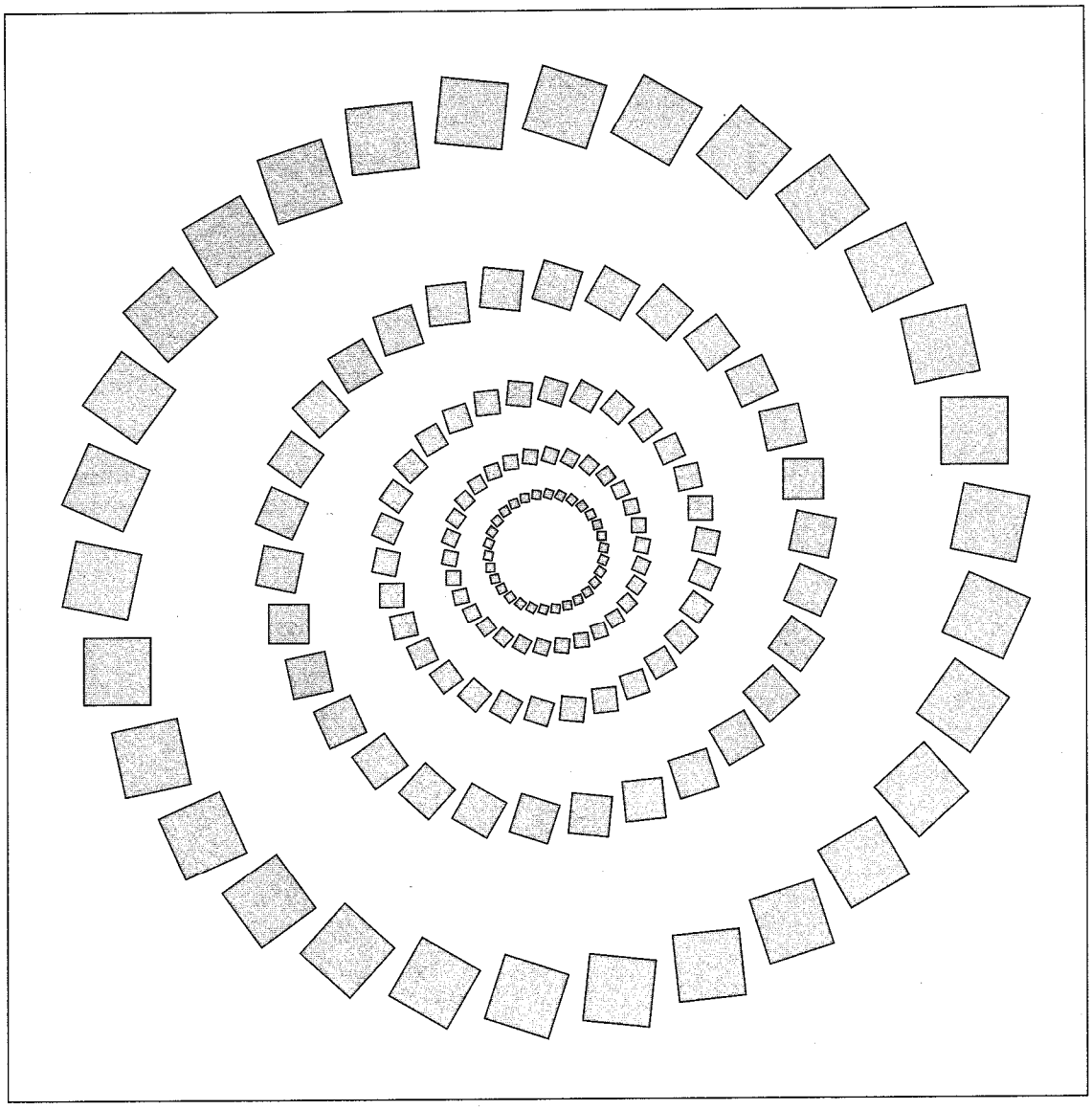
MEASUREMENT

In this chapter students measure the attributes of length, area, mass, and angle.



