CHAPTER 14
Measurement

Lesson 14–1 page 206 People Students answer teacher questions about wrist lengths.	Lesson 14–2 page 207 People Students place body measurements in order by size.	Lesson 14–3 page 208 People Students review methods of organizing body measurement data.	Lesson 14–4 page 209 People Students check the accuracy of body measurements taken of them by other students.
Lesson 14–5 page 210 People Students find ratios of length that exist between various parts of the body.	Lesson 14–6 page 211 People Students find new ratios of length that exist between various parts of the body.	Lesson 14–7 page 211 People Students measure contents of desks with body parts.	Lesson 14–8 page 212 People Students measure items in the room with body parts.
Lesson 14–9 page 212 People Students use measurements to place objects in order by size.	Lesson 14–10 page 214 People Students test student-created rules for measuring objects with body parts.	Lesson 14–11 page 214 Desks Students use measuring rules to place desk measurements in order by size.	Lesson 14–12 page 215 Desks Students test new measuring rules to place desks in order by size.
Lesson 14–13 page 216 Same-size Objects Students retest measuring rules by putting same-size objects in order by size.	Lesson 14–14 page 216 Map Students make sectional maps of the classroom to be fit together.	Lesson 14–15 page 217 Map Students test student-created rules for making sectional maps of the class.	Lesson 14–16 page 217 Straws, Sticks, Cubes Students measure objects with other objects and place them in order by size.
Lesson 14–17 page 218 Toothpicks, Tiles, Paper Students measure objects and put them in order by size.			

Prerequisite chapters:

None

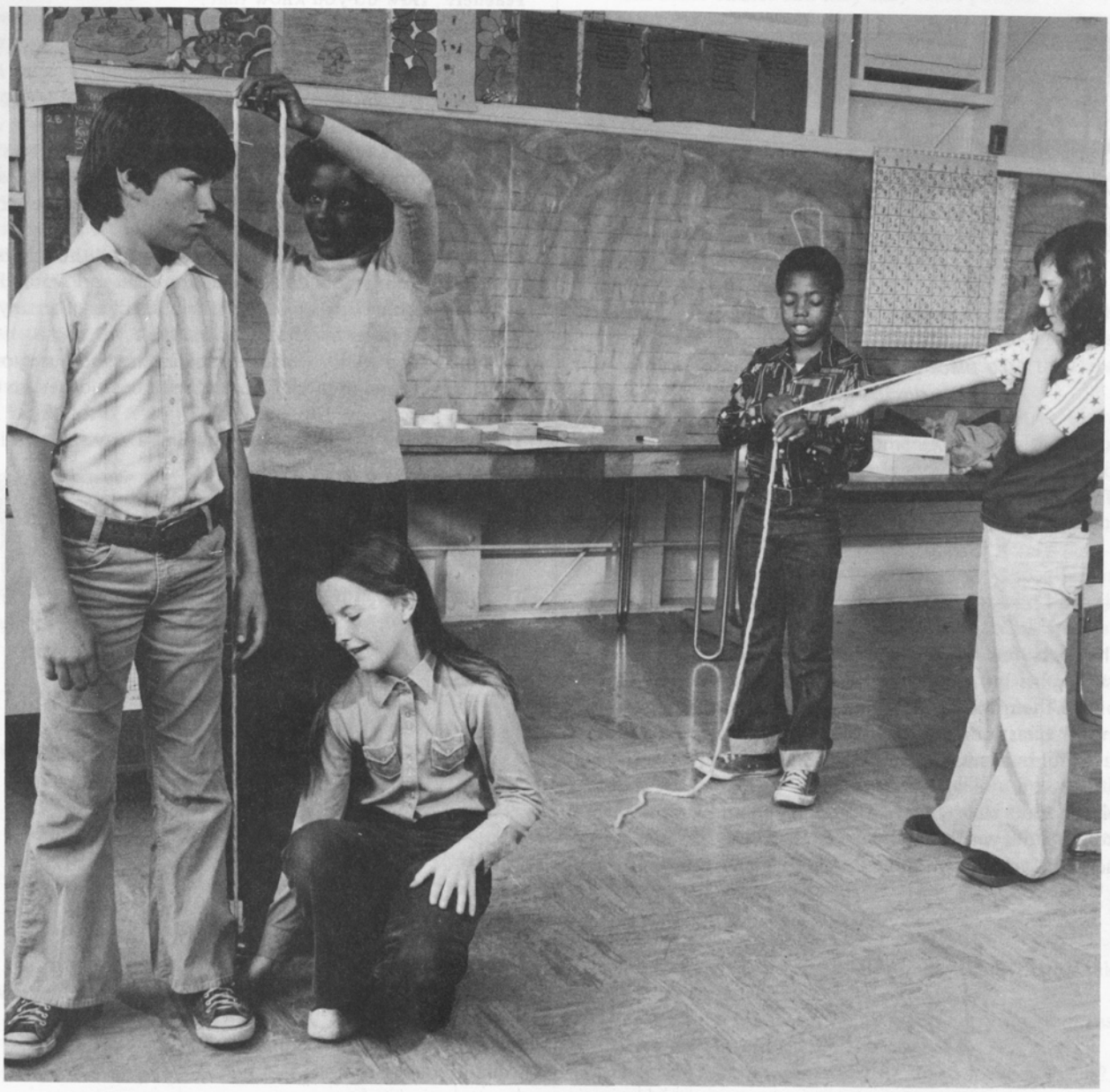
MATERIALS

For overhead projector:

None

Student materials:

Individual blackboards _____ Materials chapter, page 294
Spelling notebooks _____ Materials chapter, page 296
Paper
Scissors
String
Glue
Masking tape
Tiles, toothpicks, cubes, sticks, straws
Objects available inside and outside the classroom



The lessons in this chapter provide students with an understanding of the kinds of problems that historically lead to the development of standardized systems of measurement. Civilization felt the need for standard units of measure because to be without such units was a constant source of frustration. The following activities expose students to a few of these frustrations. Neither metric nor U.S. Customary units of measure are introduced now, since the point of exposing students to these frustrations is to create in them a need for such standardized units of measure. This need isn't satisfied until a later chapter.

