

PLACE VALUE

INTRODUCTION

A firm understanding of place value is a prerequisite for all work in arithmetic. Students who do not understand the concept of place value can not progress through the four basic operations without difficulty. They “learn” the operations by memorizing an increasing number of seemingly unrelated facts and procedures. Even the most capable students become confused when the load becomes too great, usually at the point when they must learn multiplication and division. Mathematics becomes increasingly mysterious for these students, who have little hope of understanding the content of secondary-level mathematics courses. It is essential that place value be given major emphasis in the primary grades and that students have frequent experiences with manipulative materials that demonstrate place value.

1985 California Mathematics Framework, p.23

In order to effectively implement the place value objectives outlined in the 1985 California Mathematics Framework (above) in the primary grades, the mathematics curriculum must include place value experiences that:

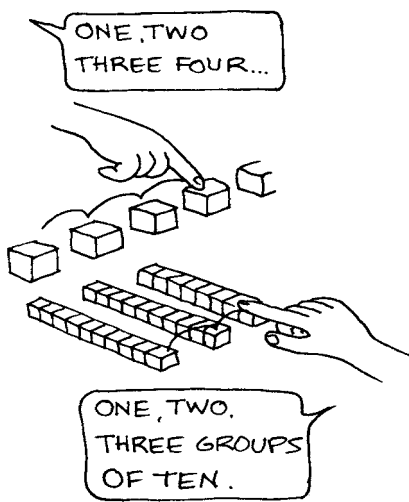
- use developmentally appropriate manipulatives.
- use a variety of bases to teach the place value system.
- explore place value patterns (linear and matrix).
- provide regrouping and non-regrouping multi-digit addition and subtraction experiences simultaneously.

PLACE VALUE MANIPULATIVES

Place value manipulatives fall within two distinct developmental levels: concrete and representational. Concrete place value experiences must come first. Representational activities are appropriate later in the elementary grades — after children have engaged in a variety of experiences with concrete materials.

Concrete Materials

Concrete materials — beans and cups and Unifix cubes — allow the children to construct groupings. Experiences with these materials stress one-to-one correspondence between the number and the material it represents.



Kathy Richardson addresses the issue of one-to-one correspondence at the place value level in her book, *Developing Number Concepts Using Unifix Cubes* (Chapter 5, p. 133):

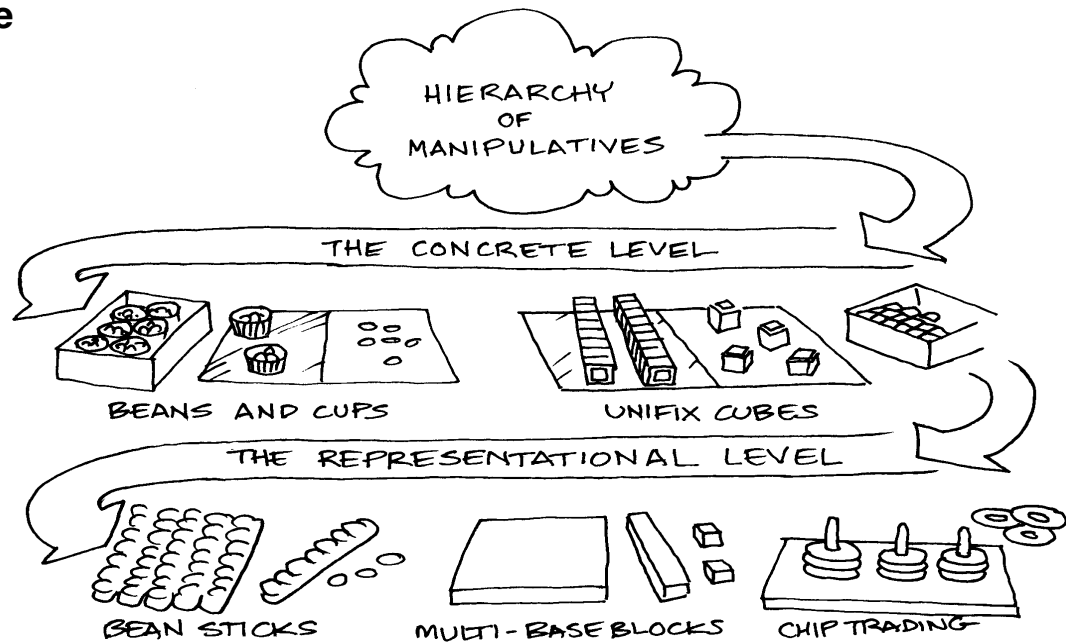
Counting groups [of objects] requires a different kind of thinking from counting single objects. Children's first counting experiences require an understanding of one-to-one correspondence. They learn that one number word goes with one object. But, when dealing with numbers above ten, children are required to count groups as though they were individual objects. The question "How many tens in thirty-four?" assumes the child can conceive of ten objects as one entity. The question "How many hundreds in 346?" assumes the child can conceive of one hundred objects as one group (all the while remembering that each hundred is also ten groups of ten).

Representational Materials

Bean sticks and multi-base blocks are "fixed" materials. The materials impose a ready-made structure on the student. Although children manipulate materials — e.g., chips, multi-base blocks and bean sticks — they are required to think abstractly. A child with insufficient experiences using concrete materials may be confused when a number is represented by multi-base blocks. They must already understand that one block in base ten represents either ten flats, a hundred longs, or a thousand units. Rather than understanding that a number like 362 is represented by three flats, six longs, and two units, a child may only see eleven separate blocks of varying sizes.

Chip trading is also representational but at a more abstract level. Children must be able to understand the "abstraction" that one chip could represent a group of ten or a hundred chips (chips can also represent groupings in other bases). The necessity for children to be surrounded with an abundance of concrete experiences before they move on to representational materials cannot be emphasized strongly enough.

Choose Appropriate Manipulatives



Because mathematics is made by human beings and exists only in their minds, it must be made and remade in the mind of each person who learns it. In this sense mathematics can only be learned by being created.

Author Unknown

Teachers often overlook the limitations set by representational materials. One Math Their Way teacher told how she made bean sticks with her second grade class one year. The class had experienced place value activities earlier in the school year and in the first grade, using concrete materials. Once the sticks and rafts were constructed, the children used them to make addition and subtraction problems. The children loved the bean sticks. They didn't appear to have any problems with the "fixed arrangement" of the sticks and rafts.

The teacher kept the bean sticks and used them with classes after that year. The new second graders didn't seem to relate to the materials in the same manner as the first group. In retrospect, the teacher felt the reason the bean sticks were more successful the first year was because the children participated in the construction of the sticks and rafts. They understood what the beans on the sticks and rafts represented. Even though they had similar classroom experiences to prepare them for the activity, the children who used the pre-assembled materials the following years didn't experience the creation of the sticks and rafts. The pre-assembled (representational) materials "imposed a structure" that subsequent classes of second grade children were not ready to accept.

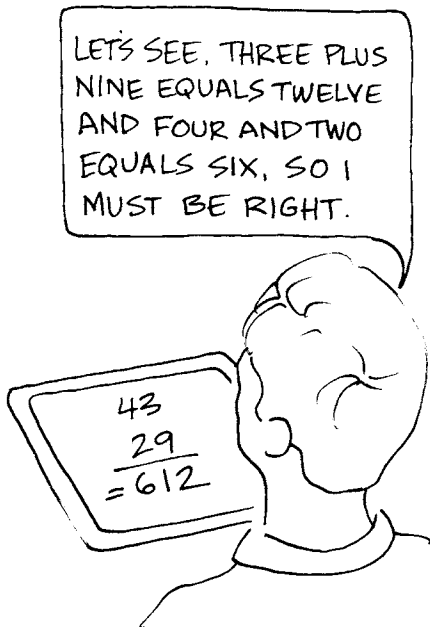
Be careful not to move prematurely to the representational level. Allow children time to construct their own understanding of place value by engaging in experiences with an assortment of concrete place value materials.

