CHAPTER 15

Graphing—Pictorial Representations

Prerequisite chapters:

Chapters 13 and 14

MATERIALS

For overhead projector or teacher demonstration use:

Transparencies	Graph paper, 1.7 cm squares	Worksheet 5
Buttons		Materials chapter, page 297
Cubes		Materials chapter, page 295
Large cardboard box (screen for graphs)		Materials chapter, page 298

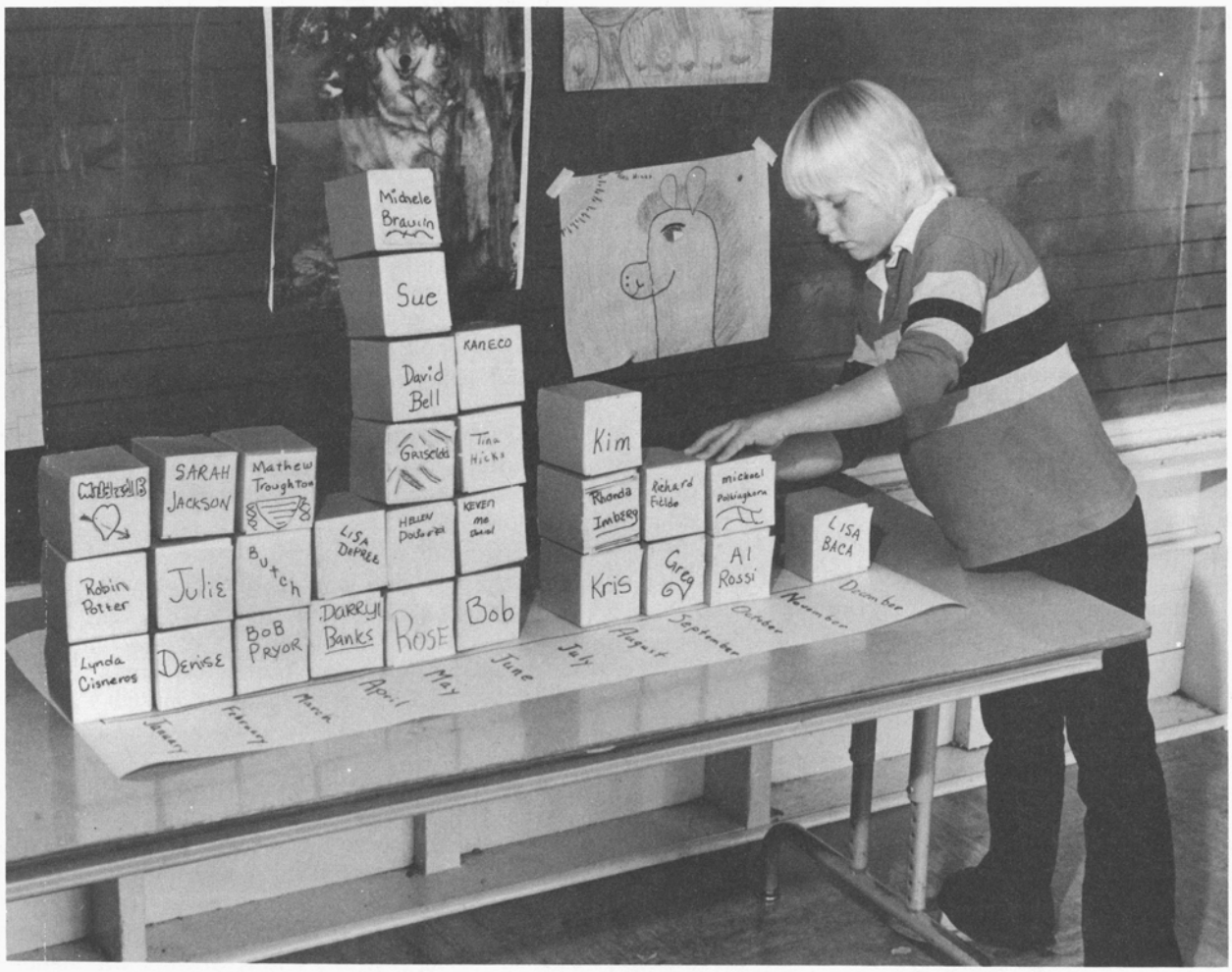
If no overhead projector is available:

Make charts in place of transparencies	Materials chapter, page 294
Button-shaped cutouts	Materials chapter, page 297
Square shapes	Materials chapter, page 295

Student materials:

Dittos	Graph paper, 1.7 cm squares
Name boxes	Materials chapter, page 298
Individual blackboards	Materials chapter, page 294
Spelling notebooks	Materials chapter, page 296

Cubes
Scissors
String, yarn, or ribbon
Butcher paper
Glue or tape
Paper



Graphing organizes data so patterns and relationships that might otherwise be missed stand out clearly. The lessons in this chapter present a series of activities to help students learn how to represent relationships between objects and events on a graph.

The graphs assembled display information pictorially through symbols such as *x*'s or squares to represent the events themselves. The students are led to the use of abstract symbolization through a series of steps designed to make clear the process of representing people, objects, or events with squares or marks.

This chapter's activities also provide students experience in creating their own graphs, which are then used as a reference for answering questions.

LESSON 15-1

TWO-GROUP GRAPHING WITH OBJECTS

PURPOSE:

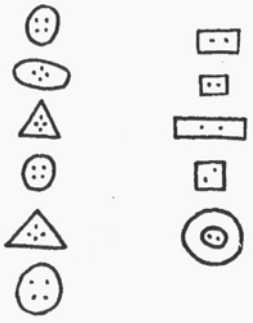
To answer more-than—less-than questions about objects displayed on the overhead

MATERIALS:

1. *If no overhead projector is available, button-shaped cutouts*
2. *Buttons*
3. *Individual blackboards*

These first few lessons allow students to explore ways of arranging materials. The purpose of the explorations is to provide a basis for making comparisons between various ways of displaying information so the students may judge which ways make questions about the material more easily answered.

Teacher: Here is a pile of buttons. I will sort them as you watch.... Can you tell me what my rule might have been?



Student: Two holes and four holes.
Teacher: Does that rule fit all the buttons?
Student: Yes.
Teacher: Okay. Then that could have been my rule. How many more four-hole buttons are there than two-hole buttons?
Student: Six.

