CHAPTER 17
Probability

Lesson 17-1 page 243 Dice Game Students learn to play a dice game.	Lesson 17-2 page 244 Dice Game Students use data from the dice game to answer teacher questions.	Lesson 17-3 page 244 Dice Game Students play a modified version of the dice game and answer teacher questions.	Lesson 17-4 page 245 Coins Students graph flips of a single coin and observe the outcome.
Lesson 17-5 page 246 Coins Students graph flips of two coins and observe the outcome.	Lesson 17-6 page 247 Coins Students graph flips of three coins and observe the outcome.	Lesson 17-7 page 247 Coins Students graph flips of four coins and observe the outcome.	Lesson 17-8 page 247 Coins Students predict the shape of a five-coin graph before flipping.
Lesson 17-9 page 248 Dice Students graph rolls of a single die and observe the outcome.	Lesson 17-10 page 248 Dice Teacher records two dice combinations. Students graph two dice totals.	Lesson 17-11 page 250 Dice Students record three dice combinations before graphing three dice totals.	Lesson 17-12 page 250 Dice Students record four dice combinations before graphing four dice totals.
Lesson 17-13 page 251 Assorted Graphs Students compare earlier graphs with dice graphs and answer teacher questions.			

Prerequisite chapters:

Chapter 15

MATERIALS

For overhead projector:

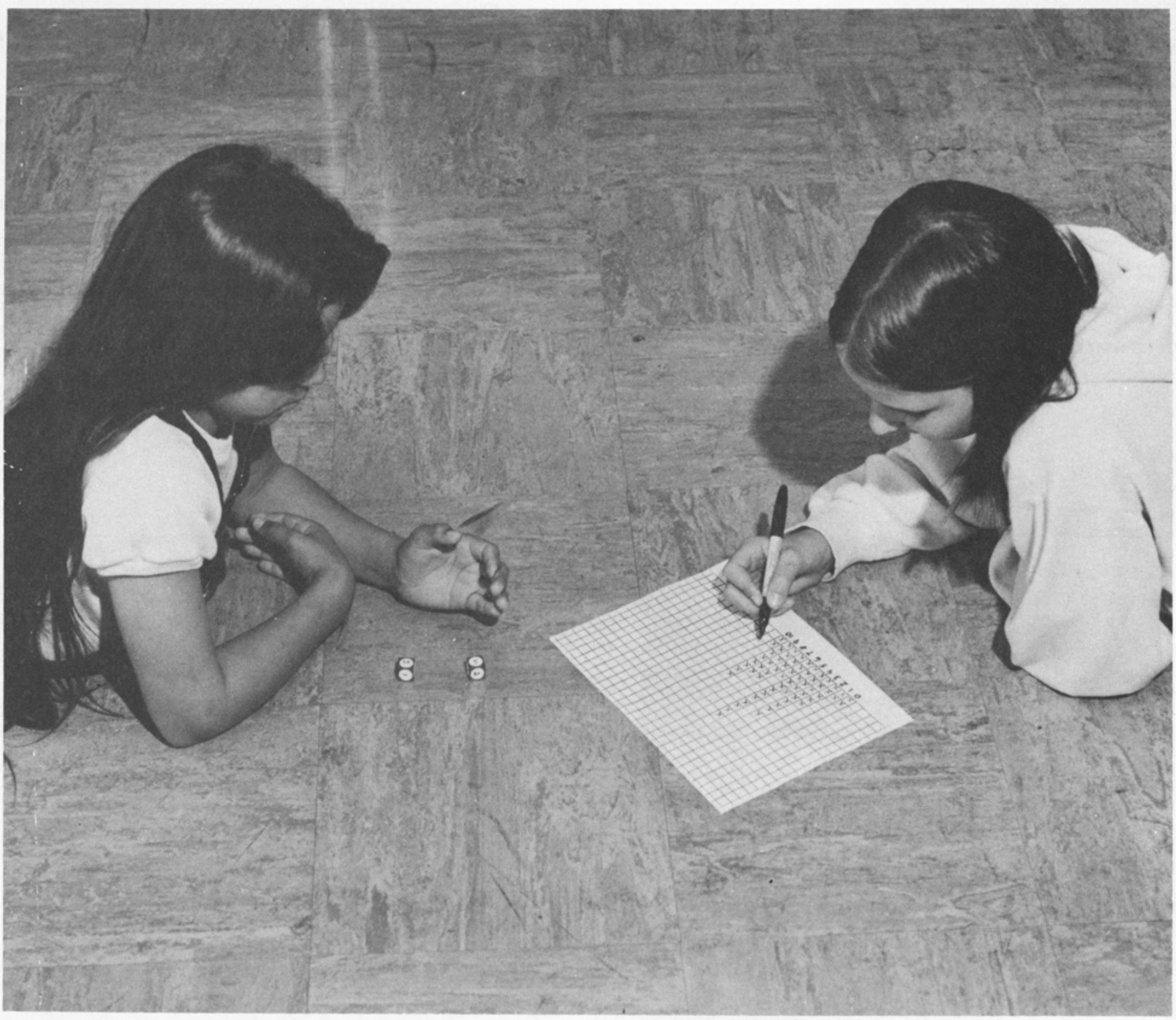
Transparencies — Graph paper, one cm squares	Worksheet 24
Overhead projector dice	Materials chapter, page 296
One bean	

