CHAPTER 18
Coordinate Graphing

Lesson 18–1 page 254 Number Machine Students predict answers for the number machine.	Lesson 18–2 page 255 Number Machine Students make sets of cards for use in the number machine.	Lesson 18–3 page 256 Number Machine Students predict answers for a drawing of the number machine.	Lesson 18–4 page 256 "T" Students predict answers when number machine is not present.
Lesson 18–5 page 257 Coordinate Tic Tac Toe Students learn to play coordinate tic tac toe.	Lesson 18–6 page 259 Coordinate Graph Students plot numbers from the number machine on coordinate graph paper.	Lesson 18–7 page 260 Coordinate Graph Students graph numbers first from a number machine drawing and then from a "T".	Lesson 18–8 page 261 Coordinate Graph Students predict two group combinations for tiles from a graph.
Lesson 18–9 page 262 Coordinate Graph Students compile graphs from teacher-suggested topics.	Lesson 18–10 page 264 Coordinate Graph Students compile graphs from their own suggested topics.		

Prerequisite chapters:

Chapters 14 and 15

MATERIALS

For overhead projector or teacher demonstration use:

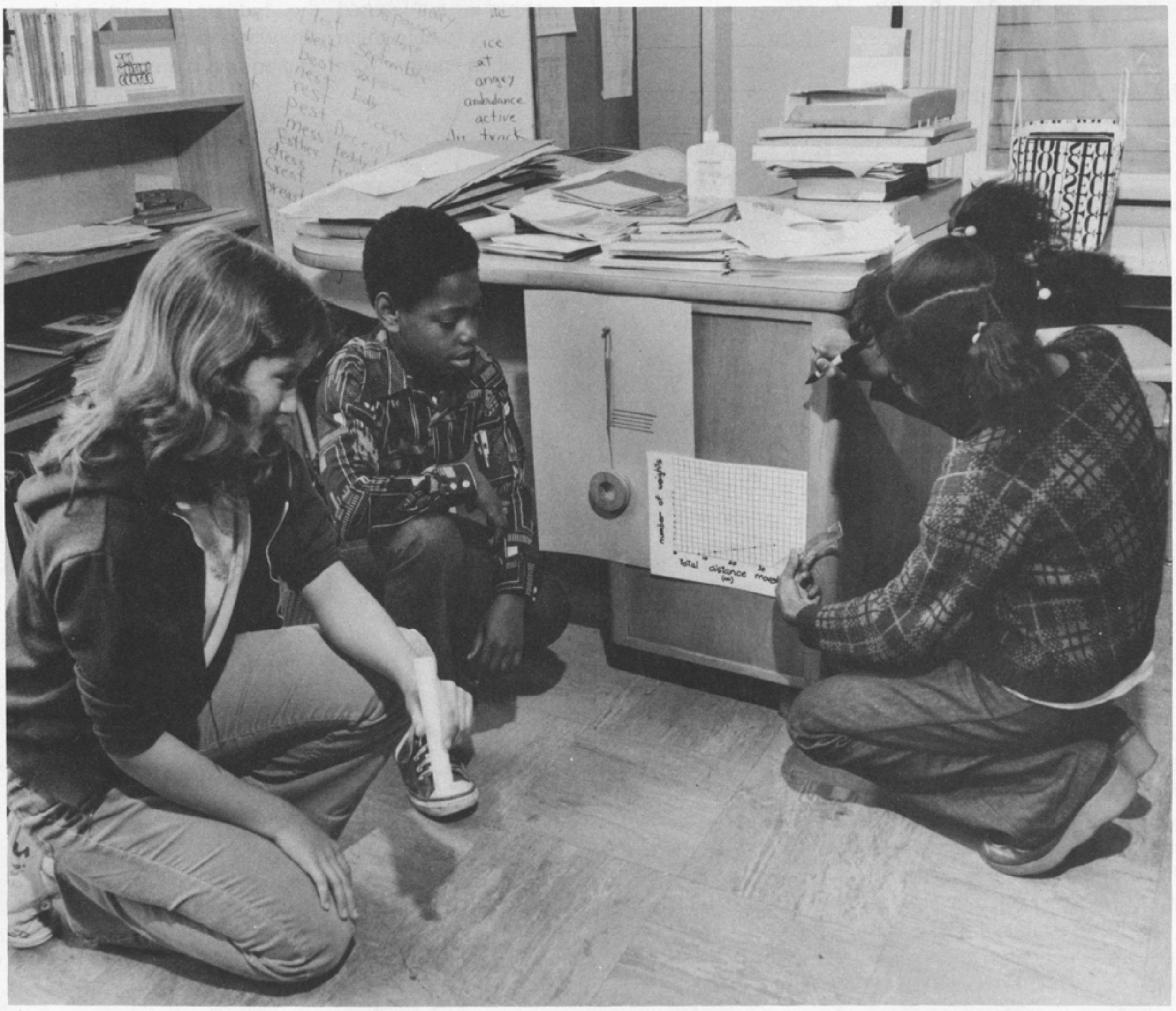
Transparencies Coordinate graph paper	Worksheet 25
Tiles	Materials chapter, page 294
Number machine and cards	Materials chapter, page 298

If no overhead projector is available:

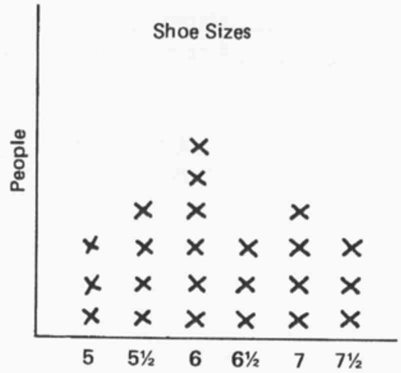
Make charts in place of transparencies	Materials chapter, page 294
Square shapes	Materials chapter, page 295

Student materials:

