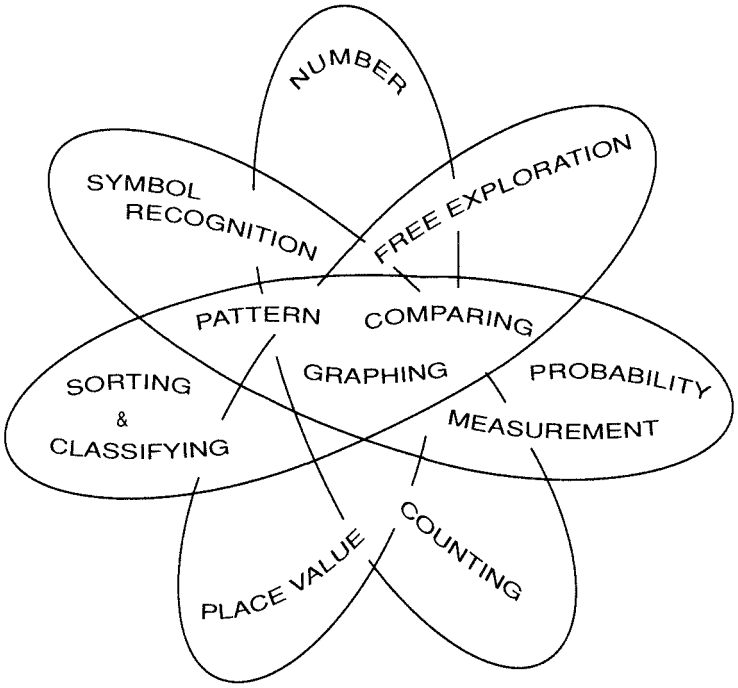
CHAPTER 10:

NUMBER OPERATIONS



> **"The child is the principal agent in his or her own education and mental development. Development and growth occurs through an enormously complicated and continuous process of interaction with the environment. Through activity, the person discovers understanding by re-inventing what he or she wants to understand. This process takes time."**
>
> **Jean Piaget**

It's important to provide classroom opportunities and experiences which build upon the children's existing knowledge. Most children possess a considerable amount of number vocabulary before they enter school. Number symbols are all around in their environment (i.e., numerals on the television dials, clock faces, telephone dials, street addresses on houses, signs on the highway…). Children are familiar with number names. Most have learned to count by rote through nursery rhymes and songs such as, "One, Two, Buckle My Shoe" and "This Old Man", and/or folk stories: "The Three Pigs" and "Snow White and the Seven Dwarfs". Some can even perform simple addition and subtraction within the context of concrete events or objects.

Computational skills (arithmetic) should not be taught in isolation. Arithmetic is only one piece of the mathematical puzzle. If we use an analogy to reading, arithmetic is like the decoding skills — phonics; mathematics is like the literature. It is critical that children learn how and when to use various computational skills in the context of their work (e.g., word problems, graphing, comparing activities), rather

than memorizing meaningless "math facts". Studies are now indicating that the strong emphasis on isolated computation is the main factor which contributes to why American children are ranked thirteenth in the world in mathematics. To change this trend, we must reevaluate our existing math programs and adapt them to meet the children's needs. A related newspaper article was published in the Spring of 1986:

U.S. Pupils' Math Skills Are Lagging

LOS ANGELES (AP) - Students from the United States are lagging behind those of a dozen other industrialized nations on mathematics studies, and in many skill areas. U.S. students only outperform students from Third World countries.

The average score of U.S. students in an international sample was lower than the overall average of all the participants, ranking above only Swaziland, Nigeria, Thailand and Sweden.

Among eighth graders, students from Japan, the Netherlands and Hungary scored the highest on tests covering arithmetic, algebra, geometry, statistics, and measurement.

In a separate 10 nation comparison in which the top 5 percent of 18-year-olds were tested in algebra and calculus, American students were last.

The study found American students are receiving more hours of instruction in mathematics than students from other nations, but the math curriculum in Japan and the top-scoring European nations was more intensive.

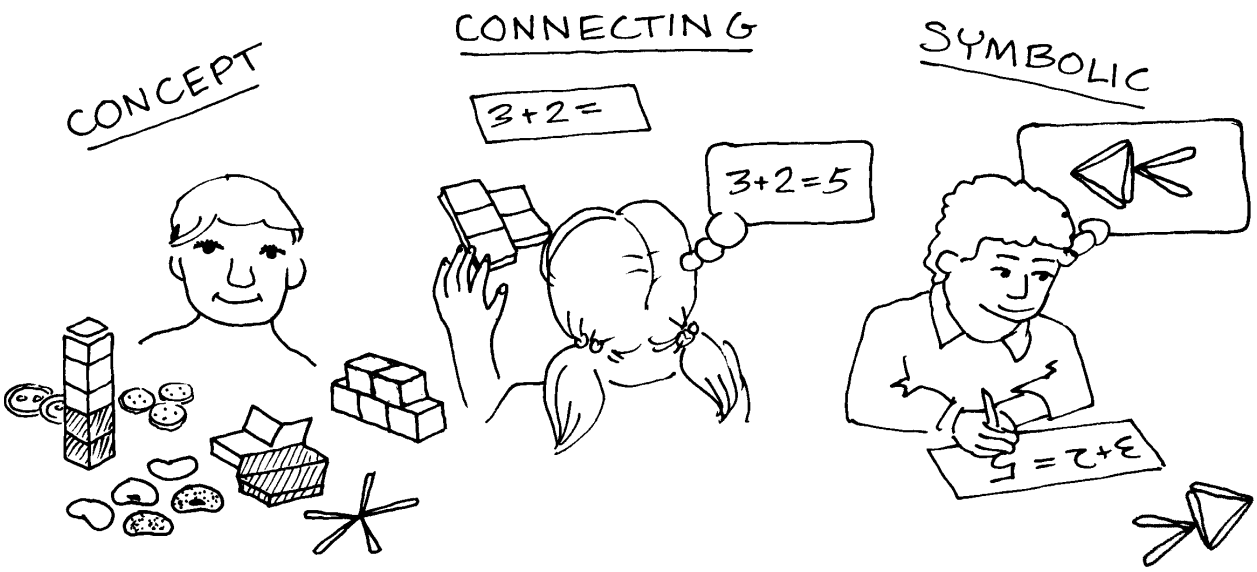
"By grade seven, the Japanese and European students have computation behind them and are moving into algebra, geometry and mathematical problem solving," said Joseph Crosswhite, professor emeritus at Ohio State University and a consultant on the study. "In the United States and Canada, the curriculum at grade eight is still dominated by arithmetic computation," he said.

One of the goals of *Mathematics Their Way* activities is to surround children with meaningful concrete experiences which build a true understanding of mathematical patterns, language and symbols needed to perform simple number operations. Children progress through three levels of abstraction: concept, connecting and symbolic, as they work — depending on the degree of difficulty the task at hand requires of them.

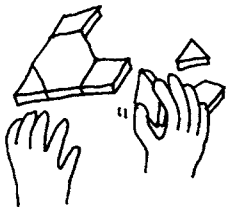
Schools are supposed to work for children, not children for schools.

John Holt

LEVELS OF ABSTRACTION



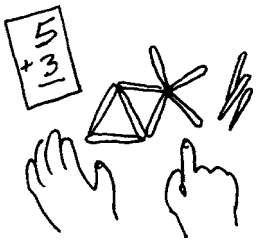
Concept Level



Children develop an understanding of mathematical language as they explore number patterns under ten in the context of real events and/or concrete materials.