USE A VARIETY OF BASES

Place value is merely a system of organizing large numbers. Teachers tend to think that the base ten system is the only base we use in the real world. For this reason, they often question why other bases are taught in the lower grades. It's true that our society frequently uses base ten, but we also use other bases every day without thinking about it. Every time we bake, sew, measure with a ruler, or tell time, we are working with other bases. The computer operates on a binary system or base two. Yet, textbooks tend to teach isolated place value skills (e.g., borrowing and carrying) in base ten only. Children will develop a better understanding of place value if they are allowed opportunities to compare and contrast relationships (using real materials and real-life experiences) in other bases.

TEACH REGROUPING AND NON-REGROUPING ADDITION AND SUBTRACTION TOGETHER

The 1985 California Mathematics Framework, p. 22, emphasizes the importance of teaching multi-digit addition and subtraction in a way that students understand the process, rather than viewing each algorithm as a separate, isolated function:



The practice of a skill in isolation is seldom effective in developing the understanding required to make the skill useful. Instruction in computational algorithms should emphasize understanding the procedures that are being used.

For example, it is now common to teach two-place addition and subtraction without regrouping before introducing regrouping. This approach leads students to focus on separate procedures and hinders their understanding of the basic operations. Most students view a problem such as $\begin{array}{r} 43 \\ +29 \end{array}$ as two one-place problems pushed together ($\begin{array}{r} 4 \\ +2 \end{array}$ and $\begin{array}{r} 3 \\ +9 \end{array}$) and do not think of adding 40 and 20. Because students get a great deal of practice in this kind of problem, they assume that $\begin{array}{r} 43 \\ +29 \end{array}$ can be calculated in the same way and see nothing wrong with the answer of 612. After learning a new procedure in which 1 is carried or put on top of the ten's column every time, students are confused when presented with a mixture of problems that require regrouping and problems that do not. The child who asks, "Do I regroup on this problem?" has almost no understanding of the concept of two-digit addition or subtraction. This dilemma can be avoided by teaching multi-digit addition and subtraction with and without regrouping simultaneously, using manipulative place value materials, and relating the process to realistic situations. When manipulative materials are used, it is easy to demonstrate when regrouping is needed; and there is no need to teach regrouping as a separate algorithm.

EMPHASIZE THE IMPORTANCE OF PATTERNS

Children should have opportunities to explore both linear and matrix place value patterns in a variety of bases. An understanding of patterns encourages children to see relationships between the parts and the whole. An understanding of the process develops from experiencing numerous patterns in different bases. Bob Baratta-Lorton wrote in *Mathematics...A Way of Thinking*, p. 81:

When students search for patterns in groupings of three, four, and five, and they then see these same patterns repeat for groupings of ten, they achieve a far greater understanding of borrowing and carrying than is possible from studying base ten in isolation.

PLACE VALUE SCHEDULE

Choose the activities that best fit your class's needs. In general, the place value focus for each grade level includes the following:

Kindergarten

Kindergartners do not formally explore place value concepts. They are exposed to the place value board and the notion of organizing large numbers of objects into groups of ten through the following types of activities:

- Estimating and counting groups of objects in containers (see NL, Measurement pp. 7.12 - 7.15). The place value board is used when the objects are counted and grouped into ten's.
- Opening activities, NL Chapter 4 (e.g., Straw Count, Tally Count, Number Line).

First Grade

Place value concepts are formally introduced and explored at the concept and connecting levels late in the school year.

- Estimating and counting groups of objects in containers (see NL, pp. 7.12 - 7.15). The place value board is used when the objects are counted and grouped into ten's.
- Opening activities, (e.g., Straw Count, Tally Count, Number Line).
- Place value activities using other bases (concept and connecting level).
- Base ten station activities (concept level — recording is not necessary).

Second Grade

Place value is the main focus of second grade mathematics. Second graders explore all levels of place value throughout the school year.

- Estimating and counting groups of objects in containers. The place value board is used when the objects are counted and grouped into ten's.
- Opening Activities, (e.g., Straw Count, Tally Count, Number Line).
- All place value activities using other bases (concept and connecting and symbolic level).
- All base ten activities (station and independent).

EXPLORING PLACE VALUE IN A VARIETY OF GROUPINGS

The initial place value activities in this section provide children a framework for understanding place value while adding and subtracting in different groupings. Children acquire a better understanding of the process of regrouping (borrowing and carrying) when they have experiences with groupings of four, five, and six — as well as with base ten. Independent base ten activities are introduced once the children demonstrate a clear understanding of the place value process. The urge to focus on base ten too quickly should be resisted.

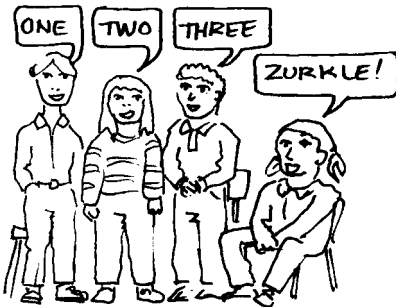
Each grouping used (except base ten) is given a nonsense name. The children are asked for suggestions. The nonsense name for each grouping remains the same for the remainder of the year. A class chart of the nonsense names is displayed. Each grouping name is added to the chart after it is introduced through the Counting Game.

CONCEPT LEVEL: GROUP ACTIVITIES

Searching for Patterns

Counting Game (MTW, pp. 98-99)

Practice counting by the groupings using the nonsense word.



Class Count

Choose a nonsense name for the grouping to be experienced. ("Zurkles" will be used instead of four in this description). Ask the class to stand up and count off by the grouping chosen. Each person who says the nonsense word must sit down (e.g., "One, two, three, zurkle ... one, two, three, zurkle..."). The game continues until one person remains standing.

When half the class remains standing, ask the children if they can predict who will be the last person standing.

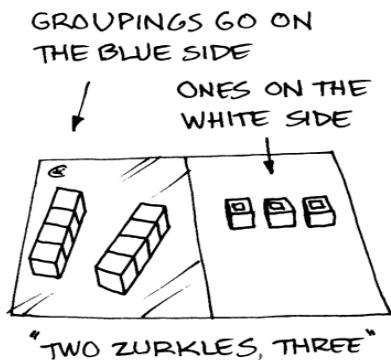
Group Count

Once the rules are established, split the class into small groups. Ask them to count off in their groups. Use the following questions to challenge the groups:



- Predict who will be the last person standing.
- Does it make a difference where you start counting?
- If you start with the same person twice, will the last person standing be the same twice? Why? Why not?
- Start with the person standing to the right (left) of the first starter. Predict who will be the last person standing.
- Add a person to the circle. Predict who will be the last person standing. Continue adding a person; is a pattern developing?
- Count in another grouping. Do you think the outcome will be the same? If not, can you predict who will be the last person standing?

Note: *Mathematics Their Way*, (Ch. 12) says to read amount of materials on the place value board with “and” (two zurkles and three). This is mathematically in-correct. “And” signifies a decimal point. The correct way to read the amount is “two zurkles, three.”



Introduce Addition and Subtraction on the Place Value Board

Materials: Place value boards (MTW, p. 364); manipulative materials (e.g., Unifix cubes or beans and portion cups)

Before beginning: Review how to use the place value board. The groupings (e.g., Zurkles) are placed on the blue side and the one's are placed on the white side. If the children use Unifix cubes, they snap the cubes together into a stick before they place them on the blue side. If they use beans, they place the grouping of beans in a cup and move the cup to the blue side.

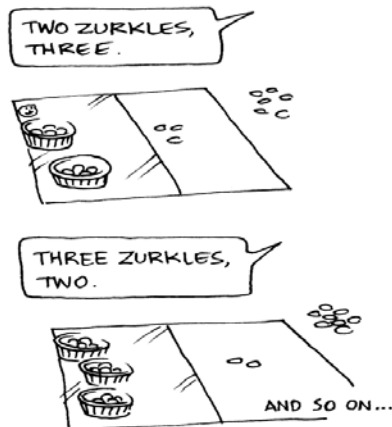
Addition: Begin by adding one object. Explain to the children that each time you say “plus one”, they add a cube (or bean) to their place value board. Once the material is added to the board, the whole class says the number. The group continues to add to the largest two-digit number possible, regrouping whenever necessary. For instance, in zurkles (groupings of four), the largest number would be three zurkles and three.

