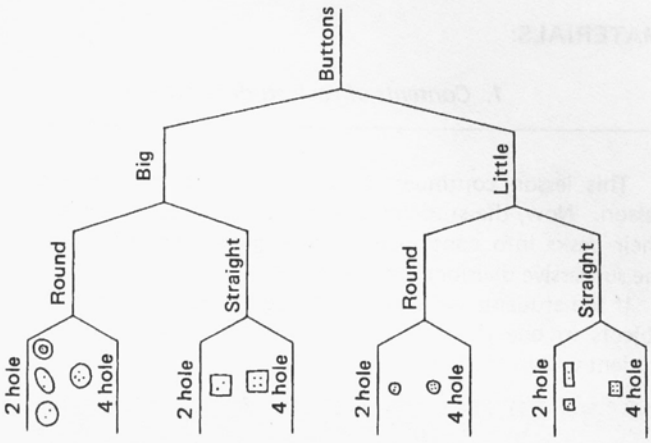
Teacher: I'll write "round" and "straight" on these branches of the sorting tree and then move the buttons to the correct branches.



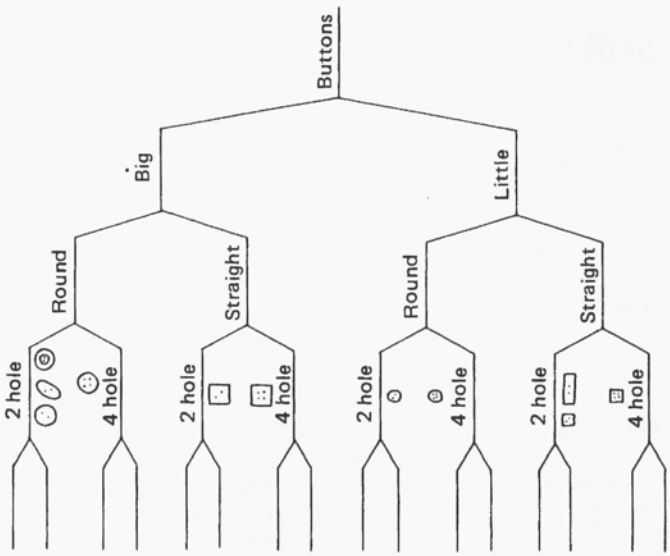
Another way?

Student: Two holes and four holes.

Teacher: Okay. I'll write that on the branches first, and sort the buttons by two holes and four holes.



Are there any other ways we sorted the buttons?



The process continues until the students run out of ideas for dividing the buttons, there is no more than one button on each end branch, or there is no more room on the tree. Then, they divide piles of buttons on their own copies of the sorting tree. Students who wish to, may work together and record their joint efforts on a single sorting tree. Any words they need spelled are written in their spelling notebooks.

The assignment is to divide the buttons into continually smaller groups until no more space remains on the tree or there is no more than one button on each end branch. Each new division is recorded on the appropriate branches.

As the students work, the teacher asks the following questions:

Does sorting the objects in a different order make any difference? Would the smallest piles on the overhead have turned out differently if we had sorted the buttons by two holes and four holes first, then by round and straight, and last by big and little? If it makes a difference, why? If it doesn't, why not?

Can you describe the objects on a branch by reading the attributes written on all the branches that lead up to it? Look at the tree. If I read the top branch and all those along the far right-hand side, each branch in turn says "buttons," "little," "straight," and "four holes." Are the objects on the bottom branch little buttons with four holes and straight sides?

Can you use the branches to help you describe what an object might look like for an empty branch? Could you find an object or draw a picture of an object that might go there?

LESSON 13-18

SORTING INTO PILES OF ONE

PURPOSE:

To use a system for recording successive sortings by attributes as the basis for answering teacher-generated questions about objects and their attributes

MATERIALS:

1. *A variety of small objects*
2. *Dittoed copies of sorting tree*
3. *Spelling notebooks*

The activities for this lesson are essentially the same as those in the previous one, the only difference being the objects sorted.

