# CHAPTER 19
## Negative Numbers

**Prerequisite chapters:**

Chapter 18

## MATERIALS

**For overhead projector:**

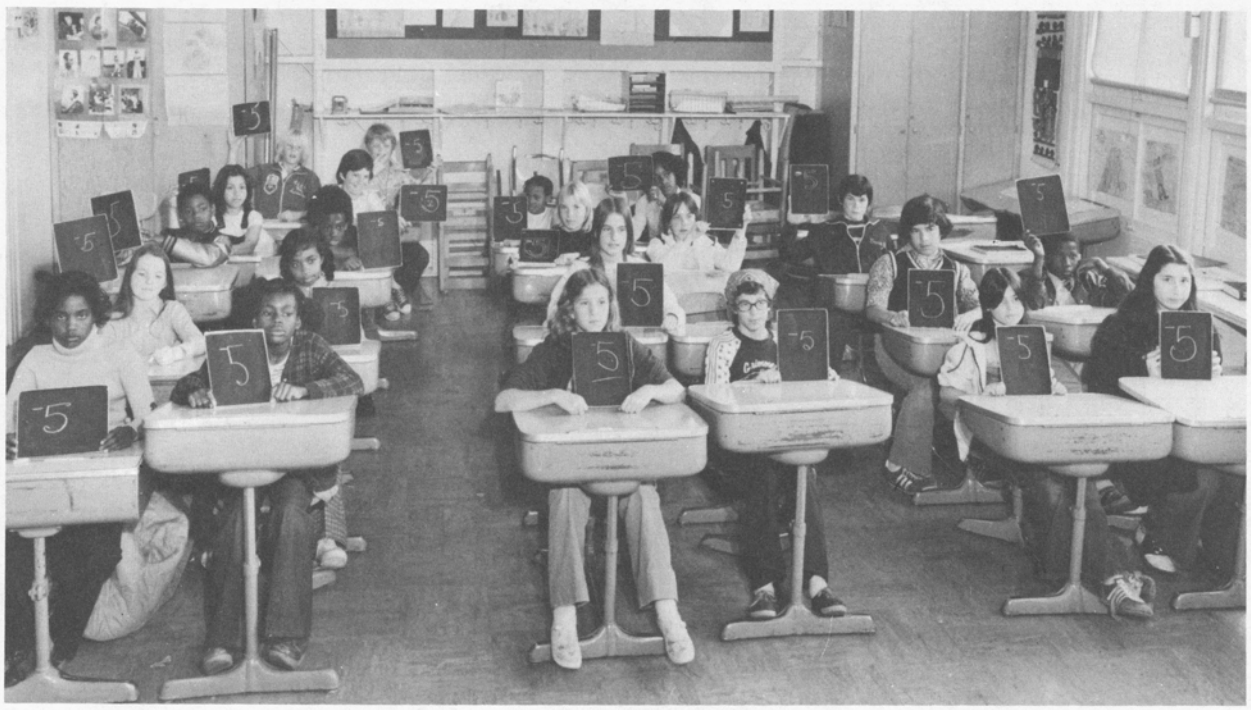
Transparencies Coordinate graph paper_____Worksheet 25

**If no overhead projector is available:**

Make charts in place of transparencies _____Materials chapter, page 294

**Student materials:**

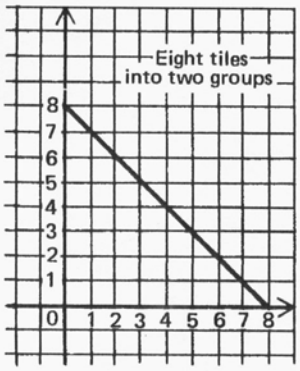
Individual blackboards _____Materials chapter, page 294

