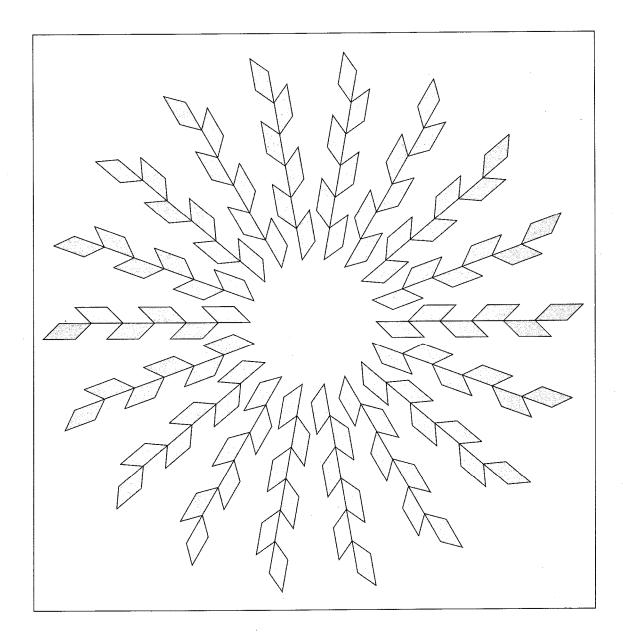
GEOMETRY

The activities in this chapter encourage students to acquire the vocabulary to describe shapes in terms of their geometric properties. The activities also encourage students to explore the concepts of symmetry, congruency and similarity.



POLYGONS: IDENTIFYING SIDES

Materials:_____Power Blocks

Unlined paper

Centimeter cubes or beans

Purpose: _____To identify the sides of a shape

Activity: _____Teacher: Today we are going to study the sides of a shape. I want to mark each side of R5 with

a centimeter cube. How many sides do you think R5 has?

Student: Four.

Teacher: I will mark each side by putting a centimeter cube as close to its middle as I can. How

many cubes have I put around R5?

Student: Four.

Teacher: I want you to mark the sides of as many Power Blocks as you can.

When students can identify the sides of Power Blocks, they will learn to identify the sides of more elaborate polygons.

Teacher: I have built this shape with R5 and T5. We are going to identify its sides. The sides are the outside edges of this shape. Do not look inside the shape for sides. Pretend the shape is made from one piece of plastic. How many sides do you think it has?

Student: Five.

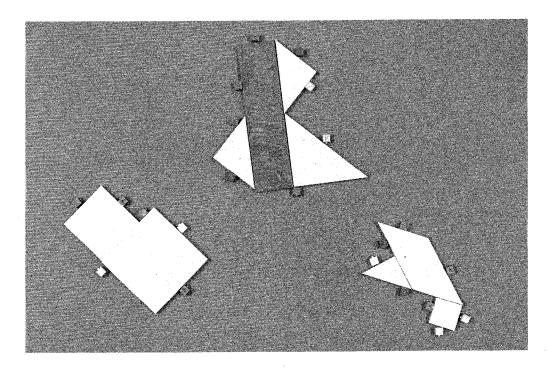
Teacher: How did you get five?

Student: I counted them.

Teacher: How did you know what to count? **Student**: I touched each side and counted.

Teacher: How did you know where the next side was? **Student**: When the side turned, then it was a new side.

When students understand what is required, they work together building polygons. They mark the sides with centimeter cubes or beans. They may trace their polygons and mark sides with a symbol of some kind.



Questions to explore with students:

- Can you make a shape with five sides?
- Can you make a shape with six sides?
- What is the shape with the fewest sides?

POLYGONS: CONVEX AND CONCAVE POLYGONS

Materials:_____Power Blocks

Unlined paper

Centimeter cubes or beans

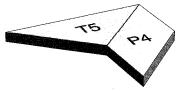
Purpose: _____ To identify shapes that are convex or concave

Activity: _____Teacher: Here are two shapes P4 and T5. When I put them together like this, they make a

trapezoid.



Now watch what happens when I change the shape.



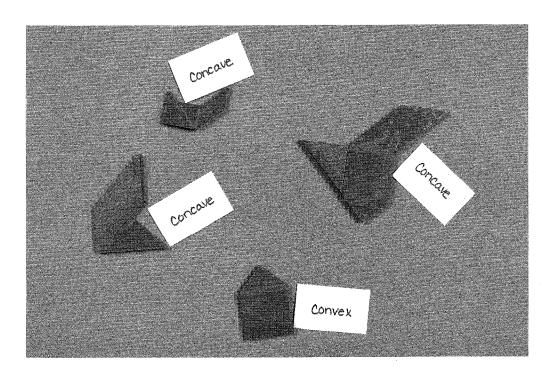
If I put T5 and P4 together like this, the polygon has a dent in it. If a polygon has sides like the trapezoid, then it is convex. If it has sides that push in (make a dent) then it is concave.