Questions involving the concepts of "more than" or "less than" often give students difficulty. What the students think the teacher wants to know is which group has more or less in it and how many are in *that* group.

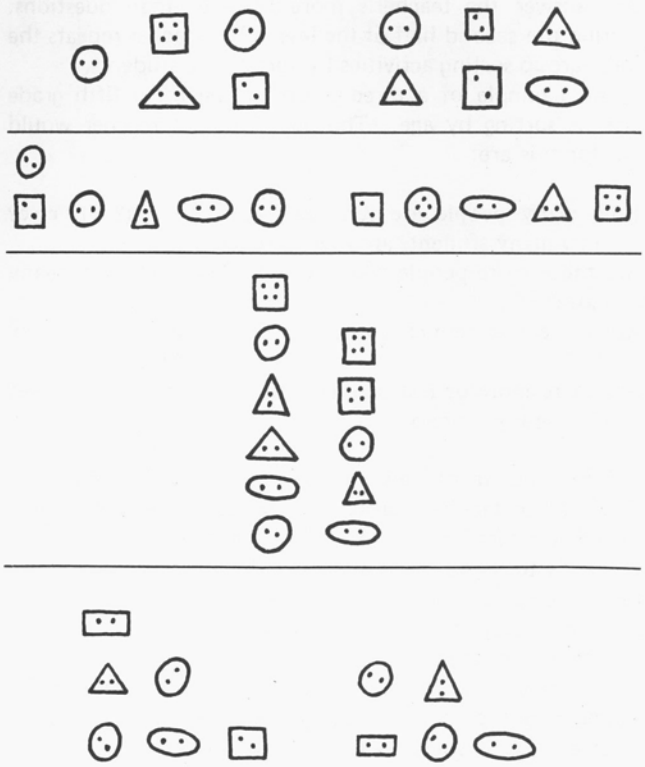
Teacher: How many buttons have four holes?
Student: Six.
Teacher: How many buttons have only two holes?
Student: Five.
Teacher: Which group has more buttons in it?
Student: The group of four-hole buttons?
Teacher: How much bigger is it?
Student: What do you mean?
Teacher: How do you know that the four-hole button pile has more buttons in it than the two-hole pile?
Student: We counted.
Teacher: How many more buttons are in the four-hole pile than the two-hole pile?
Student: Six.
Student: One.
Teacher: If I wanted to have the two-hole button pile be the same size as the four-hole pile, how many more two-hole buttons would I need to get?
Student: One.
Teacher: Okay. When I ask how many more four hole buttons there are than two hole buttons, what I mean is, how many more buttons would I need to make the two piles even. How many more would I need?
Student: One.
Teacher: So, how many more four-hole buttons are there than two hole buttons?
Student: One.

When the students can state which column has more or less buttons in it and how many more or less, the teacher sorts the buttons into two new groups and repeats the same questions.

By what rule might I have sorted the buttons?
Does that rule fit all the buttons?
How many buttons are in the _____ column?
How many buttons are in the other column?
Which column has more (or less) buttons in it?
How many more (or less) does that column have?

Once the students understand what is meant by the questions, the teacher sorts the buttons using a different rule. For each new rule, the buttons are arranged in a different way. Examples of other ways the buttons might be displayed can be seen in the figures on the next page. The buttons are arranged differently with each new sorting to acquaint students with a variety of ways information may be displayed.

The students will decide in later lessons which ways are more useful in answering the teacher's questions.



All graphing can do is help students organize data into potentially more useful displays of information. The more complex the data, the more useful graphs become as organizers of that data. But no one way of displaying data is inherently superior to another. What determines the method selected is the kind of questions the graph is expected to assist in answering.

The teacher continues to sort the buttons into two groups and asks students more-than–less-than questions throughout the time available for the lesson.

LESSON 15-2

TWO-GROUP GRAPHING WITH PEOPLE

PURPOSE:

To answer more-than–less-than questions about students in the classroom

MATERIALS:

1. Individual blackboards

Teacher: Everyone who is wearing white please stand up. How many people are standing? . . . How many sitting? . . . How many students are there altogether?
Are more people sitting or standing? . . . Less? . . . How many more people are sitting than standing? . . . Less?

The same kinds of questions the students were asked for groups of buttons in the previous lesson they are now asked for people. When they have answered the questions for the first two-group sorting, the teacher sorts the students in a different way. For the second sorting, students are also asked to stand so the information they represent is displayed differently.

Teacher: I want each person who is wearing blue to go stand by the desk of someone who isn't wearing blue. Johnny, you are wearing blue, so I want you to stand by Sammy's desk. Brenda, go stand by Gregory. Anyone else who is wearing blue, look around and find the desk of someone who isn't wearing blue and stand by it. Only one person wearing blue to a desk, please!
How many people are standing? . . . How many people are still sitting? (And so on.)

When the students have answered the same basic questions for the second grouping as they did for the first, the teacher asks them:

When I had the people wearing blue stand by the desk of someone who wasn't wearing blue, did it make any of my questions easier to answer? . . . Why? . . . Why not?
Is there another way we could arrange ourselves that might make my questions easier to answer? Or doesn't it make any difference how or where we stand?

The students are asked to suggest different ways to arrange themselves so the teacher's questions for subsequent groupings will be easier to answer. Each new suggestion is tried in turn.

Suggested methods may range from having everyone remain seated while one group raises its hands to having everyone assemble in matching columns. Each suggestion is tried out on a different sorting task so the students may judge its ability to make questions easier to answer.

When students decide a way of organizing themselves that makes the questions either easier or more difficult to answer, the teacher asks them to think about why this is so.

For the specific questions asked for each new sorting, some methods of grouping are more effective than others. Students should become aware that there is a wide variety of ways information can be displayed. They should also begin to distinguish between useful and useless methods.

The teacher continues the activities throughout the time available.