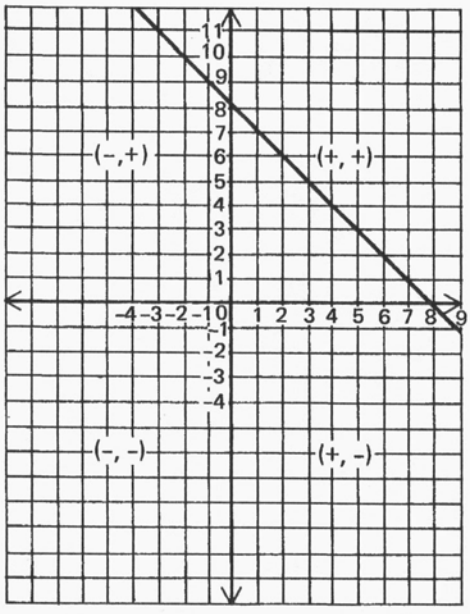



Negative numbers are a part of the real world. They appear on the television screen as a space rocket approaches the time for lift off. The nightly weather report on the news includes negative numbers every time a city's temperature drops below zero. The mini calculators now on the market include negative numbers as a part of their display.

Negative numbers are also useful when constructing coordinate graphs. They allow students to project their linear graphs as far as they wish, and then speculate on the meaning of *all* the points on the line.

As an example, look at the graph made in the preceding chapter for ways to divide eight tiles into two groups. With the addition of negative numbers, the line of the graph may be extended as in this figure. Negative one and nine are now represented by a point on the line. Do these two numbers represent ways to make eight? Are all the points on the line ways to make eight?




Mathematics is more than performing basic arithmetic operations—mathematics is a way of thinking. Negative numbers are a tool students can use to expand their thinking.

The activities used to introduce negative numbers in the lessons that follow are patterned after the "Postman Stories" in *Discovery in Mathematics,* by Robert Davis (Addison-Wesley, 1964).

# LESSON 19-1

## ADDITION OF SIGNED NUMBERS

PURPOSE:

*To learn a format for adding signed numbers*

MATERIALS:

*1. Individual blackboards*

The next few lessons introduce students to addition and subtraction of signed numbers and provide them with a knowledge of negative numbers that will be used to expand their coordinate graphing capabilities in later lessons.

**Teacher:** Today I will give you some arithmetic problems to work out, all centered around a mail carrier named Sam. Sam is no typical mail carrier. When I tell you about him, you may wonder why anyone lets him deliver mail at all. Sam is a little strange, because sometimes after he has delivered the mail he comes back and takes it away again.

Before we talk about his mail deliveries, I'll tell you how to write down the mail he brings. Sam only delivers two kinds of mail: checks and bills. The bills he brings are the kind you get when you owe somebody money. If he brings a check, for three dollars, I want you to write that like this

$^{+}3$

and I'll read it *positive three.* Notice where I put the sign: near the top of the three and not in the middle.

If Sam brings a bill for two dollars, you write that like this

$^{-}2$

and I'll read it, *negative two.* You can see that I wrote the negative sign in the same place as I wrote the positive sign for positive three.

Now I'll write some numbers on the overhead and I want you to tell me whether they are checks or bills?

**Teacher:** $^{+}4$
**Student:** Check.
**Teacher:** $^{-}5$
**Student:** Bill.
**Teacher:** $^{+}1$
**Student:** Check
**Teacher:** $^{+}5$
**Student:** Check.
**Teacher:** $^{-}1$
**Student:** Bill.
**Teacher:** Now, suppose you were sitting at home and Sam brought you a check for three dollars. Would you be richer or poorer?

**Student:** Richer.
**Teacher:** By how much?
**Student:** By three dollars.
**Teacher:** So if Sam brings you a check, you are richer by the amount of the check. If Sam brings you a bill for three dollars, would you be richer or poorer?
**Student:** Poorer.
**Teacher:** By how much?
**Student:** By three dollars.
**Teacher:** Suppose Sam brought you a check for two dollars and a bill for three dollars. Would you be richer or poorer?
**Student:** Poorer.
**Teacher:** By how much?
**Student:** By a dollar.
**Teacher:** Now, I'll give you a problem and show you how to write it down. Sam brings you a check for three dollars, then another for two dollars. Are you richer or poorer?
**Student:** Richer . . . by five dollars.
**Teacher:** Okay.

