CHAPTER 21 Problem Solving



Prerequisite chapters:

To be decided by the teacher

MATERIALS

For overhead projector:

Student materials:

To be decided by the teacher

To be decided by the teacher and the students



Each mathematics lesson in the preceding chapters involved some form of problem solving, focusing on specific skills. As the students acquire a working knowledge of these basic skills, they can be introduced to problem solving that is not structured around any specific mathematical skills. Participating in a wide variety of problem-solving activities allows them to apply the mathematical skills they have learned.

Before students are ready to explore an increasing variety of problem-solving activities, they must be ready to work independently with the limits imposed by the presence of 30 or so classmates and a teacher. Students need to accept responsibility for their own actions, and to know that answers can come from themselves and not the teacher. They must also know how to use their classmates and teacher as resources for ideas. They need a minimum knowledge of certain mathematical skills, necessary tools in any problemsolving situation, and to know how to combine all the skills and resources available to them in working out, to their own satisfaction, solutions to the problems they face. If these needs are met, the transition from teacher-directed lessons to student-initiated explorations can be made smoothly.

As students complete the series of activities presented in the preceding chapters, problem-solving activities become the central focus of the math period. Each student or group decides what to measure or explore and conducts its investigations independently of the remainder of the class.

There is no definite time when a class becomes ready for problem-solving activities; each class is different. Only the teacher can judge when a student, a group of students, or a whole class is ready.

The single lesson in this chapter represents a model for independent explorations.

JESSON 21-1

PROBLEM SOLVING

PURPOSE:

To apply any and all of the mathematical skills gained from the activities in earlier chapters to a variety of problem-solving situations

MATERIALS:

1. To be decided on by the students and teacher, according to the situation

When the teacher feels individuals or groups of students are ready to begin unstructured problem-solving activities,

he or she suggests topics or questions to pursue. The topics are only suggestions, however, and each student is free to propose any other area of exploration. New topics are suggested any time a student or group completes an investigation on an earlier topic. Time is set aside at the end of each math period to permit those who have finished working on a project to share the results of their investigations. Examples of investigations the teacher might suggest that take a considerable amount of time to complete are the following:

- What does it cost to live in a house? How much for rent? How much for everything else?
- How much garbage does your neighborhood make in a week? A month? A year?
- Take a census of the school. What questions should be asked?
- What does it cost to run a school?
- Plan a trip. Where would you go? How much would it cost?
- Who is of average height in our classroom? Can you find a person of average weight?
- Make a million of something. How many things around school do we have a million of?

How much money does it take to support a mother, a father, and two children for a month?

Make a map of the school.

Construct the school out of tagboard or light cardboard.

- Make a model of your house.
- Make a map of the neighborhood. Show all the houses.
- Make a kite. Which kinds of kites fly best?
- Which paper airplanes fly best?
- Make a puppet stage for our puppet plays.
- Which grocery store in the neighborhood has the best prices on produce? On dairy products? On canned goods? On meat? On all food?

Examples of investigations that do not take much time on any given day, but which must be conducted over a series of days are:

How fast does your hair grow?

Catch an insect. What can you find out about it?

- Plant bird seed. Will it grow? Into what? How many different kinds of plants will grow?
- How much food does our pet rat eat each day? Each week? Each month? Each year? How much weight does it gain each week?
- What is the temperature outside each day at noon? How does the temperature change every half hour during a school day? Is this the same each day? Each month?
- Pick a stock on the New York Stock Exchange. Keep track of how it does each day for a month.
- How many letters come to your house each day? Each week? Each year? To your block? To school?
- Do newspapers have more square centimeters of ads in them on one day of the week or is each day the same?
- What is the best price you can find in the newspaper for sugar? Coffee? Soap?

Examples of questions that only take a period or two to explore, but which may lead to many more questions are:

What is the most frequently used letter in the alphabet? Does it make any difference what book, magazine, or newspaper you use to count from?

How big is an orange peel?

- How fast can you read? Write? Run?
- How do you spend your time each day?
- How big are you? In how many ways?
- How many pages do you read each day?
- What's the best way to fold a newspaper for throwing?
- How fast is your pulse? When does it speed up? Slow down? Does everyone's pulse beat the same?
- Do objects weigh the same underwater?
- What is a batting average? Can you find your kicking average?
- Is our kickball diamond the right size?
- How far can you throw a ball? Does practice increase the distance?
- How does a bricklayer know how many bricks to use?
- How are fingerprints used to tell people apart?
- Are the ads on television true?
- How far could you walk in a day?
- How much does a sheet of paper weigh?
- How thick is a page in your dictionary?
- How many times do you breathe each day?
- If you could have your weight in coins, which coin would you choose? How much would you be worth?
- How many parents, grandparents, great grandparents, etcetera do you have? Keep counting back for 2,000 years. How many ancestors do you have? How many people were alive 2,000 years ago?
- How much does a newsperson's load of papers weigh?
- Take a piece of construction paper and make the largest box you can-it doesn't need a top. How do you know its' the largest?
- How much milk do they use in the cafeteria each day? Each week? Each year?
- Take an opinion poll in our class. Can it be used to predict the results of a school-wide opinion poll?
- What is the average age of our class in years, months, and days? Is our average the same as for other fifth-grade classes in the school?