LESSON 15-3

THREE-GROUP GRAPHING WITH OBJECTS AND PEOPLE

PURPOSE:

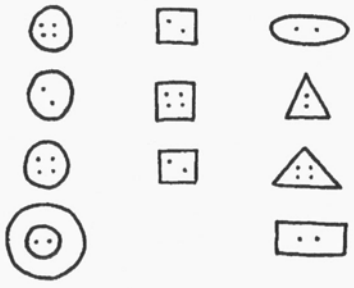
To answer more-than–less-than questions about objects on the overhead and students in the classroom

MATERIALS:

1. *If no overhead projector is available, button-shaped cutouts*
2. *Buttons*
3. *Individual blackboards*

The activities for this lesson are essentially the same as those in the previous two lessons. The only difference is the buttons and students earlier sorted into two-group categories are now sorted into three-group categories.

An example of a three-group sorting for buttons can be seen in this figure.



The questions the teacher would ask are among the following:

How many round buttons?
How many square buttons?
How many buttons are neither square nor round?
How many buttons altogether?
Are there more round or square buttons?
How many more round than square ones?
Are there more square or other buttons?
How many less square than other buttons?
Are there less round or other buttons?
How do you know?

Although the structure of the situation is nearly the same as for two groups of buttons, there is much more information with which the students have to deal. Now, if they are to tell how many more round buttons there are than squares, they must look specifically at the rounds and squares, *not* at the pile of other buttons. For the first time, the students must decide to selectively exclude information they have.

For each new sorting, the teacher places the buttons in different arrangements. The students then offer suggestions as to which sorting categories the teacher might have used, and answer the teacher's more-than–less-than questions. During the second half of the lesson the teacher repeats the three-group sorting activities by sorting the students.

An example of a three-group sorting for a fifth grade class is sorting by age. The questions the teacher would ask for this are:

How many people are ten years old? Eleven? Twelve? How many students are there altogether?
Are there more people who are ten or eleven? How many more?
Are there less people who are ten or twelve? How many less?
Are there more or less people who are eleven than twelve? How many more or less?

The problem of how the people information might be displayed to facilitate answering the questions is presented to the students immediately. The teacher asks all the ten-year-olds to stand, then sit, then the eleven-year-olds and then twelve-year-olds. The students are asked how they wish to arrange themselves to answer the more-than–less-than questions the teacher is about to ask.

The teacher next asks a new three-group question; for example, grouping blue eyes, brown eyes, and all other color eyes. The students again organize themselves to make the questions easier to answer.

For three groups of people, the contrasts between practical and impractical methods of organizing the information are much more pronounced than for two. The teacher asks the students why their way of organizing obscures the answers or makes them easier to come by.

LESSON 15-4

FOUR-GROUP GRAPHING WITH OBJECTS AND PEOPLE

PURPOSE:

To answer more-than–less-than questions about objects on the overhead and students in the classroom

MATERIALS:

1. *If no overhead projector is available, button-shaped cutouts*
2. *Buttons*
3. *Individual blackboards*

The activities in this lesson are the same as those in the previous lesson. The only difference is the buttons and the students are now sorted into four categories rather than three.

An example of a four-category sorting for buttons might be: curved edges with four holes, curved edges with two holes, straight edges with four holes, and straight edges with two holes. An example of a four category sorting for people might be: boys wearing red, boys not wearing red, girls wearing red, and girls not wearing red.

LESSON 15-5

TWO-GROUP REPRESENTATIONAL GRAPHS

PURPOSE:

To learn a process of recording data using objects to represent people

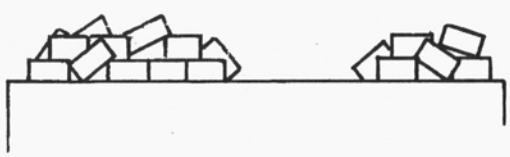
MATERIALS:

1. *Name boxes*
2. *Individual blackboards*

The graphs in the first four lessons were formed from the questions about buttons or people. In the next few lessons *name boxes* are used to show students how graphs about people may still be used to answer questions when there are no people in the graph.

Teacher: Today I want you to sort yourselves into two groups. Instead of standing up or raising your hands, though, you'll use your name box. I'll explain what I want you to do once I tell you what two groups we will start with today.

I want to know who has on long pants and who is wearing a dress. Think about which group you belong to. If you are wearing long pants today bring your name box up and place it on the counter. If you are wearing a dress today, bring your name box up and place it on the counter, but keep it separate from the other boxes.



Which group has more?

Student: Pants.
Teacher: Which group has less?
Student: Dresses.
Teacher: Are more people wearing pants today or dresses?
Student: Pants.
Teacher: Are less people wearing pants or dresses?
Student: Dresses.
Teacher: How many people are wearing pants? The rule for this game is, you can't get out of your seat to answer the question.

Some students may attempt to count the name boxes in the pants section form their seats; others start looking around the room and count those who they can see have on pants. Others ignore the question because they can't see from their seats how many are in the pile.

The teacher asks the students to think of different ways the boxes might be arranged so they can tell from their seats how the class has divided itself. The teacher follows the students' instructions, arranging the boxes accordingly. For each new way, the students are asked to answer the same comparative questions of how many more are in one pile than the other, and if that way of arranging the boxes makes the question easier to answer.

When the class has found an appropriate way(s) of organizing, the teacher redistributes the boxes and asks another two-group question. Now, the students decide how they will group their boxes before they bring them to the front counter. At this point, the students are asked to predict the outcome: which pile will be bigger or smaller, and by how much? When the boxes have been placed on the front counter, the teacher asks the same kinds of more-than–less-than questions as for earlier two-group graphs.

Initially, the two-group topics are based on physically observable traits. Next, the teacher asks the students to sort themselves into two groups using attributes that are not visibly evident:

Would you rather eat lunch at school or at home today?
Would you rather go to the beach or to the park tomorrow?
Which is more fun for you, math or reading?
Would you rather have a car or a motorcycle?
And so on.

The students assemble as many two-group graphs as time permits.

LESSON 15-6

TWO-GROUP REPRESENTATIONAL GRAPHS

PURPOSE:

To use a process of recording data to find answers to student-generated questions

MATERIALS:

1. *Name boxes*
2. *Individual blackboards*

This lesson continues the activities in the previous lesson. Now, the topics graphed come from the students themselves. The only conditions placed on the topics is that there must be only two choices in the question and, if appropriate, no topics are permitted that might hurt or offend someone.

If the teacher decides to reject a suggestion, the reasons should be matter-of-factly explained to provide the students with a framework for understanding what the teacher will and will not accept.

When the students have an opportunity to make suggestions for possible graphs, graphing quickly becomes a way to find answers to questions of real interest.