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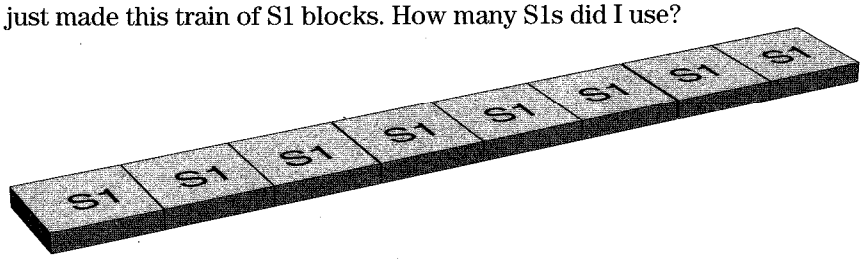
LENGTH: NONSTANDARD UNITS

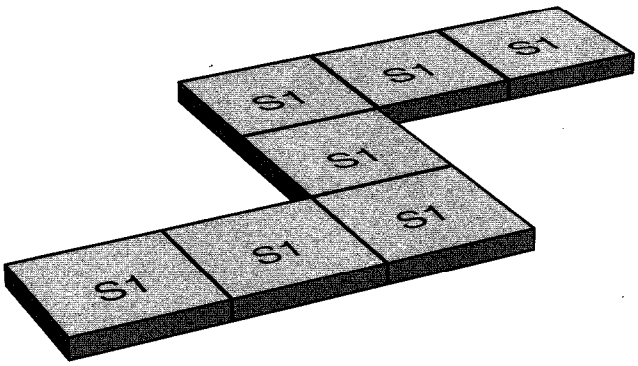
Materials: Power Blocks
Unlined paper

Purpose: To measure length using S1 blocks

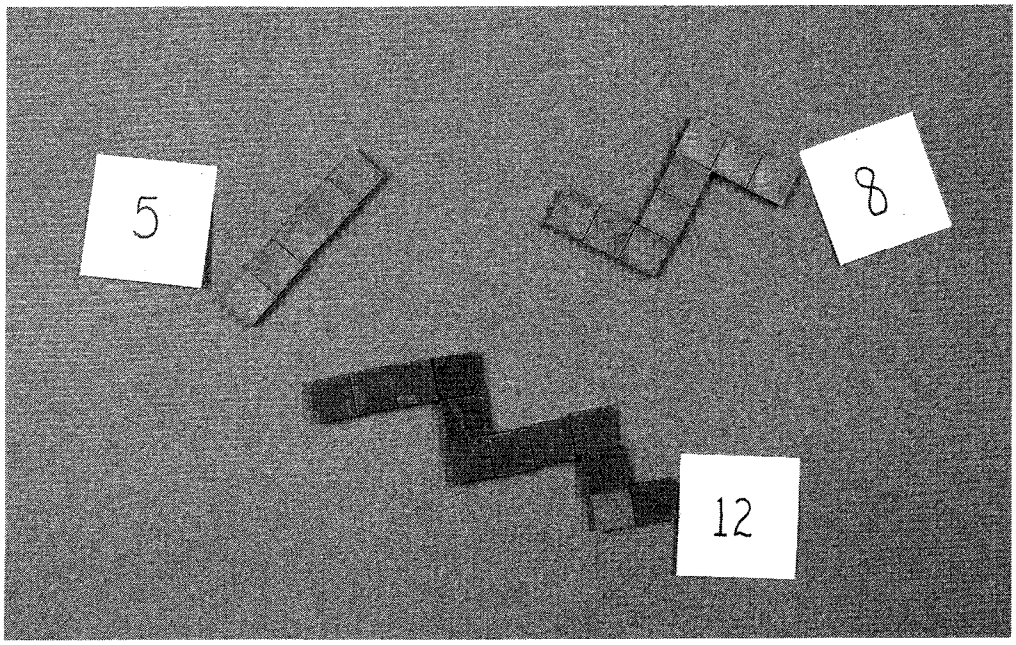
Activity: **Teacher:** I have just made this train of S1 blocks. How many S1s did I use?



Student: Eight.
Teacher: Okay. The train is eight S1s long. I have made another train. How many blocks did I use to make this train?
Student: Seven



Teacher: Right. So we can say the train is seven blocks long. Please make your own trains now. Use no more than two handfuls of S1s to make each train. The blocks must be connected so we can imagine your blocks as a train. Record the length of your train on a piece of paper and place it beside your train.



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When students can measure the length of their trains in terms of S1 blocks, they may use the S1 blocks to measure other objects in the room.

Teacher: I want you to work with a partner. One person on each team get a book and put it in front of your partner. Together, estimate how many S1 blocks long your book is. Take out that number of S1s. Record your estimate. Try to make your estimate within plus or minus one block of the actual measurement. Now measure the length of the book using S1s and record your measurement. Was your estimate greater than, less than, or equal to your measurement?
Student: Greater.
Teacher: Was it within plus or minus one block of the measurement? Use symbols or words to record this information.

