CHAPTER 20

Tangrams — Logical Thinking

Lesson 20–1 page 276 Tangrams Students tear the tangram puzzle from paper and fit the pieces back together.	Lesson 20–2 page 277 Tangrams Students fill in shapes on a matrix.	Lesson 20–3 page 278 Tangrams Students make and solve *yes–no* tangram task cards.	

Prerequisite chapters:

None

MATERIALS

For overhead projector or teacher demonstration use:

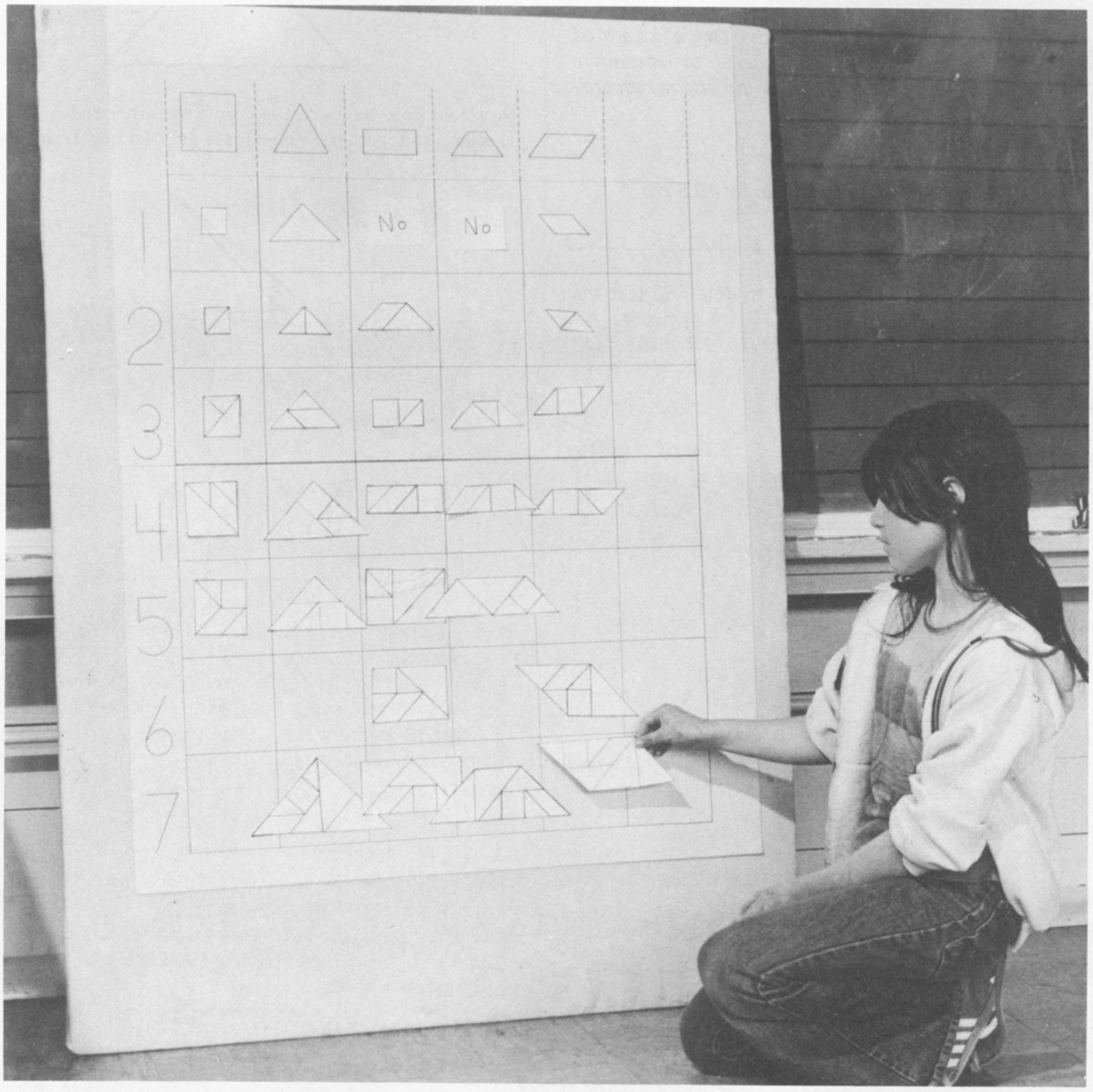
Transparencies — Tangram task card triangles _____ Worksheet 26
Square of paper — 10.5 cm by 10.5 cm

If no overhead projector is available:

Make charts in place of transparencies _____ Materials chapter, page 294

Student materials:

Dittos — Tangram task card triangles
Tangram puzzles _____ Materials chapter, page 299
Squares of paper — 10.5 cm by 10.5 cm _____ Materials chapter, page 297
Unlined paper
Scissors



In this chapter, students learn to make nonverbal task cards to use with a seven-piece set of shapes called the *tangram puzzle.* The task cards give them practice in problem solving and logical thinking. The last lesson is meant to provide students practice in logical thinking on a weekly basis throughout the school year.