LESSON 15-7

REPRESENTATIONAL GRAPHING USING THREE OR MORE GROUPS

PURPOSE:

To use a process of recording data to find answers to specific questions

MATERIALS:

1. *Name boxes*
2. *Individual blackboards*

The activities in this lesson are essentially the same as those in the previous two lessons, but now the graphs are not limited to two categories per topic.

The teacher begins by having the students heap their boxes in three separate piles on the front counter in response to the questions, "Who has brown eyes? . . . blue eyes? . . . any other color eyes?" The students try to answer the teacher's more-than—less-than questions using the unorganized boxes; they then suggest how the boxes might be arranged to facilitate answering. For the next topic, the students are no longer limited to three categories or visible attributes. An example of such a topic is, among these four colors, which is your favorite?

After the initial topic, the students suggest how the boxes are to be arranged to facilitate answering the questions before they bring their boxes to the front of the room. They may use the same system or try new arrangements as the number of categories changes. The decision is a product of student discussion and vote.

The teacher presents the first five or six topics to be graphed; after this, the students suggest topics. No limit is placed on the number of categories generated by the topics.

LESSON 15-8

TWO-GROUP REPRESENTATIONAL GRAPHING

PURPOSE:

To learn how to ask questions about graphed date

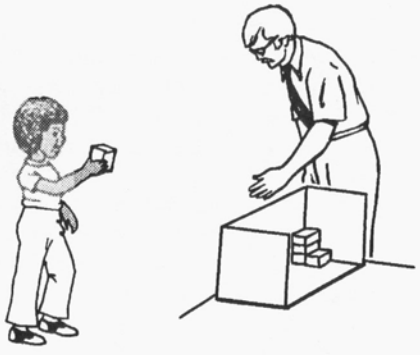
MATERIALS:

1. *Large cardboard box cut as a shield for the name boxes*
2. *Name boxes*

The students can now assemble graphs and use them as the basis for answering the teacher's questions. In this lesson the students learn how to ask their own questions.

Teacher: Today I want you to tell me which place you go more often, McDonald's or Jack-in-the-Box. You will come up one at a time and whisper to me into which pile to put your box. When you give me your box, I will put it behind this screen.

Row one, bring me your boxes please.



The teacher assembles the graph behind the shield so no one sees what the graph looks like.

Teacher: So far, when we have made graphs together, I have always asked you questions about the graphs and you answered them. This time, I want you to ask me the questions. I'll write each question on the overhead. After you've asked all the questions you want, I'll answer

them. Once you say you're through asking questions and I start answering the ones on the overhead, you can't ask any more, so make sure you ask all the questions you want to know.

Who has any questions?

Student: How many boxes in the McDonalds pile?
Student: How many boxes in the Jack-in-the-Box pile?
Student: How many boxes altogether?
Student: Which had more, McDonalds or Jack-in-the-Box?
Student: How many more or less did each one have than the other?
Teacher: Any more questions? Is that all you want to know?
Student: Yes.
Teacher: Are you sure?
Student: Yes.
Teacher: Okay.

The teacher then reads the questions on the overhead one at a time and looks at the boxes behind the shield to find the answers. After answering the last question, the teacher disassembles the boxes without letting the students see the graph.

Student: Hey! Wait! We want to see the graph!
Teacher: Why?
Student: I want to see which pile my box was in.
Teacher: But you know which pile you are in because you told me where to put your box.
Student: But I want to see where everybody else was, too!
Teacher: Oh! This graph is for practice in asking questions. You only get to learn about what goes on behind the screen by asking questions. I thought you had asked all the questions you wanted to.
Student: But I thought you were going to turn it around.
Teacher: Why would I need to do that? You already learned everything you wanted about the graph by asking me questions.
Student: "But I like to see where everybody is. I'd like to know how everybody voted. Did more boys or girls pick McDonalds?
Teacher: If you wanted to know that, how could you have found out?
Student: Ask a question?
Student: I didn't know you meant *that* kind of question!
Teacher: I meant any kind of question you wanted me to answer for you.
Student: I didn't know that's what you meant.
Teacher: Well, for the next graph, make sure you ask me everything you want to know about the graph, because the next one will be made behind the screen, too.

The teacher returns the boxes to the students, asks another two-group question, and the process is repeated.

From now on, the students suggest their own topics, with the only condition being that the topics form a two-column graph.

LESSON 15-9

REPRESENTATIONAL GRAPHS WITH THREE OR MORE GROUPS

PURPOSE:

To learn how to ask questions about graphed data

MATERIALS:

1. *Large cardboard box cut as a shield for the name boxes*
2. *Name boxes*

The activities for this lesson are the same as those in the previous lesson, but no limit is placed on the number of categories included in the topics explored.

The teacher selects a topic for the class to graph, such as, What television shows do you like to watch on Saturday morning? The process continues as in Lesson 15-8. Then, a second topic is graphed. After the second example, the students suggest the topics to be graphed.

LESSON 15-10

REPRESENTATIONAL GRAPHING WITH ANY NUMBER OF GROUPS

PURPOSE:

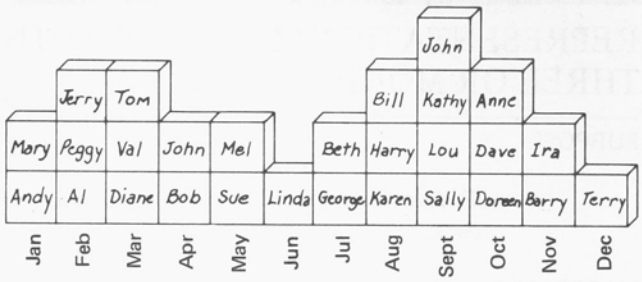
To record graphs for later use.

MATERIALS:

1. *Name boxes*
2. *Butcher paper*
3. *Glue*

Each graph made in the previous lessons had to be disassembled before a new graph could be made. The lessons that follow present a series of activities providing students with systems of recording graphs in more lasting forms and showing them why a square or mark on a piece of paper can be used to represent a person, object, or event.