If no overhead projector is available:

Make charts in place of transparencies	Materials chapter, page 294
Dice cards in bag	Materials chapter, page 296

Student materials

Dittos — Graph paper, one cm squares	
Individual blackboards	Materials chapter, page 294
Cardboard coins	Materials chapter, page 298
Dice	Materials chapter, page 296
Beans	Materials chapter, page 295



This chapter introduces students to the study of probability. The understanding they are expected to gain is that some events are more likely to occur than others. The students compile their own data, search for patterns that help them predict probable outcomes, and use the patterns they see to generate rules for making such predictions. The students are not required to translate their rules or patterns into the formal ratios usually associated with probability, since the results they obtain will not be precise enough to convert to meaningful ratios. Simple statements about the range of answers anticipated in a given situation are all that is expected.

The observations made by students in the probability activities increase their ability to draw meaning from the graphs they assembled in earlier lessons and use these same graphs as the basis for makinq predictions about the shapes of graphs yet to be constructed.

LESSON 17-1

PROBABILITY IN DICE GAME SITUATIONS

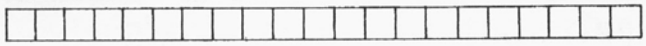
PURPOSE:

To learn a dice board game in preparation for using game data for answering teacher-directed questions

MATERIALS:

1. *Strip of graph paper on a transparency or drawn on the blackboard*
2. *Overhead projector dice, or dice cards in a bag*
3. *Beans*
4. *Dice*
5. *Strips of graph paper*

The activities in the first three lessons cause students to think about why some events seem to occur more often than others. They are not expected to know why this happens, only to recognize this is so and search for an explanation. Why events occur more frequently will be the subject of later lessons.

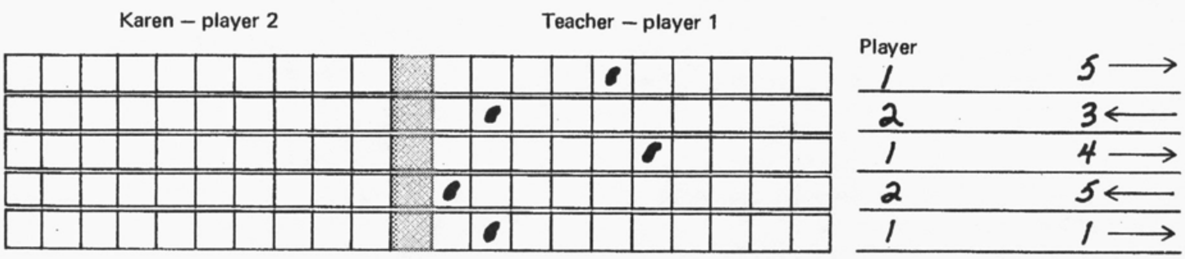


Teacher: I have a strip of graph paper on the overhead. How many squares long is it?
Student: Twenty-one.
Teacher: I'll color in the middle square and put a bean on the colored square. I now have a game board for a game I want to teach you.

Karen, would you be my partner, please. You and I will show everyone else what to do. I'll roll first . . . what did I get?
Student: Five.
Teacher: Then I move the bean five spaces in my direction. Karen, you roll. . . . What did Karen get?
Student: Three.
Teacher: Karen moves the beans three spaces in her direction, starting from where the bean was at the end of my turn.
My turn . . . I got a four, so I move the bean four spaces in my direction.
Karen's turn . . . she got a five–what should she do?
Student: Move the bean five spaces in her direction.
Teacher: My turn. What do I do first?
Student: Roll the die.
Teacher: Okay . . . what did I get?
Student: One.
Teacher: What do I do?
Student: Move the bean one square in your direction.
Teacher: Now what?
Student: Karen's turn. . . .