LESSON 20-1

TANGRAM PUZZLE

PURPOSE:

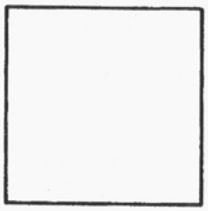
To learn how to fold and tear a square of paper into a tangram puzzle; to reassemble the torn pieces back into the original square

MATERIALS:

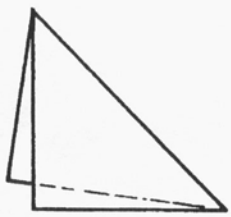
1. *Paper squares 10.5 cm by 10.5 cm*
2. *Tangram puzzles*

In this lesson, students learn how to fold and tear the pieces of the tangram puzzle from a square of paper. Arranging the puzzle pieces into a square is a difficult task; tearing the pieces from a square first convinces students the square *can* be made.

Teacher: Today I will show you how to make a puzzle called a tangram puzzle out of the square of paper I have given each of you.

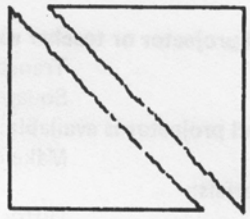


First, fold your square in half, like this, so the fold line makes two triangles.



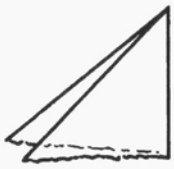
Check with your neighbor to see if I am making my instructions clear.

Now, tear apart the two triangles along the fold line. Some people have found if you lick the fold first, it tears more easily.



Hold up one of the two triangles and put the torn edge at the bottom. One point should be pointing up.

Now fold your paper so the points on either end of the torn sides are together. You should get two new triangles.

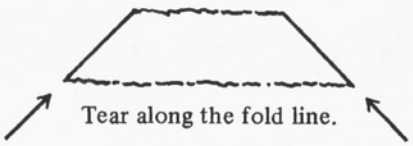


Tear them apart along the fold line and put them on your desk.

Pick up the other big triangle. Hold it with the torn edge down and one point up like you did the first one, but don't fold it the same way. Take the top point and fold it down until it touches the middle of the bottom edge. It should make a little triangle on the top part of the big triangle.

Now, tear along the fold line. Put the little triangle with the other two little triangles.

Hold the long piece by both ends at the bottom and fold the ends until they touch. Each side will look something like a shoe.

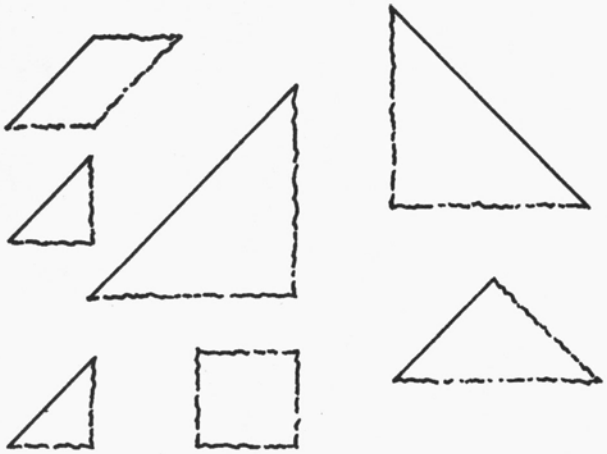


Hold up one of the shoes. Put the longest side at the bottom. Fold the longest side so that you get a square on one side and a triangle on the other. Now, tear the square and the triangle apart and put them with the small triangle.

Pick up the other shoe and hold it in one hand by the heel, with the long side at the bottom. Do you see where the laces would tie if it were really a shoe? Fold it so the heel touches where the shoelaces would tie. You should have a small triangle on one side and a parallelogram, (it looks kind of like a squashed diamond), on the other.

Tear the two pieces apart and put them with all the other shapes.

How many pieces do you have altogether?



Student: Seven.

Teacher: That's the tangram puzzle. See if you can put your pieces of paper back together to form the square you started with.

As the students attempt to reassemble the square, the teacher gives each a more durable set of tangram pieces with which to work. They are the same size as the paper pieces so the students know they can be formed into a square. Students who manage to reassemble the square are asked to form a triangle, then a rectangle, then a parallelogram using all seven pieces. Those who complete each of these shapes may spend the remainder of the period making whatever other shapes they wish.