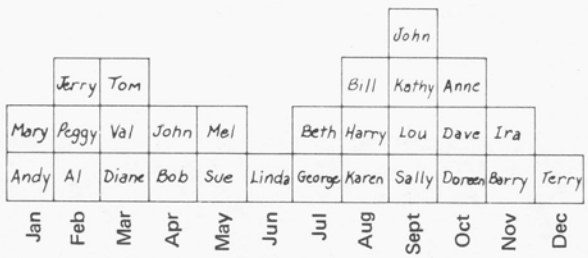
Teacher: We won't build today's graphs behind the shield. Everyone born in January, bring your box up and put it in this pile . . . everyone born in February in this pile . . . March in this pile. . . .



Each student places his or her box in the appropriate pile.

Teacher: We'll refer to this graph in later lessons, so we need to save it. I want everyone whose box is in the January pile to take your name card out of the pocket on your box and glue it above the "January" on this piece of paper.
Now, the February pile. . . .

The process continues until all the name cards in the boxes have been transferred to the paper.



Teacher: You have had a chance to ask questions about graphs you couldn't see, because they were behind a screen. Now I want you to decide what questions could be asked about a graph you *can* see. Look at this graph and tell me what questions you think the graph could be used to answer.

The teacher writes each question the students ask directly on the paper.

It is common to accompany graphs with lists of statements that tell what the makers of the graph have discovered. A more open-ended procedure is to accompany the graph with questions. When questions are included with the display of data, students who didn't make the graph may use it for answering the questions—this helps them learn how to use graphs to find answers. It also helps students appreciate the value of graphs in answering questions.

When the first graph is assembled and five or six questions recorded, the teacher issues the students new name cards for their name boxes. The teacher then selects a topic and a second graph is assembled in the same manner as the first.

The third graph is assembled without the use of the boxes. Instead, the students glue their name cards directly onto the paper. Again the teacher decides the topic and assigns the categories. After the students glue their name cards on the paper, they suggest several questions for the teacher to write on the graph.

Beyond the third graph, the students suggest all the topics. For each new graph made, questions are recorded by the teacher directly onto the graph itself.

LESSON 15-11

REPRESENTATIONAL GRAPHING

PURPOSE:

To learn an alternate system of compiling data in a graph; to decide topics to be graphed in a future lesson

MATERIALS:

1. *Unifix cubes*
2. *Individual blackboards*
3. *Spelling notebooks*
4. *Lined paper*

Teacher: Who are some of your favorite monsters on television or in the movies?
Student: King Kong.
Student: Godzilla.
Student: Frankenstein.

The teacher writes each monster's name on the overhead, then gives each student one Unifix cube.

Teacher: Okay, everyone in the first row bring your cube up and put it next to the name of your favorite monster. Second row, bring up your cubes. . .

Once the monster graph is assembled on the overhead, the students ask the questions they think it could be used to answer, and the teacher writes the questions next to the graph. The students then use the graph to answer their own questions, one at a time, on their blackboards. This process is repeated for two more graphs.

Teacher: I want you to divide yourselves into groups of three or four and decide something you could graph that you would like to know about the people in our class.

When you decide what you want to ask, write it on a piece of paper, sign all your names to it, and bring it up to me. If you need a word spelled, bring your spelling notebook.

When each group has submitted its topic, the lesson is complete for that day. After school, the teacher writes each topic on a ditto master in the form of individual questions. A copy is made for each student to use in the following lesson.

1. What is your favorite TV show?
   _____
2. What kind of ice cream do you like best?
   _____
3. Do you like school?
   _____

LESSON 15-12

REPRESENTATIONAL GRAPHING

PURPOSE:

To compile data in graphs; to decide topics to be graphed in a future lesson

MATERIALS:

1. *Dittoed copies of the topics from lesson 15-11*
2. *Unifix cubes*
3. *Spelling notebooks*
4. *Scissors*
5. *Lined paper*

Each student is given a copy of the list of questions developed by the class in the preceding lesson. The teacher reads each question aloud. Students who wish to may read ahead, but this should not be required of any student during the math lesson. (A rationale for this may be found in the "spelling notebook" section of Chapter 23, Materials.) The teacher's stated reason for reading each question aloud is to insure that everyone can read the teacher's handwriting. The students write answers next to each question on the paper.

When all the questions have been answered, the students cut them apart. The teacher uses numbered pieces of paper to designate areas on the front counter where each question is to be placed. Each row of students comes up one at a time to place the question by its corresponding number on the counter. This puts each group's question in a separate pile.

Teacher: When I say so, I want one person from each group to pick up your group's question and everybody's answers to it.

When you have your answers, make a graph using cubes to show what people said. Your group may decide how it wants to show the answers with the cubes. Also, think of questions your graph might be used to answer.

Okay. One person from each group pick up your questions, please.

Since the teacher has not said how the cubes are to be arranged, the students may form their graph in any way they wish.

When the cubes are assembled, each group in turn displays its findings. The members state the topic graphed, and make their graph visible to their classmates in some way. The students who made the graph then ask the rest of the class the questions they have formulated. The class answers the questions by referring to the cubes.

Group: We did our graph on who is more popular, the Osmonds or the Jackson Five. Look at our graph and we'll ask you some questions.

Who is more popular?

Class: We can't see your graph, hold it up!

Class: Who has the Osmond Brothers pile?

Group: Eddie does.

Class: Then the Jackson Five is more popular.

Group: By how many more?

Class: Hold the cubes up so we can count them . . . no, not Eddie's! Both piles!

Cubes are often inconvenient for graphing. They fall over easily, they are not easily seen by the whole class when placed on someone's desk, and there isn't a convenient way to write what each pile stands for so the whole class can see it. The cubes are particularly awkward for the groups who have a topic that has generated more categories than the members can hold up at once. If, for example, some group has twelve months of the year represented and has only three group members, when the class asks that all the cubes be held up, the group has problems.

The name boxes used in earlier lessons form a necessary link in the minds of some students between a concrete event or experience and the transferring of a record of that experience to paper. Without the intermediate step of the name box, some students are unable to grasp the association between marks on a graph and the event the marks purport to represent.

Cubes serve the same transitional function as the name boxes. The added attraction of the cubes is their inconvenience. Their awkwardness helps students appreciate the system of graphing offered in place of the cubes, the subject of the following lesson.

When each group has had its turn, the teacher has the groups write new topics to be graphed in the next lesson. After school, the teacher again dittos the individual questions.

