The purpose of this chapter is twofold: first, to set forth a procedure for introducing materials to students that insures proper use, second, to describe in detail the materials needed for each lesson.

Students learn more quickly and with greater understanding when they are able to “see” the concept they are to learn. Use of materials offers students this opportunity. To be truly practical in a classroom setting, however, the materials must not add to the classroom management problems of the teacher and they must be inexpensive enough for classrooms operating on the lowest of budgets. The materials described in this chapter have been selected with these criteria in mind.

Three steps may be taken to insure the ready acceptance of materials as aids to learning by students in a classroom.

The first step is to use the material as the basis for lessons involving all students. Many of the students who would benefit most from using concrete aids will refuse to do so if the material is in any way perceived as a form of remediation. If manipulative aids are reserved for only the slower students, they are apt to feel they are conceding their own ignorance by using the materials. If all students in class use the same material, it cannot be identified as remedial.

The second step is to explain to the students the learning value of each new material introduced for the first time. The materials the teacher has selected are designed to make the students learning easier; therefore, they will be able to learn more this year than in any previous year. Tiles, for example, are used to help everyone learn about decimals, fractions, area, multiplication, and division. Geoboards help everyone learn about geometry, which most people don't study until high school. The beans and cups are used to teach arithmetic in other bases, and some of the materials may be used to teach algebra. Most people, again, don't learn these things until high school.

The students will be able to learn all these things because these materials make learning so much easier. Many teachers do not use them because they feel their students are not ready to accept the responsibility that accompanies the use of a material. The students in this class, however, will decide for themselves if they are ready to accept this responsibility, which will help them learn more than they ever have before.

The third step is to ask each student individually if he or she is ready to learn and therefore ready to accept responsibility for the correct use of the material. Before the teacher distributes a material for the first time, all the prestigious uses to which it will be put are described in glowing terms. The students are then told what the responsible use of that material entails. For example, the rubber bands on the geoboard are to be used to learn about multiplication, fractions, decimals, geometry, and so on. Students who use them for snapping or shooting at others have decided they are not yet ready to learn using geoboards and rubber bands.

As the teacher hands the new material to each student individually, each student must state whether he or she is ready to learn using the material and, therefore, ready to accept responsibility for the correct use of material. If the student is ready to learn, he or she is given the material to use. If a student mishandles it, that student has decided that he or she is not yet ready to learn, and the material is withdrawn from that student alone. When the student decides that he or she is ready to learn, the material is reissued.

Students are anxious to use materials well if they see them as prestigious and evidence of a desire to learn.

**MATERIALS**

**Overhead Projector** Access to an overhead projector is not a requirement for any of the lessons in this book. A blackboard supplemented by a giant pocket chart (described below) serves equally well.

If an overhead projector is available, it is much more convenient than a blackboard. An overhead projector screen can be set up so it blocks no one's view, but because higher angles of overhead viewing distort the top of the picture, the screen should be hung at an angle that eliminates the distortion. An example of such a set-up can be seen in this figure. Overhead projector screens may be made by painting a large piece of cardboard white or covering it with an old sheet.
save for later use. Third, anytime the students are given a ditto with which to work, an exact copy can be made into a transparency and used as a visual aid on the overhead. Fourth, if lines or squares are needed on the overhead screen, a lined or graph paper transparency produces this effect.

**Giant Pocket Chart** If the teacher does not have access to an overhead projector, the inadequacies of a blackboard may be counterbalanced by supplementing it with a giant pocket chart. Instructions for making the pocket chart are included in this figure.

![Diagram of Giant Pocket Chart](chart-diagram.png)

- **Cardboard** 61 cm x 80 cm
- **Acetate** 61 cm x 85 cm
- **Pocket chart**

Once the pocket chart has been made, the next task is to make replicas of all the necessary charts and matrices included in the black lines for which a transparency would have been made for use on the overhead.

Each chart or matrix is used by inserting it behind the acetate on the giant pocket chart. The pocket chart permits the tagboard replicas to be placed behind the acetate sheet. All appropriate markings may then be made on and erased from the acetate sheet without marking on the replica itself, so it needn’t be remade (see drawing above right).

**Individual Blackboards** Many of the lessons involve the whole class at once. Use of the individual blackboards permits everyone the opportunity to respond to questions without having to compete with each other for the teacher’s attention.

The blackboards may be made by painting the inside of nylon stocking box lids with blackboard paint, which can be obtained at most paint stores. The lids make excellent blackboards and the boxes themselves are good containers for chalk and the towel scraps which may be used as erasers. The stocking box blackboards are durable enough to last an entire school year.