Subtraction: Start with the largest two-digit number possible in the grouping. Ask the children to subtract one object at a time until they reach zero — regrouping whenever necessary.

Extension: Add or subtract by two or three objects at a time.

Random Practice

It is important that children have opportunities to randomly add and subtract.



Building Numbers

Materials: Beans and cups or Unifix cubes; place value board

Tell the children a number to build on the place value board (e.g., 2 zurkles, three). Then tell them a new number (e.g., 3 zurkles, 2). The children build the new number on the place value board with materials. Repeat the process several times.

Observation: Observe how the children find their answers. Do some children clear their boards every time they build a new number? Do others add on or subtract the appropriate objects? Ask the children to explain how they made the new number. Ask the children if they think the new number is larger or smaller than the previous number. Ask them to explain their answer.

Add or Subtract

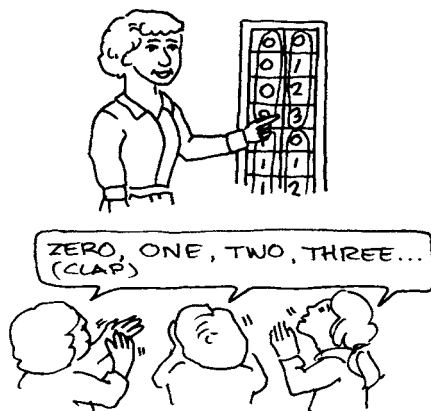
Tell the children a number to build on their place value board. Now ask them to add or subtract a certain number of beans.

Observation: Watch to see if the children can regroup when it is necessary.

CONNECTING LEVEL: GROUP ACTIVITIES

The goal of following activities is to acquaint the children with the written symbol that represents the number on the place value board. The class repeats the addition and subtraction activities introduced at the concept level. This time the teacher records the numerical patterns either on a long strip or in a matrix frame, as the children add and subtract the materials on the place value board.

Searching for Patterns



Vertical Patterns

Materials: Place value boards (MTW, p. 364); adding machine tape; manipulative materials (Unifix cubes or beans and portion cups)

Procedure: After the numerical patterns are recorded on the strip, the class searches for patterns and then interprets the patterns in each column. Ask the children to look for a pattern in the blue column. Then ask them to alternate standing and sitting each time the pattern sequence changes as they read the patterns in unison. Loop the patterns as the children interpret each pattern.

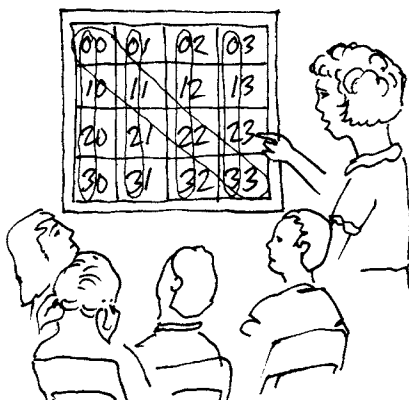
Ask the children to look for patterns in the right column (plain side). This time, ask the children to stamp their foot each time a zero is read. Loop the repeating sequence as the children interpret the pattern.

Extension:

- Repeat the number strip activities for different counting sequences (by one's, two's, three's,...) in one grouping. Continue to search for patterns in the one's and cup's column until the whole counting sequence begins to repeat. How many numbers did you have to count before this occurred? Is there a pattern?
- Repeat the activities in a variety of groupings.

Matrix Patterns

Materials: Place value boards (MTW, p. 364); class size matrix grid; crayons; manipulative materials (Unifix cubes or beans and portion cups)



Children should have opportunities to search for patterns on matrices as well as number strips. Encourage the children to compare the matrix patterns and make predictions from the various patterns.

Procedure: The teacher records on a matrix grid the same size as the base (e.g., base 4 would be a 4 box by 4 box grid) instead of a number strip. The class searches for patterns after the sequence is recorded. The teacher loops and shades the patterns with crayons as the children discover the patterns.

Extension:

- Repeat the matrix activity for different counting sequences (by one's, two's, three's,...)
- Change the size of the matrix grid (e.g., from a 4 box by 4 box matrix to a 5 box by 5 box matrix).

Comparing Patterns on the Matrices and Number Strips

After the teacher has recorded a variety of sequences on matrix grids and number strips, the class compares, discusses and makes predictions using the information.

- ❑ Compare different counting sequences (by one's, two's, three's,...) in one grouping on number strips.
- ❑ Compare different counting sequences (by one's, two's, three's,...) in one grouping on matrix grids.
- ❑ Compare a counting sequence in different groupings on the number strips.
- ❑ Compare patterns of a counting sequence recorded on a matrix grid and a number strip.
- ❑ The teacher records the counting sequence in a different size matrix grid. The class searches for patterns in the new grid and then compares the two grids.
- ❑ The teacher records different groupings in the same size matrix grid.

ZURKLE PATTERNS

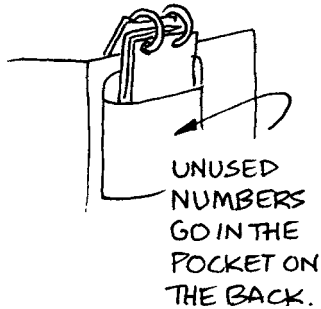
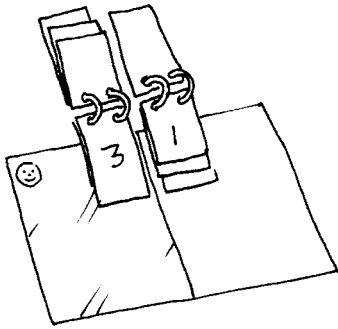
+1	+2	+3	ZURKLE+1	ZURKLE+2	ZURKLE+3
0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	
0 0 1	0 0 2	0 0 3	0 1 1	0 1 2	
0 0 2	0 1 0	0 1 2	0 2 2	0 2 0	
0 0 3	0 1 2	0 2 1	0 3 3	0 3 2	
0 1 0	0 2 0	0 3 0	1 0 0	1 0 0	
0 1 1	0 2 2	0 3 3	1 1 1	1 1 2	
0 1 2	0 3 0	1 0 2	1 2 2	1 2 0	
0 1 3	0 3 2	1 1 1	1 3 3	1 3 2	
0 2 0	1 0 0	1 2 3	2 0 0	2 0 0	
0 2 1	1 0 2	1 3 2	2 1 1	2 1 2	
0 2 2	1 1 0	2 0 1	2 2 2	2 2 0	
0 2 3	1 1 2	2 1 0	2 3 3	2 3 2	
0 3 0	1 2 0	2 1 3	3 0 0	3 0 0	
0 3 1	1 2 2	2 2 2	3 1 1	3 1 2	
0 3 2	1 3 0	2 3 1	3 2 2	3 2 0	
0 3 3	1 3 2	2 3 3	3 3 3	3 3 2	
1 0 0	2 0 0	3 0 0	0 0 0	0 0 0	
1 0 1	2 0 2	3 0 3	0 1 1	0 1 2	
1 0 2	2 0 2	3 0 3	0 1 1	0 1 2	

WHEN WE ADD BY ONE, THE PATTERN REPEATS AFTER ADDING FOUR TIMES IN THE ONES COLUMN, BUT NOT 'TIL SIXTEEN IN THE ZURKLES COLUMN.

THE PATTERN IN THE ONES COLUMN IS THE SAME WHEN YOU ADD TWO BEANS OR ONE ZURKLE, TWO BEANS.

WHEN YOU ADD ONE ZURKLE, ONE, THE PATTERN IN THE ONES COLUMN AND ZURKLE COLUMN ARE THE SAME.