Student: Are those the only two ways polygons can be?

Teacher: Let's find out. I want you to make as many polygons with your blocks as you can. If it is convex, don't do anything to it. If it is concave, put a centimeter cube at the point of the dent.

When students understand what is required, they work together building polygons. They identify them as either convex or concave. They mark the concave sides with cubes or beans. They may trace and label their polygons to make a record of what they have done.

- Can you make a convex shape with five sides?
- Can you make a concave shape with five sides?
- Which are easier to make with the blocks, convex or concave shapes? Why?
- Can you make a concave shape with three sides?
- If you can, how did you do it? If you can't, why not?
- Make a convex shape with as many sides as you can. How many sides does it have?
- Make a concave shape with as many sides as you can. How many sides does it have?



POLYGONS: IDENTIFYING INTERIOR ANGLES

Materials:____Power Blocks

Unlined paper

Centimeter cubes or beans

Purpose: ______ To identify the interior angles of a shape

Activity: _____Teacher: Today

Teacher: Today we are going to learn about interior angles. When two straight lines meet, angles are formed. Watch what happens when I trace T5 like this. Notice how the sides meet at a point. The space between the two sides and the point they meet is called an angle. Take out R5. Put a centimeter cube on top of each of the shape's interior angles. How many angles did you find?

Student: Four.

Teacher: Good. I want you to mark the interior angles of as many Power Blocks as you can.

When students can identify the interior angles of Power Blocks, they will learn to identify the interior angles of more elaborate polygons.

Teacher: I have built this shape with S5 and T5.



How many interior angles do you think it has?

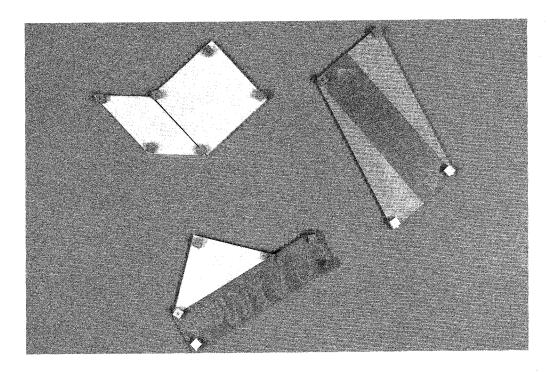
Student: Four. **Teacher**: Show me.

Students: I put a cube on each corner like this:



When students understand what is required, they work together building polygons. They mark the interior angles with cubes or beans. They may trace their polygons and mark the interior angles. The complexity of the polygons they make can be controlled by limiting the number of blocks they use to build a given shape. The interior angles of concave shapes may prove a little more difficult to identify, but with practice they will identify them as quickly as they do the other angles.

- Can you make a shape with five interior angles?
- Can you make a shape with six interior angles?
- What is the shape with the fewest interior angles?
- What is the largest number of interior angles you have in one shape?
- Can you make a shape with one more interior angle than your previous shape?



POLYGONS: IDENTIFYING EXTERIOR ANGLES

Materials: Power Blocks

Unlined paper

Centimeter cubes or beans

Purpose: _____ To identify the exterior angles of a shape

Activity: _____Teacher: Today we are going to study exterior angles. Watch me as I put a centimeter cube at

each exterior angle of R5. How many cubes have I put beside R5?

Students: Four.

Teacher: If you had to tell friends how to find the exterior angles of a polygon, how would you

tell them?

Student: I would tell them that the exterior angles are the corners.

Teacher: I want you to mark the exterior angles of as many Power Blocks as you can. Put a

centimeter cube at each exterior angle.

When students can identify the exterior angles of Power Blocks, they learn to identify the exterior angles of more elaborate polygons.

Teacher: I have built this shape with R5 and T5. Estimate with centimeter cubes how many exterior angles you think it has...



Let's see. I will place a cube at each angle. How many cubes are there?

Students: Six.

Teacher: Which angle do you think is the hardest one to remember to count?

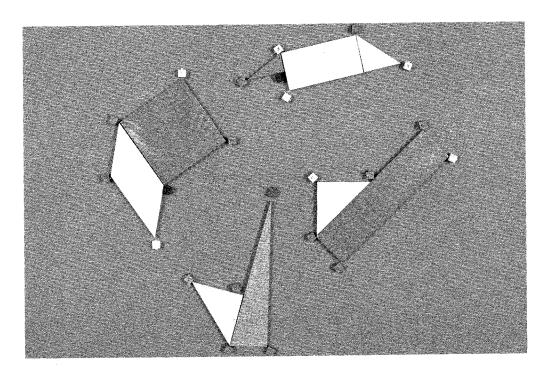
Student: The one where the sides bend in.