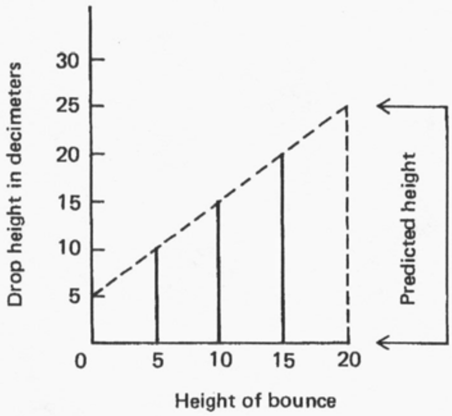
Dittos Coordinate graph paper	
Individual blackboards	Materials chapter, page 294
Blank cards for number machine	Materials chapter, page 298
Tiles	
Unlined paper	



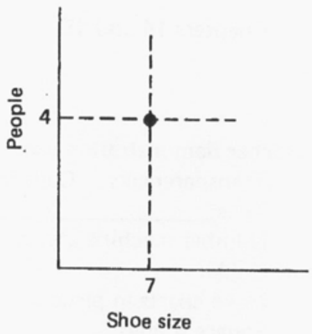
A graph is a drawing designed to illustrate a relationship between two or more sets of values or variables. There are two uses for the graphs students make. The first is to display data so its separate components may be compared more readily. The graph of shoe sizes in this figure, for example, provides the students with a display of information that facilitates answering questions such as which is the most common shoe size.



The second use of graphs is to predict specific future events. For example, a graph of heights for the first bounce of a ball as it is dropped from successively higher points may be used to make a specific prediction for how high the ball will bounce when dropped from a height of 25 decimeters.



Coordinate graphs use ordered pairs of numbers to designate a single point on a grid that represents two variables at once. The two variables for the shoe graph would be shoe size and the number of people who wear each size. The four people who wear size seven is represented by the ordered pair of numbers four, seven. Four represents the number of people wearing that size; seven represents the shoe size. The two variables for the bouncing ball graph are the height from which the ball was dropped and the height of the first bounce. On a coordinate graph a bounce height of ten decimeters is represented by the ordered pair of numbers ten, fifteen. Ten represents the height of the bounce; fifteen represents the height from which the ball is dropped.



The data from any graph students make can be displayed using ordered pairs of numbers on a grid, but not all data is usefully displayed in such a manner. Only data that can be used to make specific predictions for future events is best represented and interpreted in coordinate graphs. Why this is so will be made clear in the following lessons.

The activities in this chapter teach students the basic techniques of assembling coordinate graphs. Once these techniques are learned, the students are guided through experiences to help them learn when to use a coordinate graph and when another form of graphing is more appropriate.

LESSON 18-1

THE NUMBER MACHINE

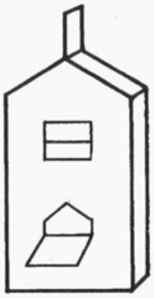
PURPOSE:

To learn to use the number machine; to predict answers for numbers placed in the machine

MATERIALS:

1. *Number machine*
2. *Number machine cards*
3. *Individual blackboards*

Coordinate graphs are best used to depict functional relationships, in which a first number is related to a second so that for every value of the first number there is a unique corresponding value of the second. The next few lessons use a *number machine* to introduce students to ordered pairs of functional or interrelated numbers they will later learn to graph.



Teacher: This is a number machine. When I put numbers in this machine, it changes them, then spits the numbers back out. The machine changes each of the numbers in exactly the same way.

Here is a stack of cards with numbers on them. I will put the cards in the machine one at a time. Watch carefully and figure out what the machine does to the numbers.

The first number I'll put in is three.

The teacher shows the class the three on one side of the card before sliding the card, three side up, into the top slot of the machine. Once inserted, the card slides down the cardboard chute inside the machine, and out, bottom side up.

Teacher: What came out?
Student: A six.
Teacher: This card has a five on it. In it goes . . . what came out?
Student: An eight.
Teacher: This card has a six on it. In it goes . . . what came out?
Student: Nine.
Teacher: Do you think you know what the machine will do to the next number I put in? This time I'll put in a seven. Write on your blackboards the number you think the machine will give back to me.
Hold up your boards so I can see your predictions. . . . Most of you think the machine will give me a ten. In goes the seven . . . what came out?
Student: Ten.
Teacher: Let's try another one. This time I'll put in a card with a one. Write on your blackboards the number you think the machine will give back to me . . .

The teacher continues this process through a full stack of cards. For a second stack, the class is notified the teacher is adjusting the machine so it will make a different change in the numbers. The game is then played again.

Rather than telling the teacher the rule the machine uses to change the numbers, the students demonstrate their understanding by predicting the number that comes out. At the start, when the examples are easy, stating rules may also seem easy. As the examples grow more difficult in the following lessons, the ease with which rules may be stated quickly diminishes. When the students learn the skills of coordinate graphing, they will be able to make accurate predictions even when they cannot state the rule. For this reason, predicting output when input is known is all that is asked of them.

The teacher continues the activity throughout the time available for the lesson.

LESSON 18-2

THE NUMBER MACHINE

PURPOSE:

To make sets of cards for use in the number machine; to predict answers for numbers placed in the machine.

MATERIALS:

1. *Number machine*
2. *Blank cards*
3. *Individual blackboards*

For this lesson the class divides itself into groups of two or three students. Each group is given a set of blank cards and asked to write numbers on them for use in the machine. The students work in groups so they can help each other decide how the numbers are to be written, since the teacher does not explain this. They may examine the sets of cards prepared by the teacher for clues as to how their own cards might be numbered.

Were the teacher to instruct the students in how to prepare cards, the exercise would be a lesson in following the teacher's directions. By placing the responsibility on the students, the teacher causes them to theorize what makes the cards work in the machine.

Once the students finish making a set of cards, the teacher runs each stack separately through the number machine. The students predict the numbers the machine will give for each new pile on their blackboards. They will not be able to make accurate predictions for some card stacks, either because the cards present inconsistent rules or because a rule used by some students may be too advanced for the class as a whole to comprehend.

The test for each new set of cards is: can the answers the machine gives back be predicted from the numbers put in? The teacher places those cards for which most of the class can predict accurately in one group, and those that represent a rule inconsistently applied or too advanced in another. Both groups of cards are saved for use in Lesson 18-6.