Random Practice



Place Value Number Flips

Materials: Place value boards; place value number flips (MTW, p. 363); manipulative materials (Unifix cubes or beans and portion cups)

Show the children how to attach the number flips to their place value boards. Only the numbers used in the base being used are on top of the place value board. All the other numbers are tucked into the pocket on the back of the board.

Building Numbers

Tell the children a number to make with the place value number flips. Then ask them to build that number on the place value boards with materials (either beans and cups or Unifix cubes). Repeat the process with a different number.

Add and Subtract

Ask the children to make a two-digit numeral with their place value number flips. Then ask them to build the number on their place value boards with materials. Now ask them to add or subtract a certain number of objects. Once they have added or removed the designated number of materials, ask them to change the place value number flips to represent the new quantity of objects.

CONNECTING LEVEL: INDEPENDENT ACTIVITIES

Random Practice

Materials needed for each child: portion cups; beans; bowl; place value board; 1-3 dice

The following activities can be played in any grouping. Wooden cubes can be used to make the dice. Some teachers choose to make a different set of dice for base 4, 5, and 6. Other teachers make a set of base 6 dice (numbered zero to five). When they play the game in groupings of 4 or 5, they cover the inappropriate numbers with a gummed label. If the children roll a blank side when playing the games in either groupings of 4 or 5, they repeat the number they previously rolled.

The children work in pairs. They take turns rolling the dice and building the material on the place value board.

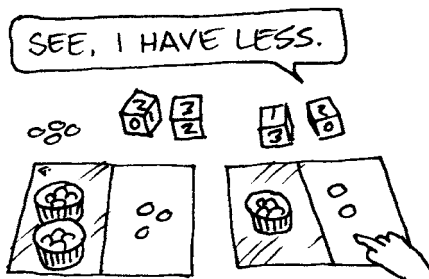
Dice Arrangement

Note: Before the game begins, the players must decide how the winner will be determined: either the smallest or largest number.

Procedure: The first person rolls two dice. He or she arranges the dice to make the largest (or smallest) number possible. Then the person builds the number with beans on his or her place value board. The second child rolls the dice and builds the number with beans and cups on his or her place value board.

Once the numbers are built on the place value board, the children compare their numbers to determine which number is larger (or smaller).

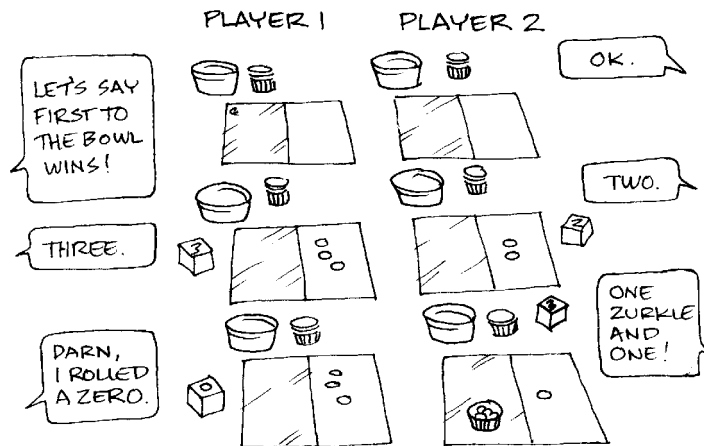
Extension: For children who may need to be challenged, give them three dice to arrange.



Race to a Bowl

Note: Before the game begins, the partners decide how the winner will be determined: either the first person to reach the bowl column or the last person to reach the bowl column.

Procedure: The players start with empty place value boards. Each player will take a turn rolling the die and adding that number of beans to his or her place value board. The players continue rolling the die and adding beans to the place value board until the first person reaches the bowl column.

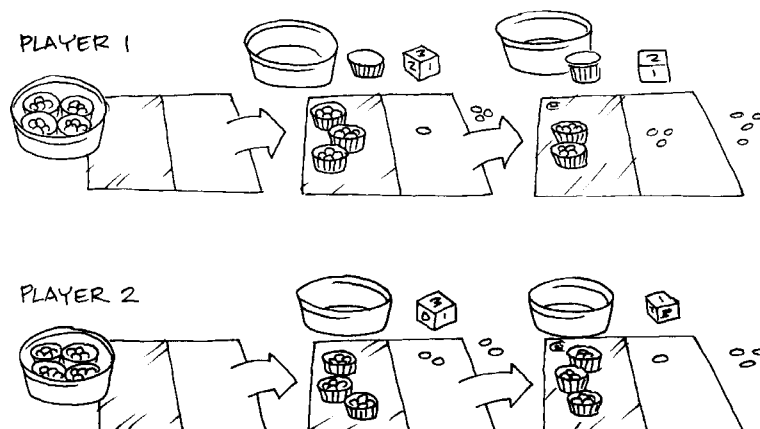


Race to Zero

Note: Before the game begins, the players must decide who the winner will be: either the first person to reach 0 or the last person to reach 0.

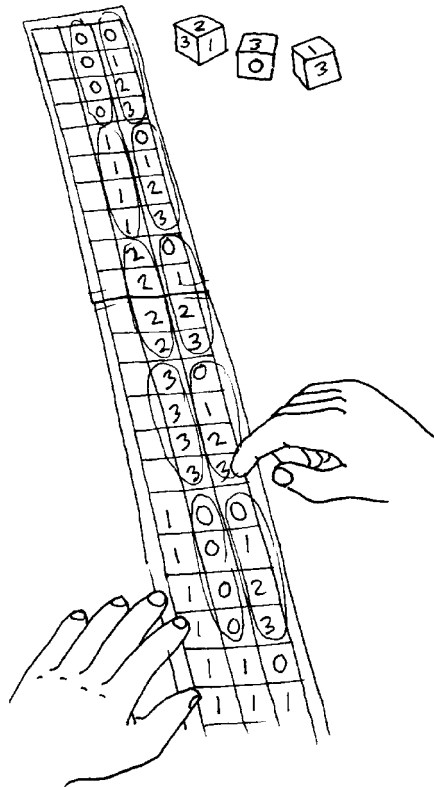
Procedure: Start with the bowl column filled (e.g. in Zurkles, the bowl would contain four cups with four beans in each cup). The players take turns rolling the die and subtracting the beans until one player reaches zero. Sometimes the children add the rule that the exact number must be rolled on the last roll.

Observation: Watch how the children regroup the first time they take the beans away.



SYMBOLIC LEVEL: INDEPENDENT ACTIVITIES

Searching for Patterns



Vertical Patterns

Materials needed for each child: Place value strips (NL Blackline #47); beans; portion cups; 2-3 small envelopes; a large plastic zippered bag; clothespin; 3 dice with numerals in the predetermined grouping

The class works together at this activity when it is first introduced. They decide which grouping (e.g., Zurkle) to use. To determine how far to add or which number to start subtracting from, three dice are rolled and arranged into a three-digit number.

This process may take more than one day. When the child finishes for the day, he or she rolls up the place value strip and clips it with a clothespin. The place value strip, place value board, portion cups, and envelopes of beans are stored in a large zippered plastic bag with the child's name on it. The next day the child builds the last number on the place value strip and continues: adding or subtracting materials on the place value board; recording the numerals on the place value strip; and looping the patterns in each column.

Addition Procedure: The class works at this activity together when it is first introduced. They add materials on their place value board and record the numerals together to the end of the first strip. Then, each child loops the patterns in each column on his or her strip and attaches a new place value strip. The class works together for approximately two and a half strips. Then they continue working at their own pace until they reach the number predetermined by the dice roll. When the child has added to the third column, the beans in the portion cups are placed in an envelope and sealed.