$$^{+}3 + ^{+}2 = ^{+}5$$

Now, here is another problem: Sam brings you a bill for five dollars, then another for four dollars. Are you richer or poorer?
**Student:** Poorer . . . by nine dollars.

$$^{-}5 + ^{-}4 = ^{-}9$$

**Teacher:** New problem. Sam brings you a check for four dollars and a bill for five dollars. Are you richer or poorer?
**Student:** Poorer by a dollar.

$$^{+}4 + ^{-}5 = ^{-}1$$

**Teacher:** Okay. What do you think this might mean?

$$^{+}2 + ^{+}4$$

**Student:** Sam brought a check for two dollars and then he brought another for four dollars.
**Teacher:** Would you be richer or poorer?
**Student:** Richer by six dollars.
**Teacher:** How about this one?

$$^{-}5 + ^{-}2$$

**Student:** Sam brought a bill for five dollars and then another for two dollars . . . and we were poorer by seven dollars.
**Teacher:** Try this one.

$$^{+}4 + ^{-}2$$

**Student:** Sam brought a check for four dollars and then a bill for two dollars, but we were still richer by two dollars.
**Teacher:** Okay. I will tell you the words for some mail carrier problems. Write the numbers on your blackboards for each word problem I give, then write how much richer or poorer you think you'd be.

The problems fit into four formats:

1. The mail carrier brings a check for _____, then he brings another check for _____ (written + _____ + + _____).
2. The mail carrier brings a bill for _____, then he brings another bill for _____ (written - _____ + - _____).
3. The mail carrier brings a check for _____, then he brings a bill for _____ (written + _____ + - _____).
4. The mail carrier brings a bill for _____, then he brings a check for _____ (written - _____ ++ _____).

The students write numbers for the teacher's problems throughout the time remaining in the lesson.

Although the effect of a negative number added to a positive number is reminiscent of subtraction, the problems presented in this lesson are addition problems. The problems in the next lesson introduce the students to subtraction of negative numbers.

Note: If any students have difficulty deciding if they are richer or poorer as a result of the mail deliveries, checks and bills may be cut from paper and each problem may be physically enacted until the process becomes clear.

# LESSON 19-2

## SUBTRACTION OF SIGNED NUMBERS

PURPOSE:

*To learn a format for subtracting signed numbers*

MATERIALS:

*1. Individual blackboards*

**Teacher:** So far the mail carrier has been pretty good. Sam has always brought the right mail. Now I want you to think about a new situation.
If the mail carrier brings you a check, are you richer or poorer?
**Student:** Richer.
**Teacher:** What happens if he takes a check away from you?
**Student:** Then we're back where we started.
**Teacher:** Okay. But are you richer or poorer than before he took the check away?
**Student:** Poorer.
**Teacher:** If the mail carrier brings you a bill?
**Student:** We're poorer.

**Teacher:** What happens if he takes the bill away?
**Student:** We're richer.
**Teacher:** When Sam brought you a check for three dollars, we wrote it like this.

$$+^{+}3$$

What do you think this means?

$$-^{+}3$$

**Student:** Sam took away a check for three dollars.
**Teacher:** And this?

$$-^{-}3$$

**Student:** Sam took away a bill for three dollars.
**Teacher:** I will write some problems on the overhead. I want you to tell me what the numbers indicate Sam did.

$$^{+}3 - ^{+}4$$

**Student:** He brought a check for three dollars and then he took away a check for four dollars.
**Teacher:** Are you richer or poorer?
**Student:** Poorer by one dollar.

$$^{-}3 - ^{+}4$$

**Teacher:** Try this one.
**Student:** He brought a bill for three dollars and took away a check for four dollars.
**Teacher:** Are you richer or poorer?
**Student:** Poorer by seven dollars.
**Teacher:** Think about this one before you tell me what the numbers mean Sam did.

$$^{+}3 - ^{-}4$$

**Student:** He brought a check for three dollars and he also took away a bill for four dollars.
**Teacher:** Richer or poorer?
**Student:** Richer by seven.
**Teacher:** This one?

$$^{-}3 - ^{-}4$$

**Student:** He brought a bill for three dollars, but he took away a bill for four dollars . . . so we are now richer by one dollar.
**Teacher:** Okay. I'll tell you the words for some mail carrier problems. Write the numbers for each word problem on your blackboards. After you write the numbers, write how much richer or poorer you think you'd be.