Teacher: How can we identify the exterior angles?

Students: They are at the corners.

When students understand what is required, they work together building polygons. They mark the exterior angles with cubes or beans. They may trace their polygons and mark the exterior angles.

- Can you make a shape with five exterior angles?
- Can you make a shape with six exterior angles?
- What is the shape with the fewest exterior angles?
- What is the largest number of exterior angles you have in one shape?
- Can you make a shape with one more exterior angle than your previous shape?



POLYGONS: PREDICTING SIDES AND ANGLES

Materials:_____Power Blocks

Unlined paper

Centimeter cubes or beans

Purpose: ______To identify the relationship between the number of sides, interior, and exterior angles

Activity: _____Teacher: Write these headings on your paper.

Sides Interior Exterior

I have made this polygon from R4, S3, and T3.



Teacher: How many sides does it have?

Student: Six.

Teacher: Record it like this.

Sides Interior Exterior 6

How many interior angles does it have?

Student: Six.

Teacher: Record it like this:

Teacher: How many exterior angles does it have?

Student: 6

Teacher: Record it like this:

Use your set of blocks to make as many polygons as you can. Record the number of sides, interior angles, and exterior angles on your paper.

When students have gathered their data the teacher consolidates it into a table.

Sides	Interior	Exterior
6	6	6
3	3	3
7	7	7
9	9	

Teacher: Do you see a pattern? If you knew the number of sides a polygon has, could you predict the number of interior angles it has?

- Can you predict the number of exterior angles if you know the number of sides?
- Can you predict the number of interior angles if you know the number of exterior angles?

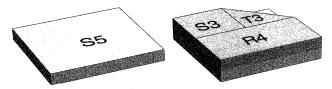
CONGRUENT POWER BLOCKS

Materials: _____Power Blocks Unlined paper

Purpose: _____ To create congruent Power Block shapes

Activity: _____Teacher: Today

Teacher: Today we are going to make shapes that are congruent. Congruent shapes are copies of one another. If you look in a mirror, the reflection you see is congruent to you. (Technically, for shapes to be congruent their internal structures must also be identical. However for now, we are defining congruent as shapes having the same outline and same size.) I want you to build another shape that is the same size and shape as S5. Build it right on top of S5. Watch me. I will cover S5 with R4, S3, and two T3s.

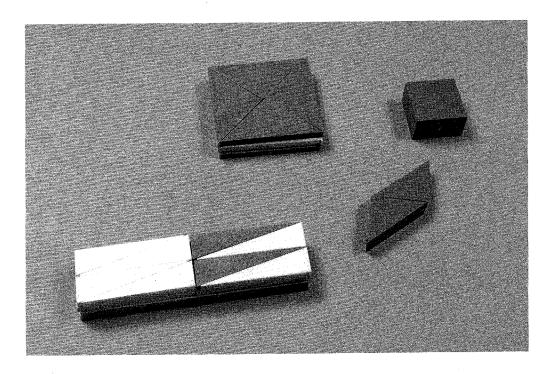


Can anyone think of a different way to cover S5? **Students**: Can anything hang over the sides?

Teacher: No. How many different ways can we make S5?

When students have explored S5, they build congruent versions of the other blocks. Students may trace their shapes to make a record of their work.

- How many different ways can you make a shape?
- Is there a relationship between the number of T1s it takes to make a shape and the number of ways it can be made?



RECORDING EQUATIONS FOR SHAPES

Materials: _____Power Blocks Unlined paper

Purpose: _____ To record congruent shapes as equations

Activity:

Teacher: We are going to study congruent shapes and make a record of what we find out. Take out P4 and trace it like this. I will build a congruent shape on top of the tracing. I will remove one block at a time and make a tracing of where it was on my tracing of P4. I will write an equation that tells how I made my shape. When I covered P4, I used three blocks. I can write that like this:

$$93 + 213 = P4$$

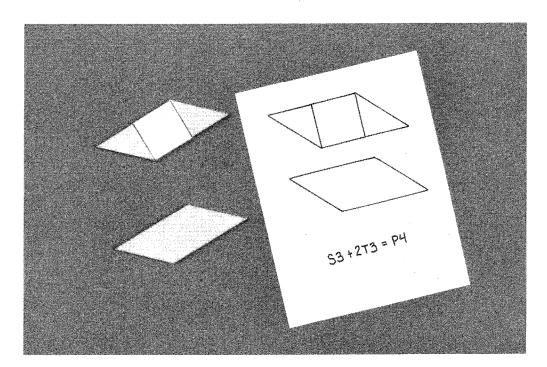
What does this mean?