Subtraction Procedure: The children begin by building the number determined by the dice with materials on their place value board. The objects for the third column are put in envelopes and placed to the left of the cups column. Then the children record the number at the top of their place value strips. Next, they subtract one object at a time and record the number on their place value strips. When they have completed one strip, each child loops the patterns in each column of the place value strip and attaches a new strip. The class works together for approximately two and a half strips. The children continue working at their own pace until they reach zero. When the child subtracts all the objects in the cups and units columns of the place value board, he or she opens an envelope of objects in the third column and regroups them in portion cups before continuing to subtract.

Note: Pattern is a self-correcting tool and whenever a child finds an error, rather than erasing, the child merely cuts off the place value strip where the error occurred and attaches a new strip.

Do not be surprised about how many children forget to write in the numbers at the regrouping stage. This error corrects itself as soon as the children realize the value of pattern. They begin to find their mistakes by looking for the recorded evolving patterns.

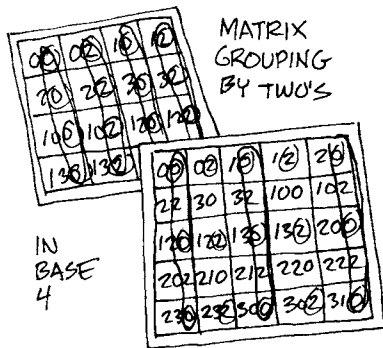
Extension: Ask the children to search for patterns while adding or subtracting by two (or three) objects each time.

Matrix Patterns

Materials needed for each child: matrix frames of various sizes (NL Black-line #46); materials to count; portion cups; place value board; colored pencils, crayons, or markers to loop patterns

Same Grouping — Different Size Frames

The children can work in partners or alone. They begin by adding objects to the place value board one at a time and recording the counting sequence for the chosen grouping in the matrix frame. They record the numerals in sequence until the matrix frame is filled.



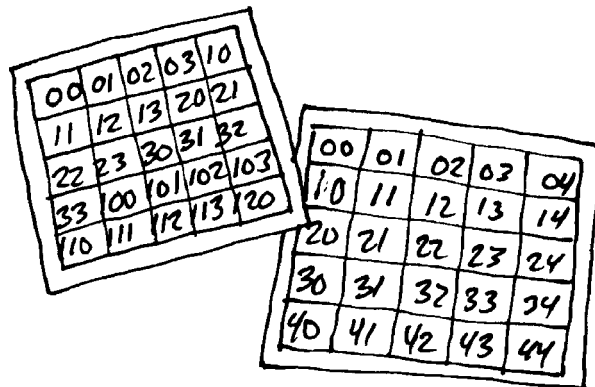
Extensions:

- ❑ When the children are experienced with recording in a matrix frame, they can record one numeral at a time until they begin to see patterns developing in the columns and rows (vertically, horizontally and diagonally). They complete the matrix by recording the patterns as they discover them. If they're unsure of a pattern, they can check it by using concrete materials.
- ❑ Once one matrix has been completed, the children can try a different size matrix frame. Some of the patterns in the first matrix may be useful when the children predict patterns in the new frame.
- ❑ The children can record patterns by adding by two or three numbers at a time. Concrete materials can be used to confirm the matrix patterns.

Same Size Frame — Different Groupings

The pair chooses a matrix frame (e.g., 5 boxes by 5 boxes) and a grouping. They record until they begin to see patterns developing in the columns and rows (vertically, horizontally and diagonally). They record the matrix patterns as they are discovered. When the frame is completed, the pair searches for additional patterns. They loop and/or shade patterns with the side of a crayon.

When the pair is finished searching for patterns in the first grouping, they choose a different grouping and record it in the same size frame. They compare the patterns in the two groupings in the same matrix grids.



BASE TEN WORK

GROUP ACTIVITIES

Searching for Patterns

Materials: class-size (10 by 10) matrix frame or overhead transparency of NL Blackline #45; class-size hundred's chart or an overhead transparency of NL Blackline #44

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15					
21		23		5					
32			34	5					
41				5	56				
51						67			
61					5		78		
71						5		89	
81							5		
91								5	

Searching for Patterns on the Hundred's Chart

Procedure: Record the counting sequence on a class-size 10 by 10 matrix frame as the children count by one's. Complete approximately one and one-half rows in sequence before asking the children if they see any patterns developing (vertically, horizontally or diagonally). Complete the matrix as the children discover the patterns.

Extensions:

- Using the same procedure, the class searches for patterns in other counting sequences (by two's, three's, five's...) as you record the pattern sequences on a large class-size (10 by 10) frame.
- The class searches for patterns while subtracting. Begin the activity by recording 100 in the first box. The class counts backward, one number at a time, as you record the numerals on the matrix. Complete approximately one and one-half rows. Ask the children to search for patterns. Record the patterns as the children discover them.
- A completed class-size hundred's chart is used for this activity. Ask the children to search for pattern sequences. Ask the children if they see the same sequence anywhere else on the chart. Loop and/or shade the patterns with a colored marker or crayon. Ask the children to search for a different pattern sequence. Continue to search for patterns in this fashion.

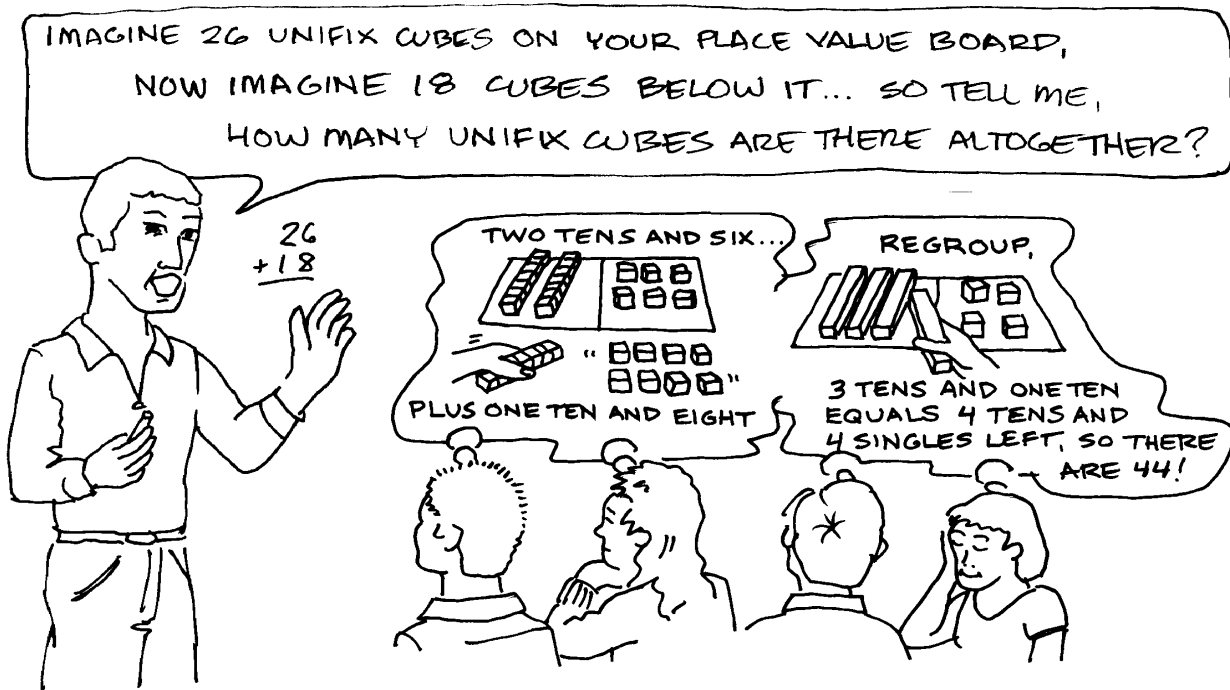
Random Practice

Visualization

After the children have had extensive manipulative experiences regrouping while adding and subtracting large numbers in base ten, allow some time for the children to visualize the regrouping process. Visualization enables children to transition from their concrete experiences to a more abstract level. This is necessary for them to be successful in school testing situations.

Procedure: Write a problem on the board. Ask the children to close their eyes and imagine a certain type of manipulative on the place value board as the teacher records the action numerically. The children should experience visualizing both addition and subtraction equations.