When students understand what is required, they work together estimating the length of objects found in the room. After making their estimates, they measure the length of the objects with S1 blocks. They record the results and compare the estimated length with the measured length.

Questions to explore with students:

- What would happen if we changed the S1 block to a larger block?
- What would happen if we changed the S1 block to a smaller block?
- Would the measurement change?
- If the measurements changed, how would they change?

LENGTH: PERIMETER NONSTANDARD UNITS

Materials: Power Blocks
Unlined paper

Purpose: To measure perimeter using S1s

Activity: **Teacher:** Take out an S1. I am going to say the side of S1 is one unit long. How many sides does S1 have?
Student: Four.
Teacher: The sum of the lengths of the sides of a shape is its perimeter. What is the sum of the lengths of the sides of S1?
Student: Four.
Teacher: Yes. The perimeter of S1 is four. Take out two S1s. Put them together so they make one shape. When you put them together, their sides and corners must line up. What shape did you get?
Student: A rectangle
Teacher: Did anyone get a different shape?
Student: I don't see any.
Teacher: Okay. What is the perimeter of your shape?
Student: Six.
Teacher: How many different shapes can you make using three S1s? Make sure the sides and corners line up. Don't make shapes that just have the corners of the pieces touching.
Student: I made two different shapes.
Teacher: How long are their perimeters?
Student: Eight S1s long.
Teacher: Take out four S1s and make as many different shapes as you can using four S1s. Record the perimeter of each shape on a piece of paper.

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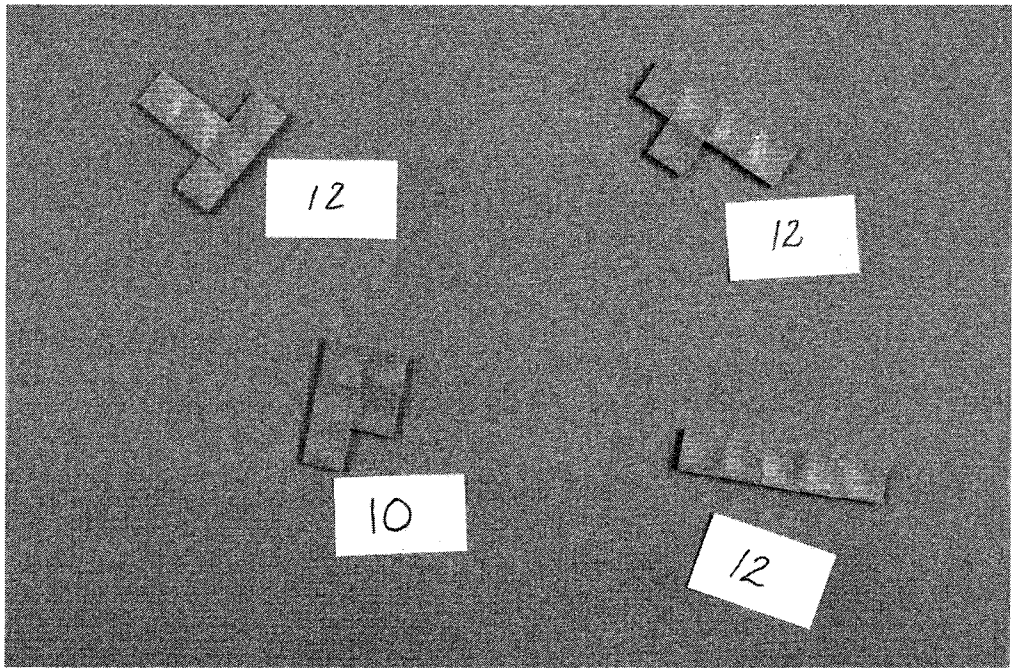
When students understand what is required, they work together to explore the perimeters possible for a given number of S1s. They may make a record of their work by tracing their shapes and recording their perimeters on a piece of paper.

Questions to explore with students:

- Is there more than one way to make a shape with the same perimeter?
- Do you see a pattern that would help you predict the longest perimeter for a given number of S1s?
- Do you see a pattern that would help you predict the shortest perimeter for a given number of S1s?
- Is there a pattern that would enable you to predict the number of different perimeters that are possible for a given number of S1s?