Student: An S3 and two T3s equals a P4.

Teacher: Make a tracing and write an equation for each way you can make a shape that is congruent to P4.

When students understand what is required, they work together building shapes that are congruent to specific blocks. They record their results as described above.

- Is it possible to have the same equation for two different ways to make a shape?
- How many different ways can you write an equation for the same block?
- Is there a relationship between the number of T1s it takes to make a shape and the number of ways it can be written as an equation?



CONGRUENT SHAPES

Materials:_____Power Blocks

Power Block triangle paper on legal size sheets

Purpose:

To make student generated task cards for building congruent shapes

Activity:

Teacher: Watch how I draw a polygon on the triangle paper. Notice that I must always trace on top of the lines of the triangle paper. I can go in any direction I want, as long as I don't cross over the top of the line I have just traced. I keep tracing until I get back to the point I started. Each polygon should be equal to at least sixteen little triangles. I want you to make as many polygons as you can in the next fifteen minutes.

Students: Can we draw more than one polygon on a each side of a piece of paper?

Teacher: No. I want you to limit it to only one polygon on each side of paper.

When students have finished creating polygons (about five or six per student) the teacher collects the papers. These will be used as task cards.

Teacher: Shortly, I will give you some of the drawings that you have just made. When you get them, count the number of little triangles it takes to make the polygon. Each little triangle is equal to T1. In my puzzle, there are 49 T1s. I record it like this:

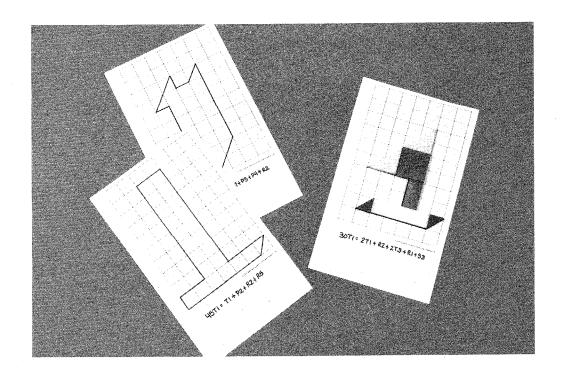
$$49T1 =$$

I will try to cover the polygon with as few blocks as possible and then record what I have done like this:

$$49T1 = 3T1 + 2T3 + P1 + R1 + R4 + P4$$

When you get a task card, I want you to make the polygon outlined on your paper using as few blocks as possible. After you have covered your polygon, record the results. Pass the paper to a neighbor. Your neighbor is to try to make the shape using the blocks indicated in your equation. After he/she has done it your way, see if he/she can find a different way to cover the polygon.

After three or four students have tried each polygon, collect the task cards. Save them for the next activity.



SIMPLIFYING EQUATIONS

Materials: Power Blocks

Task cards from previous lessons

Table of relative areas

Lined paper Calculators

Purpose: _____To simplify student generated equations

Activity: ______ Teacher: Yesterday you wrote equations to describe polygons. The equation for the polygon I made was written as:

$$49T1 = 3T1 + 2T3 + P1 + R1 + R4 + P4$$

If T1 equals one, how much is the left side of the equation?

Student: Forty-nine. **Teacher**: Why?

Student: Because forty nine ones is forty nine.

Teacher: O.K. I will record it like this:

$$49 = 3T1 + 2T3 + P1 + R1 + R4 + P4$$

How much are three T1s?

Student: Three.

Teacher: Record it like this:

$$49 = 3 + 2T3 + P1 + R1 + R4 + P4$$

How much are two T3s?

Student: Eight.

Teacher: O.K. Record it like this:

$$49 = 3 + 8 + P1 + R1 + R4 + P4$$

The process continues until the equation looks like this:

$$49 = 3 + 8 + 2 + 4 + 16 + 16$$

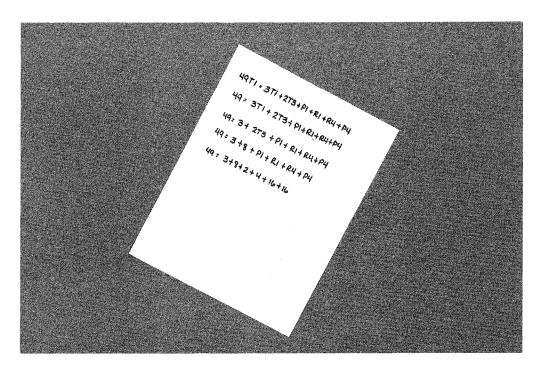
Use your calculators to check to see if this is true?

Student: It's true. Forty-nine equals forty-nine.