Children demonstrate their understanding at this level by:
- building concrete models with a variety of manipulatives.
- describing what they have created using mathematical language.

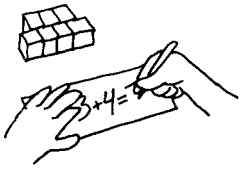
Connecting Level



Children usually have not encountered mathematical symbols in the context of their natural environment. Often, their first experiences occur in school-related activities. Numerical and mathematical symbols are introduced at the connecting level. Mathematical symbols (e.g., +, –, =) are even more abstract than the mathematical language they represent. Both vertical and horizontal equations should be experienced.

Children visualize symbols as they solve number problems using manipulatives. They show their understanding at this level by:
- building concrete models with various types of manipulatives to match written equations.
- relating equations to manipulatives or to a word problem they have created. The teacher records the mathematical symbols.

Symbolic Level



At this stage, children record equations to represent concrete number patterns. Eventually they will develop an ability to record equations by visualizing concrete experiences. Children record on their own when they are comfortable writing numerals. This occurs when they have developed the necessary fine motor skills. Many kindergartners are not ready for this level.

Visualization

One way children transfer the number concepts learned at each level is through visualization. Once children have internalized an abundance of concrete number experiences, the teacher may tell number stories that encourage children to visualize the process.

A first grade teacher explained how her children visualized word problems and wrote them on their individual chalkboards. She told the following number story: "Close your eyes and think of 5 rocks in the domino pattern. Put them right into your head so you can see them. Pretend that the Big Red Rockeater came and stole one rock out of the center of the pattern. Now write a story of what happened on your blackboard."

The next day, the mother of one of her students came to her and said, "You'll never believe what my daughter said last night. She asked me what (5 – 1) was and I said 4! Then she said, "Mom, do you have rocks in your head?"

WORD PROBLEMS

Word problems provide opportunities for children to learn number operations in the context of meaningful situations. Before children can successfully solve a word problem, they must be able to:

- ❏ Interpret and organize the information in the problem.
- ❏ Decide what information is important.
- ❏ Choose the number operation(s) necessary to solve the problem.
- ❏ Apply the operations necessary to solve number problems.

The opportunity to solve word problems should be a common occurrence in all classrooms. Teachers can take advantage of real classroom situations (e.g., lunch count, attendance, choice of snack) to model word problems (see MTW, pp. 204-205). The following sequence suggests ways to incorporate word problems at the concept, connecting, and symbolic levels.

Note: The experiences could be scheduled either as a whole class or a small group activity.

ACTING OUT SITUATIONS

Concept Level

Procedure: Tell a number story while a group of children acts out the verbal directions. The number story could be real or fantasy. The answer to the problem is not important at this point. The children demonstrate their understanding of the number operations described in the story by interpreting and acting out the actions in the story. After the children have acted out a situation, have them describe the story in their own words to further reinforce the experience.

Addition Example:

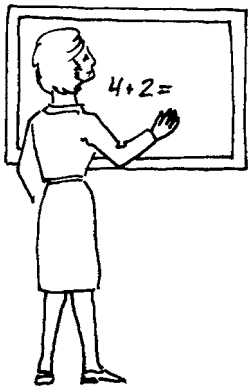
Carlos, Adi, Molly and Peter were frogs. They hopped into the pond. How many frogs are in the pond? Susan frog and Steve frog hopped into the pond, too. How many frogs came? How many frogs are there altogether in the pond?

Subtraction Example: (Continue the story, this time subtract some of the frogs.)

Peter frog, Carlos frog, and Molly frog jumped out of the pond to bask in the warm sun. How many frogs were there altogether? How many frogs jumped out of the pond? How many frogs are still in the pond?

Extension: Vary the procedure by telling the entire story first, without having the children act it out step by step. Ask the children to predict the result before it is acted out. Their predictions can later be verified by having them act out the situation.

Extension: Use number rhymes and songs as a framework for number stories. A group of children can act out the number problem as they sing the song or the rhyme is being recited (see Newsletter Chapter 12).

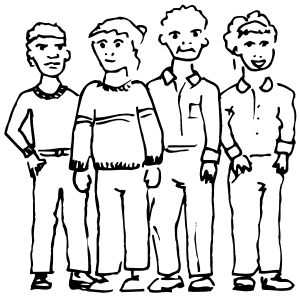


Connecting Level

Procedure: Record the number sentence as a group of children act out a word problem. Model both horizontal and vertical formats.

Addition Example: Ask six or seven children to stand up in front of the class. Have them sort themselves by a physical attribute (e.g., shirt sleeve length). Record the equation as the story progresses.

"How many children are wearing long sleeves?" ("Four.")
"How many children are wearing short sleeves?" ("Two.")
"Figure out in your head how many children there are altogether. " ("Six")
"Which is the largest group? Let's start with the group of four."
("Four, five, six, seven.")
"Does it work the other way?" ("Yes. Two, three, four, five, six.")
"Let's read the number sentence together." ("Four and two equals six.")



Subtraction Example: (Either begin with a new story or continue with the same story. Teacher records the equation as the story progresses.)

"How many children do we have altogether?" ("Six.")
"Everyone with long sleeves, sit down. How many children sat down?" ("Four.")
"And how many people are still standing?" ("Two.")

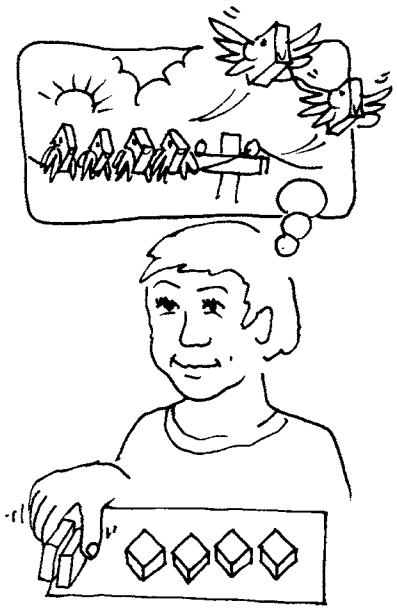
Symbolic Level

Materials: plain paper; pencils and crayons; equation cards; chalkboard, chalk and eraser

Procedure: The class makes up number stories and acts them out as before. The children record the number sentence on their paper or chalkboards as it's being acted out by a group of children. Next, the children can either draw a picture or write a story that reflects the word problem and equation.

USING MANIPULATIVES AND A WORK SPACE

Note: The scene of the story might vary just by changing the color of the work space paper. For instance, if the paper is black, the children might generate stories which occur in the dark, in a cave, in the dirt…; blue paper might generate stories which occur on a body of water or in the sky; green paper might generate stories which occur in the grass or a tree….



Concept Level

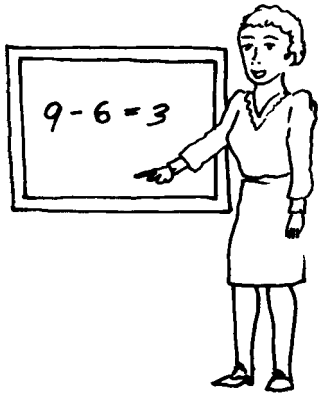
Materials: A collection of small objects (e.g., blocks, Unifix cubes or junk, fish crackers) to represent whatever is referred to in a word problem; a paper work space to help the children focus on the task.

Procedure: Tell the children a number story. Ask them to act it out with the objects on the work space as you go.

Example Story: (The work spaces are blue. The children decide it's the sky.)

"There are six pigeons sitting on a telephone wire. Two pigeons flew away. How many pigeons are sitting on the wire now?" ("Four")
"Then what happened?"