The activities used to focus student attention on the need for standardized units of measure center on linear measurements. This should not be construed to mean students confine themselves to linear measurements. The full extent of the measuring activities made available to students will be seen in Chapter 21, Problem Solving.

LESSON 14-1

MEASURING WITH NONSTANDARD UNITS

PURPOSE:

To place selected measurements in order by size; to discuss ways to make them more useful in helping answer teacher-directed questions

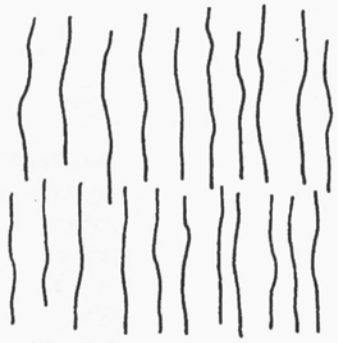
MATERIALS:

1. *String*
2. *Scissors*

The next few lessons familiarize students with taking measurements by using lengths of string. The activities also introduce them to the ideas that measurement may be used to answer questions and some displays of measurements are more useful in answering questions than others.

Teacher: Take the string I have given you and wrap it once around your wrist, then cut off the piece of string that measures your wrist and bring it to me.

Now, I'll lay out as many of these strings as I can on the overhead projector. What did these strings measure?



Student: Wrists.
Teacher: Are everybody's wrists the same size?
Student: No.
Teacher: How do you know that?
Student: Because all the strings aren't the same size.
Teacher: Does *anybody* in class have a wrist the same size as anybody else?
Student: Match some up and see:
Teacher: How should I match them?
Student: Take one and hold it up against all the others.
Teacher: Okay. These two seem to match, so I guess at least two people in the room have the same size wrists. How many other matches are there?
Student: Try matching another one up to all the others.
Teacher: Well, I could, but that would mean I would have to match the strings one at a time, 32 times. Can anyone think of a faster way?
Student: You won't have to do it 32 times, because you've already found two that match with each other, so you don't have to use those again.
Teacher: That's true. But can you find a way so I wouldn't have to hold up the strings at all?
Student: Try putting the strings in order, form the biggest to the smallest. Then we could see which strings matched with other strings, because the ones that are the same size would be next to each other.
Teacher: Okay, I'll try that and see if it works . . . Can we tell which wrists are the same size when all the strings are in order?

Student: Yes.
Teacher: Are there any the same size?
Student: Yes.
Teacher: How many?
Student: It looks like nine people have the same size wrist as somebody else.
Teacher: Which nine people?
Student: We can't tell.

Teacher: Why?
Student: Because we don't know whose string is whose.
Teacher: Can we find a way to tell whose string is whose?
Student: Yes.
Teacher: How?
Student: Write their names on the string.
Teacher: How could we write names on the strings?
Student: We could write the names on paper and stick the strings on the paper, or we could write the names on masking tape.
Teacher: We might try that method to see what would happen . . .

The teacher's questions should cause students to think about how the string measurements could be made more useful. The teacher asks the students to look at the strings and answer questions about wrists. Some questions may be answerable, some may not. For any question not easily answered, students are asked to think of a way the data might be changed so the question *could* be answered.

Students may not suggest placing all the strings in order from largest to smallest, but insist the teacher match each string against every other to find ones the same length. They may also invent other ways. The teacher should try whatever methods the students suggest.

Shortly, the students will be asked to take string measurements and place them in order. This lesson gives students the opportunity to think out in advance what methods of operation might make their data more meaningful. The teacher assists them in this by either physically carrying out their suggestions with the strings, or asking them how to restructure the exercise so the questions are more easily answered. The real test of the effectiveness of their think. ing, however, is putting those thoughts to use. This is the basis for the next lesson.

LESSON 14-2

MEASURING WITH NONSTANDARD UNITS

PURPOSE:

To develop and test methods of placing selected measurements in order by size

MATERIALS:

1. *String*
2. *Scissors*
3. *Glue*
4. *Paper*
5. *Masking tape*

Teacher: Today you will divide into groups of three or four people. Each group will decide what body part to measure, then measure it using string.
For each different part of the body measured, I want to know who's biggest who's smallest and who's the same size as somebody else?
You already measured your wrists, but I couldn't answer those questions with the strings you brought me, because I couldn't tell who the strings belonged to. Some groups may do wrists again to answer those questions. What other things might you measure with your string?
Student: Arms, hand, legs, necks, heads, waists . . .
Teacher: I don't think I will accept waists. Sometimes people get teased about their waists, and I don't want people measured if that might be used to tease them. So waists are out.
Student: We won't tease anybody!
Teacher: I believe you, but we may share what we find with other classes and they may tease people.
Student: How about if we only measure the waists of people who want to have them measured?
Teacher: Well . . . okay. We'll try that and see what happens. What else can you measure?
Student: Feet, fingers, ankles, chests, height . . .
Teacher: Okay. Decide who you will team with and then tell me what your group plans to measure. Each group will measure something different, so we can do as many different things as possible. If a person does not want to be measured, respect that person's decision.

The teacher lists on the overhead each part of the body suggested for measurement. As the groups make their selections, the parts chosen are circled to indicate they are no longer available.

Once a group has chosen, its members are free to move about the room taking measurements. The initial problem with this is that everyone else is moving around the room taking measurements. Some students may be overwhelmed by being measured while trying to measure people who are being measured by others.