When the students can direct the teacher and opponent through the steps, they are ready to make their own game boards and begin playing in groups of two. The winner of the game is the student who is first able to move the bean off the end of the game board. When one game is finished, another is started.

The students continue playing the dice board game throughout the time available for the lesson.



LESSON 17-2

PROBABILITY IN DICE GAME SITUATIONS

PURPOSE:

To compile data from the dice board game to answer questions

MATERIALS:

1. *Graph paper on a transparency or on a large tagboard*
2. *Graph paper*
3. *Strips of graph paper*
4. *Beans*
5. *Dice*
6. *Individual blackboards*

Teacher: In the dice games you played yesterday, did the person who started first usually win or lose?

Student: The starter wins, because Eddie started first most of the time when we played and he kept winning.

Student: No . . . because Debbie started first most of the time when we played and I kept beating her.

Teacher: I've given each of you a piece of graph paper. Today when you play the dice game, I want one person to keep a record of when the starter wins or loses. The other person keeps track of how many turns it takes to win each game. Does each game take the same number of turns to play?

About halfway through the period, the teacher stops the game and records the data on the overhead.

Teacher: Write on your blackboard how many games the starter won, and hold it up. Now, write how many games the starter lost and hold it up.

The teacher records the data on the overhead in the form of a graph.

Teacher: Was it better to start first or second? Why?

Deciding why one person wins more often is not easy. It might be argued that the person who rolls first wins because of a head start. If so, they why doesn't the first person always win. What else may make a difference in the outcome?

It should not be expected that the students will produce clear-cut answers. They should begin thinking why one person wins and another loses.

Teacher: Look at the record you kept for how long it took to play each game and write the answers to the questions I ask on your blackboard.
How many games did you play that finished in one turn?
How many games finished in two turns?
Three. And so on.

The teacher graphs the data on the overhead one step at a time as the students respond.

Teacher: What was the shortest game? . . . The longest? . . . Which length game occured most often? . . . Why?

The students attempt to provide explanations for the fact that a substantial number of games took about the same number of turns to complete. Can any explanation also explain why not all games took that same number of turns to play?

The students discuss possible rationales for the shape of the graph, then resume playing their dice board games and make a new record. When only five minutes remain, the teacher again stops the games. The students provide the additional data they have gathered and the teacher adds it to the graphs on the overhead.

The teacher and the class then discuss the following questions:

Was it better to start first or second? Does the graph still show what it showed earlier?
What was the shortest game? The longest? Are the answers now the same as or different from the ones you gave me earlier? Why?
Which length game occured most often? Is this the same as before?
Why are the results on the graph the same as (or different from) the results we got earlier?
Would the answers to my questions change if you had time to play more games? Why? Why not?

LESSON 17-3

PROBABILITY IN DICE GAME SITUATIONS

PURPOSE:

To compile data from a variation of the dice board game to answer questions

MATERIALS:

1. *Graph paper on a transparency, or on a large tagboard*
2. *Graph paper*
3. *Strips of graph paper*
4. *Beans*
5. *Dice*
6. *Individual blackboards*

Teacher: I have given you new strips of graph paper on which to play your dice games today. How many squares did your old strips have?
Student: Twenty-one.
Teacher: How many squares do your new strips have?
Student: Forty-one.
Teacher: When you've colored in the middle square, you may begin playing. The rules for playing the game are the same as before, although the paper is longer.

Once the students have had the opportunity to play a few games, the teacher asks them the following questions:

Does it take the same number of turns to play a game now? Why? Why not?
Does it make any difference now who starts first?
What is the shortest game anyone has played?
Could there be a shorter one?
How about the longest game?
Is there any group who has not yet had a winner?
Could there be a game with no winner?

For the most part, the students' answers are based on speculation. Very few games have been played, and no formal system of recording the data from the games has been introduced. After a brief discussion of possible answers, the students are given sheets of graph paper and asked to keep track of how many turns it takes to produce a winner and if the winner started first or second.

About halfway through the lesson the teacher stops the games and compiles the data. The teacher repeats the same questions asked at the start of this lesson, in addition to the following:

How would our graph change if we made our playing strip even longer?
What would happen to the length of time it takes to play a game if we made our strip of paper really short, say, only 11 squares on a strip?
Why does a game on a longer strip take so much longer to play?