The children take turns adding to the story — all the time modeling the story with manipulatives. (A child might say, "Three robins flew all the way from the south and sat on the wire to rest. Now there are seven birds.")



Connecting Level

Materials: plain paper; pencils and crayons; equation cards; chalkboard, chalk and eraser; objects (e.g., Unifix cubes, junk, beans,…)

Procedure: Start by writing an equation (e.g., 9 – 6 = 3) on the board. Ask the children to tell a story in their own words as they act out the story with manipulatives on their work spaces.

Extension: Divide the class into groups. Write a number sentence on the chalkboard. Ask each group to think of a number story to go with the number sentence. Give each group an opportunity to share their story.

Extension: Ask the children to draw pictures to represent the number story they develop. Allow time for the children to share their pictures. The pictures could be stapled together to make a class book for that equation.

Symbolic Level

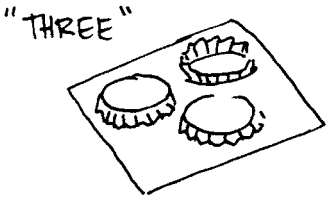
Materials: plain paper; pencils and crayons; equation cards; chalkboard, chalk and eraser; objects (e.g., Unifix cubes, junk, beans,…)

Procedure: The children write story problems for classmates to solve using manipulatives.

JUNKJOB SEQUENCE

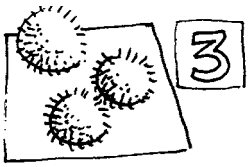
Counting: Exploring Numbers (0 - 9)

Materials: junk; 8 paper work spaces per child



Concept Level

The children count the same predetermined number of objects onto each counting area.



Connecting

The children count the appropriate number of counters onto each work space to match each numeral.

Symbolic Level

The children record on a piece of paper the total number of objects on each counting area.

Note: It's important that the teacher models a variety of ways to verbalize equations. For instance, an equation (4 + 3) could be verbalized, "Four *plus* three." or "Four *and* three."; (5 – 2) could be verbalized, "Five *take away* two." or "Five *minus* two."

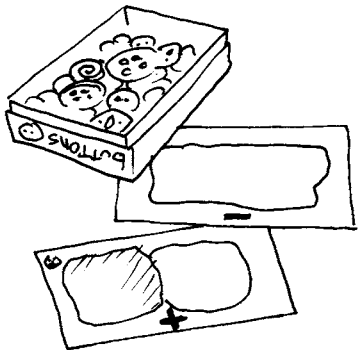
Exploring Number Operations on Addition and Subtraction Boards

Concept Level

Teacher-Directed Lesson

Materials: junk; one addition/subtraction board per child

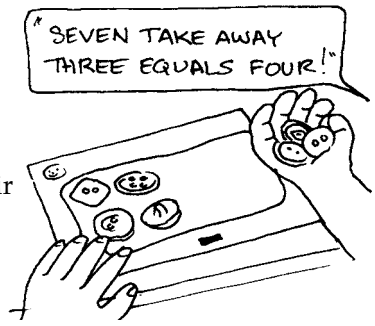
Procedure: Verbalize an equation to the class. Ask the children to choose the appropriate side of the addition/subtraction board and solve the equation using junk. The children verbalize their actions.



Addition

Teacher says: "Three and two".

Children: Place the appropriate number of objects on each side of their addition board. The group verbalizes the equation ("Three plus two equals five.").



Subtraction

Teacher says: "Seven take away three".

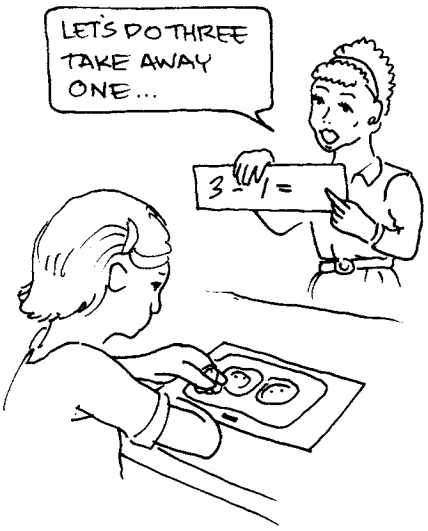
Children: Place (seven) objects on their subtraction boards and then remove (three) objects from the board, keeping them close to the board. They verbalize the equation. ("Seven take away three equals four.")

Connecting Level

Teacher-Directed Lesson

Materials: 1 addition/subtraction board per child; equation cards; junk.

Procedure: Mathematical symbols (+, –, =) are introduced. Children do not record at this level. Show the group an equation card and read the equation. The children add or subtract junk on the appropriate side of the board to show an understanding of the equation.



Addition:

Teacher: Shows equation card and reads the equation. ("Four and two.")

Children: Place objects on the appropriate side of the addition board. They verbalize the equation. ("Four and two equal six.")

Subtraction:

Teacher: Shows card and reads the equation. ("Three take away one.")

Children: Place (three) objects on their subtraction boards and verbalize the equation, ("Three take away one equals two.") as they remove (one) object(s).

Independent Activity

Children work independently using 6 or 8 addition/subtraction boards and equation cards as soon as they understand the process.

Materials each child needs: 6–8 addition/subtraction boards; junk; addition and subtraction equation cards

Procedure: At this level, the children place an equation card next to each addition or subtraction board and perform the operation specified on the board.

Symbolic Level

Teacher-Directed Lesson

The children determine sums and differences and write the equations.

Materials each child needs: 1 addition/subtraction board; junk; chalk, chalkboard, eraser

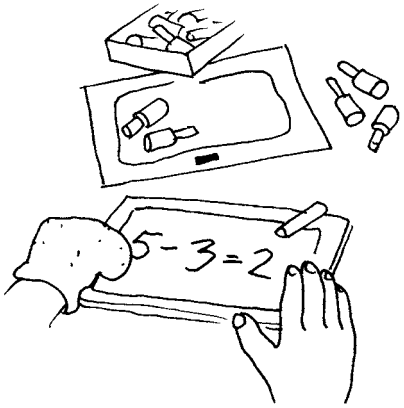
Addition:

Teacher says: "Three and two equals?"

Children: Place appropriate number of junk on each area of the addition/subtraction board and say the answer, ("Three and two equals five.")

Teacher: Demonstrates how to write the equation. (3 + 2 = 5)

Children: Write the equation (3 + 2 = 5) on a chalkboard.



Subtraction:

Teacher says: Five take away three is…?

Children: Place (five) objects on the subtraction board and remove (three). They verbalize the action ("Five take away three equals two.").

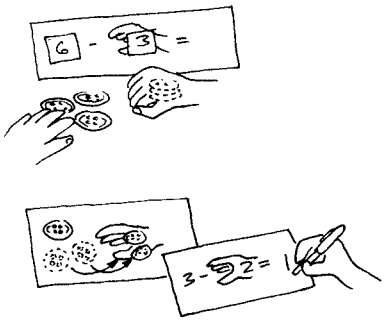
Teacher: Models how to write the equation
Children: Record the equation on their chalkboards.

Independent Activity

Each child needs: equation cards appropriate for the child's level; 6-8 addition/subtraction boards; junk; (2" by 6") newsprint to record equations

At this level the children match an equation card to each addition/subtraction board and perform the operation specified. After the child is finished arranging the junk on the boards, he or she records the equations, including the answers, on pieces of newsprint. When the child is finished working, he or she can staple the equations together into a little book. Horizontal and vertical equations should be in separate books to avoid confusion.