LESSON 18-3

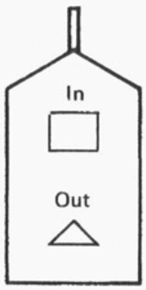
THE NUMBER MACHINE

PURPOSE:

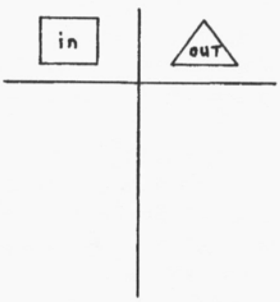
To substitute a drawing of the machine for the number machine itself; to predict answers for numbers placed in the machine

MATERIALS:

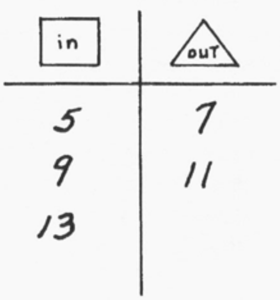
1. Individual blackboards



Teacher: Today we will use this number machine. Since this is a drawing, I can't put any real cards in it, so I'll write the numbers for the cards on this table, which I shall call a *T*.



I put in a five . . . the machine gave me back a seven. I put in a nine . . . I get back an eleven. I put in a thirteen. Write the number on your blackboards you think the machine will give back.



The teacher notifies the class the machine is being adjusted so it will make different changes in the numbers put in, by drawing a new *T* on the overhead. Each new *T* is the equivalent of introducing a new stack of cards for use in the machine.

The teacher continues the process as time permits. For variety, the students tell the teacher what numbers they would like put in; the teacher then writes the numbers the machine would give back. This allows the students more control over the information provided on the *T*. For example, if the teacher had put in a two, then a five, then an eight, the class could ask to see the results of putting in one, three, four, six, and seven. The students may want to see all the "in" numbers in succession rather than in random order.

The teacher may vary the activity further by telling the students what numbers the machine puts out and asking them to decide the numbers that went in.

LESSON 18-4

THE NUMBER MACHINE

PURPOSE:

To predict in or out answers for out or in numbers placed on a T

MATERIALS:

1. Individual blackboards

The activities for this lesson are essentially the same as those in the previous lesson. The two differences are no number machine is drawn on the overhead, and the *T* no longer includes the words "in" and "out."

Given one number and having figured out a rule for what happens to it, the students are now able to state a second number that follows from the first. At this stage, they still have to figure out what is happening to the first to get the second. Identifying this relationship is not always possible. When the students encounter a situation for which a rule cannot be uncovered, a special kind of graph can assist them. The next lessons provide students with their first introduction to this new kind of graphing.

LESSON 18-5

COORDINATE TIC–TAC–TOE

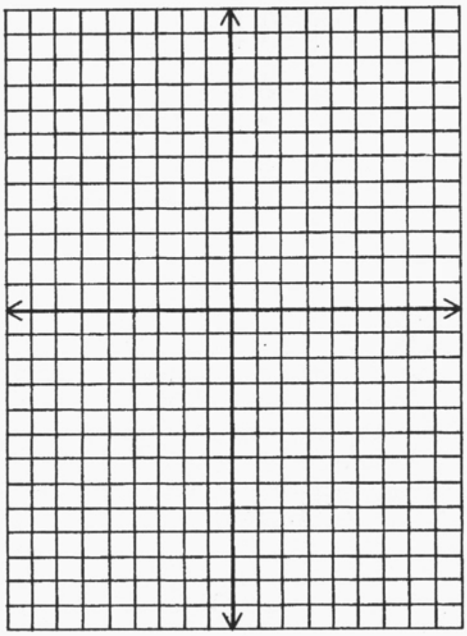
PURPOSE:

To learn how to play coordinate tic-tac-toe

MATERIALS:

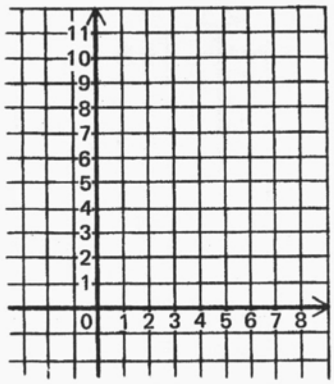
1. *Coordinate tic-tac-toe grid on a transparency, or a large tagboard*

The purpose of the coordinate tic-tac-toe game is to show students how ordered pairs of numbers can be used to indicate a single point on a grid. This skill is a necessary component in the graphing activities introduced in later lessons.



Teacher: Today, we will play a game called *coordinate tic-tac-toe.* This is the playing board. It is divided into four sections. For today's game you will only play in the upper right-hand section. Later, I will show you how to play this game in the other three sections, too.

Before you play, I will number the lines that divide the sections–they are called axes. Where the two lines cross, I'll put a zero. On the part of the axis that starts at the zero and goes to the right, I'll write numbers on each line crossing the axis. Notice the numbers are on the lines and not in the spaces. On the part of the axis that starts at the zero and goes to the top I'll also write numbers on the crossing lines.



The students learn the use of the grid more quickly if they have an opportunity to observe the teacher place the numbers on each axis.

Teacher: For today, the right half of the class will play against the left half. The right half will be *O*'s, the left will be *X*'s. To win, your team needs four marks in a row.

To make a move, tell me two numbers. The first number tells me where to start on the box axis (that's this horizontal line) and go up. The second number tells me where to start on the triangle axis (that's this vertical line) and go across. The mark goes where the lines for the two numbers meet. If you don't understand how to play, watch closely. I can't tell you any more and no one on your team can tell you what to do when it's your turn: But if you watch, you will be able to figure out the game.

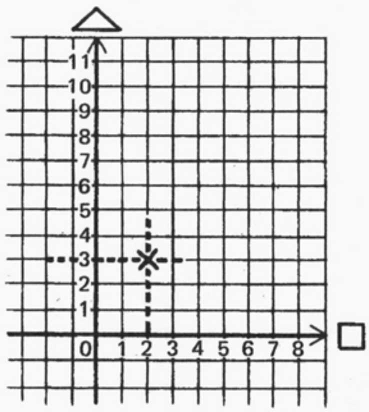
Johnny, your turn is first.