Sample Teaching Strategy – Visualization



INDEPENDENT ACTIVITIES

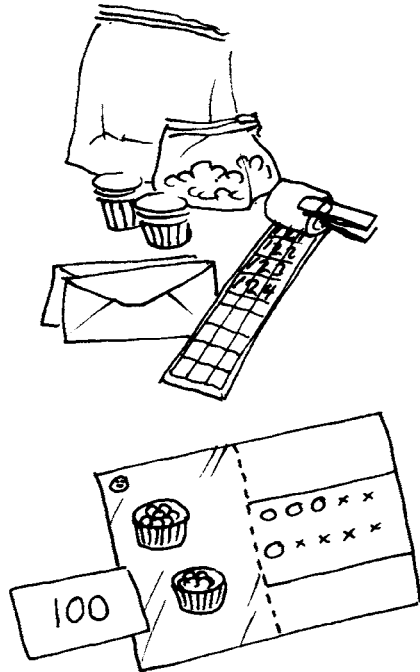
Searching for Patterns

Before You Begin: Run off NL blackline #43 for each child. The master has two rows with five stars in each row.

Place a copy of the star blackline on the one's side of each child's place value board. It is difficult to tell at a glance (without counting) how many counters are on the board in base ten. The stars give the child an organizing tool for work in base 10. By covering each star with the material, children will think in terms of number combinations. Without the addition of the star working area, they may count by ones and miss the opportunity to reinforce the addition and subtraction combinations.

Vertical Patterns

Materials needed for each child: Place value strips (NL Blackline #47); a small cylinder made of tagboard; a clothespin; beans; 10 portion cups; 2-3 envelopes; large zippered bag (to store the materials)



Procedure: The children practice adding to or subtracting from a large three-digit number in base ten. It's helpful if the children work in pairs. Each pair chooses a small container from the comparing station and fills it with beans. The beans are placed in a zippered bag for storage.

Addition: The children count by one's — one person records the sequence as the other person adds the material to the place value board. They search for and loop patterns every time an individual place value strip is completed. Once the patterns are looped and any mistakes are snipped off, a new place value strip is added and the pair continues adding beans. When they reach 100, they empty the ten cups of beans into an envelope and seal it. The envelope is placed off the place value board (to the left of the 10's column).

Subtraction: This time, the container full of beans is grouped into hundred's, ten's and one's. The hundred's are sealed in an envelope. The total is recorded at the top of the place value strip. The pair subtracts (one bean at a time), records, and searches for patterns on the number strip until they are out of beans.

Extension: Children should have experiences adding and subtracting by two, three, four... objects at a time. (see MTW p. 314).

Matrix Patterns

Materials needed for each child: hundred's chart (NL blackline #44); blank (10 by 10) matrix frame (NL blackline #45)

Procedure: The children independently search for and record patterns on a matrix. See NL, page 11.15 for a detailed description.

Random Practice

Dice Games

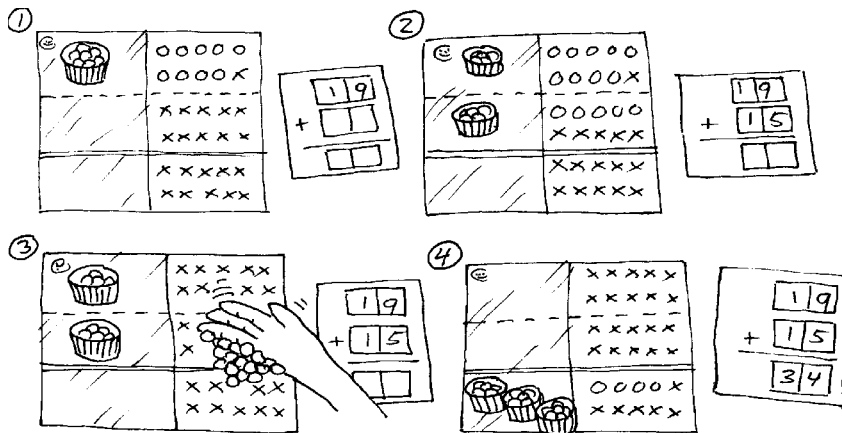
Play Dice Arrangement (NL, p. 11.11), Race to a Bowl (p. 11.12), Race to Zero (NL, p. 11.12) in base ten.

Bob's Ditto

Addition

Materials: small objects; portion cups; addition board* (NL Blackline #39); recording sheet (NL Blackline #38)

(*) Note: Large place value addition boards can be made with 9" x 11" file folders. (see directions p. 11.23)



Step One: Take a handful of junk and count it into counting cups. Place the junk on the top place value boxes of the addition board. Record the number on the recording sheet.

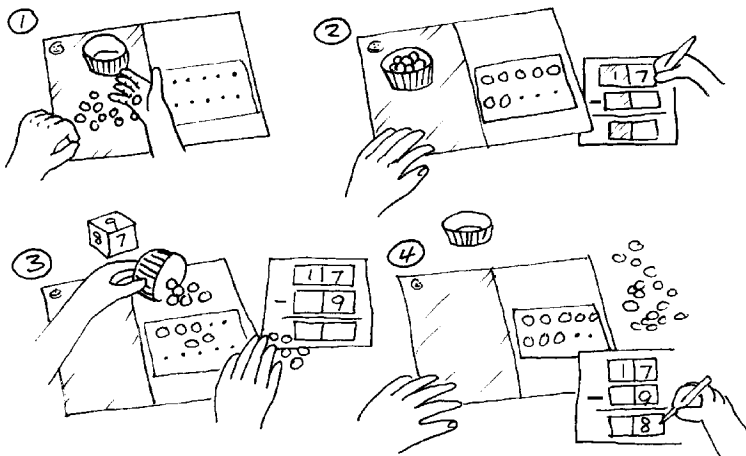
Step Two: Take another handful and count it into counting cups. Place the junk in the middle place value boxes. Record the number on the recording sheets.

Step Three: Move the junk down to the bottom place value boxes.

Step Four: Regroup to find the total. Record the total on the recording sheet.

Subtraction

Materials: place value board; small objects; portion cups; a wooden cube numbered 4, 5, 6, 7, 8, 9; recording sheet (NL Blackline #38)



Step One: Take a handful of objects and regroup into ten's if possible. Place the objects on the place value board.

Step Two: Record the amount of objects on the recording sheet.

Step Three: Roll the die. Record the number on the paper. Subtract that many objects from the place value board.

Step Four: Record the answer.

PLACE VALUE STATION ACTIVITIES

The place value station activities fall into two categories:

- Counting Activities
- Addition and Subtraction Activities

Place Value Counting Activities

The counting activities provide children opportunities to rebuild in base ten. The counting stations should be introduced before the addition and subtraction stations.

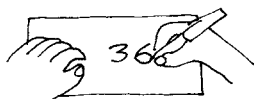
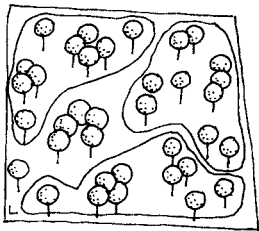
Measuring	MTW, p. 307
Estimating and Checking	MTW, p. 308
Counting Jars of Objects	MTW, p. 310
Perimeter	MTW, p. 315
Unifix Stack	MTW, p. 320
Magazine Pictures	NL, p. 11.19

Place Value Addition and Subtraction Activities

The following activities provide children opportunities to regroup while adding and subtracting. Add these stations when the children demonstrate a clear understanding of regrouping while counting in base ten.

Determining Prices	MTW, p. 312
The Store	MTW, p. 317
Inside-Out	NL, p. 11.20
Bob's Ditto	NL, p. 11.18

Place Value Activities Not Found in *Math Their Way*



Magazine Pictures

Materials: Magazine pictures, old wrapping paper and/or stickers scattered on a page of paper; water-soluble pen; wet rags; NL Black-line #42

Note: Find either magazine pictures, wrapping paper, or create pictures by sticking stickers on a page. Each picture should have between 12-100 objects that are easily countable. Label each picture with a different letter. Cover the pictures with acetate.