LESSON 15-13

REPRESENTATIONAL GRAPHING

PURPOSE:

To compile data in graphs; to decide topics to be graphed in a future lesson

MATERIALS:

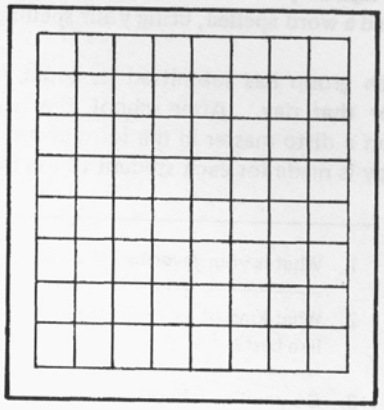
1. *Dittoed copies of the topics from Lesson 15-12*
2. *Graph paper 1.7 cm squares on an overhead transparency, or graph paper on a large tagboard*
3. *If no overhead projector is available, paper squares*
4. *Unifix cubes*
5. *Spelling notebooks*
6. *Scissors*
7. *Dittoed copies of 1.7 cm graph paper*
8. *Lined paper*

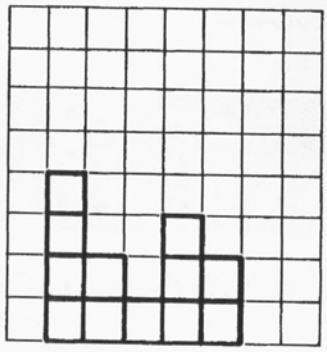
This lesson begins as did the previous one. The teacher reads aloud each question on the dittoed sheet. After the students write their answers, they cut the questions apart and stack them on the appropriate numbers on the front counter. Each group then uses its pile of questions as the basis for a cube graph.

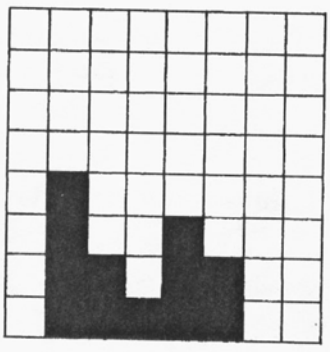
Teacher: Before I ask each group to show us its cube graph and ask questions about it, I want to show you a way of recording the information so you can save the graphs you have made.

I have made a cube graph on the edge of the overhead. Here is a sheet of graph paper whose squares are the same size as the cubes. I'll lay each one of my stacks of cubes on the graph paper. Next, I'll remove one row of cubes at a time and color in the exact number of squares on my graph paper the row of cubes covered up.

I now have a copy of my cube graph on my paper. Take a sheet of graph paper and record your cube graphs, too.







Usually one example by the teacher is enough because the students are working in groups. Groups learn faster than individuals, because the individual students in the group share their understanding with other members.

Once the graphs have been recorded on paper, the teacher has each group in turn display its findings. The members of the group state the question they graphed and hold up their graph paper recording. The students who made the graph ask their classmates questions the group thinks can be answered by looking at the graph.

When each group has displayed its data and asked the class questions, the teacher has them write down the new topics about which they would like to graph in the next lesson. After school the teacher makes a ditto of the questions.

LESSON 15-14

REPRESENTATIONAL GRAPHING

PURPOSE:

To compile data in graphs; to decide topics to be graphed in future lessons

MATERIALS:

1. *Dittoed copies of the topics from Lesson 15-13*
2. *Unifix cubes*
3. *Spelling notebooks*
4. *Scissors*
5. *Dittoed copies of 1.7 cm graph paper*
6. *Lined paper*

In the next few lessons the students' skills in asking questions about graphs are combined with their skills at recording graphs on paper to allow them to discover ways to make their graphs more useful in displaying information.

This lesson begins in the same manner as the previous two lessons.

Teacher: Yesterday, each group held up its graph and asked the rest of the class about it. Today, I want you to think of all the questions your graph could be used to answer and write them down. You may write the questions directly on your graph paper, or use a separate sheet of paper. If you need a word spelled, bring me your spelling notebook.

When the groups finish writing their questions, the teacher collects the papers. One of the purposes of the written questions is to focus student attention on the information that needs to be contained on a graph to make it useful in answering questions. To eliminate the students temptation to add orally information they see is needed after a question is read, the teacher rather than the students reads the questions to the class.

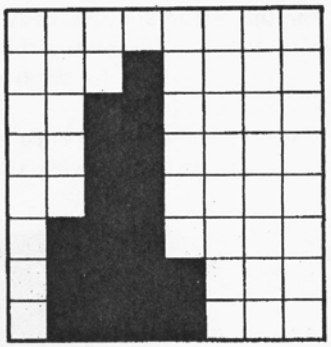
Teacher: Look at this graph. I want you to use it to answer some questions I will read aloud. Only the people who didn't make this graph are to answer.

How many boys like kickball?

Student: Which column is for kickball?

Teacher: You have to look at the graph and use it to answer the question.

Student: We can't answer that question. It doesn't say which column is for which thing. It doesn't even say what question the group made the graph for, or even who made the graph.



1. How many boys like kickball best?
2. How many boys like baseball best?
3. How many boys like football best?
4. How many boys like volleyball best?
5. Which is the most popular?
6. Which is the least popular?
7. How many boys voted?

Teacher: Well, I know I asked the people who made the graph to remain silent, but I'll add a rule. Any group that wants to add more information to its graph may come up at any time and get it from me, even if it is the one I am talking to the class about at that moment.

The knowledge students gain while trying to use other people's graphs to answer other people's questions helps them learn what information is critical for each graph to contain. The more the students hear their own questions read, the more they are able to anticipate the needed information before the graph and its questions are submitted.

When each group's graph has been shown to the class and its questions read and answered, the teacher has the groups write new topics they would like to graph in the next lesson. The teacher makes a ditto of the questions.

LESSON 15-15

REPRESENTATIONAL GRAPHING

PURPOSE:

To compile data on graphs; to decide topics to be graphed in future lessons

MATERIALS:

1. *Dittoed copies of the topics from Lesson 15-14*
2. *Spelling notebooks*
3. *Scissors*
4. *Dittoed copies of 1.7 cm graph paper*
5. *Lined paper*

The activities for this lesson are essentially the same as those in the previous lesson. The only difference is each group records its graph directly onto graph paper rather than building it first with Unifix cubes.