The problems the students are given fit into four basic formats:

1. The mail carrier brings a check for _____, and takes away a check for _____ (written + _____ − + _____ ).
2. The mail carrier brings a check for _____, and takes away a bill for _____ (written + _____ − − _____ ).
3. The mail carrier brings a bill for _____, and takes away a check for _____ (written − _____ − + _____ ).
4. The mail carrier brings a bill for _____, and takes away a bill for _____ (written − _____ − − _____ ).

# LESSON 19-3

## ADDITION AND SUBTRACTION OF SIGNED NUMBERS

PURPOSE:

*To practice adding and subtracting signed numbers*

MATERIALS:

1. *Individual blackboards*

The activities for this lesson combine those of the previous two lessons. The teacher alternates presenting word problems for addition with those for subtraction to provide students additional practice in using signed numbers.

# LESSON 19-4

## COORDINATE GRAPHING WITH SIGNED NUMBERS

PURPOSE:

*To learn to play coordinate tic-tac-toe using all four sections*

MATERIALS:

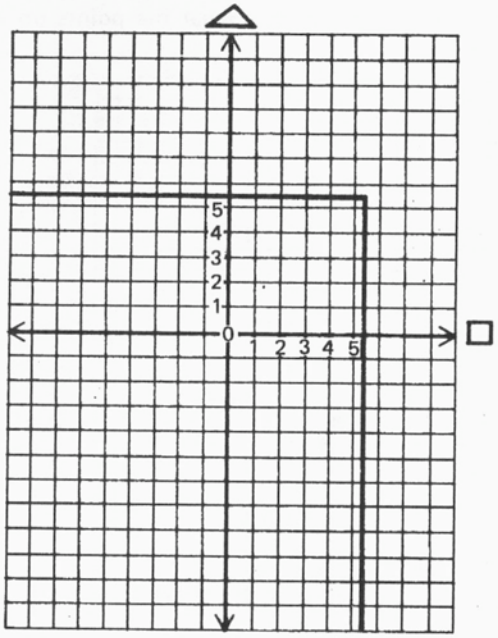
1. *Coordinate tic-tac-toe grid on a transparency or a large tagboard*

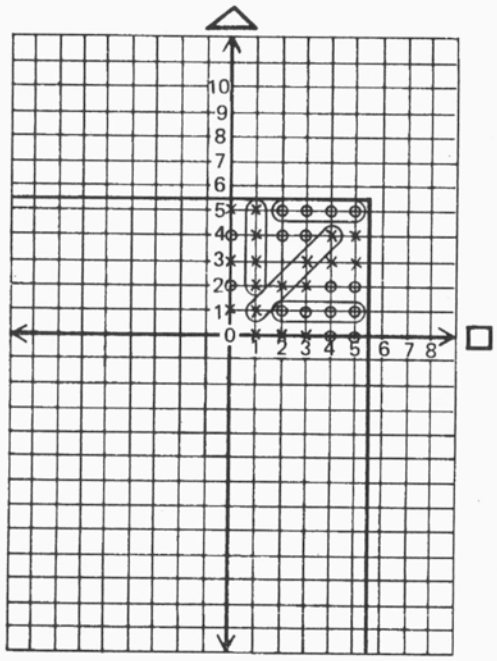
In this lesson, students discover the negative numbers they used in the previous three lessons can also be used to designate points on a grid.

The game of coordinate tic-tac-toe was first played in Lesson 18-5. To introduce the students to negative numbers on a coordinate graph, a modified playing area is used. The teacher draws lines on a grid. The following rule is added to the game: each team must place all its marks

(*o*'s or *x*'s) to the left and below the boundary lines drawn by the teacher. Marks may be recorded anywhere on the grid within the boundaries.

The students quickly find they can no longer make additional moves in the section where they first learned to play the game.