When the students have predicted answers to these questions, those who wish may try varying the length of their papers to see what difference it makes. The rest of the class continues playing on the 41-square strip of graph paper.

Near the end of the lesson, the teacher again stops the games, adds the new data to the graphs, and asks the same questions. Students who have explored the effects of varying the length of the playing strip add their knowledge to the discussion.

The students' answers are still based on speculation. They do not yet have a framework to assist them in providing rational explanations for the graphs. This speculation is important, for it causes them to begin thinking about why some things seem to happen more often than others. The activities in the next lessons provide a potential frame of reference for understanding this phenomenon.

LESSON 17-4

PROBABILITY FOR "COIN" FLIPPING

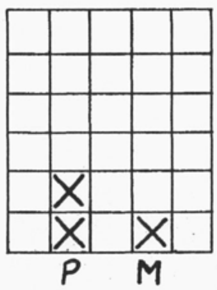
PURPOSE:

To record and observe the results of flipping single squares of cardboard

MATERIALS:

1. *Graph paper on a transparency or on a large tagboard*
2. *Graph paper*
3. *Cardboard squares*
4. *Individual blackboards*

Teacher: You each have a cardboard square marked on one side and plain on the other. Today you will flip the squares as if they were coins and make a graph of which side comes up for each flip.
Before you begin, I'll demonstrate. First, I toss my square . . . it landed with the plain side up. On my graph paper I put an x above the p, which stands for plain. Now, I toss my square again . . . the plain side is up again. What do I do?
Student: Put another x in the plain column.
Teacher: Okay. Now what?
Student: Toss the square again.
Teacher: Okay . . . the marked side is up. What do I do?
Student: Put an x over the m.
Teacher: Okay. Now what?



As soon as the students can direct the teacher in tossing the cardboard square and recording the results on the graph, they begin flipping their own squares and recording the results.

The students toss their squares until one column reaches the top of their paper, then start another graph on the same sheet. The process continues until about 15 minutes remain in the period.

Teacher: I want to make a graph on the overhead of all the results so I can use them to ask you questions. If I had each of you tell me how many x's you had on your papers, there would be too many for me to write.

So I can record all your results on my graph, I only want you to tell me for each of your graphs which won, marked or plain. Write on your blackboards how many times the marked column reached the top of your paper first. . . . Now, how many times did the plain column win?

The teacher graphs the winners at the overhead. Even though adults might anticipate marks should be winners about as often as plains, this is often not the case. Regardless of what the teacher's graph looks like, the question is always the same. Why?

Teacher: Which column is the tallest?
Student: The marks.
Teacher: Why?
Student: Marks are luckier.
Student: People put the marked side up more often when they tossed them. The side you put up before the toss wins the most.
Student: The marked side is heavier so it lands up.
Student: No, the plain side is heavier, so it lands down.

If possible, the explanations offered should be investigated. For example, the hypothesis that marks came up more often because more people flipped the square starting with the marked side up can be tested by having everyone start with the marked side up for all tosses for a period of about five minutes. At the end of that time the winners are graphed on the overhead.

If the experiment proves the theory was "right," it should be accepted as one explanation for the results. The students may then decide if all future tosses should be made marked side up, or whatever other way a "proven" theory has shown makes a difference.

A proven theory remains valid as long as no later evidence contradicts it. At this time, the students cannot know that almost any explanation they give will not be borne out by their experiences in the lessons that follow.

LESSON 17-5

PROBABILITY FOR "COIN" FLIPPING

PURPOSE:

To record and observe the results of flipping two squares of cardboard

MATERIALS:

1. *Graph paper on a transparency or on a large tagboard*
2. *Graph paper*
3. *Cardboard squares*
4. *Individual blackboards*

Teacher: Today you will toss two squares at a time and keep track of the sides landing face up. Before you start, please take a few practice tosses with your squares, and tell me the different ways the squares land face up.
Student: Two marks face up . . . two plains face up . . . and a mark and a plain face up.
Teacher: Any more ways?
Student: No.
Teacher: How many columns will you need on your graph paper to keep track of the different ways?
Student: Three.
Teacher: Which three?
Student: *PP*, *MM*, and *MP*.
Teacher: When you've written the three column headings on your paper, you can begin.