At the end of the lesson, the groups once again prepare lists of topics to be graphed on the following day.

LESSON 15-16

REPRESENTATIONAL GRAPHING

PURPOSE:

To compile data on graphs using a variety of student-created methods

MATERIALS:

1. *Dittoed copies of the topics from Lesson 15-15*
2. *Spelling notebooks*
3. *Scissors*
4. *Whatever materials or paper the students elect to use*

The activities begin as in the previous lesson. The purpose of this lesson, however, is to allow students to expand their thinking about the ways data may be displayed on a graph. As the students prepare to convert the information from their topics into graphs, the teacher says:

Teacher: You made all your graphs for the last few days by coloring in squares on graph paper. Can you think of some ways of recording the information from your questions besides coloring in squares?

Student: What do you mean?

Teacher: When we made the graphs from the name boxes, we glued your name cards to the paper. That was another way of making a graph. Could we put something in the spaces of your paper besides colored squares?

Student: People's names?

Teacher: Okay. What if I made a graph of who likes baseball and who likes football?

Student: We could put baseballs in some squares and footballs in others.

Teacher: How about for the kinds of pets you have at home?

Student: We could draw pictures of the kinds of pets.

Teacher: When you were experimenting with different ways to arrange the buttons or arrange the name boxes so that you could answer my questions more easily, did you always put the buttons or name boxes in columns?

Student: No.

Teacher: Do you think you could find different ways to record the information from your topics besides placing all the colored squares in rows or columns?

Student: Maybe.

Teacher: Do you think you always need to record your information on the same graph paper? Could you use a different size graph paper? Do you even need to use graph paper at all? What if you used a piece of colored construction paper?

Today, I want you to think of a way to display the information for your topic so we can all clearly see what you found out. You may do this in any way you can think of, or with ways we have already used. The only requirement is that we be able to answer your questions when we look at your graph.

When the groups finish making their graphs, they write the questions to go along with them, then the teacher collects them. The measure of the effectiveness of each group's graph, regardless of how it has been made, is if it can be used by the class to answer the accompanying questions. If the graph needs to be changed so it is more useful, the group is free to change it. The goal is graphs that communicate information clearly.

There is a fine line between encouraging diversity and discouraging participation. The statement "John found a *better* way to graph" condemns methods used by other students. The statement "John found a *different* way to graph" provides the potential for other students to continue to feel their work has value. This potential can only be realized, however, if teachers maintain an awareness of how important it is to let every student know that his or her work is accepted. No method of graphing that displays information clearly should be valued over any other.

LESSON 15-17

REPRESENTATIONAL GRAPHING

PURPOSE:

To write and answer questions for a new kind of graph

MATERIALS:

1. *String, yarn, or ribbon*
2. *Scissors*
3. *Glue or tape*
4. *Paper*

Teacher: Today you will make a graph of all the measurements of your body you can think of: head, wrist, arm, finger, anything! Put all your measurements in order from largest to smallest, then write some questions for your graph like you have done for other graphs.

You may work with someone else if you want, so you can measure each other, but I want a graph from each of you with only your own measurements recorded.

Use string, yarn, ribbon, or whatever else you think might work.

Graphing is one way of recording measurements. Some measurements may tell how many (three pet fish, five people who like red); these are countable.

The earlier string measures (see Chapter 14, Measurement) that were taped or glued to paper are also a form of graphing. String or ribbon graphs involve a problem not present in counting graphs.

Teacher: Can you tell me how much longer the string that measured your leg is than the one that measured your arm?

Student: I can *show* you.

Teacher: Yes, but when we made graphs using cubes you could *tell* me in numbers how much bigger or smaller one pile was than another. Can you think of a way to tell me how much longer or shorter one piece of string is than another?

How is string counted? Can students tell how much longer a leg is than an arm? This type of question will appear on many of the students' graphs although they don't realize the question cannot be answered from the graph. When the students were making counting graphs using cubes, boxes, or pictures, more-than and less-than questions were easily answered. Is it possible now to answer these questions?

When the students have finished writing questions for their string graphs, the teacher collects the papers, then reads the questions. If the students cannot find the answers from the graph, the student who made it can try to change it so it *is* useful for answering all the questions.

Left to their own devices, students can produce an assortment of techniques for converting measuring graphs to counting graphs. Two of the more common techniques are (1) placing the string measurements on graph paper and using the squares to establish the string's length, and (2) converting all the string measurements to appropriate lengths of cubes and displaying a cube graph instead.

The students' potential for solving this kind of problem will increase when they have been introduced to metric units of measure, but they need first to undergo one more experience with the inconveniences of their present range of measuring tools. That inconvenience is the basis of the following lesson.

LESSON 15-18

REPRESENTATIONAL GRAPHING

PURPOSE:

To measure ten objects and place the measurements on a graph in order from largest to smallest

MATERIALS:

1. *The students decide the materials to use for this activity*

The graphing activities prior to this lesson always controlled the units of measure to be graphed. In this lesson, for the first time, the students experience the difficulties encountered when the units to be graphed are not carefully selected.

Teacher: Select ten items around the room you wish to measure. You may use any measuring devices you wish . . . string, cubes, straws, anything. Keep a record of what you measure and how big it is. When you've recorded all the measurements, make a graph showing the measurements placed in order from the largest to the smallest. You may make any kind of graph you wish to show your results. Then write questions you think your graph can be used to answer.

You may work with one or two other people or by yourself. If you choose to work in a team, the team measures ten objects and submits only one graph.

When most of the graphs are in, the class uses each one in turn to answer the questions that accompany it. Students who measured using single units such as cubes or lengths of string will find the assignment easy. Students who measured using all sticks or all straws will have a slight difficulty deciding how to transfer their measurements to paper. Students who measured each new object with a new measuring device, and whose measurements now include cubes, tiles, toothpicks, straws, and whatever else was available find their graph almost impossible to assemble in any meaningful way.

Regardless of the ease or difficulty students encounter while assembling their graphs, the measure of the effectiveness of any graph is, as usual, its ability to be used to answer the questions accompanying it.

Students may change their graph to make the data more useful, if necessary. If they cannot, the class as a whole may offer suggestions. This enables everyone to reach the goal of producing graphs that display information in a useful form.

The teacher asks the class to think about the following question:

What makes some graphs useful and some not for answering questions?