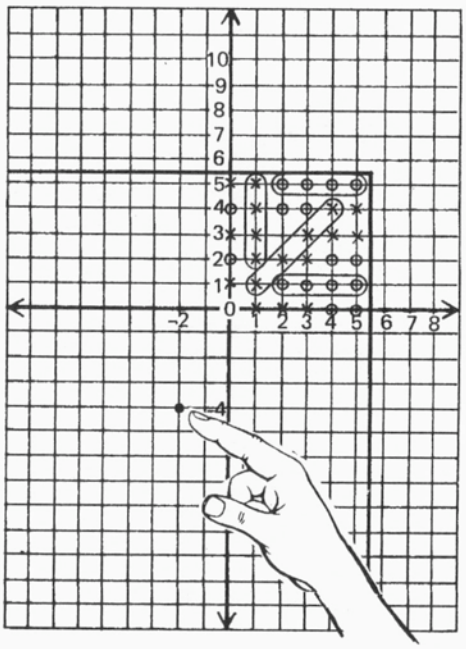



**Teacher:** Richard, your turn.
**Student:** Two and one.
**Teacher:** I'm sorry, there's already a mark at two and one . . . Gregory, your turn.
**Student:** Five and six.
**Teacher:** By the rules of this game, I can't put a mark at five and six, because that point is above the boundary line.
**Student:** Can we make moves in the other sections?
**Teacher:** Within the boundaries, yes.
**Student:** How do we do it? There aren't any numbers in the lines.
**Teacher:** There are numbers that go on the lines. I haven't written them yet, because I want you to think about what they might be. When you tell me the numbers that give a point in another section, I'll write them on the line.
Lynn, your turn . . .

Experience shows that once the students have been exposed to negative numbers, they are fully capable of discovering negative numbers can be used to describe points on a grid. Not all students will make this transfer independently, but it is enough for a single student to say:

**Student:** Negative two and negative four.
**Teacher:** Negative two is on the box axis and negative four is on the triangle axis, so that point would be here.




The teacher writes the numbers on the box and triangle axes that correspond with the numbers given. As more points are added, more numbers are added to each axis. Because the teacher has not told the students what numbers go on each axis, they accept finding the missing numbers as a challenge.

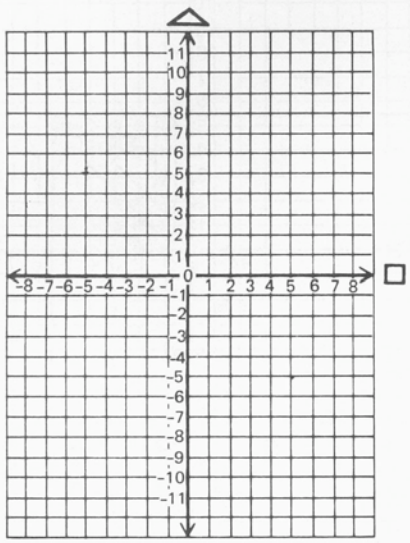
Once one student shows the way, it is common for almost all students to be able to identify points on a grid using both positive and negative numbers on their next turn. It is also common for *all* students to have reached this point of mastery by their second turn.

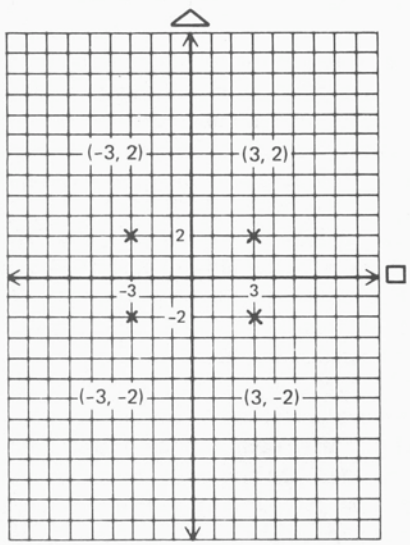
If, on the first day they play coordinate tic-tac-toe, no students discover a method of describing points in the three empty sections after each has had one attempt, the game is over for the day. On the next day a new game is started using a reduced playing area, and the students once again face the problem.