When students are comfortable with the process of simplifying equations, pass out the previous lesson's task cards. Students copy the equations from the task card on to lined paper. They record the process of simplification. If they have equations that do not balance, they may record the results like this:

$$38T1 = 3T1 + R1 + T3 + S3 + S1 + T5$$

 $38 = 3 + 4 + 4 + 8 + 2 + 16$
 $38 \neq 37$



REFLECTION SYMMETRY

Materials:____ Power Blocks

Unlined paper

Mirrors Rulers

Purpose: ______To identify lines of reflection symmetry in Power Blocks

Activity: ____

Teacher: Take out S5. Can you place your mirror on the block so the image in the mirror is exactly the same as the part of the block behind the mirror?

Student: Yes.

Teacher: How many different ways were you able to put your mirror on S5 so the reflection was

identical to the part of the block that was hidden?

Student: There are two ways to put the mirror from corner to corner and two ways to put the mirror from side to side.

Teacher: Can anyone find a different way?

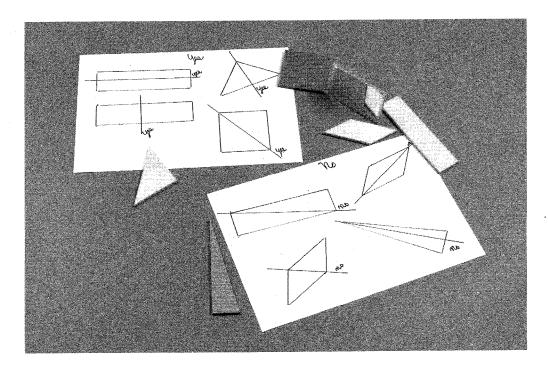
Trace S5 and use your ruler to draw where you put the mirror. That is the line of reflective symmetry. A shape has reflection symmetry when its mirror image is an exact copy of the part hidden by the mirror. S5 has four lines of reflection symmetry. Take out P4. Does it have lines of reflection symmetry?

Student: I can't find any.

Teacher: Why not?

Student: No matter how I place the mirror, what I see in it is different from the part behind it.

- Which Power Blocks have reflection symmetry?
- Which shapes do not?
- The square has four lines of reflection symmetry. Does the rectangle?
- Do you see any objects in the classroom that have reflection symmetry?
- Can you think of any objects at home that have reflection symmetry?



REFLECTION SYMMETRY

Materials:_____Power Blocks

Mirrors Rulers Yarn

Purpose: _____ To build shapes that have reflection symmetry

Activity: ______ Teacher: I need someone to volunteer to work with me. I will choose a Power Block, and set it in front of us on one side of the yarn. My partner will take the same kind of block and place it beside mine on the other side of the yarn.

Student: Is there any special way that you want me to put it against your block?

Teacher: Yes. Pretend there is a mirror on the line between our two blocks. If you were to look into the mirror, you would see a reflection of my block. Place your block so that it is a reflection of my block.

Student: I get the idea.

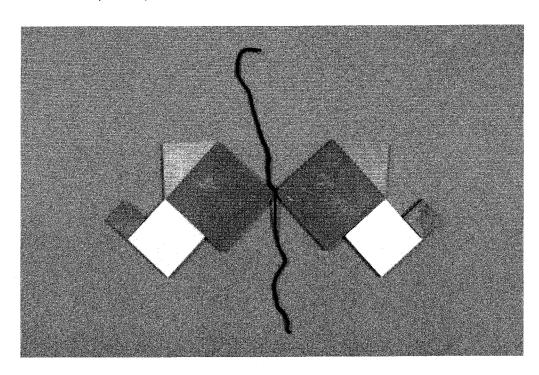
Teacher: Let's continue to build our shape, always being careful to make sure that they are symmetrical. How could we check to be sure they are symmetrical?

Student: Use a mirror.

Teacher: Work with a partner to make shapes or designs that have reflection symmetry. Use a mirror to make sure your shapes are symmetrical.

When students understand the task of building symmetrical shapes and designs, they may make a record of what they have done by tracing their polygons and drawing in the line(s) of reflective symmetry.

- Remember how the square had four lines of symmetry. Can you make another shape that has four different lines of reflective symmetry?
- What shape can you make that has the most lines of reflection symmetry?



3D REFLECTION SYMMETRY

Materials: _____Power Blocks

Yarn

Purpose: ______To build three-dimensional shapes that have reflection symmetry

Activity:

Teacher: Up until now we have been studying two-dimensional shapes. Two-dimensional shapes are flat. They have length and width. I want you to build three-dimensional shapes. These are shapes that have length, width, and height. I am going to build a shape in three dimensions. The shape is going to have at least one line of reflection symmetry. Does anyone see a line of reflective symmetry?