![Diagram of Nylon Stocking Box Blackboard](blackboard-diagram.png)

- **Nylon Stocking Box**
  - **Bottom**—for storage of chalk and towel scrap used as eraser.
  - **Top**—inside painted with blackboard paint.

When a group answer is expected, each student writes an answer on the blackboard and turns it face down on the desk. The teacher gives the signal to hold up the boards when all have been turned face down. This gives everyone a chance to answer the teacher’s questions.

When each student knows he or she will have a chance to respond to the teacher’s questions, it becomes much more worthwhile to follow the lesson.

The blackboards are also the teacher’s source of a class answer, the one appearing most frequently as the boards are held up.

**Ceramic Tiles** (2.5 cm square) The ceramic tiles are the kind used to tile floors or walls, most often coming in sheets of 144. It is often possible to obtain incomplete sheets free at construction sites or at a reduced price from tile wholesalers.
Tile wholesalers (listed in the yellow pages under Tile, ceramic) may allow teachers to purchase at discount odd lots of tiles or tiles from incomplete sheets. It is important when asking for a discount for the teacher to identify himself or herself as a teacher and explain what use will be made of the materials. Builders or sellers will sometimes provide a certain amount of materials free if they know the goods are for use in teaching.

If possible, between 100 and 144 tiles should be obtained for each student in class. These amounts are not mandatory, however, and if a lesser number of tiles is available, the same activities may be explored by having the students work together in teams.

The students may remove the tiles from their backings. The tiles should be stored in stocking boxes, cloth bags, or plastic containers to make them easier to transport and store.

If there are no funds available for the purchase of tiles and none have been donated, an adequate substitute may be made by cutting out 2.5 cm squares of cardboard. Shirt cardboard from a laundry is usually thick enough to work well.

The tiles double as their own visual aid on the overhead projector, since they silhouette nicely on the screen. If no overhead projector is available, visual aids may be cut from paper. These square shapes should be large enough for all students to see easily. They may be pinned to the bulletin board or used on a flannel board.

When the teacher is showing students what to do with a material, the visual aid supplementing the words should resemble the material as closely as possible, to avoid confusion. The aid needn’t be made of exactly the same material the students use, but it must have as many of the same properties as possible. Squares of paper on a bulletin board or flannel board are closely akin to tiles on a student’s desk. Drawings of squares on a blackboard are not. Although students who abstract easily will be able to make the connection between drawings on a blackboard and tiles on their desks, those who have difficulty in learning or who do not understand English become lost and confused.

Unifix Cubes  Unifix cubes are the only material recommended for use for which an adequate substitute cannot easily be made. Unifix cubes are plastic shapes 1.7 cm high which snap together to form columns. They may be purchased from a number of mathematical supply houses, a sample listing of which is found on page 299.

An important supplement to the Unifix cubes are the Unifix grids, which are used to permit individual columns of cubes to be linked together to form solid walls or blocks.

A good starting number of cubes for a classroom is between 500 and 1,000. The long-range goal, however, should be to acquire as many as 3,000 cubes over a period of years.

The cubes may be stored in a plastic bucket or cardboard box. In many schools, the cleaning solvents and waxes used by the custodian come in durable plastic buckets, usually discarded when empty. Most custodians will be happy to save these buckets for use in the classrooms.

Beans, Cups, Bowls, and Tin Cans  The lessons that use beans and cups, and later add bowls and tin cans, may be done with a variety of materials. Beans are used because they are relatively cheap and available in large supply. Any
other small objects readily available would do as well: pebbles, shells, seeds, bottle tops, small washers, cut-up bits of cardboard, paper clips, brass fasteners.

The cups, bowls and tin cans are meant to be containers for the beans or other small counting objects. When the cups alone are used, the only criterion for the containers is that they all be the same size. Cut-open milk cartons, baby food jars, empty match boxes, shaving lather lids or any other containers available in quantity would serve as well.

As bowls and tin cans are introduced, the new criterion is that each new size container must be larger than the container in use before it. If small milk cartons were used in place of cups, cut-off half gallon milk cartons might be used instead of bowls, and gallon milk cartons instead of cans. The containers used should be a function of what materials are readily available in the school or in the students’ homes.

Any counter will also be a good visual aid for itself on the overhead. If no overhead is available, the teacher should cut paper shapes similar in outline to the counters. These shapes may then be used either on the bulletin board or flannel board to supplement the teacher’s oral instructions.