How can you use an object that rolls to measure length?

- Do children who eat breakfast weigh more than those who don't? Are they taller? How about those who drink milk and those who don't?
- Is it cheaper to travel by car, bus, or plane from here to New York City?
- What does a broken window cost the school to replace?
- How long would it take an insect to crawl across the floor from the door to the opposite wall?
- How long is a million seconds? Minutes? Hours?
- What things have no weight?
- How much weight can a string take before it breaks?
- How many hairs do you have on your head?
- How high is the auditorium ceiling? The flagpole? The roof of our classroom?
- Do things get lighter or heavier when they are cooked?

Do you weigh more standing up or lying down? How big was Noah's ark?

Science investigations provide a steady source of problemsolving activities for students. Examples of the kinds of explorations the students may make are:

- What happens to the water level as objects are immersed in a jar of water? Which objects make the water level rise? Which make the level fall?
- Put some objects in both pans of the pan balance. What can you do to make the objects balance? Does it make any difference where they are placed in the pan? How can you tell if one is heavier than another? Can you put a group of objects in order by weight?
- Make a pendulum by tying a weight on a piece of string and taping the string to the edge of your desk. Can you get two pendulums to swing together for ten swings? For twenty swings? Can you make one pendulum swing twenty times while another swings only ten? What can you do to a pendulum to change the way it swings? Does the length of string make any difference? Does the weight of the pendulum bob make any difference? How long will it take for the pendulum to stop swinging?
- Plant some seeds. How can you tell if what you planted is a seed? Is there a tiny plant already inside the seed? Will half a seed grow? How much water should the seed get? Will it grow in all water or no water? Will a seed grow in the dark? Which is more important to a plant, leaves or roots? What parts of the plants grow? How could you get the next seeds you plant to grow taller?

In addition to providing topics for investigation, any question asked will spark other questions. If the question is asked, Will half a seed grow? then the question, Does every-thing that grows start from a seed? can also be asked. If the teacher asks how much milk is used in the cafeteria each day, he or she can also ask how much milk is used at home, how much milk is spilled, how much money is spent on milk, and so on. Good resource materials for questions that should be made available in the school are: mail order catalogs, phone books, magazines, newspapers, television guides, travel folders, menus, bus, train and plane fare and time schedules, travel guides with motel rates, world almanacs, atlas and road maps, *Ripley's Believe It or Not, Guiness Book of Records*, math puzzle books, children's cookbooks, and encyclopedias (available in the school library).

The teacher should also be alert to situations that occur naturally in the course of school events. A cupcake sale to raise money for physical education equipment can be used to explore such things as:

What does it cost to make a cupcake?

How much should they be sold for?

- How many cupcakes should be made to earn \$1, \$10, and \$100?
- Can a poll be taken to predict the number of cupcakes that would sell?
- Does the price of a cupcake make any difference in how well they sell?

Does advertising help sell cupcakes? How could more be sold next time?

How many cupcake sales will be needed to raise enough money to buy the equipment?

Is it cheaper to make cupcakes from scratch or from a mix? Is there a better way to raise money?

The problem-solving activities arising from natural situations will differ from school to school, but the opportunities are everywhere.

How many parents are members of the P.T.A.? Are more parents of younger students or of older students members?

How much does it cost to have a class party?

- Is it cheaper over a ten-year period to have a playground covered with grass or blacktop?
- Why do schools put up portable classrooms instead of building new rooms?
- How much does it cost to run a school bus each day? Each year?

Why do people have to wait so long in the lunch line? Where is the best place for each class to go in a fire drill?

Is that the same place the class is supposed to go now? Which meals in the cafeteria are more popular? Are nutritional foods more popular than others?

Problem solving allows students to use their mathematical skills to find answers to things *they* want to know about. The topics the teacher provides at the start expand student awareness of the wide range of subjects that may be explored. There is no need to limit the problems to those obviously mathematical in scope. Mathematics is more than a reliable approach to knowledge—it is the key to understanding the patterns in nature.

Mathematics itself is not composed of separate bits of knowledge—the skills from the preceding chapters. Problemsolving activities, in a framework of independent exploration, let students merge all their separate skills into the most important mathematical skill of all ... thinking.