Student: What do I do?

Teacher: Give me two numbers between zero and eight.

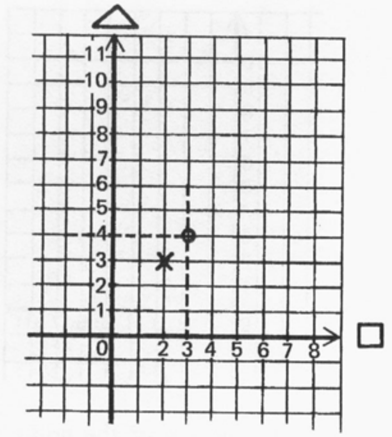
Student: Two and three.

Teacher: Johnny's move is two and three. That means I find the two on the box axis and the three on the triangle axis. Where the two lines cross, I mark an *X*.

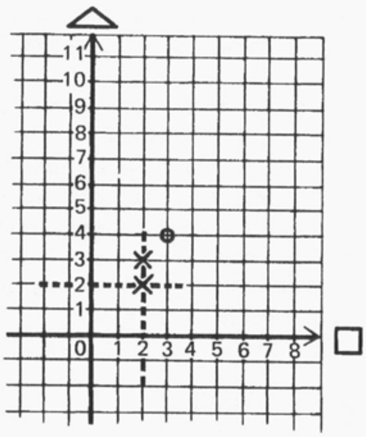


Sally, your turn. Give me two numbers.

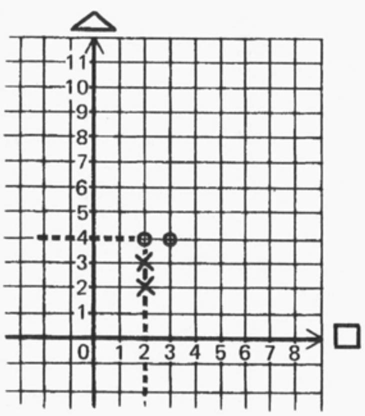
Student: Three and four.



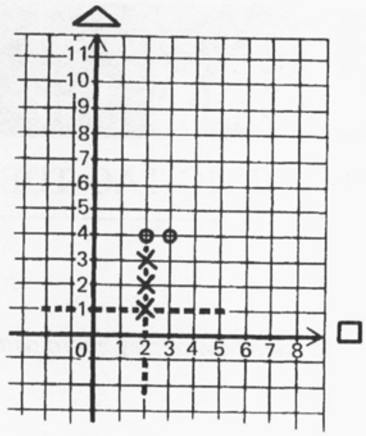
Teacher: Three and four go here. Susie, you're next.
Student: Two and two.



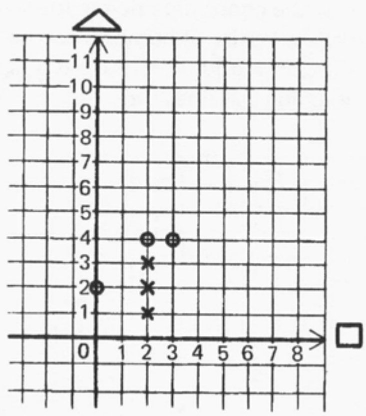
Teacher: Emily.
Student: Two and four.



Teacher: Eddie.
Student: Two and one.



Teacher: Sally.
Student: Zero and two.

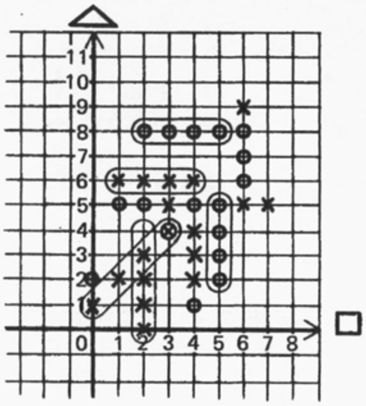


Teacher: Okay.
Student: I didn't want it there! I wanted it under the *X*'s!
Teacher: The rule for this game is, I have to put your mark where the numbers you give me tell me to put it. The first number always tells me where to start on the box axis. The second number always means the triangle axis. You said zero and two, and this marks zero and two.
David, your turn.
Student: Two and zero.
Teacher: That's one for the *X*'s. Brenda, your turn.

The students watch the teacher mark points on the grid for successive pairs of numbers. This observation combined with their need to know so they can make correct moves causes students to learn quickly how to provide ordered pairs of numbers to designate a specific point on a grid.

When one team places four marks in a straight line, it earns a point. The game does not start over, however. All the marks already made are left on the grid, eventually causing the playing area to become very crowded. The purpose of this intentional over-crowding will be explained in Lesson 19-4 in the chapter on negative numbers.

The students continue to add circles and *X*'s to the grid throughout the time available for the lesson. The winning team is the side of the class with the most points.



LESSON 18-6

GRAPHING COORDINATES

PURPOSE:

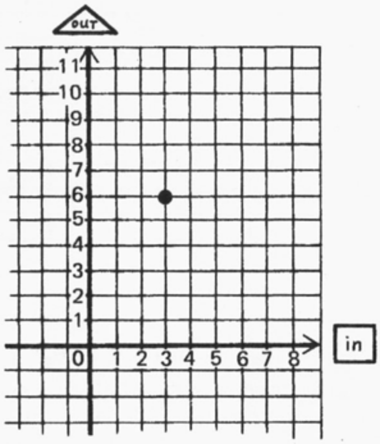
To plot in and out numbers from the number machine on coordinate graph paper; to use the plotted points to predict future in or out numbers for the number machine

MATERIALS:

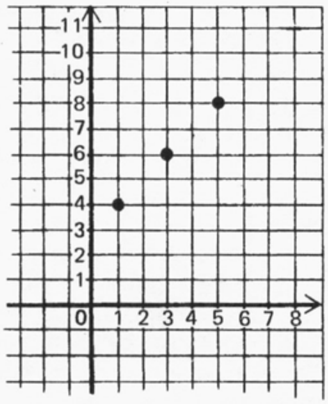
1. *Number machine*
2. *Number machine cards made by teacher and students (see Lesson 18-2)*
3. *Coordinate graph paper on a transparency or a large tagboard*
4. *Dittoed copies of coordinate graph paper*
5. *Individual blackboards*

The activities in the next few lessons show students how to combine what they know about marking points on a grid with the in and out numbers for a number machine to predict numbers when the machine's rule is not known.