If nickles and dimes were used instead of squares of cardboard, some students might see four possibilities exist and not three, as they suppose. For coins it might be seen that heads for the nickel and tails for the dime is a different result than tails for the nickel and heads for the dime. As it is, the squares of cardboard effectively mislead the students into thinking they have only three outcomes to consider.

When about 15 minutes are left in the period, the teacher constructs a winners' graph on the overhead from the students' results. For this graph, the MP column will be significantly taller than either the *MM* or *PP* columns—the teacher then asks the students to explain why.

Some students might figure out that *MP* occurs more times because it is a composite of the two possibilities *MP* and *PM* but it is most likely that no one will be able to provide an explanation. The students should still have the opportunity to try.

Teacher: Why is the *MP* column so much taller than the other two columns on my graph?
Student: You didn't give us enough time. If we'd made more tosses, the other two columns would have caught up.
Student: The squares don't like to come up the same color.
Student: It's magic.
Student: People didn't shake the squares up enough before they tossed them.

Any testable explanations offered are tested.

The object of asking the students why, apart from providing them a continuing opportunity to think, is to focus their attention on the possibility that since something seems to be happening, there may be a reason for it. The squares tossed in the next lesson only deepen the mystery.

LESSON 17-6

PROBABILITY FOR "COIN" FLIPPING

PURPOSE:

To record and observe the results of tossing three squares of cardboard

MATERIALS:

1. *Graph paper on a transparency or on a large tagboard*
2. *Graph paper*
3. *Cardboard squares*
4. *Individual blackboards*

Teacher: Today you will toss three squares at a time and keep track of the sides landing face up. Which ways can they land face up?
Student: Three plains, three marks, two plains and a mark, and two marks and a plain.
Teacher: Any more ways?
Student: No.
Teacher: How many columns will you need to make?
Student: Four . . . *PPP, PPM, PMM,* and *MMM.*
Teacher: Okay. Toss the squares and record your results.

When about 15 minutes remain, the teacher compiles the student's information in a winners'graph on the overhead.

The *MMM* and *PPP* columns are apt to have few, if any, *x's.* The *PPM* and *PMM* columns, on the other hand, contain a substantial number of *x*'s.

The students are still unlikely to provide an explanation, but now they have no difficulty deciding something *does* need explaining.

LESSON 17-7

PROBABILITY FOR "COIN" FLIPPING

PURPOSE:

To record and observe the results of tossing four squares of cardboard

MATERIALS:

1. *Graph paper on a transparency or on a large tagboard*
2. *Graph paper*
3. *Cardboard squares*
4. *Individual blackboards*

The students now graph the results when tossing four squares of cardboard at a time under the column headings *PPPP, PPPM, PPMM, PMMM, MMMM.*

As each additional square is added to the number of squares the students toss, the difference between columns becomes more pronounced. For four squares, these differences are obvious on the students' graphs as well as on the teacher's composite graph.

Although the students still are not able to provide a rationale they can substantiate through testing, they are at least ready to demonstrate their awareness of the phenomenon they have been observing. The next lesson provides a test of this awareness.

LESSON 17-8

PROBABILITY FOR "COIN" FLIPPING

PURPOSE:

To predict in advance and then check the results of tossing five squares of cardboard

MATERIALS:

1. *Graph paper on a transparency or on a large tagboard*
2. *Graph paper*
3. *Cardboard squares*
4. *Individual blackboards*

Teacher: Today you will toss five squares. Before you begin, draw a graph you think your graph will end up looking like. You don't have to guess the exact number of *x*'s–just predict which columns will be the lowest and which the highest.

The students make their predictions by filling in columns of *x*'s on a piece of graph paper, then begin tossing the squares and compiling the actual graphs. As they work, the teacher circulates among them observing the predictions. Students who have correctly anticipated the columns that will be high or low are beginning to accept, at least on an intuitive level, what is likely to happen.

Before the teacher makes the winners' graph on the overhead, the students are asked to predict the form it will take. Students who predict the center column will represent almost all of the winners and the outside columns will be nearly empty are beginning to recognize that the data on their graphs is likely to reflect the data on everyone else's graph as well.

Students need not know why a graph is the shape it is to use what they do know to predict the shapes of graphs. But in order for them to apply this predicting ability to areas beyond tossing squares of cardboard, understanding the reasons *is* essential.

The students can now anticipate the shape of a graph before the graph is assembled. They also know that something seems to be happening that makes some columns rise higher than others. In the lessons that follow, the students will learn what this is.