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LENGTH: PERIMETER STANDARD UNITS

Materials: Power Blocks
Unlined paper
Centimeter cubes

Purpose: To develop a sense of the magnitude of a metric unit by measuring the perimeter of the blocks with centimeter cubes

Activity: **Teacher:** Today we are going to measure the perimeters of the blocks with centimeter cubes. The length of each side of a cube is one centimeter. Take out S5. How many sides does S5 have?
Student: Four.
Teacher: Put out the number of centimeter cubes you think it would take to go around the outside of the block. Try to make your estimate come within a range of plus or minus four cubes of the measured perimeter. Snap them together.
Student: Okay. My estimate is thirty- two centimeters.
Teacher: Use a different set of cubes to measure the length of each side of S5. When you have finished, snap the cubes together. How did your estimate compare to your measurement? Was it within the range of plus or minus four cubes?
Student: It was eight cubes too short.
Teacher: Make a tracing of S5 and record your estimate and your measurement. You may record the information like this:

Estimate	Comparison	Measurement
32	is less than	40

Or you may write it like this using symbols:

Estimate	Comparison	Measurement
32	<	40

Students estimate the perimeters of other blocks by setting out the number of cubes they think are equal to the perimeter. They surround the block and count the total number of cubes used. They compare the two numbers and describe the relationship between them with words or symbols. The results of their measurements for a given block will vary. Some of the sides of the shapes are not evenly divisible by cubes. The cubes will be a little too short or a little too long to exactly match the length of a side. Students will vary their approach to solving this problem. The issue is how close is close enough. It is important to discuss it because it is a problem common to all measurements.

When students can measure the perimeter of a single block with cubes, they make polygons using several blocks. When making polygons, it is easier for students to trace the shape first, then measure it. After they have traced their polygon, they record their estimate of the perimeter. They measure the perimeter, record their results, and compare the measured results with the estimate. They may pass their tracing to a neighbor who re-measures the polygon to confirm the perimeter.

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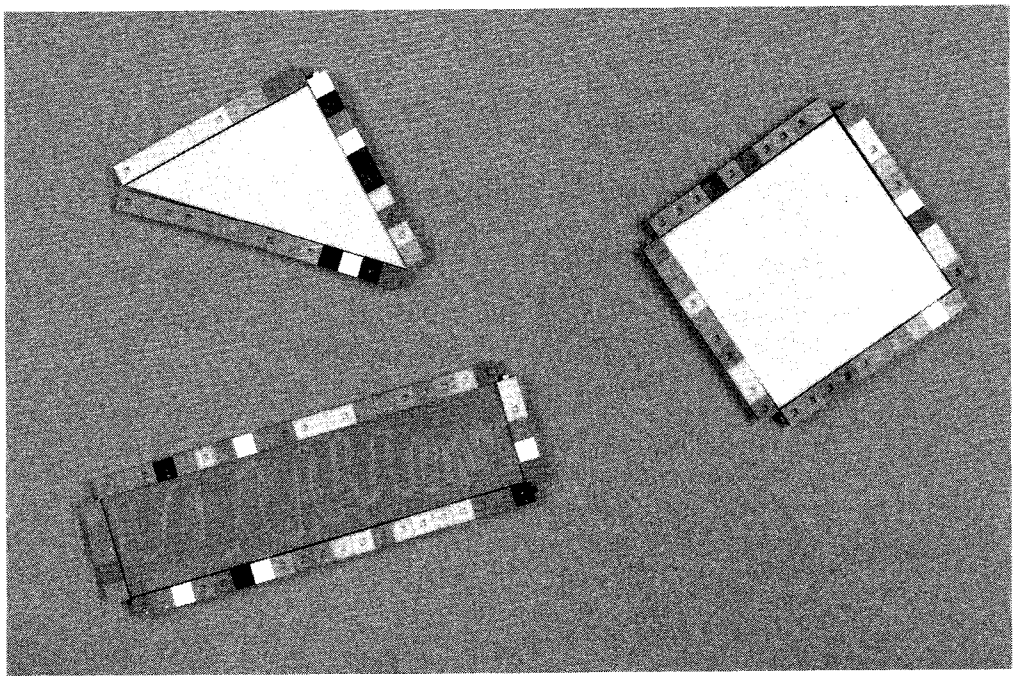
Questions to explore with students:

- How close were your estimates of a block's perimeter to the measured length? Were they within an acceptable range?
- How close do you think they should be? What is close enough?
- Can you sort your blocks based on their perimeters?