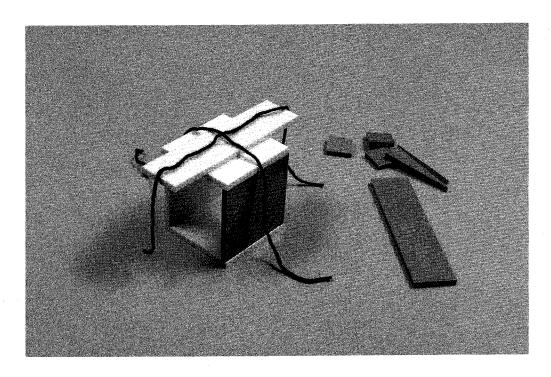
Student: Yes.

Teacher: Use this piece of yarn to show where we would put a mirror.

Student: Don't we need mirrors?

Teacher: No, in this situation we will use an imaginary mirror. You can show where the mirror would cut through your shape by laying a piece of yarn across it. Build as many three-dimensional shapes with reflection symmetry as you can.

- Can you make a three-dimensional shape that has two different lines of reflective symmetry?
- Can you make a three-dimensional shape with more than two lines of reflective symmetry?
- Do you see any three-dimensional objects in the classroom that have reflection symmetry?
- Can you think of any objects at home that have reflection symmetry?



BUILDING SHAPES FROM REFLECTIONS

Materials:_____Power Blocks

Activity:

Unlined paper

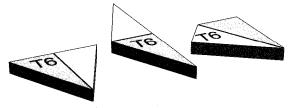
Mirrors

Purpose: ______ To explore reflections and lines of symmetry

made. Be sure to draw a line where the mirror was. Do this for each one of the sides of T6. What can you say about what happened? **Student**: Sometimes you get a triangle, sometimes you don't.

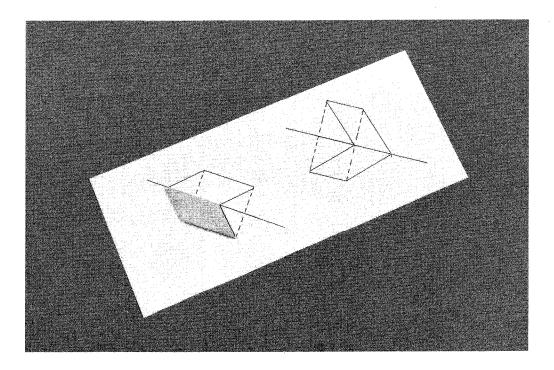
Teacher: Take T6 and place a mirror along one of its sides. Place a block behind the mirror so that it is in the same position as the image. Remove the mirror and outline the shape you have

Teacher: Do this for other shapes. Write about the differences and similarities of the reflections you create.



Questions to explore with students:

Reflect a parallelogram over each side. Draw a dotted line that connects corresponding vertices (the end points of corresponding angles). Measure the distance from each vertex to the line of reflection. What is true about these distances? Do the same for other shapes.



SIMILAR POWER BLOCKS

Materials: _____Power Blocks
Unlined paper

Purpose: _____ To identify similar geometric shapes among the Power Blocks

Activity:

Teacher: We are going to search for similar geometric shapes among the Power Blocks. Place
T5 flat on your desk. Hold T3 in your hand and stand in front of T5. Hold T3 so it is just above
T5. Align two sides and an angle of T3 with those of T5. You have to get your head right over the top of the triangles. Close one eye. Slowly bring T3 toward your other eye. At the same time you are doing this, you must keep the sides and angle of the two triangles aligned. What happens to

Student: As you bring T3 up, it covers more and more of T5.

Teacher: Is there ever a point when T3 appears to be congruent with T5?

Student: I don't understand.

Teacher: Is there ever a point when T3 appears to cover T5 exactly with nothing left to cover and nothing hanging over?

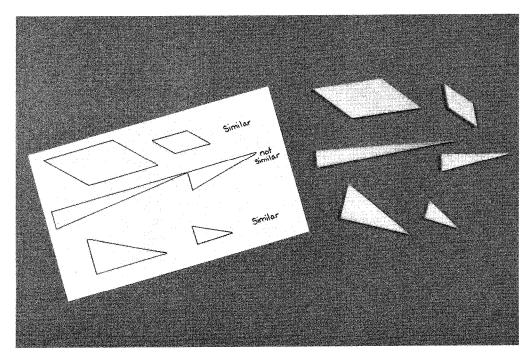
Student: Yes.

T5 as you do this?

Teacher: If it appears to be congruent, then the two shapes are similar. Are there other Power Block shapes that are similar to each other?

When students understand how to do this, they make a record of what they have found by tracing the two shapes and noting whether they are similar or not.