Activities Not Found in *Mathematics Their Way*



Cave

Materials: junk; 6-8 cave cards per child (NL Blackline #35); equation cards (NL Blacklines #22-34); cave recording sheets (NL Blackline 36)

Connecting Level

Procedure: The children work in pairs. One child reads an equation card, while the other child proceeds to do the problem with junk on the cave card. Both children verbalize the equation together.

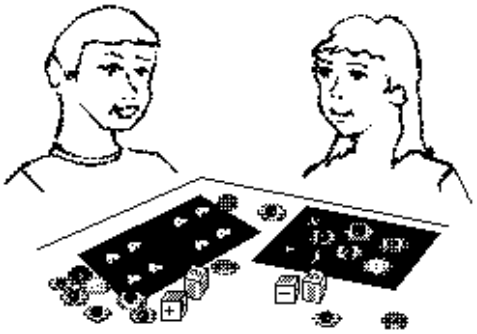
Symbolic Level

Procedure: The child reads an equation card and performs the problem with junk on a cave card. He or she then records the equation and answer on the recording sheet.

Listen and Count ("Impossible")

Connecting Level

Materials each child will need: junk; chalkboard, chalk, eraser; 1 die with (+) and (-) written on it; 1 die with (0, 1, 2, 3, 4, 5) written on it.



Making the gameboard: Each child has his or her own chalkboard. The children choose a number they want to add up to and make that many marks on their chalkboards. The marks can be anything (e.g., the player's first name initial, hearts, "X's", triangles, stars…). They can be arranged any way, even randomly around the chalkboard.

Note: The nice thing about using chalkboards as gameboard is that the game can vary without making different workspaces with marks for every number. Children have an opportunity to make some decisions about the game. They decide the number of marks, create the kinds of marks to draw, and decide how they want the marks arranged on the chalkboard. Once a game is over, they can erase the chalkboard, choose a new number to roll up to and arrange new marks on their chalkboard.

Procedure: The children work in pairs. The pair determine in advance who the winner will be; either the first to cover the marks on the chalkboard with junk or the last person to cover the marks with junk.

The first player rolls both the plus and minus die and the numeral die. If the child rolls (+) (3), then he or she places 3 pieces of junk on top of the markers on the chalkboard. The second player takes a turn. The players continue to take turns until one player covers all the marks on his or her chalkboard.

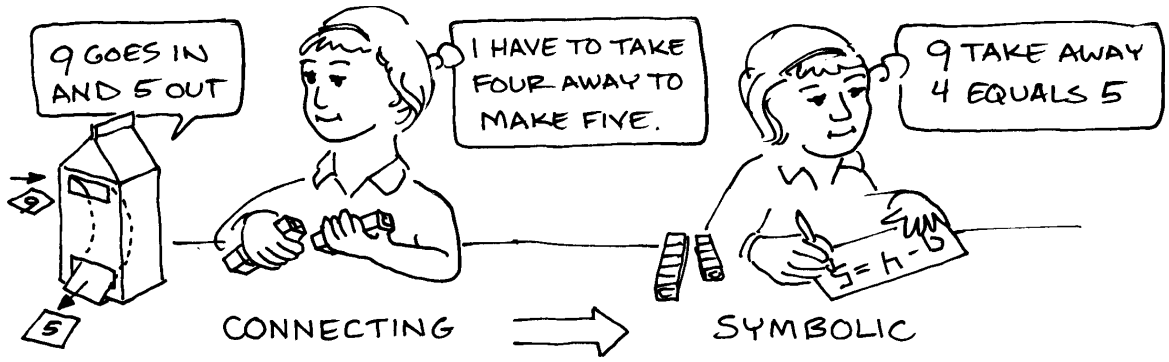
Variation: Start by covering all the marks on the chalkboard with junk. The object to this game is to remove the junk off the chalkboard. The children decide if the first or last person to remove all the junk off his or her *chalkboard is the winner before starting the game.*

"Impossible" Examples: Some plus / minus and numeral dice rolls will be impossible to add or subtract because of the perameters of the game rules. For example, if a child rolls a (-) and any number the first time when adding up to a certain number, he or she will have to lose a turn, because there's no junk on the board to take away. Let's say child has five of eight marks covered with junk on his or her board, and he or she rolls

a (4). He or she would lose a turn because there are only three more marks to cover. Four is too many. The child says "Impossible" when he or she cannot add or subtract the number indicated on the die, leaves materials as they are and loses a turn.

Magic Box

Materials: Magic box; magic box cards; paper to record the equation; manipulatives (e.g., Unifix cubes, junk, wooden cubes).



Connecting Level

Procedure: The child chooses a magic card. He or she counts out that many objects. If the child uses Unifix cubes, they should be snapped together.

The child inserts the magic card into the magic box. (The numeral which was counted out should be facing up.) He or she reads the numeral that comes out at the bottom of the box and makes that number by either adding or taking away some of the manipulatives. If the number is larger, objects need to be added to make the new number. If the number is smaller, manipulatives need to be taken away to make the new number.

Symbolic Level

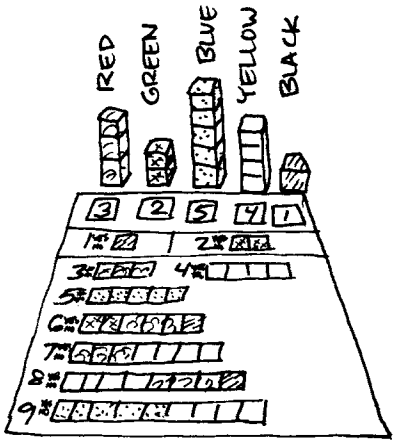
The child repeats the connecting level procedure (above). This time, he or she records the equation which describes what was done to the objects.

Equation Dice Toss

Symbolic Level

Materials for each person: NL Blackline #20; one die with dots; one die with numerals; pencil

Procedure: The child needs a recording sheet, two dice (one with dots and one with numerals) and a pencil. He or she rolls the dice. The child reads the numbered die first and then adds the dotted die number. Some children count the dots to find the total. Once the child has found the total, he or she finds that number on the grid and records the equation in a box in its column. The child continues until a column is completed.



Addition with Unifix Cubes

Materials: five groups of different colored Unifix cubes (5 per color group); NL Blackline 19; crayons the color of the Unifix cubes

Concept Level

Before beginning: The child builds five stacks of Unifix cubes using a different color for each stack. The stacks can be the same or different heights. He or she then records the number in each stack in the box at the top of the paper (see MTW, p. 246) and then lightly colors the box the color of the cube(s).

The objective of this activity is to determine whether a combination can be made for each number on the recording sheet. Different colored stacks can be combined to make combinations, but a single color cannot be broken apart.

Procedure: The child decides whether he or she can use the premade Unifix stacks to make a combination for each number on the recording sheet. If the number can be made, he or she circles the word "yes" and records the color(s) of the cube(s). He or she circles "no" if a combination cannot be made with the premade Unifix cubes. Sometimes there is more than one way to make a number. The child chooses only one of the ways.

Symbolic Level

The child repeats the connecting level procedure (above). This time, he or she records the number equation below each picture of the Unifix cubes. If the number is made with only one color, then the second

Geoboard Designs (Inside-Outside)

Materials: MTW Blackline #61; approximately 80 Unifix cubes in two colors; 5-6 geoboards; 3 place value boards

Concept Level

The children make a design on one geoboard with <u>one</u> geoband. They cover the nails on the "inside" that are <u>not</u> touched by a geoband with one color Unifix cube and the "outside" nails with the second color. They snap the two colors of cubes from the geoboard into groups of ten's and one's.