The containers for a variety of objects are placed on the front counter. The students divide themselves into groups of two or three. Each group sends one person to the front of the room to select a container, then the group records all the successive divisions for it on a single sorting tree. The process is repeated throughout the time remaining for the lesson.

As the students work, the teacher asks them to answer the questions from the previous lesson.

LESSON 13-19

SORTING INTO PILES OF ONE

PURPOSE:

To use a system for recording successive sortings by attributes as the basis for answering teacher-generated questions about objects and their attributes

MATERIALS:

1. *Contents of each student's desk*
2. *Dittoed copies of sorting tree*
3. *Spelling notebooks*

This lesson continues the activities of the previous one. For this lesson, the students record sortings of the contents of their desks, working in teams if they wish. The same questions are asked.

LESSON 13-20

SORTING INTO PILES OF ONE

PURPOSE:

To use a system for recording successive sortings by attributes as the basis for answering teacher-generated questions about objects and their attributes

MATERIALS:

1. *Paper bags*
2. *Objects from outside the classroom*
3. *Dittoed copies of sorting tree*
4. *Spelling notebooks*

This lesson continues the activities of the previous two lessons. Now, the students record sortings of the contents of their paper bags, alone or in groups.

LESSON 13-21

SORTING INTO GROUPS OF ONE

PURPOSE:

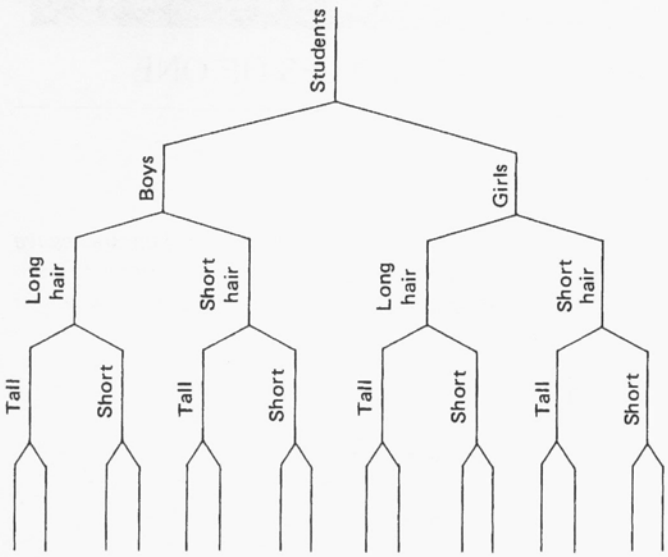
To use a system for recording successive sortings to distinguish students in the classroom from one another

MATERIALS:

1. *Sorting tree on transparency, or on a large tagboard*

2. Dittoed copies of sorting tree
3. Spelling notebooks

Teacher: What was one way you used to divide yourselves into groups of two?
Student: Boys and girls.
Teacher: Okay. Boys to the front, girls to the back. I'll write each group's name on my tree on the overhead. What is another division you can make?
Student: We could use long hair and short hair again.
Teacher: Okay. Decide where the new groups will stand, and I'll write the new categories. What's another way?
Student: Tall and short.
Teacher: Okay. Another?



The process continues until the teacher has written a sorting description on each branch of the tree. The sorting tree has only 16 branches at its base, so the students will not have divided themselves into groups of one before the teacher is through writing categories on the overhead. The students assignment is to discover ways to sort themselves so each person in class has his or her own branch on a tree with 32 branches at one end.

Although this assignment is relatively simple to present, its solution may be more difficult to find. For this reason, the teacher encourages the students to work in teams of twos or threes.

In searching for a solution, the students will learn a great deal about the attributes that make them the same as and different from everyone else in class. Even if no solution is found, the knowledge gained is the real worth of the activity.