LESSON 20-2

TANGRAM PUZZLE

PURPOSE:

To provide practice in assembling shapes with the puzzle pieces; to allow each person to succeed in assembling shapes

MATERIALS:

1. *Tangram puzzles*
2. *Unlined paper*
3. *Scissors*
4. *Yarn*

In the previous lesson, some students may not have been able to reassemble the square with their puzzle pieces. This lesson is designed to provide them practice in assembling a variety of shapes with their puzzle pieces while allowing each to control the level of difficulty at which he or she is working. Thus, each student has the opportunity to complete a task and feel successful.

Teacher: I have drawn a matrix on the overhead and a larger version on the bulletin board. I will help you begin to fill in both matrixes.

Can you make a square using one tangram piece?

Student: Yes.

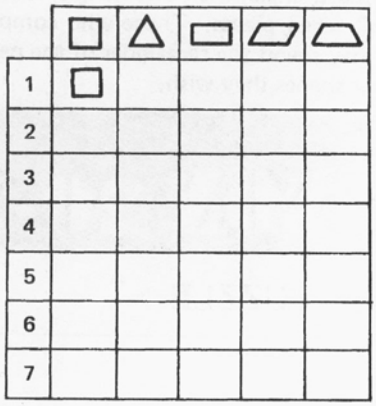
Teacher: Show me how.

Student: This piece is one piece and it's a square.

Teacher: Okay. Since you can make a square with one piece, I'll write "yes" on the overhead underneath the square and across from the one.

Johnny, would you please trace the square onto a piece of paper, cut it out, and pin it up in the appropriate space on the bulletin board . . .

	□	△	▭	▱	⏢
1	YES				
2					
3					
4					
5					
6					
7					



Can you make a triangle with one?

Student: Yes . . . here's a triangle with one piece.

Teacher: Then where will I write yes on the overhead?

Student: Underneath the triangle and across from the one.

Teacher: Lia, please cut out that triangle and put it on the bulletin board.

Can you make a rectangle with one?

Student: No.

Teacher: It's okay if you can't do it, but I can't write "no" on the overhead just because you say no. When I asked you to make a square with all seven tangram pieces some people were unable to make it, right?

Student: That's right.

Teacher: Does that mean the square can't be made?

Student: No.

Teacher: When you tell me you can't do something with the tangram pieces, I believe you, but that doesn't mean no one else can. If I write "no" on the overhead, you must tell me why no one in the world could do it. It's okay if you can't think of a reason, but I can't write no unless I do have such a reason.

Can you tell me why no one could make a rectangle with one piece?

Student: Because there are no one-piece rectangles in our tangram puzzles.

Teacher: I'll accept that reason . . . and write no underneath the rectangle on my matrix . . .

Although squares are, by definition, rectangles they are treated as separate entities for this exercise.

Teacher: Can you make a parallelogram with one piece?

Student: Yes, because we have that piece in our set.

Teacher: And where would I write yes on the overhead?

The teacher directs the students in providing yes or no answers for each of the empty spaces on the overhead until the matrix has been filled in halfway across the second row. For each yes answer, a student is selected to cut out that shape and post it on the bulletin board. No answers are written when the students explain to the teacher's satisfaction why no one could ever make that shape with that number of pieces.

Teacher: You've seen how to fill in a yes or no on the overhead. Now you may each decide which shape you want to make and how many pieces to use. When you've made a shape, come up and write yes in the appropriate space on the overhead, then cut it out and pin it on the bulletin board.

If someone writes yes in a space before you do, you may still cut out your shape and pin it to the bulletin board. We are interested in seeing if each of you made the shape the same way.

If you want to write no in a space, you must tell me why you think that particular shape is impossible for anyone to make.

Are there any questions? Then you may begin.

The students make and trace shapes throughout the time remaining. When the lesson is over, the bulletin board matrix is left in position. Students who wish to, continue their efforts to complete the matrix during their free time.

LESSON 20-3

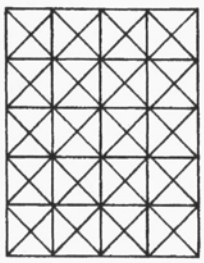
TANGRAM PUZZLE

PURPOSE:

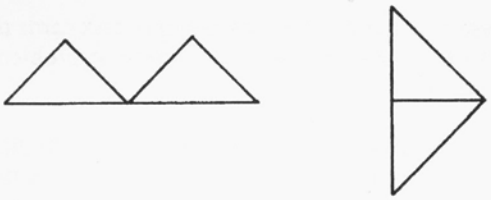
To make nonverbal task cards for use with the tangram puzzles; to use them as an exercise in logical thinking

MATERIALS:

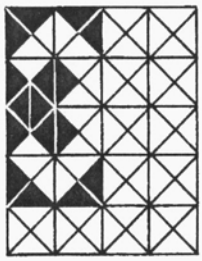
1. *Tangram task card paper on a transparency or a large tagboard*
2. *Tangram puzzles*
3. *Dittoed copies of tangram task card paper*
4. *Scissors*



Teacher: You each have a page of triangles like the one I have on the overhead. Cut out shapes containing exactly 16 triangles. Each triangle must touch another at either a corner, like this, or on a side, like this.