The students may enjoy this confusion; if so, there is no reason for the teacher to stop the students and have them reorganize. But if the situation seems to be frustrating them, the teacher has them return to their seats and presents the problem to the class for suggestions of a method for resolving it.

Each measurement activity the students face will present them with different problems to be solved. While the teacher may assist by leading the discussion, any solutions must come from the students themselves. If they are to become problem-solvers, they must be allowed to pose their own solutions to existing problems and experience the consequences of their decisions.

Teacher: I can see some people are having difficulty taking their measurements. Why?
Student: There are too many people trying to measure each other at once. It's too crowded.
Teacher: What do you suggest we do to make it easier?

Student: Let each group take all the measurements the other groups want and then give them the measurements.
Student: No. Let one person in each group take all the measurements for that group. Everyone else just sits still and gets measured.
Student: No. . . .

The solution is selected by class discussion and vote.

Other problems may face the students. Some will collect all the appropriate measurements and then discover they forgot to note which string came from which person. Others may have used bits of paper to label each new string but neglected to attach the strings to the papers. Some may get the masking tape labels stuck to all the string. Students who think they have gathered all their strings, correctly labled, may find there are not enough strings for everyone in class. Students measuring longer lengths like arms, legs, or height may discover they don't have enough room on their paper to put all the string. The string itself doesn't lie there patiently waiting for a solution, but insists instead on tangling itself up into knots, either alone or in conjunction with any and all other string around it.

We might think our job is to shelter the students from confusing situations, but in life there is no shelter. The most we can hope to do is to help them learn how to make problems manageable and, hopefully, how to solve them.

The students continue to collect data, overcoming each new obstacle in turn. If they complete the data collecting, they use the data to answer the three questions that were the basis of the assignment: Who's biggest, who's smallest, and who's measurement is the same size as someone else's?

Each group decides how to assemble its data so the questions may be answered. Just how successful each group is at completing the task is the topic for the next lesson.

LESSON 14-3

MEASURING WITH NONSTANDARD UNITS

PURPOSE:

To discuss methods of placing selected measurements in order by size

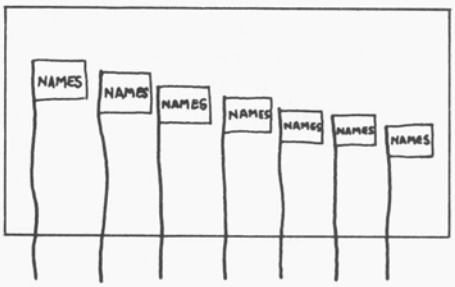
MATERIALS:

1. Any data collected for Lesson 14-2

The teacher leads the class in a discussion of everyone's results from the previous lesson—how far each group progressed, and what each group discovered.

If a group had difficulty completing the assignment, the teacher asks why, not to place blame, but to find out what went wrong so the class may discuss remedies. The rest of the students should not have to make the same mistakes.

The emphasis of the discussion of the completed work is on how the problems inherent in the task were resolved. The actual data is discussed, but is of secondary importance. What is most important is the process used to collect and display the data in a workable manner.



Teacher: Here are the string measures from Rhonda, Lonnie and Russell's group. Can you tell from this paper who's the biggest?
How did you collect all the strings? How did you decide to put them in this order? Did you have any troubles making this? What did you do to take care of the troubles? And so on.

Although displaying the data is of secondary importance, the teacher can help the students make their presentations more meaningful.

Teacher: If I took this paper into the teacher's room and showed my teacher friends this chart, could they tell me which was the biggest?
Student: Yes.
Teacher: The biggest what?
Student: Head.
Teacher: How could they tell these strings measured heads?
Student: Because we measured heads.
Teacher: But how would they know you measured heads without your telling them?
Student: We could write what we were measuring on the paper.
Teacher: That might work. Whose heads did you measure?
Student: Everybody's.
Teacher: Even all the teachers'?
Student: No! Everybody's in the class!
Teacher: How would the teachers know whose heads you measured?
Student: We could write it on the paper.
Teacher: What else do you think might be useful to write?
Student: The names of who made it?
Teacher: Okay. Anything else?

The teacher can assist the class in discovering other information by holding up a paper and asking questions about

it that the group who assembled the data is not allowed to answer. Information needed but not included readily occurs to students who *didn't* prepare the display.

Each group's results are discussed in turn. The data from the groups who completed the assignment is posted on the bulletin board for all to examine, and will be the basis for the following lesson.

LESSON 14-4

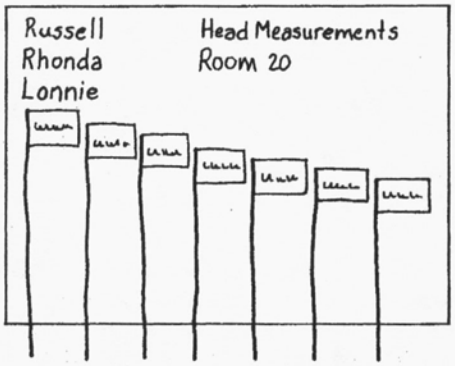
MEASURING WITH NONSTANDARD UNITS

PURPOSE:

To verify and discuss ways to improve the accuracy of specific measurements taken

MATERIALS:

1. *Data from the groups who completed the assigned task in Lesson 14-2*
2. *String*
3. *Scissors*



Teacher: Please look at this. What is it?
Student: It's the page with all the head measurements in class on it.
Teacher: I will read each of your names off the string labels, and point to the string that stands for your head measurement. As I do, see where your string is and decide if that's where you thought it would be. If you are the same size as someone else remember who it is. . . .

I want all the people who are the same size as someone else to get some string and measure each other to see if you really are the same size.

Those of you who think your heads are bigger or smaller than the measurements up here say, get some string, measure yourself, and then bring the string up here and check to see if you agree with this measurement.