- Did you find any shapes that were similar to more than one shape? If so, can you put them in an order that illustrates their similarity?
- Are all the triangles similar?
- Are all the rectangles similar?
- If you found a series of shapes that were similar, can you make the next shape in the series by putting Power Blocks together?



SIMILAR SHAPES

Materials:____Power Blocks

. Unlined paper

Clear vinyl $8 \times 11 \times 20$ mils thick

Purpose: _____To build similar geometric shapes with Power Blocks

Activity: _____Teacher: Make this shape using T2, S3 and R2



Can you make another shape similar to the one I have just made?

Student: Yes.

Teacher: What blocks did you use?

Student: T3, S4, and R4.

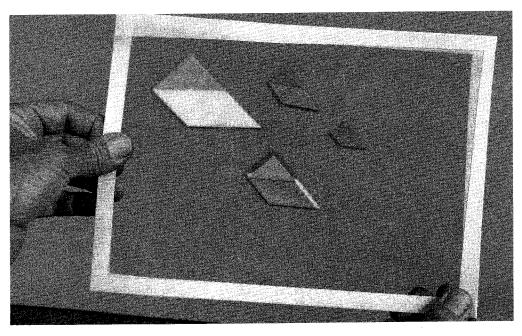
Teacher: Take T2, S3, and R2 and build the shape on a sheet of vinyl. Hold the shape on the vinyl over the shape you just made with T2, T3, and R2 and check to see if it is similar to the first shape. Is it?

Student: No.

Teacher: What could you do to make it similar? Can you make another shape that is similar to the first?

When students understand how to do this, they make a record of what they have found by tracing the shapes on a piece of paper. They may stack their tracings in order of increasing size and staple them together.

- How far were you able to extend your series of similar shapes?
- If you could make a series of shapes using two blocks at each step, could you do a series using three blocks at each step?
- How many T1s did it take to make each of your shapes? Is there a pattern that would enable you to predict the number of T1s for any step in the series of shapes?



SIMILAR POWER BLOCKS

Materials: Power Blocks

Protractors Rulers Calculators Lined paper

Purpose: _____ To explore the properties of sides of similar shapes

Activity: ______ Each group of students needs to have families of power blocks they have identified as similar, for example: T1, T2, T3, T4, T5.

Teacher: The vertex of an angle is the point where the lines that form its sides meet. Label the vertices of the triangles A, B and C. make C the vertex of the right angle. I want you to make a table like this:

Shape | Side AC | Ratio | Side BC | Ratio | Side AB(hypt.) | Ratio

Take T1 and measure its sides in millimeters. Record your measurements in the table like this:

Shape | Side AC | Ratio | Side BC | Ratio | Side AB(hypt.) | Ratio | T1 | 25 | 35

Take T2 and measure its sides. Record its measurements in the Table like this:

Shape	Side AC	Ratio	Side BC	Ratio	Side AB(hypt.)	Ratio	
Tİ	25		25		35		
T2	35		<i>3</i> 5		50		

Use your calculator to determine the ratio of the sides of T2 to corresponding sides of T1. How much is 35 divided by 25?

Student: About 1.4

Teacher: T2s side is 1.4 times longer than T1s. Do this for each of the other sides and I will record your data...Here is what you came up with:

Shape	Side AC	Ratio	Side BC	Ratio	Side AB(hypt.)	Ratio
TÍ	25		25		35	
T2	35	1.4	35	1.4	50	1.4

Do you see a pattern with respect to how the sides of T2 grew with respect to T1?

Student: They are about the same.

Teacher: What does it mean if they are the same?

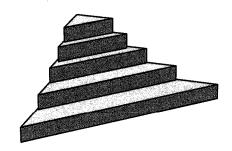
Student: Each side of the larger triangle is 1.4 times larger than the smaller triangle. **Teacher**: Measure the corresponding angles with a protractor. What did you find out?

Student: They are equal.

Teacher: Explore the relationship between the corresponding sides of two other similar shapes. Record your data in a table like the one above. Measure the corresponding angles. Do you see a pattern? Can you write a definition for similar shapes?

Questions to explore with students:

- What would happen if you traced a shape on top of a similar shape so that they have one corresponding angle in common? Continue to do this for each shape in the family of similar shapes. What can you say about similar shapes based on this drawing?



TANGRAMS

Materials: _____Power Blocks

Student made tangram puzzle

Clear vinyl sheet 8 X 11 20 mills thick

Purpose: ______To create tangram puzzles from Power Blocks

Activity: _____ The activity that follows should be preceded by the following lessons from Mathematics...A Way Of Thinking: Lesson 20 – 1, Tangram Puzzle, in which students fold and tear a square of paper to create their own puzzle. Lesson 20 – 2, Tangram Puzzle, which provides practice at assembling tangram pieces into specific shapes.