AREA: NONSTANDARD UNITS

Materials: Power Blocks
Unlined paper

Purpose: To measure the area of squares and rectangles using S1s

Activity: **Teacher:** Take out R5. We are going to measure its area. A block's area is the number of square units it takes to cover it. Estimate how many S1s it would take to cover R5. Try to estimate within plus or minus one S1 of the actual area. What did you estimate?
Student: Twenty-five.
Teacher: Measure the area of R5 by covering it with S1s. How many S1s did it take to cover R5?
Student: Sixteen.
Teacher: Okay. We say that R5 has an area of 16 square units. I want you to work with a partner to measure the areas of the squares and rectangles in your set of blocks. Try to make your estimates fall within two blocks of the measured area.

When students have measured the area for all the squares and rectangles in their set of blocks, they may make additional squares and rectangles by putting together combinations of blocks. In each case, they trace the perimeter of the shape they have created. They do not draw the internal structure. They estimate its area. Then they measure its area with S1s. Finally, they compare their estimated area with the measured area.

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Teacher: Make a table with three columns. Label the columns like this:

Length	Width	Area

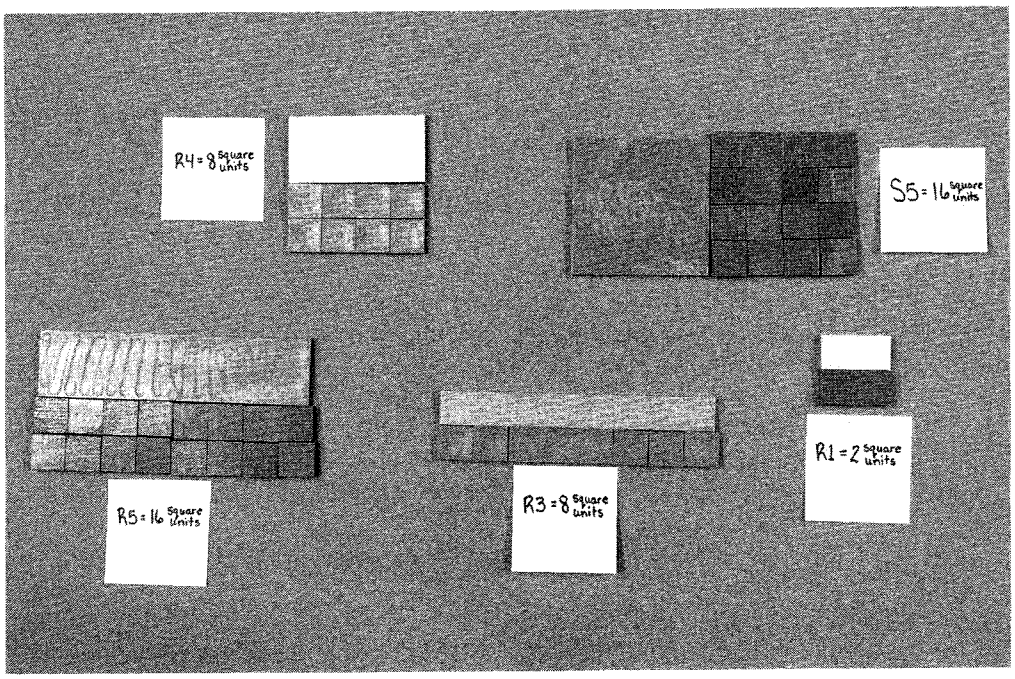
If S1 is one square unit of area and the length of its sides are one unit of length, measure the length and width of the rectangles and squares you have covered. I have collected some of your data. Here is how I want you to record the information:

Length	Width	Area
1	1	1
8	2	16
4	4	16
2	4	8

As you record the information see, if you can see a pattern that would help you predict the area of a square or rectangle if you knew its length and width.

Questions to explore with students:

- What is the area of your paper, desk, favorite book, or a friend's favorite book?
- Could you predict the area of a rectangular or square object without using S1s to cover it?



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AREA: NONSTANDARD UNITS

Materials: Power Blocks
Table of Relative Areas (blackline master)
Unlined paper
Calculators

Purpose: To record the area of the blocks systematically
To use this information to measure the area of student generated shapes

Activity: **Teacher:** Take out as many T1s as you can find in the next two minutes. How many T1s does it take to cover an S1?
Student: Two.
Teacher: Okay. Two T1s cover an S1. If T1 is equal to one square unit of area, what is the area of S1?
Student: Two square units.
Teacher: Record this information in your Table of Relative Areas. Next to T1, write the number 1, and next to the S1 write the number 2.
Student: Okay.
Teacher: I'm going to say T1 is one unit of area. How many T1s does it take to cover T2?
Student: Two.
Teacher: What is the area of T2?
Student: Two square units.
Teacher: Record it in your table next to T2. Please find out how many T1s cover each of the other blocks. Record your answers in your table.