Teacher: Number the box and triangle axis on your coordinate graph paper the same way they were numbered when you were playing coordinate tic-tac-toe. What number is on the top side of this card?
Student: Three.
Teacher: Three goes in the number machine . . . what came out?
Student: Six.
Teacher: A three went in, a six came out. Mark that point on your coordinate graph paper.
Student: How?
Teacher: How would I have marked three and six for you when we were playing coordinate tic-tac-toe?
Student: You didn't tell us which axis to use for which number.
Teacher: The number that goes in the machine is the first number and the number that comes out is the second one. I'll write "in" in the box on my graph paper and "out" in the triangle. Now, mark three and six on your graph paper. I'll mark the point for three and six on my graph paper, too. Does your mark agree with mine?

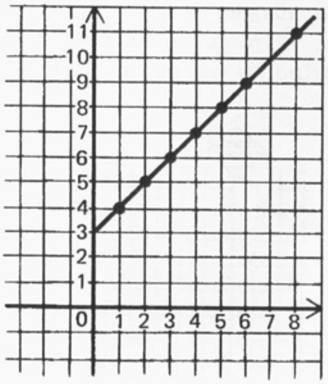


The next card has a five on the top side. In it goes . . . what came out?
Student: Eight.
Teacher: What are the numbers I want you to use to mark the next point?
Student: Five and eight.
Teacher: Why not eight and five?
Student: Because five is the in number and eight is the out number. You said to mark the in number first.
Teacher: Mark the points for five and eight on your graph paper and then look at mine to see if we agree.
The next card has a one on the top side. Write your predictions on the blackboards.
Most of you predict four. I'll put the card in and see. Yes, a four came out. What numbers do you use for marking the point?
Student: One and four.
Teacher: Mark it please.



This process is continued until five or six points have been plotted.

Teacher: Draw a line through all points you have marked so far, like this.



Do you think the next card I put in the number machine will also have numbers on it that will be on the line? . . . Can you look at the line and tell me what some in and out numbers might be before I show you the card that goes in the machine? . . . Will all the cards in this pile give us numbers on this same line? . . . Why? . . . Will all the piles of cards I put in the machine have all their points in a straight line? . . . Or will some make zigzag lines or squares or triangles?

The teacher places all the cards from the first pile in the machine one at a time. The students plot a point on their graphs for each new set of in and out numbers as they investigate the answers to the teacher's questions. When all the cards from one stack have been through the machine, the class is told the machine is changing what it does to the numbers and a new stack of cards is begun.

First, the teacher-made cards from Lesson 18-1 are used. The second and third groups are student-made cards from Lesson 18-2. The second group contains those cards for which most students were able to predict an out number given the in number; the third group contains those too advanced or made by students who did not understand the assignment.

The first two sets of cards will yield points on the graph which, for each set of cards, are all in a straight line. The third set may yield sets of points that zigzag or curve.

Not all graphable events yield linear patterns. Shortly, the students will begin marking points on their graphs to record events not as carefully controlled as the teacher's cards. The cards are used to assist the students in learning how to plot points on a coordinate graph, and to show that some activities recorded on a coordinate graph yield points all in a line. The third group of cards keeps the students from thinking this is true for *all* activities.

LESSON 18-7

GRAPHING COORDINATES

PURPOSE:

To plot in and out numbers, then box and triangle numbers on coordinate graph paper; to use the plotted points to predict future numbers

MATERIALS:

1. *Coordinate graph paper on a transparency or a large tagboard*
2. *Dittoed copies of coordinate graph paper*
3. *Individual blackboards*

The teacher draws a number machine on the overhead, then provides the students in and out numbers as in Lesson 18-3. The numbers for each new rule are recorded on a *T*. The students use these numbers to plot points on their coordinate graph paper.

Halfway through the lesson time, the teacher erases the machine from the overhead and provides the students in and out numbers as in Lesson 18-4. The students use these to plot points on their coordinate graph paper.

As the students work, the teacher asks them:

If I tell you the in number, can you use the line of points on your graph to tell me what the out number might be?
Could you tell me the in number if I tell you the out number?
Do all the points on the line fit the rule for that line?
Could you draw the line long enough so some of the points don't fit the rule?
Are there any numbers that fit the rule I am using that are *not* on the line?

So far, the process has involved marking single points for pairs of numbers and then using the lines of points to predict future numbers that follow the same rules. The students now have the skills necessary to plot points on a graph for successive pairs of numbers. In the next lesson, these skills will be used to graph actual events and make predictions for them.

LESSON 18-8

GRAPHING OF COORDINATES

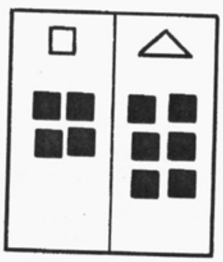
PURPOSE:

To use a coordinate graph to predict answers to specific problems

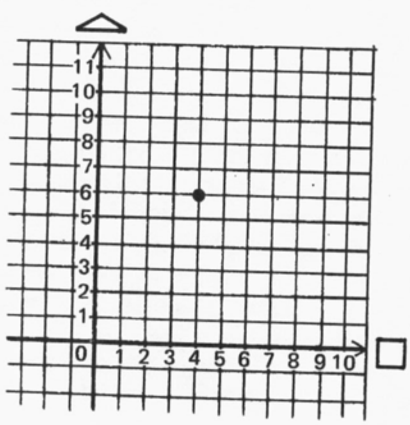
MATERIALS:

1. *Coordinate graph paper on a transparency or a large tagboard*
2. *If no overhead projector is available, cut out squares of paper*
3. *Dittoed copies of coordinate graph paper*
4. *Tiles*
5. *Unlined paper*
6. *Individual blackboards*

Teacher: Put a line down the middle of your paper. Draw a box at the top of one side and a triangle at the top of the other side. Take out ten tiles. What is one way to divide the ten tiles into two groups?
Student: Four and six.