The students continue working until they find a way or the lesson time is consumed. They keep track of their efforts on dittoed copies of the sorting tree. Although they may not have found a solution when the time is up, the following lessons may provide them additional insights into solving such a problem. The sorting trees help students know which methods have been tried and how close they came to a solution, so they may try again.

LESSON 13-22

SORTING INTO GROUPS OF ONE

PURPOSE:

To write descriptions distinguishing each student from all others.

MATERIALS:

1. Lined paper
2. Spelling notebooks

The activities in the lessons that follow test each student's knowledge of his or her own unique attributes through an attribute game; the goal is for students to heighten awareness of sameness and differences in themselves and to practice thinking logically.

Teacher: I want you to write a description of yourself on your paper, but do not write your name. Describe yourself so no one else matches your description. If you need a word spelled, I'll write it in your spelling notebook.

The teacher allows the students about ten minutes before collecting the descriptions.

Teacher: When I ask you, I want you all to stand up. I will read a description—as soon as it no longer fits you, sit down. The person who wrote the description should be the only one left standing when I get through reading. Everyone please stand up. The first line on *this* description says "tall." Who should sit down?
Student: Everyone who is not tall.
Teacher: Okay. Everyone who is not tall, sit down.
Student: Johnny's still standing and he isn't tall.
Student: Yes I am!
Teacher: How tall do you have to be to be tall?
Student: I don't know but Johnny isn't tall!
Teacher: I'll add a rule to this game, then. If you think someone is standing who should sit down, or if a person is sitting who should still be standing, tell me and I'll have all of you vote. Raise your hand if you think Johnny should stand . . . if you think he should sit. Okay, more people think Johnny should sit down this time. The next line in this description says "wearing blue pants." Who should sit down now?
Student: Anyone who is not wearing blue pants.
Teacher: Okay . . . is anyone still standing who should be sitting?
Student: No.
Teacher: Is anyone sitting who should still be standing?
Student: No.

Teacher: The next line says "wearing a white shirt." Who should sit down?
Student: Anyone not wearing a white shirt.
Teacher: Is anyone standing or sitting who shouldn't be?
Student: No.
Teacher: The next line says "long hair."
Student: Long hair for a boy or a girl?
Teacher: It just says "long hair."
Student: Frank sat down and he has long hair.
Teacher: We'll have to vote to decide . . . The last line on this description says "wearing glasses." Who should sit down?
Student: Anyone who is not wearing glasses.
Teacher: How many people are still standing?
Student: Two.
Teacher: I have finished reading the description and more than one person is standing. What could have been added to this description so only one person would have been left standing?
Student: The person who wrote it should have put whether or not they were a boy or a girl, because Eddie and Brenda are still standing. We don't know if a boy or a girl wrote it.
Teacher: Would that leave only one person standing? Let's see. Who wrote this description?
Student: I did.
Teacher: Okay, Brenda, I'll add "girl" to your description and see if that leaves you the last person standing. Now the last line says "I am a girl." Who should sit down?
Student: Eddie.
Teacher: How many people are left standing now?
Student: One, Brenda.
Teacher: Okay. If Brenda had said on her description she was a girl, she would have been the only one standing. Let's try someone else's description, and see if it leaves only one person standing at the end.

The teacher continues reading descriptions throughout the time available, and then saves them for use in the following lesson.

LESSON 13-23

SORTING INTO GROUPS OF ONE

PURPOSE:

To write descriptions distinguishing each student from all others

MATERIALS:

1. *The written descriptions from Lesson 13-22*
2. *Lined paper*
3. *Spelling notebooks*

The teacher begins by rereading some of the descriptions from the previous lesson. This may pose problems. Brenda may still be tall and wearing glasses, but she may not be wearing a white shirt and blue pants. When appropriate, the teacher and the class discuss why some of the descriptions now leave no one standing, and some leave more than one person standing. The students are then asked to write descriptions that accurately identify them today and will accurately identify them tomorrow, or at any time within the next month.