The cups, bowls, and tin cans may be simulated on the overhead by drawing circles on acetate squares. If cups, bowls, and tin cans have been replaced by other containers, the drawings should reflect the shapes and relative sizes of the containers actually in use.

If no overhead projector is available, circular cutouts used to represent the containers should be made for the bulletin board or flannel board.

Cross-Line Multiplication Flash Cards The flash cards may be made on any available paper. Twenty-two cm by 26 cm tagboard works well. Each card is made by drawing an appropriate number of lines down and across the card. Circular self-adhesive gummed labels may be placed on the intersection to facilitate counting.

The number of cards to be made depends on the multiplication facts the teacher wishes the students to study. (For a description of the use of the flash cards, see Lesson 5-14.)

Spelling Notebooks Students who have difficulty reading should not be required to handle this difficulty in their math lessons. None of the lessons in this book require students to read; however, some do require them to write. The use of the spelling notebook permits students to write who cannot yet read.

Spelling notebooks are made by stapling 26 lined pages inside a construction paper cover. The students then write one letter of the alphabet on each page.

Anytime a student needs to know how to spell any word, he or she brings the notebook to the teacher opened to the page on which the student thinks the word belongs. The teacher then writes the requested word on the page headed by the same beginning letter. Since the teacher will write any and every word a student needs in the spelling notebook, there is nothing a student cannot write.

At the start of the year, the teacher may have to write many words for some students, but this is not long-lasting. The number of words the students need diminishes as they acquire a basic writing vocabulary in their spelling notebooks. Some may not be able to read what they have written, but reading isn’t a requirement in math.

No words should be taken from the spelling notebook and used as the basis for spelling tests—this would discourage students from asking for more words.

Dice The dice may be anything from cubes cut from sheets of foam rubber and numbered with marking pens to the plastic dice sold by mathematics supply houses. The most versatile dice that can be purchased are those with blank faces whose numbers may be added by attaching self-adhesive labels on each side. Most of the activities in this book that require the use of dice recommend each die be numbered from zero to five.

Teacher-made dice may either have changeable numbers or be produced in such quantity that dice whose numbers do not go as high as five can be made as a separate set. In schools where the use of dice is discouraged, spinners may be used instead. A sample spinner is in this figure.

Materials for Spinner

Cardboard or tagboard square
Bent paper clip
Cardboard or tagboard arrow
Polk bent end of paper clip through center of square and arrow

Spinner Assembled
If dice are to be used, the teacher can make a set of overhead "dice" by cutting squares of tagboard five by five cm and punching holes in them. These squares cast dice-like shadows when placed on the overhead. The zero die is a paper square without holes. If no overhead projector is available, the same dice cards made for the overhead may be placed inside a small bag or box and drawn out one at a time to simulate dice rolls.

If spinners are used, an overhead projector spinner may be made from a piece of acetate, a paper clip and a cardboard arrow in the same manner as was demonstrated above for constructing student spinners. If no overhead projector is available, a demonstration spinner may be attached to a suction cup which is, in turn, attached to the wall.

**Chips and Chip Trading Boards** Good materials from which to cut chips are construction paper or drawing paper; a good size is four cm by four cm. Chips made from colored construction paper are ready to use once they are cut. Chips made from drawing paper must be colored in by the students first, but permit the students to add new chips to their sets in any color they need simply by coloring in additional blank chips. Chips should be available in sufficient quantity to permit students to find an answer to any problem on which they may be working, even in base ten. A sufficient number of chips for each student is 20 or 30 of each color, plus as many as 100 chips of the color used to represent the units column on their boards. The teacher should also have several hundred chips in reserve in case they are needed by any student or group working on a particularly large problem.

The teacher makes chips for the overhead projector by using marking pens of the appropriate colors to shade in acetate squares. The trading board can be drawn on the overhead by lining the necessary number of columns and shading the top of each with the appropriate color. If no overhead projector is available, strips of colored paper may be attached to the bulletin board or a flannel board to form the necessary number of columns. The paper squares can then be cut from colored construction paper and placed in the columns as needed.

**Geoboards** Geoboards are readily available commercially, and they can also be made easily. Given the necessary tools, intermediate-grade students can make their own.

The actual size of the geoboard is not as important as its relative dimension. The board’s 25 nails should be placed in 5 rows of 5, forming 16 small squares. The distance from the edge of the geoboard to the first row or column of nails should be one-half the distance between two adjacent nails.

![Diagram of geoboard](image)

Popular sizes of geoboards range from between 12 and 13 cm square to 25 cm on a side.