An important part of measuring is knowing what to measure and when it has been measured. Most students in class have not yet acquired any such knowledge; to most, a measurement of something is correct. If what they measure differs from someone else's measurement, then that person is wrong. The person who collected the data for the bulletin board, on the other hand, thinks the measurement is right.

Teacher: Sam and John, the chart says your heads are the same size. Did you measure your heads as a check?
Student: Yes.
Teacher: Are your heads the same size?
Student: No!
Teacher: Whose is bigger?
Student: John's.
Teacher: Why does the bulletin board say they are the same size?
Student: They didn't measure right.
Teacher: What did they do that wasn't right?
Student: They got the wrong measurements.
Teacher: But how? When they came around and measured you, did they do something different than you did?
Student: No.
Teacher: Then how did they get a different measurement?
Student: I don't know.
Teacher: Then how can you be sure their measurement was wrong?
Student: Because I got a different one.
Teacher: Who measured John for the bulletin board?
Student: I did.
Teacher: Did you measure him wrong, Debbie?
Student: No!
Teacher: Then why did John get a different measurement than you did?
Student: I don't know!
Teacher: Take this string and remeasure John's head and see if you still get the same length around as before. . . . Is that the same measurement?

Whether or not it is the same, the students must explain how different measures can be obtained for the same person. It is likely when Debbie remeasures John, John may protest she isn't holding the string right. It's too tight, too loose, not around the right part of his head, on crooked, her finger's in the way, and so on. The same process is repeated in reverse when John demonstrates to her how he measured his own head. Each time one student complains about an error in the other student's way of measuring, the teacher asks each how important that factor is in getting an accurate measurement. The students may not really know, but at this point each is convinced some factor caused the other student to measure incorrectly.

The purpose of this debate is to start the students thinking of all the factors that must be considered if two people are to measure an object and get the same result. The teacher should get them to spell out the steps they went through to get their "correct" answers, so someone else can use those same steps to get the same "correct" answer.

In the case of the head measurements, the original students who collected the data explain what they feel were

the important criteria for making the measurements. This will be difficult at first, since it is highly unlikely they considered whether the string should be loose or tight, high or low, et cetera.

These after-the-fact explanations won't really fit the measurements to which they refer, because the students did not really consider these elements as they made their measurements. For future activities, however, students can consider in advance how to measure, so the descriptions of what they claim to have done will more nearly match the techniques actually used.

The teacher encourages an active discussion of why there are so many discrepancies in the measurements, encouraging students to identify the factors that might have caused them. Students who disagree on a measurement may remake it to see if any differences in technique can be detected.

The students may find they know less about measuring at the end of the lesson than they thought they did at the start. The confusion they face now, however, will eventually lead them to a fuller understanding of measurement.

LESSON 14-5

MEASURING WITH NONSTANDARD UNITS

PURPOSE:

To find ratios of length that exist between the various parts of the body

MATERIALS:

1. *String*
2. *Scissors*
3. *Paper*
4. *Spelling notebooks*
5. *Individual blackboards*

The students now know most people's body measurements are different and this will be the source of much of the frustration the students face in later lessons. The next two lessons, however, permit them to make body measurements that in many instances are not different from person to person.

Teacher: You have all been measuring different parts of each other's bodies with string. Today you will only measure yourselves.

I want you to cut a length of string that measures around your wrist, use your wrist measurement to measure the rest of you. I'll show you what I mean.

Let's see . . . my neck is as far around as two wrists, my arm is five wrists long, and my middle finger is half a wrist long.

Each time the teacher completes a measurement, the results are written on the overhead.

Neck	2 wrists
Arm	5 wrists
Finger	½ wrist

Teacher: You can see what I am doing. I want you to do the same thing, and write down what you get. If you need a word spelled, bring me your spelling notebook.

If you can't figure out how many wrists long some part of you is, ask a friend to help you, or measure some other part instead. I want you to find out all the things you can measure with the length of your wrist. Don't worry about things you can't measure.

After ten or fifteen minutes, the teacher begins assembling a list on the overhead of what they have discovered. The students answer the teacher's questions by writing the appropriate measurements on their blackboards.

Teacher: Did you measure your neck with your wrist length? Write the measurements you got on your blackboards and hold them up.

Since I don't have the space to write down all the different measurements everyone got, I'll only write the one people got most often. For necks, two wrist lengths were

the most common, so by "necks" I'll write "two." That doesn't mean if your neck wasn't two wrists, you measured wrong, just that most people in this room have necks two wrists big.

When the most common neck size has been determined, the teacher and the class discuss the following questions:

Whose neck took the fewest wrists around?
Does that person have the smallest or biggest neck in the room? Why?
Why did so many people have necks the same size when we used wrists to measure?
What would a person look like who had a neck only one wrist around?
Would a person like that have a skinny or a fat neck? Why?
Whose neck had the most wrists go around it?
Is that person's neck the biggest or the smallest? Why?
If your neck was two wrists big, would it still be that big if you used the wrist measurement of someone else to measure your own neck? Why?
What would a person look like who had a neck three wrists big? Four wrists big? Why?
Do most people have necks about the same wrist size around as we found for our class? What about adults? What about babies?

These questions are intended to start students thinking about what their measurements may be telling them. They probably will not know why so many of their classmates seem to have necks two wrists around, but that is unimportant.

When necks have been discussed, the measurements for other parts of the body are tallied and discussed. For each new body part, the most common measurement is written on the overhead and the students are asked to think about the following questions:

Why do so many people have wrist measurements so similar?
Is this pattern for our class the same for most other classes?
Is it true for children and adults?
Is there even a pattern at all? Why?

Most human bodies have similar proportions. Not every neck will be exactly two wrists around; still, how many people have necks only as thick as their wrists or as thick as three wrists? Speculation about this is left to the students.