LESSON 17-9

PROBABILITY FOR DICE

PURPOSE:

To record and observe the numbers obtained when rolling a single die

MATERIALS:

1. *Graph paper on a transparency or on a large tagboard*
2. *Graph paper*
3. *Dice numbered from zero to five*
4. *Individual blackboards*

Teacher: Today you will make a graph recording the numbers that come up when you roll a single die over and over. What are the possible numbers?
Student: Zero, one, two, three, four, and five.
Teacher: How many columns will you need on your graph paper to keep a record of how often each number comes up?
Student: Six.
Teacher: Roll your die to see which number reaches the top of your paper first. As soon as you have a winner, start a new graph on your paper and see if you get the same results again.

If the dice were rolled thousands of times, the resulting student graphs might reflect the number distribution theoretical probability indicates should be the outcome for repeated rolls of a single die. In theory, no number on a die is any more likely to come up than another. Since dice are not rolled thousands of times, the resulting student graphs usually do not mirror such expectations.

Graphs compiled by individual students commonly have one, two, or three numbers clearly ahead of the others. Even the teacher's graph of winners is apt to have two or three columns higher than the others. Whatever data are actually compiled, however, the question remains, Why?

The students continue rolling their dice and graphing the results until about half the period is gone.

Teacher: I will make a graph on the overhead showing which of your columns reached the top of your paper first.
If zero won for any of your graphs, write a zero on your blackboard and hold it up. If you had time to make more than one graph, and zero won more than once, write zero on your board as many times as it won. . . .
If one was a winner, write it on your blackboard as many times as it won and hold it up. . . .
And so on.

The teacher graphs the winners on the overhead.

Teacher: Which columns are the tallest?
Student: The three and the five.
Teacher: Why?
Student: The dice were loaded.
Student: Six was almost catching up. If we'd rolled longer, six would have won instead.
Student: I don't know about three, but five is my lucky number, so I knew it would win.

Which column is taller is a function of chance—the students have been asked to explain a phenomenon that will probably cease to exist as soon as more dice are rolled to test the explanation. Nevertheless, the students are asked to provide explanations, and any that can be tested are tested during the second half of the lesson as the students continue rolling their dice and graphing the outcome.

When about five minutes remain in the period, the teacher again stops the class and adds additional winners data to the graph on the overhead. The teacher then asks:

Are the same columns ahead on my graph now that were before? Why? Why not?

LESSON 17-10

PROBABILITY FOR DICE

PURPOSE:

To record and observe the numbers obtained when rolling two dice at once

MATERIALS:

1. *Graph paper on a transparency or on a large tagboard*
2. *Graph paper*
3. *Dice numbered from zero to five*
4. *Individual blackboards*
5. *Paper*

Teacher: Today you will graph the sums you get when rolling two dice at once. For this activity, I want you to work in pairs. Each person will roll one die; roll your own in front of you so you can tell whose die is whose.

What sums can come up with two dice?

Student: Zero, one, two, three . . . nine, ten.

Teacher: I'll write those on the overhead. How many ways can the dice come up so they add up to each of the sums?

Student: What do you mean?

Teacher: How many ways to get zero?

Student: When both dice have zero on them.

Teacher: Any other ways?

Student: No.

Teacher: Then I'll write that above the zero like this.

0+0				
0	1	2	3	4...

How many ways to get one?

Student: Zero and one.

Teacher: Whose die has the zero and whose has the one?

Student: What do you mean?

Teacher: I'll use Johnny and Eddie as an example. Can Johnny's die have a zero on it and Eddie's have a one?

Student: Yes.

Teacher: Then that would be zero plus one. Can Johnny's die have a one and Eddie's have a zero?

Student: Yes.

Teacher: Then that would be one plus zero. I want you to give me all the different ways the dice can come up. That means you'll have to think about the different ways your own die can come up, too. I'll write the ways the dice can make one on the overhead.

Now, how many ways to make two?

Student: Zero and two, two and zero, one plus one, and one plus one.

Teacher: If Johnny has one on his die and Eddie has one, do the numbers on each die change if Eddie has one and Johnny has one?

Student: They could be turned around.

Teacher: True, but do the numbers change?

Student: No.

Teacher: Then one plus one is the same as one plus one—I won't write it down twice. Here are the different ways for two.

		1+1	
	0+1	2+0	
0+0	1+0	0+2	
0	1	2	3 . . .