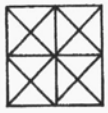


I'll color in 16 triangles on my overhead transparency to give you an idea of the kinds of shapes I want you to cut out. You have about 10 minutes to cut as many different 16-triangle shapes from paper as you can. If you finish cutting up one piece of paper before the time is up, you may begin another.

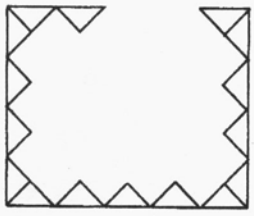


The students are not told the shapes they are cutting out will become task cards for their tangram puzzles because many would then want to place their puzzle pieces on the paper to make sure they fit evenly. This is not desirable because one of the values of the shapes is the uncertainty of whether the puzzle pieces will cover them. Why this is so will become clear as the lesson progresses.

When the time allowed for cutting is over, the teacher collects the 16-triangle shapes. The tangram puzzles have an area equivalent to the 16 triangles in each shape. If the square shape shown in this figure were cut from the paper, the 7-piece tangram square would fit exactly on top of it.



Although all the 16-triangle shapes have the same area as the tangram sets, this does not necessarily mean the tangram pieces will fit on all the shapes. The pieces would fit on the shape in the above figure, but would they also fit on the 16-triangle shape shown in this figure? For this shape, where would the square or the parallelogram be placed?



Once all the shapes are collected, the teacher reissues one to each student. The assignment is to try to fit the tangram pieces on top of the cutout. If all seven pieces fit, the student show the completed work to the teacher or to a fellow classmate, then writes a yes on the shape and pins it to a designated "yes" area on the bulletin board.

There is also a "no" area on the bulletin board, but only the teacher may write no on a cutout. Students who cannot cover a shape with their tangram pieces have three options: (1) they may continue trying, (2) they may return the shape to the front of the room and select another, or, (3) they may attempt to explain to the teacher why no one could cover that shape with the seven tangram pieces. If the teacher is convinced by the student's arguments, a no is written on the shape and it is pinned in the appropriate area on the bulletin board. If the teacher is not convinced, the student may revert to one of the first two options.

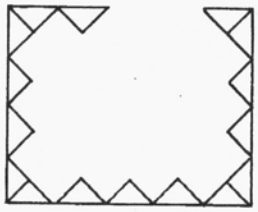
A common element of traditional tangram task cards is that each can be constructed using the tangram pieces. This means each task card can be solved with sufficient dilligence. This kind of activity gives students knowledge of shapes and their relationship to one another.

The task cards given the students in this lesson have a different purpose. When students do not know if the pieces will fit, their thinking as they move the pieces around shifts from "Can I do this? If so, how?" to "Can anybody do this? If not, why not?" Analyzing why something *can't* be done allows students to attack problems through the use of logical thinking.

Task cards that require logical thinking are also less frustrating for students to use than those that can all be solved. If a student is given a tangram task card for which everyone knows there is a solution, and cannot fit the pieces on the shape, the student has failed. If, instead, the student is given a task card which may be unsolvable, not being able to fit the pieces on the shape holds no stigma of failure.

At the start of the year most of the 16-triangle shapes will either be in the yes area on the bulletin board or returned to their storage area in the front of the room. Initially, students do not know how to rationalize a "no" shape. However, the challenge of producing explanations plus the time to develop rationales lets students find ways to justify a no for the shapes that deserve them.

The first shapes for which students are able to explain why the puzzle pieces won't fit are those on which specific puzzle pieces obviously won't fit. The shape in this figure is an example because neither the square nor the parallelogram can be placed on it.



The same logic used to justify a no for the last figure is the basis for explaining why an increasing number of shapes should be classified "no." Sometimes it is relatively simple to demonstrate that the pieces won't fit; sometimes the

process is more complex. In either case, however, the students can discover for themselves when a task card can't be covered, and when it can.

Student's ability to think logically increases in direct proportion to practice. As the activities in this lesson are repeated throughout the year, the students demonstrate the effects of this learning. Whereas, at the start shapes will be left unsolved, by the middle of the year shapes will be identified the same day they are made. This growth is a direct result of the practice the tangram task cards provide the students in approaching the solution of problems in a logical, systematic manner.

The logical approach to problems presented in this chapter is a skill students find useful in formulating solutions to problems they confront in any area of mathematics. With this skill, students are now ready for the activities in the chapter that follows.