LESSON 14-6

MEASURING WITH NONSTANDARD UNITS

PURPOSE:

To find ratios of length that exist between the various parts of the body

MATERIALS:

1. *String*
2. *Scissors*
3. *Paper*
4. *Spelling notebooks*
5. *Individual blackboards*

This lesson continues the activities of the preceding one, except the students use the circumference of their heads as the basic measuring unit.

LESSON 14-7

MEASURING WITH NONSTANDARD UNITS

PURPOSE:

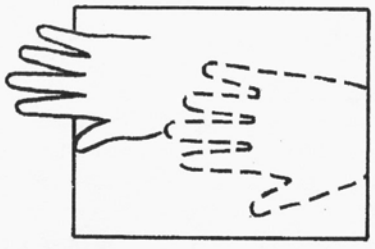
To use the body as a measuring tool

MATERIALS:

1. *The contents of each student's desk*
2. *Paper*
3. *Spelling notebooks*

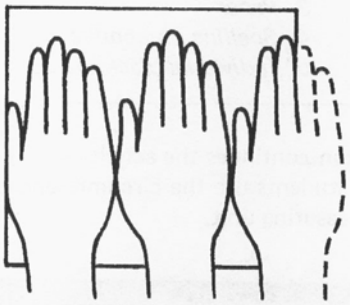
The next few lessons present a progression of activities that confront students with the problems inherent in nonstandard units of measure. They first learn to use their bodies as a measuring tool, then make measurements using body units. An element of frustration is introduced by asking them to place objects in order by size based on the measurements they have taken. Establishing rules to make such an ordering possible is the goal of these lessons.

Teacher: You have been measuring yourselves with string. Today, I want you to measure using yourselves and no string. I'll give you an example.
I will measure this piece of paper using myself as the measure. I know it is longer than my hand, but I'll use my hand to start with anyway. The page is at least one hand long . . . I can fit my thumb into the leftover space, so this paper is one hand and one thumb long.



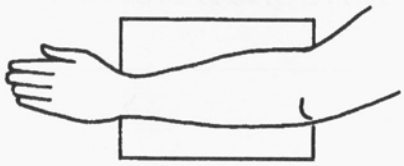
Could I have measured this paper with another part of my body?

Student: Yes, try putting your hand sideways.



Teacher: When I measure it that way, it is two hand widths and two finger widths long. Any other ways?

Student: Hold it up next to your arm.



Teacher: Now, it is as long as my arm from my elbow to my wrist. Any more ways?

Student: Can you use your head?

Teacher: The paper is longer than my head—let's see how much longer. It's longer by the length of my nose. So, this paper is a head and a nose long.

I want each of you to measure everything in your desk using your body. Use what part you think would be best; if it isn't big enough, use some other parts to fill in the extra space. Keep track of what you measured and how long or how wide it was on a piece of paper. If you need a word spelled, bring me your spelling notebook.

Most of these measurements would be of no value to determine the size of anything. Students will, for example, claim an object is the same length as an arm without first deciding what is meant by arm (i.e., is the hand included?).

Eventually, the students will discover the necessity of clarifying a unit of measurement. For now, however, they are confined to producing a wide variety of dimensions from their bodies to serve as measuring devices.

The students spend about ten minutes measuring the contents of their desks, then discuss the techniques with the teacher. The teacher lists on the overhead the different body parts used. The measurements obtained are not discussed. When all techniques have been listed, the students are asked to remeasure some things using untried ways—these too, are added to the list.

LESSON 14-8

MEASURING WITH NONSTANDARD UNITS

PURPOSE:

To use the body as a measuring tool

MATERIALS:

1. *Objects in the classroom*
2. *Paper*
3. *Spelling notebooks*

Teacher: I want you to use your body to measure something inside the classroom. There are only two conditions: first, you must stand on the floor to measure; second, if you want to measure another person, you must ask permission.

You may use any of the ways of measuring already written on the overhead or think up new ways. Be sure to write down what you measured and the result on a piece of paper. If you need words spelled, bring me your spelling notebook.

The students measure doors, windows, cupboards, tables, even the length of the floor. They may choose to measure each other or the teacher. Anything is alright to measure as long as they don't have to climb on top of something to measure it. The teacher lists all the different body parts used in measuring.

LESSON 14-9

MEASURING WITH NONSTANDARD UNITS

PURPOSE:

To use the body as a measuring tool; to create rules for placing measured objects in order by size

MATERIALS:

1. *Objects in the classroom*
2. *Paper*
3. *Spelling notebooks*

Teacher: Today you will form into groups of two or three, and each group will pick *one* thing in the room to measure. No object should be measured by more than one group. Each group may decide which ways to measure using bodies.

More problems exist for groups than for individuals, because groups must come to decisions and not everyone may agree. Students in a group must first decide what they *all* would like to measure, which and whose body units to use.

If the teacher tried to solve each potential problem for the students, many rules and instructions would have to be given. For every rule imposed, an alledged exception would have to be decided. Without "help" in rule-making, most groups overcome the problems without being aware there were problems at all. The few students who cannot resolve a difficulty may use the teacher as a resource. In this case, the teacher presents the group's problem to the whole class for discussion and they provide the decision.

When each group finishes, they write the object and its measurement on a piece of paper and submit it to the teacher, who then writes the measurements, but not the objects, on the overhead.

2 arms
Body and arm
Body, arm, 2 hands
Leg
Hand, finger
Foot, toe
Arm and hand

Teacher: Now, I would like you to help me put these measurements in order from largest to smallest. Then, we'll match them with what they measured to check if they really are in order.

The discussion can go off in many different directions. Some measurements may be easy to rank in order, however, it may not be obvious which is bigger between a foot and toe and, say, a hand and finger. Each time someone disagrees about the relative order of something on the list, he or she must state the reason. Each measurement's position on the list is decided by a vote.