What are the ways for three?

The students continue telling the teacher how each new number can be made with their dice. The number six sometimes proves confusing.

Teacher: How many ways to make six.

Student: Six plus zero, zero plus six, five plus one, one plus five, four plus two, two plus four, and three plus three.

Teacher: You are using the patterns you see in the numbers you have found already to help you know what numbers to expect next. Remember, though, the ways you give me have to be from your dice.

Is there a six on your dice?

The complete list of ways to make the numbers zero to ten with the dice is included in this figure. After the list is compiled on the overhead, the students begin rolling their pairs of dice and graphing the totals.

					3+2					
				2+2	2+3	3+3				
			2+1	3+1	4+1	4+2	4+3			
		1+1	1+2	1+3	1+4	2+4	3+4	4+4		
	1+0	2+0	3+0	4+0	5+0	5+1	5+2	5+3	5+4	
0+0	0+1	0+2	0+3	0+4	0+5	1+5	2+5	3+5	4+5	5+5
0	1	2	3	4	5	6	7	8	9	10

The teacher does not imply to the students that the overhead graph of possible combinations has anything to do with their own graphs of sums. When the students compile their graphs only a few will look like the graph on the overhead, so not many students will see a connection.

The students are given most of the period to assemble data for their graphs. When one column reaches the top of the page, they start a second graph. When about ten minutes remain in the period, the graphing stops.

Before the teacher makes a winners' graph on the overhead, the students are asked to look at their individual graphs and answer the following questions:

What do your graphs look like?

How many are higher on the ends than in the middle?

How many are higher in the middle?

Are there any mostly straight across the top?

Why do your graphs look that way?

How do you think they would look if you had more time to roll your dice?

The teacher then makes a winners' graph. The two dice graphs have eleven possible columns, and it is not likely any one of them will significantly dominate the others. What is likely, however, is that all or most of the marks on the winners' graph will fall within the sums of three and seven, with the sums from zero to two and eight to ten having no marks at all.

Teacher: Are all the columns on my winner's graph the same height?

Student: No.

Teacher: Why aren't they?

The students have been asked why many times before. This time, however, they have the *potential* of being able to see that the middle columns are higher because those columns have more ways of coming up on the dice. However, they may not yet make the connection between how many different ways a sum can be made and the number of times that sum comes up when the dice are rolled. This connection becomes increasingly obvious in the lessons that follow.

LESSON 17-11

PROBABILITY FOR DICE

PURPOSE:

To record and observe the numbers obtained when rolling three dice at once

MATERIALS:

1. *Graph paper on a transparency or on a large tagboard*
2. *Graph paper*
3. *Dice numbered from zero to five*
4. *Individual blackboards*
5. *Paper*

Teacher: Today you will use three dice at once. You will have to work in teams of three because I want each die to come from a different person—roll your own die in front of you.
What sums can come up for three dice?

Student: All the sums from zero to fifteen.

Teacher: Okay. Write the numbers zero to fifteen on your papers. How many ways are there to make zero?

Student: One.

Teacher: Which way is that?

Student: Zero, zero, zero.

Teacher: Okay. Write zero, zero, zero on your paper, like this.

0+0+0				
0	1	2	3	4...

What are the ways you can make one with three dice? Remember to count the numbers on each person's die as a different way if somebody's number is changed.

Student: Zero plus zero plus one, zero plus one plus zero, and one plus zero plus zero.

Teacher: Write that on your papers. How many ways to make two?

		0+1+0		
		1+0+1		
		1+1+0		
	1+0+0	0+0+2		
	0+1+0	0+2+0		
0+0+0	0+0+1	2+0+0		
0	1	2	3	4...

Beyond ways to make two, each group completes its own list of combinations. As soon as a group finishes its list, its members begin rolling their dice and making a graph of the sums.

Three-dice graphs are much more regular in appearance than were graphs for one or two dice. A three-dice graph of totals is likely to take on the same shape as the chart of combinations made by the students in advance.

When about ten minutes remain in the period, the students are asked the following questions about their individual graphs:

What do your graphs look like?
Are they higher at the ends or in the middle?
Do most of your graphs look the same as everyone else's graphs?
Why?
What would your graphs look like if you added more rolls of your dice to them?

The teacher then compiles the winners' graph. The number of possible columns is sixteen. It is highly likely all the marks will fall between the sums of five and ten, and that the columns on either end of the graph will have no marks.