Symbolic Level

The children repeat the process above and record the number of Unifix cubes on the inside of their designs. Then they record the number of Unifix cubes on the outside. Next the children add the two numbers together.

Note: This activity is described at the place value stations with two or more geoboards. (see NL, p. 11.20)

ADDITIONAL NUMBER ACTIVITIES

Teacher-Directed Small Group Number Activities

Concept Level

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Lift the Bowl	p.	181
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Independent Number Activities

The following independent activities can be assembled into packets and stored on a shelf in the classroom. Enough materials should be available for approximately five children to work at one time. Children can work in pairs. These activities are not appropriate for the tubbing station activities because they have only a single purpose and are not open-ended. Activities that might be included are:

Concept Level

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Subtraction Flip Cards	p.	194

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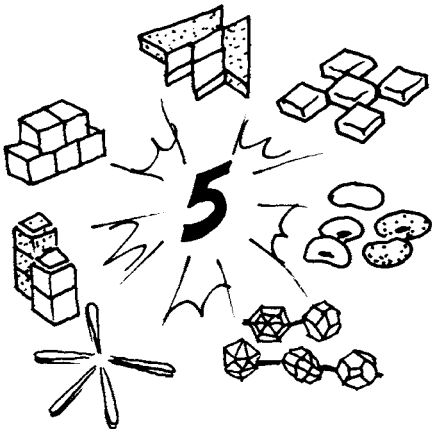
Exploring Number at the Stations



Children develop the concept of number through the various activities at the number stations. Experience with the number five, for example, helps the child to see that five can be five and still be hundreds of other things as well. It can be clusters of toothpicks arranged in an endless variety of patterns; sugar cubes stacked in every imaginable way; red and white lima beans tossed and tossed again; or Unifix cubes snapped together into so many ways that we have exhausted the possibilities. It can be nearly anything that can be counted or arranged or sorted or whatever else we want to do at the time.

All of the time the child is looking at "five", he or she is also internalizing the patterns that make up "five". Sometimes, these patterns take the form of shapes the child can recognize and give names to, like a toothpick "house" or a sugar cube "wall". Sometimes these patterns are described using numbers, as with a "1 + 2 + 2" arrangement of tiles, or a "2 + 3" cluster of jewels. The number patterns that are a part of five are what we commonly know as "number facts". In this sense, number facts are obviously a part of the number stations. The number stations help children internalize the same "number facts" that we learned by memorizing stacks of flash cards when we were students. This flash card approach to learning enabled us to put these "facts" in our memories, but it did not necessarily build a foundation for understanding arithmetic, much less mathematics.

When children in a Math Their Way classroom have internalized the concept of number combinations under ten, they are introduced to the process of grouping numbers which we call "place value". In many cases, this process of learning to group numbers progresses in parallel with the activities associated with understanding number itself. For example, from the beginning of the school year, children may use straws to tally the days they have been in school and bundle these straws in groups of ten. Whole class counting experiences involving large numbers may include the teacher assisting the class group counted objects into bowls or cups.

The fact that number stations help children learn number combinations under ten may lead teachers to believe that there should be similar Math Their Way activities to extend this learning to the larger number combinations (ten through nineteen). This is an incorrect assumption. The process of learning how to group numbers teaches children all they need to know to produce answers for problems such as 7 + 8. Seven plus eight (in base ten) on a place value board is one cup and five beans, or one "ten-stick" of Unifix cubes and five cubes left over, or any one of countless other forms of the regrouping process. Seven plus eight is also four plus five plus six, or two plus three plus ten, or as many other combinations of numbers as a child might choose to invent. Seven plus eight isn't a "fact" to be memorized for a quick answer, it is a mathematics problem that the child should be allowed the ability to solve.

In *Mathematics...a Way of Thinking*, addition is divided into two categories: Beginning and Advanced. Beginning addition is defined as any addition problem, regardless of its size, whose answer is obtained through counting. Advanced addition, on the other hand, is defined as any addition problem, regardless of size, whose answer is obtained through grouping or regrouping. Given these definitions, 7 + 8 can be seen to be both a beginning and an advanced addition problem, depending on the method employed to discover the answer. If the child counts all of the Unifix cubes or tiles or whatever, 7 + 8 is beginning addition. If the child groups the cubes or tiles, 7 + 8 is advanced addition. In neither approach is the child expected to produce an answer from memory.

When we were in school, the purpose of our arithmetic lessons was to have us provide quick answers to problems which had no meaning to us. This approach turned too many of us into people who both doubted our own mathematical abilities and hated mathematics itself. When understanding is the goal, as it is with Math Their Way, we must allow ourselves to rethink what we are asking of our students. Are we asking them to do something (like memorize the facts between ten and nineteen) because we were asked to do it as students? Or, are we asking them to learn in ways that will help them see what we may never have been allowed to see when we were children: that mathematics is a search for patterns in numbers, and mathematics is something we can all come to understand, given the time and the opportunity to learn from our own experiences. It just may be that if understanding is the goal, the quick answer really isn't any answer at all.

HOW TO BEGIN AT THE NUMBER STATIONS

The number stations provide flexibility for children of varied abilities and levels of thinking to work side-by-side with the same materials. Each child is assessed to find an approximate level of his or her number awareness. All young children, regardless of their mathematical ability, should begin working at the concept level.

- ❏ Schedule time for the class to free explore the new materials used at the number stations (e.g., two-sided beans, jewels, toothpicks…). This can be done during a separate activity time.

- ❏ Be sure the children have experienced a variety of group number activities at the concept level (see *Mathematics Their Way*, pp. 180-209, Word Problems, pp. 10.4-10.6, Junkjobs pp.10.7-10.9). Whole class and small group number lessons should continue to be scheduled throughout the year — before, during and after the number station stations are experienced.

- ❏ Introduce one material at a time during whole group and small group lessons. Familiar materials will be used in a new way. There are new ground rules, as well as language development, for the children to internalize.

- ❏ The class should experience each material as a number activity several times in group sessions before the materials are placed with the stations.
- ❏ Gradually replace the old stations with Number Stations when the class shows they understand the new guidelines. For instance, if you are presently working at the Pattern Stations, remove one station and replace it with a new Number Station until all of the Number Stations are in operation.
- ❏ Assess the children (see Number Assessment at the Concept Level, NL, p. 3.13). Determine a range of three numbers for each child to work with at the stations. Prepare range cards (NL Blackline #37) with these numbers for the children to wear while working at Number Stations.

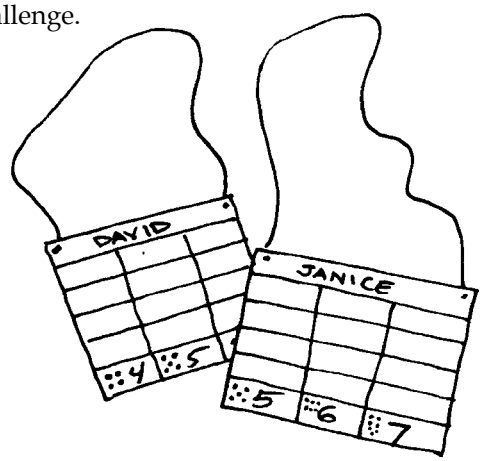
Range Card

Make an individual range card (NL Blackline #37) for each child. Each child works within a range of three numbers suited to his or her needs. For instance, if a child assessed comfortably at "five", then he or she may work on numbers four, five and six. If you feel that child needs a challenge, you may assign the child five, six, and seven. Children can wear the range card as a necklace.

Procedure: The child chooses a number on his or her range card to work on that day at the number stations and colors in the appropriate box for the number he or she has chosen. A different color is chosen each day, so the teacher can tell quickly which number each child has chosen for that day.