A student will not know what each measurement on the overhead really stands for, so the class must order the measurements by attempting to rank the relative sizes of the dimensions. The students then are asked if the list is in exactly the right order. A person who thinks the order may be wrong is given an opportunity to explain any possible errors. After all complaints have been registered, the class votes to agree or disagree with the final ordering.

body, arm, 2 hands
body, arm
2 arms
leg
arm and hand
hand and finger
foot, toe

Although each placement on the list was decided by majority vote, a majority of the class may not think the whole list is accurate. The collective minorities at each step might constitute a majority. This means that the class vote on the entire list is an informational vote, *not* an accept or reject vote, to revote each item would still lead to the same list, so it, in effect, has already been accepted.

Door – body, arm, 2 hands
Door – body, arm
Blackboard – 2 arms
Desk – leg
Counter – arm and hand
Book – hand and finger
Box – foot, toe

Once the list has been ordered by size, the teacher writes the name of each object next to its measurement and discusses with the students the accuracy of the rankings. If the relative size of any items are in doubt, they may be physically matched for comparison. Objects too difficult to move may be compared in any way the class can devise to produce an acceptable result.

One method of comparing two immovable objects is to mark the height of the object on a student, then check to see if the other object's measured dimension is above or below the mark.

Teacher: Is the list of measurements in the right order?
Student: No.
Teacher: Why?

The answers to this question depend on which items are in the wrong place. Students who earlier voted against placing an item in a particular order and now find it placed correctly are not usually willing to advance their previous reasons. On the other hand, those who said something was out of place before and were outvoted may now state with even

greater confidence why they knew all along that two toes and a hand were bigger than a little finger and a foot.

Some of the reasons given by students for why things are out of order are:

They didn't know how to measure right.
We couldn't tell how big the hand was they measured with—some hands are bigger than others.
I didn't know what they meant when they said it was one leg big. I didn't know where the leg started.
Which finger did they use?
I couldn't tell when they said hand if they meant the long way or the short way. It was too confusing.
People are not all the same size. How could we know if a hand and a finger were bigger than a toe and a foot if we didn't get to see the hand and the foot?

When students have mentioned possible causes for the difficulty in placing all the measurements in order, the teacher asks how they can make their next measurements easier to put in the right order.

It is easy to say what might have gone wrong, but not to offer workable solutions. The students' responsibility is to offer positive suggestions for correcting each complaint registered as possible grounds for earlier error. These suggestions may cause the next set of measurements to be in greater error when placed in order than the original "no rules" list. But this consequence is an integral part of the learning process.

A student suggestion that too many measurements were used the last time creates a new problem—which few measurements should be used now. The suggestion that "hand" needs to be clearly defined raises the problem of what constitutes an acceptable definition.

The problems inherent in each solution cannot be forseen by the students. If they are to understand the importance of standardized units of measure, they must experience the difficulties that follow from operating without such units. Part of this difficulty manifests itself in the problems associated with creating a set of rules to make unworkable measuring tools workable. Which rules are a benefit is something the students will learn in the following lesson.

LESSON 14-10

MEASURING WITH NONSTANDARD UNITS

PURPOSE:

To use the body as a measuring tool; to use the rules developed to facilitate placing measured objects in order by size

MATERIALS:

1. *Objects in the classroom*
2. *Paper*
3. *Spelling notebooks*

The activities in this lesson are essentially the same as those in the preceding lesson. The students divide into groups, select an object, measure it, and submit their results to the teacher. Now they apply any procedures developed in the previous lesson to making their measurements easier to place in order from largest to smallest. A good measure of how well the rules solve problems is whether the number of people who disagree with the order increases, decreases, or stays the same.

Regardless of whether the students have developed a useful set of rules, they are beginning to understand some of the elements that are important to consider if measurements are to be useful as the basis for comparisons. The following lesson should heighten this awareness.

LESSON 14-11

MEASURING WITH NONSTANDARD UNITS

PURPOSE:

To use the body as a measuring tool; to create rules that will permit measured objects to be placed in order by size

MATERIALS:

1. *Student desks*
2. *Paper*
3. *Spelling notebooks*

Teacher: I want each of you to measure across the top of your desk with one or two body measurements, then write them on a piece of paper and hand it to me.

3 hands, 1 finger
3 hands, 2 fingers
27 finger nails
One arm
3 feet
2 heads, one nose

The teacher lists ten or fifteen of the desk measurements on the overhead. The list is composed of measurements easy to compare, such as three hands and one finger, and three hands and two fingers; and measurements hard to compare, such as twenty-seven fingernails and one arm.

After the measurements are written on the overhead, the students are asked to put them in order from largest to smallest. The reactions to this assignment are apt to be diverse. Some students may begin dutifully examining each measurement to determine which is bigger and which smaller. Some students may think the question is silly. If all the desks measured are the same size, then all the measurements must be the same size, too.

Although this assignment appears to ask the students to do the same kind of ordering as in Lessons 14-9 and 14-10, it actually asks them to do something basically illogical. All the desks *are* the same size, so their measurements must be also—the differences among the measurements on the overhead are a product of imprecise measuring.

The purpose of the assignment is to force students to examine the imprecise nature of their measuring devices. Were their bodies capable of producing adequate measurements, the measurements would indicate the desks were all the same size. At any given point, some students will comment on the ridiculous nature of the assignment. As long as this group is in the minority, however, the ordering of the measurements should continue to completion.

Once the list is in the correct order, the students whose desk measurements were used are asked to place their desks in order from largest to smallest. This validates or invalidates the order of the measurements on the overhead.

If the lesson continues to the point where the list of measurements on the overhead is matched with a list of student names for desks put in order by size, the teacher asks the following questions:

Why aren't the desks in the same order as on the list? Why aren't they even close?

How could we measure the desks again to get a list of measurements that could be put in the same order as the desks?

Why was it hard to put the measurements in the correct order? What could we do to make this assignment easier?