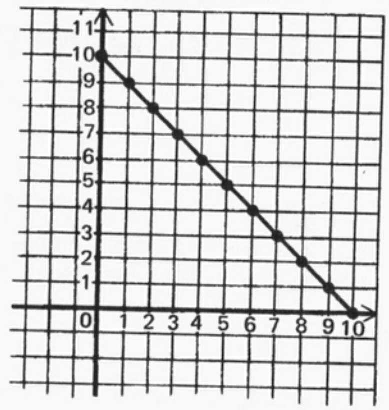


Teacher: Okay. I'll put four tiles on the box side of my paper and six tiles on the triangle side. Now, I want you to mark the point for four and six on your graphs. The four is on the box axis, the six is on the triangle axis. When you've plotted the point, look at my graph to see if we agree.



What's another way to divide ten tiles into two groups?
Student: One and nine.
Teacher: Which number should I put on the box side of my paper and which on the triangle side?
Student: Put one tile on the box side and nine tiles on the triangle side.
Teacher: Which numbers would you use to mark a point for this way of making ten on your graphs?
Student: One on the box axis and nine on the triangle axis.
Teacher: Mark that point on your graphs, then look at my graph to see if we agree.

The teacher guides the students through the plotting of two more points. They complete their graphs on their own, then provide the teacher with the numbers necessary to complete the graph on the overhead.



The class is then asked the following questions.

What about the places on the line that don't come right on the dots? Are these ways to make ten, too?
Are there any ways two groups or two numbers can be added together to get ten that cannot be found on our straight line?

Once the graph of ways to make ten has been examined, the students make a new graph on the same paper of ways to make nine, using two groups of tiles. When the graphs are complete, the students provide the teacher with the numbers necessary to draw a graph for nine on the overhead, then answer the same questions for the new graph they were asked for the old. In addition they are asked:

How is the graph for ways to make nine like the graph for ways to make ten? How is it different?
Can you make a graph for ways to make eight with two groups of tiles *before* you check your predictions with tiles?

The students predict the outcome of the graph of the combinations eight by drawing a graph on their papers. They check their predictions by dividing eight tiles into successive two-group arrangements and plotting the points for the resulting numbers.

The students now begin constructing graphs for any numbers they wish, predicting the combinations first, by drawing the graph before dividing the tiles, or dividing the tiles and then plotting the pairs of numbers obtained on the graph. They work throughout the period, and the teacher asks them to think about the following question:

The lines on your graphs for ways to make nine and ways to make ten were parallel. For which number of tiles will you *not* get a line going in the same direction when you graph the ways to divide that number into two groups?

LESSON 18-9

GRAPHING WITH COORDINATES

PURPOSE:

To use coordinate graphing skills to investigate relationships present in events or experiences suggested by the teacher

MATERIALS:

1. *Dittoed copies of coordinate graph paper*
2. *Materials available in the classroom*

Coordinate graphing is a tool; its use needn't be mastered to perfection before it is applied. Once the class understands how to plot points on a graph, the teacher provides the students an assortment of topics from which to choose so their beginning skill is put to use. The specific topics assigned by the teacher depend on the materials available in the room.

For each topic presented, the teacher and the class discuss together how the investigation might be conducted and what data to record on the graph.

Teacher: Is there a relationship between how tall something is and the length of its shadow?
Student: What do you mean?
Teacher: If you measure how tall this stick is and then how long its shadow is, could you plot one point on a graph for both numbers?
Student: Maybe.
Teacher: Could you plot a point for, say, the height of a pencil and the length of its shadow?
Student: Maybe.
Teacher: If you measure the heights of many things and how long their shadows are, could you plot points for all the measurements on a graph?
Student: Probably.
Teacher: Would all the points be in a straight line?
Student: I don't know.
Teacher: How could you find out?
Student: We could try it and see.
Teacher: What measurements would you take?
Student: How tall something is and how long its shadow is.
Teacher: How would you graph it?
Student: What do you mean?
Teacher: Which number would be for the box axis and which number for the triangle axis?
Student: The height could go on the box axis because we would measure that first. Then the shadow would be on the triangle axis.
Teacher: You might also see if it would make a difference if you put the height on the triangle axis and the shadow length on the box axis. Do you think it would change your results?
Student: I don't know.
Teacher: If all the points for heights and shadow lengths were in a straight line, could you tell how tall something was if you only knew how long its shadow was?
Student: How could we do that?
Teacher: If you got a straight line for your graphs would any numbers for heights and lengths of shadows not be on the line?
Student: I don't know.
Teacher: Well, I can see I'll have lots of questions to ask you about the graph once it is finished. You don't have enough information about heights and shadows to be able to answer my questions now.
After I finish suggesting more topics, those of you who wish may find out all you can about heights and shadows. I'll be interested to see what you discover . . .

The teacher must make sure the students have a good idea of which numbers to record on their graphs.

Although coordinate graphs are to be made now, they are subject to the same basic requirements as were the graphs made in conjunction with the activities in Chapter 15, Graphing—Pictorial Representations. Each graph is not complete until a list of questions has been written for it, to serve as the basis for the discussion that accompanies its presentation to the class.

The discussion sessions help the students who made the graphs improve their ability to present data in an intelligible form and serve as a catalyst for future graphing ideas. For example, a discussion centering around the graph of shadows and heights could lead to graphs of the measurement of an object's shadow taken at specific time intervals to see if it changes its length predictably.

When one topic has been presented and methods of creating a graph for it have been discussed, the teacher presents additional topics until the class has between five and ten ideas from which to choose. Each student selects a topic to investigate and begins the necessary measuring and recording. Students may work alone or with one or two others, and a topic may be explored independently by more than one group at a time.