The child must use only the number he or she chooses for that day during the entire math period. A different number can be chosen another day.

As soon as the range card has one column full, the child may take it home. (Be sure to send a note home to explain the range card to the parents.) Then, re-assess the child and give the child a new range card. The card will usually have at least one new number on it. For example, if a child was working on four, five, and six, then the new card's range might be five, six, seven. A teacher might decide to give two new numbers to a child who needs a challenge.



GROUP ACTIVITIES

Introduction to the Number Station Activities

(Concept Level)

Materials: one material which will be placed in the number stations (e.g., tiles, toothpicks, pattern blocks, Unifix cubes, plain wooden cubes, two-sided beans, jewels)

Gather a group of children together (on the floor or at a table). The number of children will be determined by the space and materials available for a group lesson. Ask the group to use the material and make as many different pictures as they can with a certain number of objects (e.g., five). Explain any special rules. The two basic rules are:

- When a number design is built, all materials (except junk) must touch either on a side or at a corner.
- Only one or two colors are used when designs are being built with pattern blocks and Unifix cubes. Some teachers separate these materials by color to help the children focus on the task at hand.

After the group has had time to make designs, ask them to stop and look at each other's work. Then ask, "Do you see a design that reminds you of something else?" (Children point out a design and tell what they see, e.g., "a sawhorse", "a swing set with no swings", "a roof"…) To help the others focus on the particular design, frame the design being described with your hands as the child verbalizes what he or she sees.

Once the children have had time to describe the designs, choose one of the designs and say, "If I described this design with numbers, I could say, 'two and one and two'. (Indicate how you see the number combination with your hands.) Is there another way to describe this design with numbers?"

Extension: Once the children have had time to describe the designs say, " Which designs are arranged like 2 + 3?" And so on…

Number Books

(Connecting Level)

Materials: recordings of toothpicks, pattern blocks, tiles, and junk designs the children have made at the stations; several sheets of blank white construction paper (6" by 9"); marker; construction paper for cover; stapler

Model Lesson

Procedure: Gather a small group of children together to make a book with designs representing a designated number (let's say "five"). Choose several recordings (of five) and spread them out.

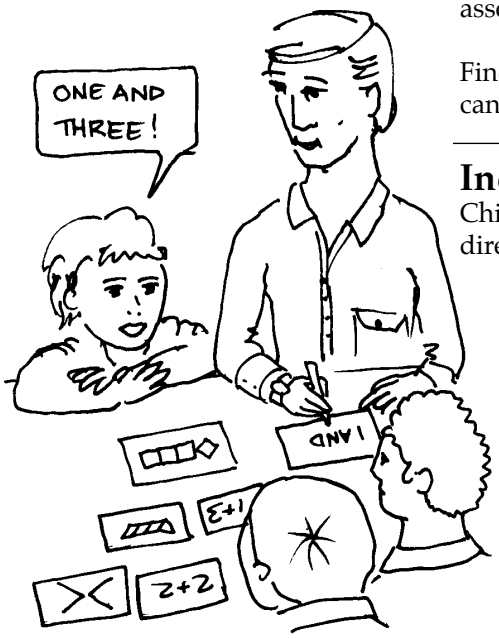
Ask: "Who sees a design that reminds you of something else?" After several suggestions, ask which description the group would like to put down on paper.

Write the word or phrase on a blank piece of paper and stack it on top of the design described. Repeat this process until all the recordings chosen have a written description.

Gradually, the descriptions will change from word descriptions to number combinations. Write the equation either horizontally or vertically, depending on how the child sees the number design.

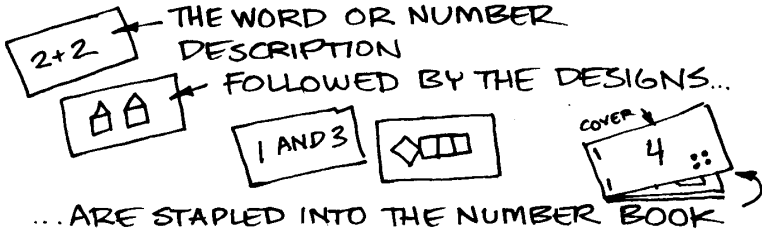
When all designs have been described, gather the pages and staple them into a book. The page with the word, sentence, or number describing the design precedes the design page. The group reads the assembled book together.

Find a spot to keep the number books in the classroom, so the children can share them with one another.



Independent Level

Children can make their own number books once they understand the directions.



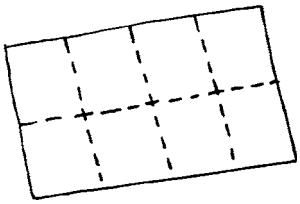
Rebuilding at the Number Stations

(Symbolic Level)

Materials: number station manipulatives; 12" by 18" newsprint; pencils

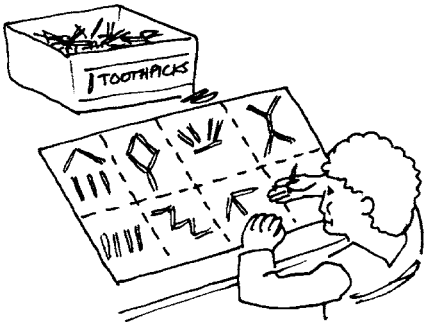
Before beginning: Give each child a piece of 12" by 18" newsprint. Ask the children to fold their paper into either six or eight boxes. They may need help with this process at first. The number tubs are placed at the stations. Taking their paper, the children choose a number station.

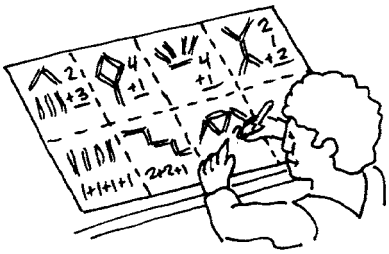
Procedure: The children build one number design in each box on the paper with the manipulatives at the station. They record a number equation that's representative of each design. The equation could be horizontal or vertical, depending on the position of the design.

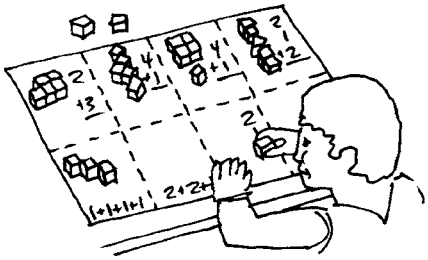


The children clear their papers and move to a new station. They rebuild each recorded number combination using the new material.

Extension: Rebuilding papers can be used over and over again. Children can exchange their rebuilding papers with each other.







Modeling Math Concepts at the Number Stations

Take advantage of opportunities as they arise to model mathematical concepts within the context of natural, uncontrived situations, rather than teaching each concept in isolation.

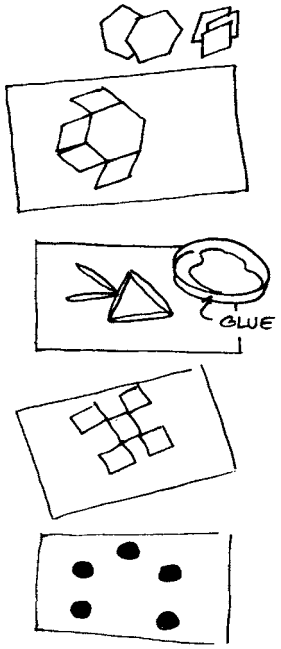
Sample Teaching Strategies

The class is working at the number stations. The teacher takes this opportunity to model counting-on in the context of the children's work at the toothpick station. The same strategy can be used for any mathematical concept.