If the majority of students indicate the assignment can't be done because all the desks are the same size, the teacher asks:

If all the desks are the same size, why did we get so many different measurements?

Is there a way to measure the desks using body measurements so we can tell all the desks are the same size just by looking at the measurements?

Examples of ways students have suggested to deal with the problem of imprecise measuring devices are among the following:

Have everybody measure with hand widths and finger widths only.

Pick one person out of each row who has the same hand size as one person out of each other row. That person measures all the desks in the row.

Cut out a lot of hand shapes all the same size and give everybody one to measure with.

Have one person go around and measure everybody's desks to see if they are all the same.

The effectiveness of any solutions offered by the students will be tested in the lesson which follows.

LESSON 14-12

MEASURING WITH NONSTANDARD UNITS

PURPOSE:

To use the body as a measuring tool; to test rules that permit measured objects to be placed in order by size

MATERIALS:

1. *Student desks*
2. *Paper*
3. *Spelling notebooks*

This lesson continues the preceding activities so the students may test the measuring rules they developed. They measure their desks and submit the measurements to the teacher, who lists a sampling on the overhead.

Whether the students measure their desks in a manner that enables them to recognize that all the desks are the same size is a function of the rules they developed in the previous lesson. The rules are a function of how clearly they were able to identify the problems inherent in body measurements and how well they overcame these problems, once recognized.

The only way students can get measurements in agreement when measuring the same size objects is to standardize the unit of measurement. Since the assignment required students to make measurements using their own bodies, they cannot get any standardized form using the allowed measuring apparatus. Their only satisfactory solution is to recognize the material with which they are measuring—bodies—is inappropriate for the task and standardize their measurements in some way.

Students may not see the basic problem is caused by each person using different size measuring tools. They may realize the problem and yet not be able to think of an effective solution. But it is possible that placed in this frustrating situation, students will both recognize the problem and come up with a workable solution.

The teacher should not tell the students what the problem is or what might be done about it. The thinking they do now will prove invaluable to their later understanding of alternate systems of measurement.

Remeasuring the desks permits students to test and refine their procedures for making body measurements that can be easily compared. If they develop a set of rules permitting them to recognize from the measurements alone that all the desks are the same size, they retest these rules in the following lesson. If the rules still produce lists of measurements indicating each desk is a different size, they should proceed to Lesson 14-14 instead.

LESSON 14-13

MEASURING WITH NONSTANDARD UNITS

PURPOSE:

To use the body as a measuring tool; to test rules that permit measured objects to be placed in order by size

MATERIALS:

1. *Objects in the classroom such as books, shelves, or chairs, all the same size*
2. *Paper*
3. *Spelling notebooks*

The activities in this lesson are essentially the same as those for the preceding one. The rules developed in the last lesson are now applied to measuring a new set of objects, all the same size.

LESSON 14-14

MEASURING WITH NONSTANDARD UNITS

PURPOSE:

To make a map of the classroom by assembling maps of six subsections

MATERIALS:

1. *Masking tape*
2. *Whatever measuring and recording materials the students decide to use*

By now, the students may have formulated rules for measuring that enable them to agree among themselves on the measured size of various objects. In developing the rules, they had to become aware of many of the factors involved in comparing measurements. The next two lessons provide a new measurement problem designed to expand that knowledge. This lesson establishes the problem and allows students to pose solutions; the next tests the solutions.

Teacher: Today you will make a map of the classroom. You will divide into six groups and each will make a map of one part of the room. Then, we'll put all six parts together and have a map of the whole room.

I have already divided the room into six sections with strips of masking tape on the floor. Your group's map will show your area of the classroom. When you measure for your map, you may use body units or anything else.

Once they begin the assignment, some students may ask questions like the following:

Do we use graph paper or drawing paper?
Do we do the floors and the walls or just the floors?
What about the desks—do we draw them too?
Is the map looking down on the room or sideways?

The students decide the answers to any questions, first in the group, then if necessary by the class.

Near the end of the available time the teacher collects the six sections and puts them together to form a map of the room.

The most common outcome is a very different map from each group. One group may have used art paper and another, newsprint; one map may be two feet by two feet and another only one square foot; one may have desks drawn top view only, another top and side, and another no desks at all. Since the students have not been taught about drawing to scale, it is likely the objects included are not proportionate.

When the sections are posted, the class can see clearly something isn't right—together, the six sections do not make a complete map.

Teacher: Did we make a map of the classroom?
Student: No.
Teacher: Why not?
Student: All the pieces are too different, they don't fit together.
Teacher: Can six different groups make sections that *would* fit together to make a map of the classroom?

The six sections provide the basis for a discussion of what each group needs to consider in drawing a new map. The students can see for themselves the differences and they use this information as the basis for deciding a set of rules for measuring that will help the maps each group draws fit together as a unit.

The considerations students sometimes set for themselves are among the following:

Only one kind of paper can be used.
We can only use this list of things to measure with.
Measure only those things stuck to the floor, and desks.
Make the map as if we were looking straight down. If you couldn't see something looking straight down, then it doesn't go on the map.
Each group has to make sure the space it draws for its map matches with the spaces for the groups around it. It's okay to go to other groups to see if some lines match, like where the wall is and where the counter is.

If the students feel it would be of assistance, the teacher makes a list of their rules for reference.

The purpose of this activity is to illustrate the importance of reaching common agreements on measuring techniques if the objects measured by different people are in any way to be related to each other. In this lesson the students have seen what happens when there are no rules in common. In the next lesson, they will see if their rules produce a better map.