Teacher: Why do the center columns of my graph have so many x's in them? Why are there no x's at all at the ends?

The students now have sufficient information available to see that the center is higher because more ways exist for the dice to make up those numbers. There is only one way for the dice to make zero, but there are 27 ways for the dice to make 7. The individual graphs look like the lists made in advance for the ways each sum could be made.

The evidence of a connection between how many ways there are for something to happen and the number of times it actually happens is mounting rapidly. If no one sees the connection, however, the teacher refrains from pointing out the relationship. The students have an additional opportunity to make this discovery for themselves in the next lesson.

LESSON 17-12

PROBABILITY FOR DICE

PURPOSE:

To record and observe the numbers obtained when rolling four dice at once

MATERIALS:

1. *Graph paper on a transparency or on a large tagboard*
2. *Graph paper*
3. *Dice numbered from zero to five*
4. *Individual blackboards*
5. *Paper*

Teacher: Today you will graph the sums for four dice. Before you begin, I want you to make a list of all the ways to reach each sum with your dice. For this list, you don't have to write zero plus zero plus zero plus one as four different ways. It can really happen four different ways, but if you really figured out every possible way, by the time you got to ways to make ten, you'd have to write nearly 300 different ways. Since I want you to have time to make a graph of the sums, too, only write ways on your lists if the numbers are different, not if the people whose dice have the numbers on them are different.

When you've made your list of possible ways each sum can come up, I want you also to take a piece of graph paper and predict what your dice graph will look like *before* you start rolling the dice. Your prediction graph doesn't have to be exact. All I want to see is whether you think the graph you get when you roll your dice will be higher in the middle or on the ends. . . .

The students make their lists, record their predictions for the dice graph, then roll the dice and record the sums. It can be expected the students' predictions will accurately reflect the graphs obtained as their dice are rolled.

The graphs have the same shape as the chart of the combinations adding up to each total. The teacher's winners graph indicates the same columns that were winners on nearly every student's graph. This phenomenon repeats with such regularity, the students can anticipate it.

The teacher has always asked why. It is now likely that the students may be able to answer the question.

LESSON 17-13

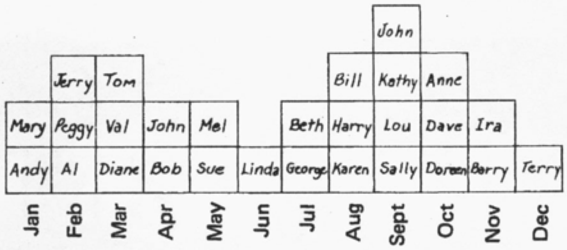
PROBABILITY IN GRAPHS

PURPOSE:

To compare graphs made earlier (Graphing Pictorial Representations, Chapter 15) with dice roll graphs; to answer questions about earlier graphs

MATERIALS:

1. *Assorted graphs from Chapter 15*



Teacher: Several weeks ago we made a graph of the months of birth of everyone in class. I would like you to look at it again. Are some columns in this graph higher than others?

Student: Yes.

Teacher: In our dice graphs, were some columns higher than others?

Student: Yes.

Teacher: Was it by accident?

Student: No.

Teacher: Could it be the months having more people born in them in our class are really months in which more people are born? If we gathered more information, would we find those columns grow further away from the rest as was true for our three- or four-dice graphs? Or, would some other column pass the higher ones, as often happened when we were using only one die?

If the columns continue to increase their lead, does that mean more people are born in some months than in others? Or are the differences we find on this graph just an accident?

The remainder of the lesson is spent reexamining all the old graphs in the classroom, including those made for flipping cardboard squares. Can the students now explain why some columns rose so much higher than others? The students do not, as a part of this lesson, collect any additional data for the graphs discussed. The purpose of the discussion is only to make them aware that more information may be contained in their earlier graphs than they realized at the time. Gathering data to find out more about the earlier graphs is something the students may undertake in conjunction with the periodic repetitions of the graphing experiences contained in Lesson 15-19 from the chapter on graphing.

The study of probability is the study of likelihoods. The students now know some things are more likely to happen than others. However, knowing this does not identify which events can be anticipated.

How does the cook in the cafeteria know how many people will buy lunch next week? Is he or she always right? How can the principal know the number of students who will come to school each fall? What other kinds of situations are there when people must plan for the future, and how do these people know what to expect?

There is a different form of graphing that may also be used to reveal secrets; this is the topic of the following chapter.