RECORDING AT THE NUMBER STATIONS

Note: Recording occurs only when the child shows an interest in having a permanent record of the design created.

Number Recordings



Pattern Blocks: 6" x 9" black recording paper; precut pattern block shapes
Procedure: Glue the pattern block shapes onto the black paper to represent the number design.

Toothpicks: 6" x 9" black recording paper; toothpicks; Elmer's glue; an old jar lid (something to place the glue on)
Procedure: Squeeze a small amount of glue onto the lid. Dip only the ends of the toothpicks into glue and place them on the black paper.

Tiles: 6" x 9" recording paper (depends on tile color); 1" paper squares same color as the tiles
Procedure: Glue the tile squares onto the paper to represent the tile number design.

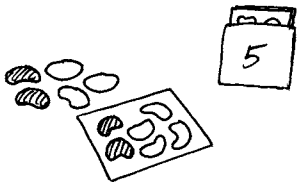
Junk: 6" x 9" white recording paper; black crayon
Procedure: Make black blobs to represent the junk design.

Tiny Number Books Blackline Recordings

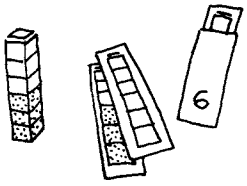
Staple individual jewel, Unifix or two-sided bean recordings together into tiny number books.

Optional: Two-sided beans, Unifix cubes, and jewel recording sheets can be cut down into individual sheets and placed into library pockets or small zippered plastic bags. That way, the child uses only the recordings needed and there's no waste.

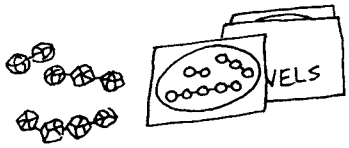
Two-sided Beans: MTW Blacklines #33-39; crayons the same color as the beans



Unifix Cubes: MTW Blacklines #28-32; crayons the color of the Unifix cubes



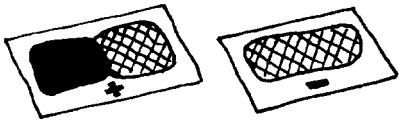
Jewels: NL Blackline #18; red, yellow, green, black, and blue crayons.
The child makes colored dots and connects them to indicate that the jewels are connected together.



Number Materials

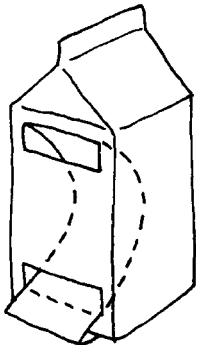
Cave Cards

Duplicate a sufficient number of boards (NL Blackline #35). Cut the tagboard sheets into thirds.



Junkjobs Cards

Duplicate a sufficient number of boards (NL Blackline #21). Cut the sheets in half. Mount the addition half on one side of each piece of tagboard and the subtraction half on the backside of the same piece of tagboard.



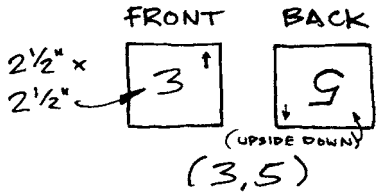
Magic Box

Materials: a box (e.g., a half gallon milk carton); 3" by 9" piece of tagboard; tape, Exacto knife; masking tape

Procedure: Cut two rectangular openings on one side of the box (one opening at the top and one opening at the bottom). Take the piece of tagboard and push it in the top opening and out the bottom opening. Tape the tagboard on the top edge of the top opening. Allow the extra tagboard to hang out the bottom opening.

Magic Box Cards

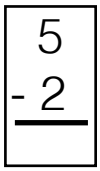
Materials: 64 tagboard cards (2-1/2" by 2-1/2"); permanent marker

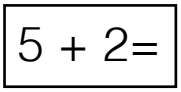


Procedure: Below are the possible number combinations for the cards. Write one number on each side.

Combinations

(0, 1)	(0, 2)	(0, 3)	(0, 4)	(0, 5)	(0, 6)	(0, 7)
(0, 8)	(0, 9)	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)
(1, 6)	(1, 7)	(1, 8)	(1, 9)	(2, 2)	(2, 3)	(2, 4)
(2, 5)	(2, 6)	(2, 7)	(2, 8)	(3, 3)	(3, 4)	(3, 5)
(3, 6)	(3, 7)	(4, 4)	(4, 5)	(5, 5)	(5, 6)	(5, 7)
(5, 8)	(5, 9)	(6, 5)	(6, 6)	(6, 7)	(6, 8)	(6, 9)
(7, 4)	(7, 5)	(7, 6)	(7, 7)	(7, 8)	(7, 9)	(8, 3)
(8, 4)	(8, 5)	(8, 6)	(8, 7)	(8, 8)	(8, 9)	(9, 2)
(9, 3)	(9, 4)	(9, 5)	(9, 6)	(9, 7)	(9, 8)	(9, 9)





Subtraction and Addition Equation Cards

Duplicate the equation cards (NL Blackline #23-34) onto tagboard. Cut apart the duplicated recordings into individual equations. Mix the addition and subtraction (horizontal and vertical) equation cards and place them in a library pocket or small zippered plastic bag. The children take any pocket and use the cards.

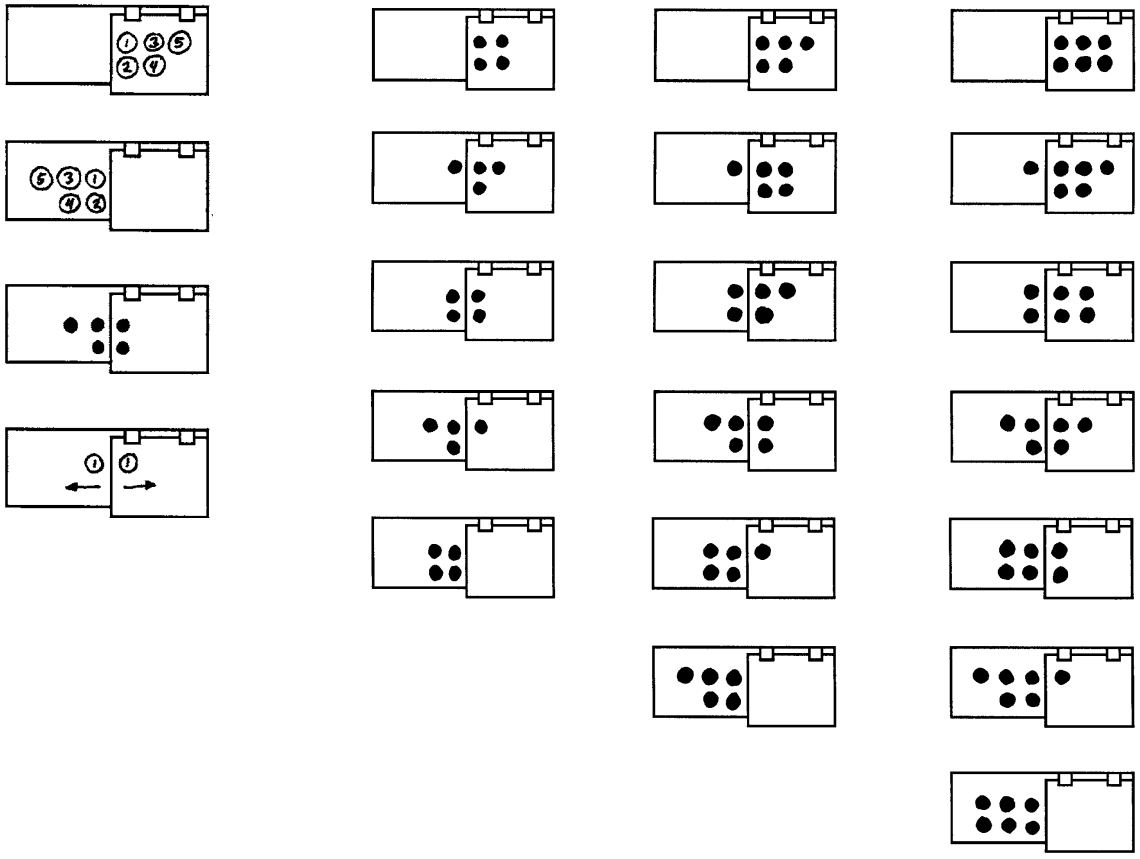
Subtraction Flip Cards

To make one Subtraction Flip Card: one piece of 3" by 9" tagboard; one piece of 3" by 4-1/2" tagboard; permanent markers or pregummed dots