LESSON 14-15

MEASURING WITH NONSTANDARD UNITS

PURPOSE:

To make a map of the classroom by assembling maps of six subsections; to test student-developed rules for map making

MATERIALS:

1. *Masking tape*
2. *Whatever measuring and recording materials the students decide to use*

The teacher divides the class into six groups, then each draws a map for its assigned portion of the room. The students use the rules from the previous lesson as their guide in drawing map sections. However, the list of rules is not likely to cover all the situations that arise. For example, the clock on the wall can be seen from above; is it to be added to the map? The potential for questions about the interpretation of the rules is endless. The answers to each question come from the students; the teacher only enters into a discussion when they are unable to resolve an issue, presenting it to the class for resolution by vote.

Near the end of the time available, the teacher collects each group's section of the map. This second six-piece map is posted beside the first. The teacher and the class then discuss the following questions:

Is the second map better than the first? How?
How is it worse?
Did the rules help?
Were the rules needed?
Would fewer rules have worked as well?
If we did it a third time (which we won't), how could we make the map even better? What rules would you add?
Could we ever get the six separate sections perfect? Why? Why not?

LESSON 14-16

MEASURING WITH STANDARDIZED UNITS

PURPOSE:

To place selected measurements in order by size

MATERIALS:

1. *Objects in the classroom*
2. *Straws*
3. *Sticks*
4. *Cubes*
5. *Paper*
6. *Spelling notebooks*

In earlier lessons students experienced difficulties in using their bodies as measuring tools. These difficulties may have been attributed to the nonstandard units of measure used. In this lesson and the next, students will measure using standard units. As they will see, standardizing the units does not necessarily eliminate all the problems.

Teacher: Today I want you to select ten things around the room to measure and write them on your paper. If you don't know how to spell a word, bring me your spelling notebook.
When you have ten things written down, measure each item and write its measurement. You may use straws, cubes, or sticks.

Student: Can we use something else?
Teacher: Tomorrow, yes, but today I want you to choose from these three things.
Student: Can we work with someone else?
Teacher: Yes. Are there any more questions? Then you may begin.

After the students have measured ten items, the teacher asks them to number the items from longest to shortest, beginning with one for the longest, then turn in the lists. The teacher selects sample lists and writes the measurements on the overhead.

4
5
7
13
21
4
8
12

Teacher: Here is a list of measurements. Are they in the right order?
Student: No.
Teacher: Why not?
Student: The eight comes after the seven. The twelve comes after the seven too, and after the eight.
Teacher: Whoever wrote these numbers should look at them and see if he or she wants to change them. This came from Eddie's paper.
Student: Mine?
Teacher: Yes, see.
Student: But those are in the right order!
Teacher: Doesn't the eight go after the seven?
Student: No! The seven was a book and the thirteen was my pencil, but the eight is how high the cupboard is . . . the cupboard is bigger than my pencil.
Teacher: But you said the cupboard was eight and your pencil was thirteen. Which is bigger?
Student: The cupboard is eight *straws* and my pencil is thirteen *cubes*!
Teacher: How are we supposed to know that?
Student: I'll write down what I used to measure on my paper.

The students who wish to label their measurements may do so.

Once the measurements are labeled, the students must still decide if the list is in the right order. Labeling does not necessarily make placing the measurements in order any easier.

Teacher: Now that the measurements have labels, can you tell me if they're in the right order?
Student: I can't tell if 21 cubes is taller or shorter than 4 straws.
Teacher: Well, how could you find out?

It may be obvious that 12 cubes is less than 7 straws, but it is not so obvious if 57 cubes is longer or shorter than 9 straws. The students need to know how the various measuring units compare with one another. The most common method suggested for determining which measurements are larger is to place those in question side by side and compare them.

As the students verify the ordering of each new list, the teacher asks them to think about the following questions:

Is there a way you could measure the objects so you could tell just by looking at the measurements if the objects were in the right order?
I didn't tell you how to measure with straws, sticks, or cubes. Are you sure you are all measuring the same way?
Is one of the materials I gave you better for measuring with than another? Why?

As the students discuss what might make the task easier, someone may suggest that the ten items should be measured using the same unit. If so, the students may remeasure their ten objects using only the unit decided on, although this might lead them to a new problem. If something small is to be measured, a straw is not appropriate; if the door or the height of a cupboard is to be measured, cubes or toothpicks make tedious measuring tools.

The students continue to verify the order of each new list and offer suggestions for ways to make the process easier throughout the time remaining. If they devise a system they believe will make the ordering of the lists easier, they may remeasure their ten objects and resubmit their lists. If not, they can always physically verify the order for each new list.

LESSON 14-17

MEASURING WITH STANDARDIZED UNITS

PURPOSE:

To place selected measurements in order by size

MATERIALS:

1. *Objects in the classroom*
2. *Straws*
3. *Sticks*
4. *Cubes*
5. *Toothpicks*
6. *Tiles*
7. *Lined paper (for measuring and writing)*
8. *Student-suggested units of measure*
9. *Spelling notebooks*

Teacher: Today I want you to pick ten objects from around the room to measure. Make five large and five small. Write each object on your paper before you begin measuring.

Measure your objects with the straws, cubes, and sticks you used yesterday. You may also use toothpicks, tiles, lined paper, or anything else you think would be good.

When you finish, number your objects starting with one by the largest. Then, turn in your list.

The same process of verifying the order of each list begun in the previous lesson is undertaken.

As the potential number of measuring units increases, so does the complexity of determining when a list of measurements is in the right order. If these increased complexities are to be dealt with effectively, however, the students must make their problems more manageable.

The measuring activities with which the students have been presented so far have had two purposes: first, to challenge their thinking, and second, to expose them to the kinds of problems that have historically lead to the development of standardized systems of measure. The second purpose provides the students with a background so they can appreciate the advantages of the metric system of measure.

Although the students now have a good basis for accepting the use of metric measures, they will not yet be introduced to more formal systems of standardized units. Instead, their measuring activities are channelled into the area of graphing. Measuring is a process of finding out about things; graphing is a method of displaying what has been measured, so the discovery may more easily be analyzed.