Each geoboard should be equipped with about four different-colored bands, so students can distinguish the various parts of the shapes they make. A good size rubber band is one that forms a square around four nails without fitting loosely or having to be stretched.

![Diagram of rubber bands](image)

A simulated geoboard may be made by outlining a board on a piece of acetate and representing the head of each nail by an X. If no overhead projector is available, a larger outline may be drawn and used in the giant pocket chart. In either situation, the rubber bands may be simulated by drawing on the acetate with marking pens.

**Objects To Be Sorted** Although the first objects sorted in the chapter on sorting and classifying are buttons, any other material that silhouettes a variety of attributes when placed on the overhead would do as well: nuts and bolts, cut-up pieces of paper, puzzle pieces, et cetera.

If no overhead projector is available, the teacher should make enlarged cutouts for the bulletin board or flannel board of whatever material the students will be using first for sorting. If buttons are to be used, then button cutouts should be made.

The sorting objects may include any items to which the teacher or the students have ready access. The students
may sort such things as Christmas cards, wrapping paper, ribbon and string, assorted jars and cans, food boxes, marbles, pot holders, pencils, paint samples, linoleum squares, pattern samples, puzzle pieces, corks, stamps, spools, metal or wood scraps, or anything else that may be found by rummaging through basements, attics, garages, old toy chests, or kitchen cupboards.

*Name Boxes* The name boxes the students use in their early work on graphing are cut-down half-gallon milk cartons with acetate pockets taped on what was formerly the bottom. The steps in constructing a name box may be seen in the figure below. Other boxes will do as well, but they must all be of the same size and stackable.

The acetate pockets are used to house the students’ names written on cards. These name cards should be available in sufficient quantity to permit students to replace them each time they are used. If acetate is not available, small pockets may be made by taping a strip of tagboard to the bottom edge of the surface of the box.

*Large Cardboard Box (screen for graphs)* Any large box or piece of cardboard may be made into a shield for graphs, or the teacher may string a rope between two chairs and drape a sheet over it.

*Metric Measuring Devices* If funds are not available for the purchase of rulers and tape measures, they may be made from cut-up strips of tagboard and adding-machine paper. Weights may be made from washers and pieces of clay. Metric measuring devices must be useful in obtaining measurements in standard metric units. Handmade measuring tools calibrated against commercial implements are as capable of producing accurate measurements as are those implements.

*Cardboard "Coins"* Probability coins may be made by coloring or spray painting one side of a piece of tagboard or cardboard, producing a marked and a plain side. The cardboard is cut into squares approximately two cm on a side. Students could also make coins by marking their own pieces of cardboard, but using the already uniformly marked coins makes it more likely no student will see by looking at the material alone there are more ways for some events to occur than others.

*Number Machine* Instructions for making a number machine from a half-gallon milk carton are included in the figure below. The machine turns cards upside down, so when a card is put in the machine top side up it comes out bottom side up.

The teacher-made cards are constructed out of five cm by eight cm pieces of tagboard. Cards are marked on one side and left plain on the other so the teacher can tell the
up side from the down side. About ten cards may be made to go with each rule the machine is using. Each stack of ten cards should also be marked so the cards belonging in one pile do not inadvertently become mixed with those in another. A convenient system of coding is to mark the top of the cards in one set red, another set blue, another set green, and so on.

The teacher first decides what number rule or equation a stack of cards will represent, then makes the appropriate entries on the cards. An example of cards numbered for the rule "add two" can be seen in this figure.

<table>
<thead>
<tr>
<th>Top</th>
<th>4</th>
<th>7</th>
<th>3</th>
<th>2</th>
<th>8</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bottom</td>
<td>6</td>
<td>9</td>
<td>5</td>
<td>4</td>
<td>10</td>
<td>3</td>
</tr>
</tbody>
</table>

Number machine cards for "add two" rule

The students' blank cards for devising rules for the number machine are also constructed out of five cm by eight cm pieces of tagboard. The students must decide how to mark their cards so they can tell top from bottom and one pile from another.

**Tangrams**

Plastic tangram puzzles may be purchased from mathematics supply houses (see list below) for about 50 cents each. They may also be cut from linoleum, cardboard, wood, or a variety of other materials. The paper squares used to introduce tangrams should be cut the same size as the actual tangram puzzle square with which the students will be working. The tangram task card triangles in the black-line masters (Worksheet 26) are designed to accompany tangrams that form a square 10.5 cm by 10.5 cm, sold by Creative Publications. Teachers using tangram puzzles of different sizes should draw a task card sheet of triangles reflecting the size of the pieces in use.