Once the teacher decides the answer to a particular question or problem is the students' responsibility, the decision should hold. The value of students learning to rely on their own thinking is lost if the teacher later intercedes by telling them the answer. If the teacher feels the students must know the answer, the students should not be mislead into thinking the answer is their responsibility. If the teacher steps in and provides the answer, will the students ever believe again that if they don't get it, it won't be given to them?

For the benefit of those readers who have not recently or never used a coordinate graph, the only knowledge necessary to correctly mark points is that the first number to be marked is found on the horizontal axis and the second number on the vertical axis. The point is the intersection of the lines leading away from each number.

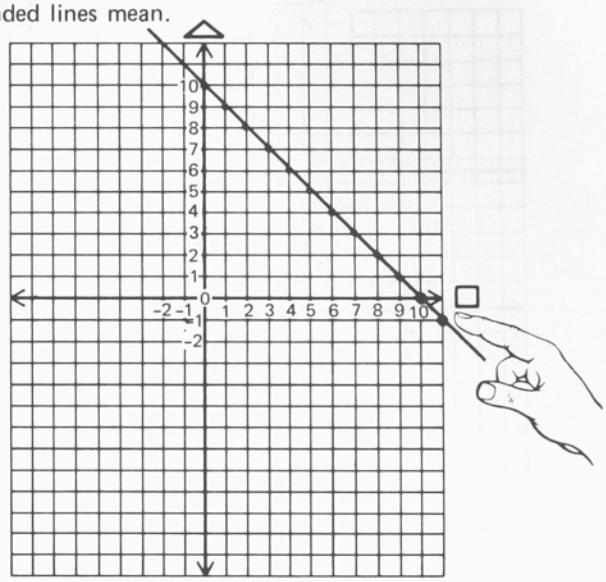
An example of a coordinate grid with each axis fully numbered can be seen in the first figure below. Examples of points marked in each of the four sections of the grid can be seen in the second figure below.






If the students teach themselves to play coordinate tic-tac-toe using negative numbers, the teacher assists them in transferring this knowledge to their work with coordinate graphs.

The teacher selects those coordinate graphs already made that contain lines extending into the negative quadrants and asks the class to contemplate what the points on the extended lines mean.



**Teacher:** This is a graph constructed for ways to make ten using two groups of tiles. I have drawn the line so that it extends into the spaces for negative numbers, too. What numbers give this point on the line?

**Student:** Eleven and minus one.

**Teacher:** Is eleven and negative one a way to make ten with two groups of tiles?

**Student:** No . . . that would be twelve tiles.

**Teacher:** If the mail carrier brought you two letters, one a check for eleven dollars and one a bill for one dollar, how much richer or poorer would you be?

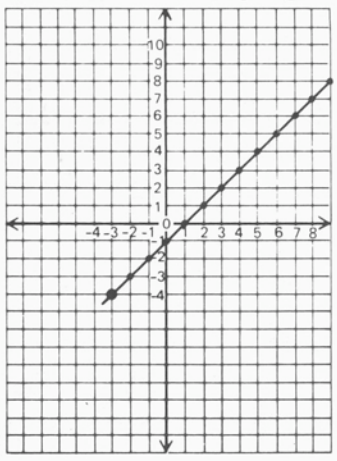
**Student:** Richer by ten dollars.

**Teacher:** Then is eleven and negative one a way to make ten?

**Student:** Yes.

**Teacher:** Can you think of how it might be a way to make ten using two groups of tiles?

**Student:** Do we have to *add* the two groups?



Students may not always be able to attribute meaning to the numbers produced by the lines on their graphs that extend into negative quadrants. In the graph for the height of bounces in the figure on the preceeding page, what is meant by the numbers negative three and negative four? How high is a bounce of negative three?

It isn't important for students always to be able to attribute meaning to everything they find. What is important is that they use all the tools available to them to learn whatever they can about the world around them. Coordinate graphing is a tool—negative numbers make it a more flexible tool. Just how useful this new flexibility proves depends on the students themselves and the problems they elect to explore.