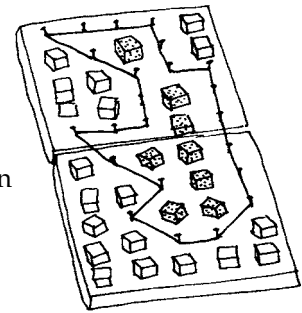
Procedure: The child chooses a picture and records the letter on the recording sheet. He or she circles as many groups of ten as possible. Next the child counts the total number of objects and records the answer on the recording sheet.

Extension:

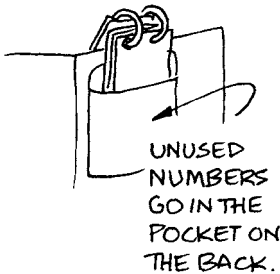
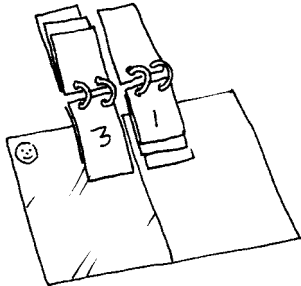
Procedure: Some teachers place several different types of rubber stamps, a stamp pad, and blank paper in this station. The child chooses a stamp and covers the paper with the stamp's image. He or she circles the stamped images in groups of ten. Next the child counts the total and records the number on the page. The recording can be taken home.

Inside-Outside

This activity is described in *Mathematics Their Way* (see Geoboard Designs, MTW, p. 316). The description (and picture illustration) uses only one geoboard. In order to provide children with regrouping experiences, there must be a minimum of two boards side-by-side.



PLACE VALUE MATERIALS



Place Value Boards

Materials for each place value board: 12" x 9" tagboard; a 5-1/2" x 8-1/2" piece of blue mimeo or construction paper; a 12" x 9" piece of white construction paper; glue stick or rubber cement; 2 library pockets (Use protective covering after the place value board is completed.)

Procedure: Glue the white paper to the tagboard. Glue the blue paper on the left side on top of the white paper. Draw a happy face in the upper left corner to provide a "right side up" signal to the children.

Place Value Flip Pockets: Glue two library pockets on the back side of the place value board. (One for the blue side and one for the white side.) Be sure to place the pockets as close to the middle of the board as possible.

Some teachers cut the place value boards in half and hinge them with tape on the back side. The folded place value boards are easy to store.

Note: If the class is going to be working a lot with 3 column numbers, you may want to attach an additional (5-1/2" x 8-1/2") piece of tagboard (with a different color of construction paper glued on top of it) to the left of the blue side. The children can fold this piece under the blue side of the place value board when they are not using it.

Star Worksheet

NL, Blackline #43; tape; place value boards

Note: The star papers are to be used with the base ten activities only. (see NL, p. 11.17 for an explanation of its purpose)

Procedure: Duplicate the star blackline. Cut the paper on the dotted line. Tape the star paper onto the white side of the place value board.

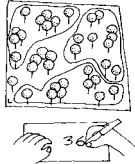
Place Value Flips

Materials: Number Flips*; chicken rings**, Chinese Jacks or small metal rings; scissors

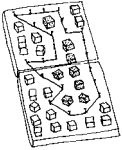
* Making Number Flips from scratch can be time consuming. We recommend that you purchase this item, predrilled and ready to assemble from the Center for Innovation in Education. If you want to assemble your own, please consult MTW, p. 363 for the directions.