The graphing activities in this and the following lesson are meant to continue over several days, so each group need not finish its graph in a single day. The end of each class period is reserved for a discussion of any completed graphs. Students who complete a graph quickly may select another topic to investigate.

The teacher's role in providing topics can be difficult. Not every graphable topic can be graphed effectively using coordinates. The teacher's search for potential topics can be narrowed, however, if it is concentrated in those areas of measurement relating to growth or change.

Rates of growth for fast-growing plants such as corn, radishes, sweet potatoes, or beans offer good graphing potential. The box axis of a graph for corn might represent days and the triangle axis, height.

If corn grows according to a predictable pattern, could its height for the next day or the next week be predicted? If its height for successive days or weeks can be predicted, could its maximum hiehgt also be predicted? Do all varieties of corn grow at the same rate? What can be done to speed up or slow down how fast corn grows?

If corn grows predictably, at least up to a certain height, do other plants grow predictably? Do animals? If classroom animals such as hamsters or rats grow at predictable rates, what factors speed up or slow down their rates of growth?

Rates of change can also be measured. As a ball is dropped from higher and higher points, does the height of its bounce change in a predictable manner? Do all balls bounce to the same height dropped from the same place? Is the graph for each separate ball and its bounce a straight line? Or, do some balls bounce less high as they are moved to successively higher points? What bouncing patterns do things other than balls have?

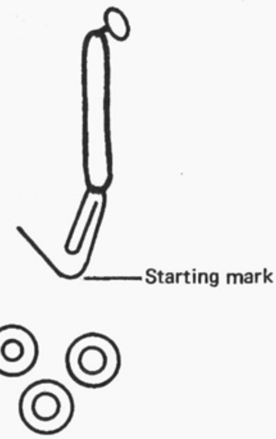
The number of questions that can follow from a single starting question is limitless. If the starting question produces a straight line on the resulting coordinate graph, the topic being investigated contains a predictable or functional relationship. If such a pattern is found, exploring what circumstances might alter it offers an excellent opportunity to ask more questions and seek more answers.

A study of balls can also lead to the study of circles. Is the diameter of a circle related to its circumference? If the diameter is known, can the circumference be predicted? What two numbers would be used to mark a point for diameters and circumferences?

Circumference is a special kind of perimeter. Do squares and rectangles of tiles have predictable perimeters as successive rows or columns of tiles are added? How would a graph for the perimeter of a rectangle with a constant number of columns and a growing number of rows compare with a graph for the same starting rectangle with growing columns and constant rows?

Perimeter can lead to area. What happens to the area of a rectangle as more rows or columns are added? Does the same thing happen to areas of other shapes? What happens to the perimeter of a rectangle if its area is kept the same but the tiles are rearranged to make long thin rectangles or short fat ones? What numbers would be graphed to show if there was a pattern? If the graph this question produces is not a straight line, can it still be used to make predictions?

Area leads to volume and capacity. Given a row of cylinders cut from a long tube of cardboard, with each cylinder of a different height, does a relationship exist between the height of a cylinder and the number of cups of rice it will hold? Could a coordinate graph be used to predict how many cups of rice a cylinder would hold if only its height were known? These cardboard cylinders change only in height. If construction paper were used to make cylinders of constant height but increasing diameter would the same kind of graph result as was achieved for the cardboard tubes with constant diameter?

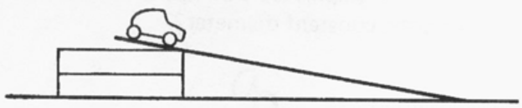


Capacity can lead to mass or weight. How can a scale give people their weight so quickly? If a rubber band is arranged as shown, can the distance it will stretch as successive numbers of washers are placed on the hook be predicted? If a graph can predict how far the rubber band will stretch, can it tell how many washers were added? Does it make any difference how thick the rubber band is? Is there a limit to how many washers the rubber band will hold? Can the graph be used to tell when this limit is near?

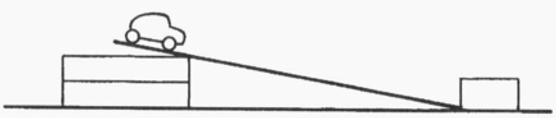
Heavy duty rubber bands can be made by cutting up old inner tubes. Would the heavy duty rubber bands act in the same predictable manner as much smaller rubber bands? What could be put on them that would pull down a little bit each time? Water or rice? If three cups of rice moved an inner tube section down as much as two cups of water, would six cups of rice move it as much as four cups of water? Could a graph be made with cups of rice on one axis and cups of water on another to predict how many cups of rice it would take to move the inner tube section as far as any number of cups of water?

Time is an area of measurement also. Pendulums move back and forth a countable number of swings in any given period of time. Can a coordinate graph be used to predict when the pendulum will finally stop swinging? Every minute the number of swings in a ten-second interval can be counted. If the box axis is for the number of swings during each interval and the triangle axis is for each successive interval, will a straight-line graph approach the triangle axis as the number of swings diminishes each interval? What does it mean if the number of swings *doesn't* diminish?

Time, too, is related to speed. Changes in speed, accelerations or decelerations, lead to graphable relationships. One apparatus for measuring changes in speed can be seen in the figure below. The toy car is released at the top of the board and allowed to roll freely down the slope. The number on the box axis indicates the amount of time the car took to make the journey. On the triangle axis are the numbers representing the books of equal size used to elevate one end of the board. After each successfully timed run at one height, the board is raised one level and the car is timed on a new run.



Is there a relationship between the time the car takes to get down the slope and the height of the board? Could a graph predict how fast the car will travel from any particular height before it is actually timed? What would happen if a longer board were used? What does make a difference? What doesn't?



The same apparatus used to measure acceleration can also be used to measure force. The only additional equipment needed is a block of wood at the end of the ramp. Can a graph be made to predict how far the car will push the block of wood as the car is released from successively higher starting points? If a higher starting point causes the car to knock the wood a predictably farther distance, what would happen if a longer board were used or if the block of wood were heavier or lighter? Do the same things cause the wooden block to be hit farther that also caused the car to travel faster in the experiments on acceleration? Why? Why not?