LESSON 15-19

REPRESENTATIONAL GRAPHING

PURPOSE:

To apply acquired graphing skill to selected graphing situations

MATERIALS:

1. To be selected by the students

The students now have a basic knowledge of how to establish a topic for investigation, gather data, and display that data in a form useful for answering questions.

Although the students' graphing skills have not yet been fully developed, they are more than adequate to permit them to begin independent exploration, and thus appreciate the value of graphing as a system of recording information.

The teacher's first responsibility has been fulfilled: the students have been helped to attain a minimum level of graphing skill. This point was reached when they were able to record cube graphs on 1.7 cm graph paper. The teacher's responsibility now is to provide the students or teams with a wide assortment of potential graphing topics from which to select. No topic is assigned to the whole class and they need not accept the teacher's suggestions. They may invent their own areas of exploration. The teacher's suggestions are meant to be sources of ideas, not binding assignments.

Almost any collection of data that can be measured, sorted, or surveyed can be graphed. The teacher may select one of these areas and ask the students to think of topics that might be explored.

Teacher: Let's try to think of something you might be able to find out about reading or books.

I wonder how many reading books are in class. Do we have more reading books or math books? Science or social studies books? Which kind of books do we have the most of? Which kind of book do people like the most? The least?

How can you tell a hard book to read from an easy one? Are the words longer in harder books?

Do all the letters in the alphabet occur the same amount of times in each book? Or do the letters at the front of the alphabet occur more often? I wonder if harder books use more letters.

Which library books do people like the most?

How many pages do most books have in them? I'll bet most have less than 200.

I wonder if our room has more books than all the other rooms in the school. Could we find out how many books are in each of the other classrooms?

Do fifth grades have more books than first grades? Or do first graders need more because they're just learning how to read and their books are shorter?

How many people think reading is fun and how many hate to read? Can you think of anything else about reading we might be able to explore?

The topics the teacher mentions are meant to be catalysts for other ideas. A discussion of reading and books may remind a student of *The Night Before Christmas* and the student may end up graphing how many people believe in Santa Claus or how many lines of the poem each person can correctly recite from memory.

The best single source of future graphs is past or present graphs; almost any topic can be expanded. A graph can be made for one class, but can the makers tell if it would look the same for other classrooms?

All aspects of school provide graphable information. Individual height and weight charts can be used to graph student growth, or size distribution within or between classes. Eye checks can provide reason to determine if the number of people who wear glasses goes up each year, or if glass wearers most often have parents who wear glasses. How many people have had measles? or chicken pox, or mumps? Are boys absent more often than girls? How many people are absent each day of school? Are teachers sick more or less often than students? Are small children absent more than big children? And so on.

Recess and P.E. are often the most popular times of the day. What do people like to do at recess? Is what they like to do the same for big kids as for little ones? How much recess equipment does each class have? Physical fitness events can be graphed either as a record of individual progress over the year, or as a class composite for each event. Scores of games are themselves graphs, and can be kept both on a cumulative basis and as a separate day-by-day graph to see if the same team does equally well each day. Individual students may elect to keep graphs of their own achievement: Number of outs, times on base, runs scored, and so on.

Any aspects of school or the school day may be considered. Who walks to school? Who rides? Who brings lunch? Who eats in the cafeteria? How many people eat in the cafeteria each day? Are some days more popular than others? What are the most popular foods? The least popular? How much money does the cafeteria make each day? How long does each class have to wait in line before being served? How much time does each class spend eating lunch? Do little kids take longer to eat than big ones? How many people cross at the traffic crossing each day? How many people jaywalk? How many people are sent to the principal each day? How many are sent to the nurse? What are the most common reasons for getting sent to the nurse? How

many people transfer out of the school each week? How many come in? Are more coming in or going out? Is the school always the same size?

Many of the graphing topics students suggest center around wanting to find out more about themselves. Height, weight, favorite television shows, who chews gum, what kind, favorite store to go shopping, favorite record, movie, comic book, favorite food, most hated food, number of brothers and sisters, been to Disneyland yet, or not, and so on.

For more ideas, the teacher may look in any student's cumulative folder to find such possible topics as the number of different schools attended, year or month of birth, most common first name in school, most common street on which people going to the school live, state in which born, who was born the farthest away, and so on.

Questions may be asked about life outside of school as well. What kinds of cars do people own? What kinds of bikes? How many miles to the gallon does each car get? How many wrecks has each student been in? What kind of car would each person like to own? How many students can cook? How many wash dishes? Grocery shop? Baby sit? How many hours can they play outside? How many hours of TV watching each day? What time is bed time? What time do people get up in the morning? Do students ever get spanked? Do boys get spanked more than girls? Who eats breakfast? And so on.

Opinion polls may also be graphed. Are girls discriminated against? Do more boys or girls think so? Should people go to the moon again? Should there be a death penalty? Do graphs for students and graphs for parents on the same opinion question look the same?

Each idea for a graph is a starting point for all the ideas that flow from it. Each idea is part of a pattern of ideas, as each arithmetic problem is part of a whole series of problems related in a patterned sequence. The goal is to look beyond the isolated idea for the whole from which it is drawn.

A single topic idea might be the number of brothers and sisters each student has. The ideas around this single topic can include: How many people in the school are related to each other? How many different families send children to the school? How many people in the room are the oldest child? The second oldest? Are there more "oldest" children in kindergarten or in sixth grade? Are the oldest children in a family more often boys or girls? How about the youngest? Do people have more brothers or sisters? Do girls have more sisters and boys have more brothers? How many years difference between the oldest child in each family and the youngest? What is the age of each brother and sister of every one in class. What is the most common age? And so on.

The authors of each graph display their results and the teacher reads to their classmates the questions written to accompany the graph. Completed graphs and the accompanying questions are then posted on the bulletin board. These posted graphs may inspire other students to gather additional information about the same topic or to think of related areas that might be investigated.

Graphing is a method of organizing data that makes it more useful in answering questions. Displaying each new graph is one way of insuring the fruits of the student's or group's efforts are shared by everyone.

Mathematics is a process of measuring, counting, observing and recording experiences in the environment. The graphing skills the students now possess offer them an effective and useful way to display a wide range of mathematical information. In later chapters students will add to their skills techniques that enable them to use these same graphs for making predictions.