** Chicken rings: From a feed store or at www.chickenrings.com

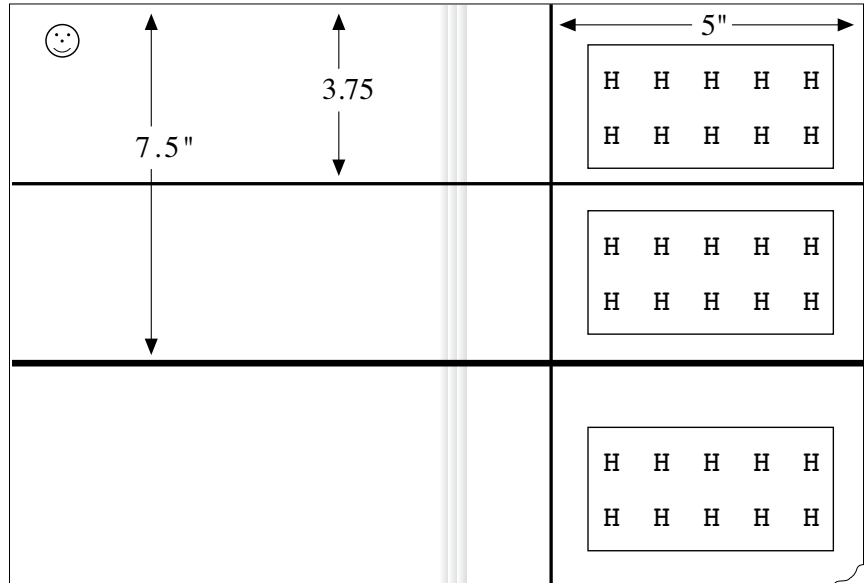
PLACE VALUE STATIONS

	COUNTING CONTAINERS OF OBJECTS	ESTIMATING AND CHECKING	PERIMETERS	UNIFIX STACKS	MAGAZINE PICTURES
Page #s	MTW, p. 310	MTW, p. 308, 360	MTW, p. 315	MTW, pp. 320, 396	NL, p. 11.19
Activity Description	<p>Materials needed:</p> <ul style="list-style-type: none"> • 6-10 small containers with small objects inside (bread tabs, macaroni, buttons, rocks, dried pinto beans, etc.) • 1 or 2 oz. portion cups • 5-6 empty margarine tubs • 4-5 place value boards <p>Procedure: The children select one container and estimate how many objects are inside. Then they place each group of ten objects in a portion cup. When there are enough to make a hundred, the ten portion cups of objects are stacked and placed inside a margarine tub. The children place objects on the place value board and read the number.</p>	<p>Materials needed:</p> <ul style="list-style-type: none"> • 7-10 empty jars or containers • 3-4 large margarine tubs of beans (each with a different type of bean, e.g., large kidney, lima beans, pinto beans...) • 1 or 2 oz. portion cups • 4-5 place value boards • 5-6 empty margarine tubs <p>Procedure: The children select an empty container and fill it with beans. Then they estimate how many beans are in the container. Then they place each group of ten beans in a portion cup. When there are enough to make a hundred, the ten portion cups of beans are stacked and placed inside a margarine tub. The children arrange the beans on the place value board and count to find the total number of beans.</p>	<p>Materials needed:</p> <ul style="list-style-type: none"> • old geoboard records (MTW, p. 177) • Unifix cubes in one color (about 80 cubes) • 4-5 place value boards <p>Procedure: The children predict the number of Unifix cubes it will take to measure the perimeter of a geoboard design. (One unit equals one side of the colored square on the recording sheet.) Then they mark each unit with a Unifix cube, snap the cubes together into a train, and divide the train into groups of ten's and one's. The Unifix cubes are then placed on the place value board.</p>	<p>Materials needed:</p> <ul style="list-style-type: none"> • 3 more/less spinners • 3 wooden cubes with numbers 0,1,2,3,4, and 5 written on them • Unifix cubes • 3 Unifix stack gameboards (MTW, Blackline #29) • 4-5 place value boards <p>Procedure: Two children take turns rolling their dice and placing a stack of cubes on their row of the gameboard. If a child rolls a zero, he or she leaves the box empty. When the rows are completed, they snap their cubes together in groups of ten's.</p>	<p>Materials needed:</p> <ul style="list-style-type: none"> • Magazine pictures or wrapping paper showing between 12-100 objects easily countable (a flock of birds, a parking lot of cars, a picket fence, a row of toys, a crowd of people, a box of chocolates, etc.) covered with acetate • 6 water color marking pens <p>Procedure: The children choose a picture, loop the objects into groups of tens and then count the total.</p> 
<p>* Recording materials are placed in the tubing stations only after the children have had sufficient time to work at the concrete (concept) level with the manipulatives. Depending on the class's experience and the material in the tubing station, the time allowed may span from several weeks to several months. Recording should be introduced as an optional activity.</p>					
* Recording Description	<p>Additional materials needed</p> <ul style="list-style-type: none"> • NL Blackline #14 stored in a small plastic zippered bag <p>Recording: The children record their estimates in the first box, count the objects into portion cups and record the actual number in the second box on the recording sheet.</p> <p>Extension: The children count the objects in the containers by 1's, 2's, 3's, 5's, 10's etc. They record the counting sequence on either a 2 column or 3 column place value strips.</p> <p>A blank 10 x 10 matrix (NL Blackline #45), 0-99 matrix (MTW Blackline #56) or hundred's chart (NL blackline #44) can also be used to record the counting sequence.</p>	<p>Additional materials needed:</p> <ul style="list-style-type: none"> • NL Blackline #14 stored in a small plastic zippered bag <p>Recording: The children record their estimates in the first box, count the objects into portion cups and record the actual number in the second box on the recording sheet.</p> <p>Extension: The children count the beans in the containers by 1's, 2's, 3's, 5's, 10's etc. They record the counting sequence on either a 2 column or 3 column place value strips.</p> <p>A blank 10 x 10 matrix (NL Blackline #45), 0-99 matrix (MTW Blackline #56) or hundred's chart (NL blackline #44) can also be used to record the counting sequence.</p>	<p>Additional materials needed:</p> <ul style="list-style-type: none"> • MTW Blackline #40 in a small plastic zippered bag <p>Recording: The children write their estimates on the recording sheets. Then they check their estimates by placing the Unifix cubes around the perimeter of the design. The children then group the Unifix cubes used to measure the perimeter into ten's and one's. They record the actual number.</p>	<p>Additional materials needed:</p> <ul style="list-style-type: none"> • MTW Blackline #61 in a small plastic zippered bag <p>Recording: Each child writes his or her name on the recording sheet. They play the game. Then each child adds his or her Unifix cubes, and records the number in the appropriate box. One child spins the more/less spinner to determine the winner.</p>	<p>Additional materials needed:</p> <ul style="list-style-type: none"> • NL Blackline #42 in a small plastic zippered bag <p>Recording: The children record the picture's letter onto the recording sheet, loop the objects in the picture in groups of tens, and record the total on the recording sheet next to the appropriate letter. (Three pictures can be recorded on one recording sheet)</p> <p>Extension: The children can make recordings to take home by stamping blank paper with a picture on a rubber stamp. Next, they circle the stamped images in groups of ten's and record the total on the paper.</p>

PLACE VALUE STATIONS

	MEASURING	GEOBOARD DESIGNS (INSIDE-OUTSIDE)	THE STORE	DETERMINING PRICES	BOB'S DITTO
Page #s	MTW, p. 307	MTW, pp. 316	MTW, pp. 317, 396	MTW, pp. 312, 396	NL, p. 11.18
Activity Description	<p>Materials needed:</p> <ul style="list-style-type: none"> Unifix cubes word and picture cards (door, chair, bookcase, table, desk, window, trash can, coat, book, paper, easel, box, record player, etc.) for the children to use to spell these words <p><i>Procedure:</i> The children select a card. (A simple picture of the object can be drawn on the card to help the children.) The children measure the object described on the card with Unifix cubes in whatever direction they choose.</p>	<p>Materials needed:</p> <ul style="list-style-type: none"> approximately 100 Unifix cubes in two colors 4-5 place value boards 6-8 geoboards  <p><i>Procedure:</i> The children make a design on two side-by-side geoboards with <u>one</u> geoboard. They cover the "inside" nails that are <u>not</u> touched by the geoboard with one color Unifix cube and the "outside" nails that are <u>not</u> touched by the geoboard with the second color. They take both colors off the geoboard, snap them together into groups of ten's and place them on the place value board.</p>	<p>Materials needed:</p> <ul style="list-style-type: none"> 10-12 objects in plastic zippered bags and priced less than 50¢ real coins (dimes and pennies only) in coin cylinders (from a coin shop) with the level marked with a marking pen. place value addition boards (NL Blackline # 39) or large place value addition boards (see p. 11.23) <p><i>Procedure:</i> The children select two items from the store to "buy". Then they put the needed coins on their addition board and compute the total.</p>	<p>Materials needed:</p> <ul style="list-style-type: none"> pennies and dimes in empty bullion tubes or coin cylinders place value addition boards (NL Blackline # 39) or large place value addition boards (see p. 11.23) cans of food (real food inside) labeled and priced less than 50¢. <p><i>Procedure:</i> The children select two cans of food to "buy". They put the needed coins on their addition board and compute the total.</p>	<p>Materials needed:</p> <ul style="list-style-type: none"> small object (e.g., elbow macaroni or kidney beans) place value addition boards (NL Blackline # 39) or large place value addition boards (see p. 11.23) portion cups 6-8 dice numbered 4, 5, 6, 7, 8, 9 <p><i>Procedure:</i> Consult NL, p. 11.18 for a detailed description of Bob's Ditto.</p>
<p>* Recording materials are placed in the tubing stations only after the children have had sufficient time to work at the concrete (concept) level with the manipulatives. Depending on the class's experience and the material in the tubing station, the time allowed may span from several weeks to several months. Recording should be introduced as an optional activity.</p>					
* Recording Description	<p>Additional materials needed:</p> <ul style="list-style-type: none"> NL Blackline #41 <p><i>Recording:</i> The children write down the name of the object measured and the number of Unifix cubes it took to measure it. (The children can measure and record three different objects on each</p>	<p>Additional materials needed:</p> <ul style="list-style-type: none"> NL Blackline #42 <p><i>Note:</i> At least two geoboards must be used so there will be enough nails inside and outside the geoboard for the children to experience addition with regrouping.</p> <p><i>Recording:</i> The children count and record the number of Unifix cubes on the inside of the geoboard. Then they count and record the number of Unifix cubes on the outside of the geoboard. They add two groups of Unifix cubes together and record the total number of Unifix cubes inside and outside the geoboard design.</p>	<p>Additional materials needed:</p> <ul style="list-style-type: none"> MTW Blackline #59 <p><i>Recording:</i> The children choose two things to "buy" and write down the names and prices. They put the money needed on the place value addition board and compute the total. Then they record the process on the recording paper.</p>	<p>Additional materials needed:</p> <ul style="list-style-type: none"> MTW Blackline #59 <p><i>Recording:</i> The children select two cans of food to "buy" and record the names of the items and prices. Then they put the needed coins on their addition board and compute the total. They record the process on the recording paper.</p>	<p>Additional materials needed:</p> <ul style="list-style-type: none"> NL Blackline #38 <p>Cut the recording sheets on the dotted line. Place the addition and subtraction sheets in separate small zippered bags.</p> <p><i>Recording:</i> Consult NL, p. 11.18 for a detailed description.</p>

LARGE PLACE VALUE ADDITION BOARD



Materials for each board: 1 file folder (9" x 11"); 1 medium black permanent marker, 3 star work sheets (NL, Blackline # 43)

Trim the file folder to 8-1/2" x 11-3/4".

Open the file. Draw two horizontal lines across the file folder. Draw the first horizontal line 3-3/4" from the top of the file folder. Draw the second horizontal line 7-1/2" from the top of the file folder. Make the second line heavier than the first line.

Draw a vertical line 5" from the right side of the file folder.

Trim the 3 star papers to approximately 3" x 4". Glue one star paper in each of the boxes in the right column.

Draw a small smiley face on top of the place value addition board.

Note: Large place value addition boards provide more room for materials on the 10's side of the board.