The teacher collects all the descriptions and, throughout the time remaining, reads them to the class to check for accuracy (leaving the person who wrote it standing). Descriptions that leave more than one person standing are changed by the class.

The teacher again saves the descriptions for the next lesson.

LESSON 13-24

SORTING INTO GROUPS OF ONE

PURPOSE:

To write short descriptions distinguishing each student from all others

MATERIALS:

1. *The written descriptions from Lesson 13-23*
2. *Lined paper*
3. *Spelling notebooks*

The teacher rereads a sampling of the descriptions from the previous lesson; the students check how well they isolate a single individual. After about ten minutes, the teacher introduces a new writing assignment.

The students are asked to identify themselves in writing using no more than five descriptive phrases. The teacher collects the written descriptions, and reads those containing five or fewer lines. If a description does not separate a single individual, the class is asked to suggest how it might be changed to describe a single person while still meeting the five-line requirement.

For each description that *does* leave only one person standing, the teacher asks an additional question:

Does it make any difference in what order I read the description? If I read each line from the bottom to the top would the same person still be standing alone at the end?

After the class discusses this, the teacher rereads the description in a new order so the class may see what happens. This process is repeated for each new accurate description.

LESSON 13-25

SORTING INTO GROUPS OF ONE

PURPOSE:

To write short descriptions distinguishing each student from all others

MATERIALS:

1. *Lined paper*
2. *Spelling notebooks*

For this lesson, the students are asked to identify themselves in writing using no more than four descriptive phrases. Those containing four or fewer lines are read aloud by the teacher. The lesson proceeds as Lesson 13-24.

LESSON 13-26

SORTING OF OVERLAPPING GROUPS

PURPOSE:

To explore possible ways of sorting people into two groups, when some may fit into both groups at once

MATERIALS:

1. *String*

All the categories the students have used so far have separated objects or people into distinct groups. Not all the applications of sorting they may encounter in life present such clearly definable distributions. The activities in the next two lessons offer students an opportunity to explore ways to sort objects or people when the categories do not lead to distinct groups.

Teacher: There are two large circles of string on the floor. I would like all the boys to stand inside this circle, and everyone who is wearing green to stand inside the other. The rule for this game is, everyone who is a boy *must* be inside the boy's circle, and everyone who is wearing green must be inside the green's circle. . . . What seems to be the problem?

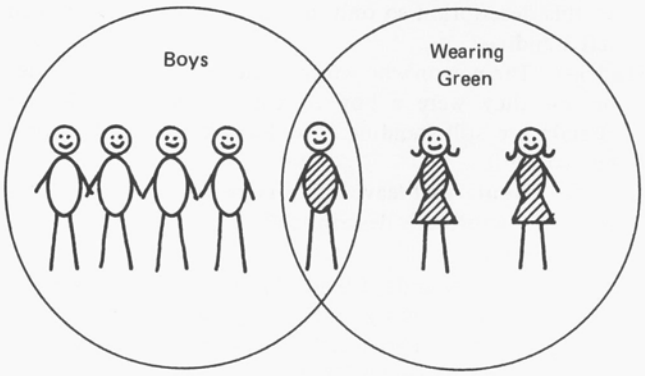
Student: Where do I go?

The teacher selects the two attributes because a boy is wearing green that day. Where he goes is a problem for the class to solve.

If the class says he goes with the boys, the teacher asks if everyone wearing green is in the green circle. If the class sends the boy to the green's circle, the teacher asks if all the boys are in the boys circle.

The class must develop a resolution to the problem consistent with the rules. The class may even choose to have him remove whatever he has on that is green. However, the next problem presented by the teacher may have all the girls in one circle and everyone with straight hair in the other.

The teacher may feel a natural temptation to lead the students to see the string circles may be overlapped, permitting a person to be in both at once. However, the point of this exercise isn't to teach a specific sorting technique, but to let the students explore a variety of possible solutions to a problem. Even if they fail to discover a workable solution, it is the process of searching that is valuable.