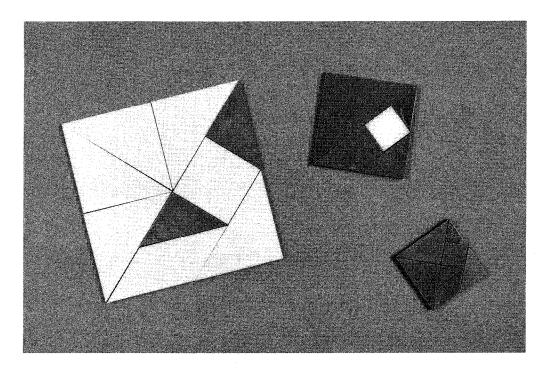
There are four standard metric tangrams in every set of Power Blocks. This tangram set is made from two T4s, one T3, two T2s, one S2, and one P2. There are other sets of tangrams that can be made with Power Blocks.

Teacher: You have created your own tangram puzzles from squares of paper. Can you make a tangram puzzle from the pieces in your set of Power Blocks?

Student: Yes. Mine is a different size than other peoples.

Teacher: I wonder how many different size tangram puzzles are in your set of Power Blocks?

- Were the different size tangram puzzles similar? Use a piece of vinyl to check your shapes.
- If you can tape blocks together, what is the largest tangram puzzle you can make with a set of Power Blocks?



TANGRAMS

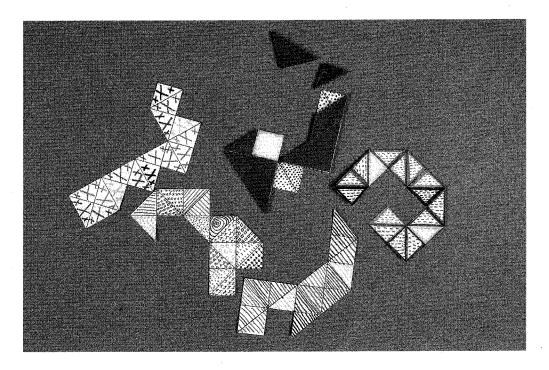
Materials:_____Power Blocks

Tangram triangle paper

Purpose: ______ To create tangram puzzles from Power Blocks

Activity: ______Power Blocks may be used for tangram Lesson 20 – 3 in Mathematics...a Way of Thinking. The tangram triangle paper in Mathematics a Way of Thinking is made for tangram puzzles 4 X 4 inches. Power Blocks are cut to metric units. The tangram triangle blackline provided in this book is made for a square that is 100 X 100 millimeters. The difference between the two is very slight, but may cause problems for some students. It is best to use paper that matches the units of the tangram puzzle students are using. Use tangram sets made from two T4s, one T3, two T2s, one S2, and one P2 when using the triangle paper provided in this book.

> You might want to read to your students *Grandfather Tang's Story* by Ann Tompert¹ to your students to motivate them. The story relates to the use of tangrams to create designs that may be made with tangrams.



PYTHAGOREAN RELATIONSHIPS

Materials:___ Power Blocks

Unlined paper Calculators

Purpose: _____To investigate the Pythagorean relationship of the sides of right triangles

Activity:

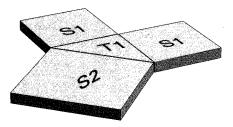
_Teacher: We are going to explore the relationship of the sides of right triangles. The side opposite the right angle is called the hypotenuse. Take out T1. See if you can surround it with squares that have sides exactly the same length as the sides of the triangle. Here is an example of what I mean.



Can you find squares to put against the other sides?

Student: Yes.

Teacher: What blocks did you use? Student: I used 2 S1s and S2.



Teacher: If S1 is one square unit of area, can you use T1s to determine the area of S2?

Student: Two square units

Teacher: I want you to record your information like this. Find all the different triangles you can find in the set of blocks that you can surround with squares. Call the smaller squares A and B, and the large square C. Record their areas.

Triangle	Square A	Square B	Square C
T1	1	1	[*] 2
T2	2	2	4
T3			

If you knew the area of square A and square B, could you predict the area of Square C?

- If you know the area of a square, how could you use your calculator to determine the length of its side?
- Which triangles in the set do not have a square that is the same length as their hypotenuse?
- If you knew the area of the square that was needed to match the hypotenuse of a block, could you use your calculator to determine the length of the hypotenuse?
- For those triangles in the set of blocks that do not have a square the same length as their hypotenuse, could you determine what size square you would need to add to the set?
- Do you see a pattern that would enable you to predict the length of the hypotenuse of any right triangle?