Almost any source of data that measures growth or change can be displayed on a coordinate graph. The topics suggested are not all inclusive, they are only examples of the kinds of investigations students may graph. Students are an additional source of topics that will be discussed in the following lesson.

LESSON 18-10

GRAPHING WITH COORDINATES

PURPOSE:

To use coordinate graphing skills to investigate relationships in events or experiences

MATERIALS:

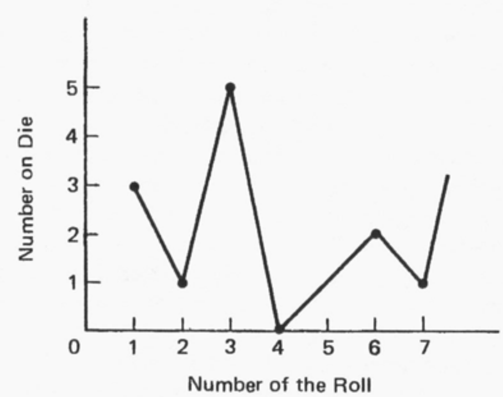
1. Materials available in the classroom

This lesson is essentially an extension of the preceding lesson; now, the students suggest topics to be explored.

Coordinate graphs are not appropriate in every situation. The students must learn through experience when they are beneficial and when another method of displaying data is more useful. The general rule for distinguishing a topic appropriate to coordinate graphing from topics better represented by another form is to determine if the graph is to be used to make comparisons (other graphs) or to predict (coordinate graphs).

Graphs of any kind are only ways of displaying information; if a student chooses to display data on a coordinate graph and it proves to be of no benefit, he or she may always switch to another form of graphing. Any rules the teacher could attempt to present for when a coordinate graph should be used would be inadequate. With the responsibility for making their own graphing decisions, the students can learn for themselves when a particular kind of graph is appropriate.

A student who decides to record successive rolls of a single die on a coordinate graph would, by a teacher's rules, be making an "inappropriate" graph. For each roll of the die the student might plot a single point on the graph representing both the number of the roll and the number on the die. An example of a coordinate graph for this can be seen in this figure.



When students are free to make their own discoveries, even "inappropriate" ones, the teacher may encourage them to analyze what has been found.

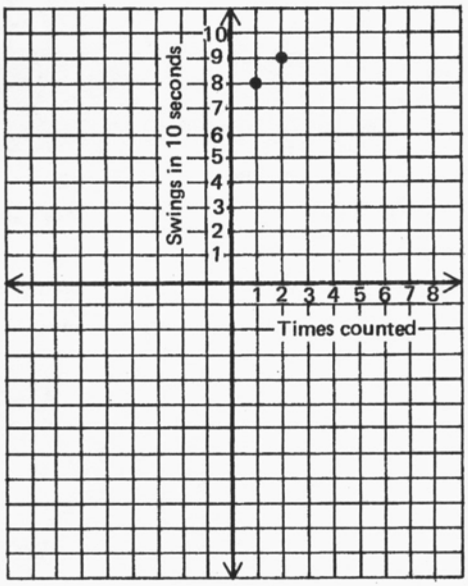
Can you predict the next roll of your die by using your graph?
Could you change your die so it produces a graph closer to a straight line?
What would a die be like that could produce a straight-line graph?
Are there patterns in the zigzags of your graph that are the same for all dice? Is it possible to compare dice by the zigzags their rolls make on a graph?

There is much to be learned from experiments that fail; students' own experiences are their best "teacher."

The teacher can, however, assist the students in deciding the numbers to record on each axis, presenting the problem to the whole class for discussion. Initially, the teacher provides most of the suggestions, as a model for how to select the appropriate numbers. This eventually leads to the students deciding the appropriate numbers for each axis.

An example of a teacher-lead discussion follows:

Teacher: Karen and Lynn have a problem for which they need our assistance . . . what is the difficulty?
Student: We want to predict when the pendulum stops—the time.
Teacher: Do you think it will stop?
Student: Yes.
Teacher: How do you know it is stopping?
Student: Because we can see it start to slow down.
Teacher: What do you see when you see it slowing down?
Student: The weight doesn't go back and forth as fast.
Teacher: Could you find a way to tell how fast it is going at some time?
Student: Yes.
Teacher: How?
Student: We could look at the clock and count how many times the pendulum swings every minute.
Teacher: You could also count how many times it swings every ten seconds, so you wouldn't have to count as long.
Student: Would that still let us predict?
Teacher: You'll have to do it and see. If it doesn't let you predict, then you can try it again using a minute.
Student: How can we graph our number?
Teacher: What numbers will you have?
Student: How many times it swung in ten seconds.
Teacher: How many times do you think you'll have to count its swings for ten seconds before you can predict?
Student: Maybe four or five.
Teacher: Okay. What's one thing you will count?
Student: Number of swings.
Teacher: Then one axis could be number of swings . . . each ten seconds?
Student: Yes.
Teacher: How many times will you count the swings?
Student: Four or five.
Teacher: Then the other axis could be for each time you count ten seconds; the first time you count for ten seconds is one, and so on. Let's suppose the first time you count you get eight, and the second time you get nine.
Student: We think we'll get less.
Teacher: Well, you can show me when you get it. Right now I'm trying to figure out if you could make a graph. Could you graph one and eight, then two and nine?
Student: Yes.
Teacher: Show me . . .
Student: Like this.



It is likely many of the topics the students suggest for coordinate graphing will correlate with their activities in science—many of these activities lend themselves readily to graphing.

As the students develop skill in coordinate graphing during their mathematics lessons, it should be integrated into the science lessons as well. The teacher indicates to the student or group when an experiment on which they are working might benefit from having its data recorded on a coordinate graph.

The students are not required to use the teacher's suggestions, but those who do, present their findings to their classmates. Discoveries in science should be discussed with the whole class, as in mathematics. From the initial few who use coordinate graphing to assist their science experiments, others take the lead.

The students now possess a beginning skill in graphing with coordinates. Their graphing so far has been confined to only one of the four sections on their graph paper. The activities in the following chapter permit them to use all four sections.