The overlapping of two circles isn't the only way the problem can be solved. The methods the students propose as possible solutions are discussed by the whole class. If the solution is consistent with the rules, it's accepted as *one* way. When one way has been found, the students are encouraged to think of others.

When the students have considered one problem for about five or ten minutes, they are given another, whether or not they have found a solution for the first. The teacher does not offer a solution. Each new problem simply involves a rewording of the original situation. For example, girls in one circle and everyone who is right-handed in the other.

It cannot be stressed often enough that the quickest way to stop students thinking is for us to do their thinking for them. Only if they have *real* responsibility for finding solutions will they rise to the challenge.

LESSON 13-27

SORTING OF OVERLAPPING GROUPS

PURPOSE:

To explore possible ways of sorting people into three or four groups, when some may fit into more than one group

MATERIALS:

1. String

This lesson is essentially the same as the previous one. The only difference is the students are asked to develop sorting systems that allow one person to be present in more than one group at a time for three groups, then for four groups. This is only appropriate if they have already developed workable solutions to the sorting problems in the previous lesson.

Teacher: There are three large string circles on the floor. I would like all the boys to stand inside this circle, everyone who is wearing short sleeves in the second circle, and everyone who is wearing glasses in the third circle, please.

Problems are the boy with short sleeves who wears glasses, the boy with glasses who is not wearing short sleeves, or the girl with short sleeves who wears glasses. Once again, it is up to the class to devise a solution or a series of solutions to the assorted problems.

If the students find workable solutions for sorting three groups, the teacher gives them four, by adding another string circle and another category.

If the students can master four groups, what about five? Is it always necessary to sort people? Could buttons be divided by brown and by two holes? Is it possible to make up headings for two overlapping groups that no one could do? Is it possible to tell that in advance? Could the students create three or four group headings for overlapping groups for other students to use in sorting objects? Is it possible to know in advance which headings or categories will produce overlapping groups and which will produce groups completely separate from one another? How?

The teacher is always responsible for the questions. Does the teacher always have to know the answer to a question before it is asked?

LESSON 13-28

SPECIFIC LANGUAGE USAGE

PURPOSE:

To think about what is meant by specific language usages

MATERIALS:

1. No materials needed

The following lesson calls on students to sort themselves by standing up or sitting down according to attributes given by the teacher. The purpose is for the students to develop a clear understanding of the meaning of specific commonly used words.

Teacher: Listen carefully to what I say and think about what it means. I want everyone who is either a boy or a girl to stand up . . . Is anyone standing who should be sitting?
Student: No.
Teacher: Why?
Student: Because you said for all the boys and girls to stand up and we all did.
Teacher: Okay. Everybody sit back down. Now, listen carefully to what I say to do this time. Everyone who is both a boy and a girl stand up.

Some may stand, some may remain seated, and many will be unsure. The statement is designed to cause students to think very carefully about what has been said.

The teacher leads the class in a discussion of the meaning of the words by asking:

Is anyone standing who should be sitting? Is anyone sitting who should be standing?

To know what to do, students must listen, think through, and often debate the statement. Language is a tool of logic, and logic is a tool of mathematical thinking. As the students become more aware of how words are used to convey meaning, their mathematical thinking is a direct beneficiary.

Other examples of language statements the teacher makes to the students are:

Everyone who is wearing *neither* a dress *nor* glasses stand up.
Everyone who is *both* tall *and* wearing shoes stand up.
Everyone who is *not* a boy and *not* wearing a dress stand up.
If you *are* wearing green and *are* a girl stand up.
Everyone who is *more than* eight and *less than* thirteen stand up.

Sorting and classification activities lead to skills of critical observation that help students think more clearly, not just about mathematics, but about everything. The students will use the thinking skills begun in this chapter throughout the entire year and, hopefully, throughout their entire lives.