Procedure: Tape the 3" by 4-1/2" piece of tagboard to the right side of the 3" by 9" piece of tagboard. Stagger the flip part of the card make it easier to flip the card.

Record the right side dots first, following the sequence in the first illustration. Next record the left side dots. There should be no visually apparent break from one side to the other.

Below are the dot arrangement patterns for four, five and six. Make similar cards for seven, eight and nine.

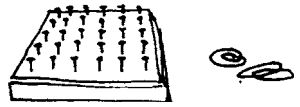
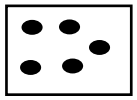
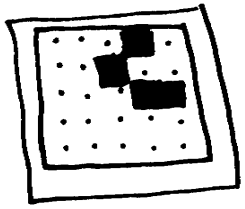


NUMBER STATIONS

	UNIFIX CUBES	PATTERN BLOCKS	WOODEN CUBES	TILES	TOOTHPICKS
Page #'s	MTW, p. 78	MTW, p. 177	MTW, p. 178	MTW, p. 177	MTW, p. 77
Activity Description	**Materials needed:** • 500 Unifix cubes separated by color Procedure: The children choose two colors of Unifix cubes to make different pattern stacks with the number being explored. The nubs on the Unifix stacks should be facing in the same direction so the children can see the reverse color patterns.	**Materials needed:** • 750 pattern blocks separated by color Procedure: The children make pattern block designs with the number being explored (either in one or two colors). The pattern blocks must touch either on the side or corner. The children should stay with the same two colors.	**Materials needed:** • 500 plain wooden cubes Procedure: The children build wooden cube designs with the number being explored. The design must touch either on a side, corner, or on the top.	**Materials needed:** • 500 one color tiles Procedure: The children build a variety of tile designs with the number being explored. The tiles must touch a side or a corner.	**Materials needed:** • 1-2 boxes of plain flat toothpicks Procedure: The children make designs with the number being explored. The toothpicks must touch either the side or corner.
*** Recording Description**	**Additional materials needed:** • Unifix cube record sheets (MTW Blacklines #33-39). Each number is duplicated, cut apart, and stored in small plastic zippered bags. • crayons the color of Unifix cubes being used. Recording: The children record their Unifix cube patterns on the recording papers by coloring the squares. The individual recordings can be stapled together into a tiny equation book. (see NL, p. 10.21)	**Additional materials needed:** • 4" by 4" work spaces (optional) • 6" x 9" black construction paper • pre-cut pattern block shapes (MTW Blacklines #2-6) • paste • wet rags Recording: The children copy their pattern block designs by gluing pattern block paper shapes onto pieces of black construction paper. (see NL, p. 10.21) 	**Additional materials needed:** • sugar cubes • white glue in a container • clear Contact paper covered piece of cardboard on which to place sugar cubes to dry • wet rags Recording: The children copy wooden cube designs by gluing sugar cubes together. (see MTW, p. 178)	**Additional materials needed:** • 1" squares of colored paper to match the color of tiles being used (or use white paper squares) • 6" x 9" drawing paper • paste • wet rags • tile template Recording: The children can either glue paper squares in the arrangements of the tiles onto a piece of paper with the tile paper shapes or trace the design with a tile template. (see NL, p. 10.21)	**Additional materials needed:** • 6" x 9" black construction paper • white glue in a small container • wet rags (for cleaning up sticky fingers) Recording: The children copy their designs by dipping each end of the toothpick into the white glue and then placing it on a sheet of black construction paper. (see NL, p. 10.21) 

*** Recording materials are placed in the tubbing stations only after the children have had sufficient time to work at the concrete (concept) level with the manipulatives. Depending on the class's experience and the material in the tubbing station, the time allowed may span from several weeks to several months. Recording should be introduced as an optional activity.**

Number Stations

	JUNK BOXES	BEANS	JEWELS	GEOBOARDS	GEOBOARD PATTERNS
Page #'s	MTW, p. 175	MTW, p. 174	MTW, p. 171	MTW, p. 176	MTW, p.241
Activity Description	**Materials needed:** • several different boxes of round junk  Procedure: The children place a designated amount of junk in an arrangement.	**Materials needed:** • two-sided beans spray painted on one side • shaker (old 35 mm film containers) Procedure: The children shake a given quantity of beans in a container and turn the beans over onto the table. They take note of the number of colored and white lima beans. This is repeated many times.	**Materials needed:** • portion cups • jewels (separated by color) Procedure: The children place a given number of jewels in the portion cups. They continue to search for as many different ways to combine the jewels as possible.	**Materials needed:** • 6-8 geoboards • squares of one color construction paper which fit between the nails of your geoboards, stored in several small plastic zippered bags Procedure: The children make patterns on geoboards with a given number of colored squares. They can make more than one shape on a board. The children can add geoboards when they need more space for square designs.	**Materials needed:** • 10-12 geoboards • geobands  Procedure: The children make a variety of shapes with sides equaling the number being explored on their geoboards. They can make more than one shape on a board. The children can add geoboards when they need space for more shapes.
*** Recording Description**	**Additional materials needed:** • 6" x 9" drawing paper • black crayons Recording: The children copy one junk design on a piece of white paper with a black crayons by making big dots in the same arrangement as the junk . (see NL, p. 10.21) 	**Additional materials needed:** • bean record sheet cut apart and stored in library card pockets with the numeral written on the outside of the pocket (MTW Blacklines #33-39) Recording: The children can make recordings of the bean tosses. They use bean recording papers for the number they've been exploring and color the number of colored beans showing after being tossed. The individual recordings can be stapled together into a tiny equation book. (see NL, p. 10.21)	**Additional materials needed:** • jewel record sheet cut apart and stored in library card pockets (NL Blackline #17) • crayons in jewel colors  Recording: The children record the jewel combinations by making small circles on their jewel record sheets with the corresponding color crayons. The children show the jewels are connected by drawing a short line between the dots. The individual recordings can be stapled together into a tiny equation book. (see NL, p. 10.21)	**Additional materials needed:** • geoboard dot paper (MTW Blackline #17) • crayons Recording: The children copy geoboard designs on the geoboard paper with a crayon. 	**Additional materials needed:** • geoboard dot paper (MTW Blackline #17) • markers and/or pencils Recording: The children copy geoboard designs onto the geoboard paper. Note: If the children record several designs with the same number of sides, they can staple them together into a book.

*** Recording materials are placed in the tubbing stations only after the children have had sufficient time to work at the concrete (concept) level with the manipulatives. Depending on the class's experience and the material in the tubbing station, the time allowed may span from several weeks to several months. Recording should be introduced as an optional activity.**