When the class has completed this task and agrees on the relative area of each block, they are ready to generate their own area task cards.

Teacher: Take out three blocks. Put them together to make a polygon.
Student: Which blocks?
Teacher: Any three blocks you want.
Student: How do you want them to go together?
Teacher: It does not matter as long as their sides are touching and they form a polygon. When you have finished making your shape, trace it. When you trace it, only show its perimeter. Do not show how the shape is made. When you finish, make another polygon using three blocks.
Student: How many shapes do we draw? May we color them?
Teacher: Make as many as you can in the next fifteen minutes. Yes, you may color them. When you are finished I will collect your work.

The teacher chooses an example of a student-generated task card to demonstrate to the class.

Teacher: If T1 is equal to one square unit area, what do you estimate the area of this shape to be?
Student: (Number is volunteered.)
Teacher: How could we measure its area?
Student: Put T1s on it until it is covered. Then count them.
Teacher: I don't think I have enough T1s to cover the shape. How can I measure it?
Student: We know the area of other bigger pieces. Put bigger pieces on first. Use the T1s after all the space has been filled in.
Teacher: Okay. I can put a S4 here and a P3 here. Now what do I do?
Student: Put two T1s in the space that is left.
Teacher: Okay. Now it is covered. How do I find the area of the polygon?

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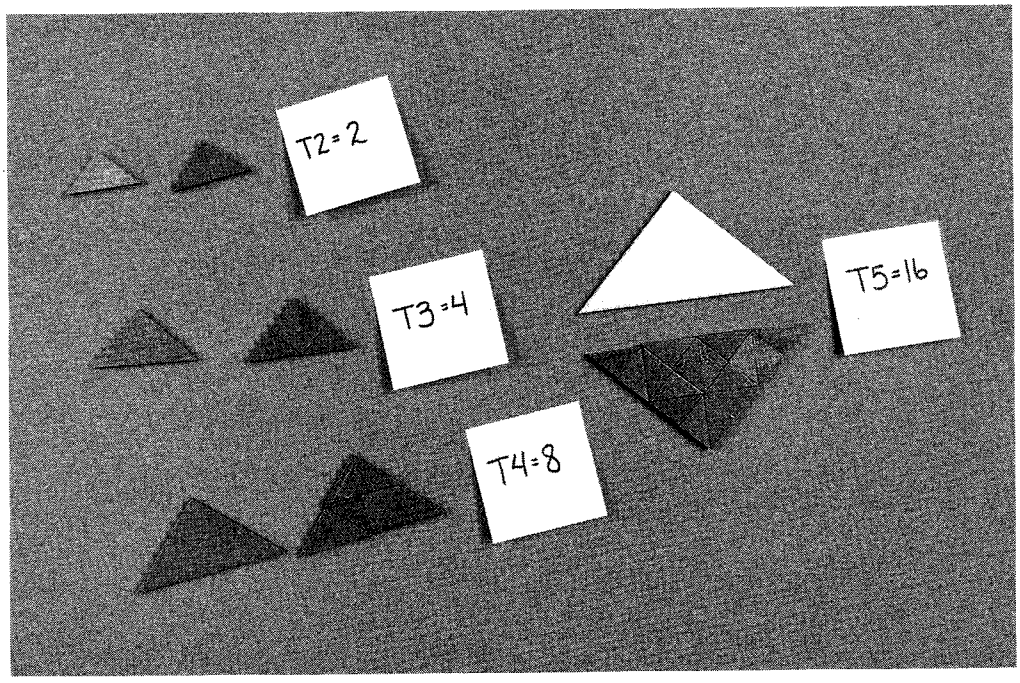
Student: Add them up. S4 equals 16. P3 equals 8, and two T1s equal 2. When I added them, I got 26.
Teacher: How did your measurement compare to your estimate?

When students understand the process of measuring the area of the polygons, the teacher passes out the rest of the cards to the class. Students work together to measure the area of polygons. They first make an estimate of the area of their shape and record it on the task card. Next, they measure the shape with blocks and record their results on the task card. They compare their estimate with the measured area.

Questions to explore with students:

- Did you use a strategy to measure the area of your shape?
- What strategy did you use?
- Were any shapes impossible to measure? If so, why?



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AREA: STANDARD UNITS

Materials: Power Blocks
Table of Relative Areas (blackline master)
Centimeter grid paper (blackline master)
Calculators

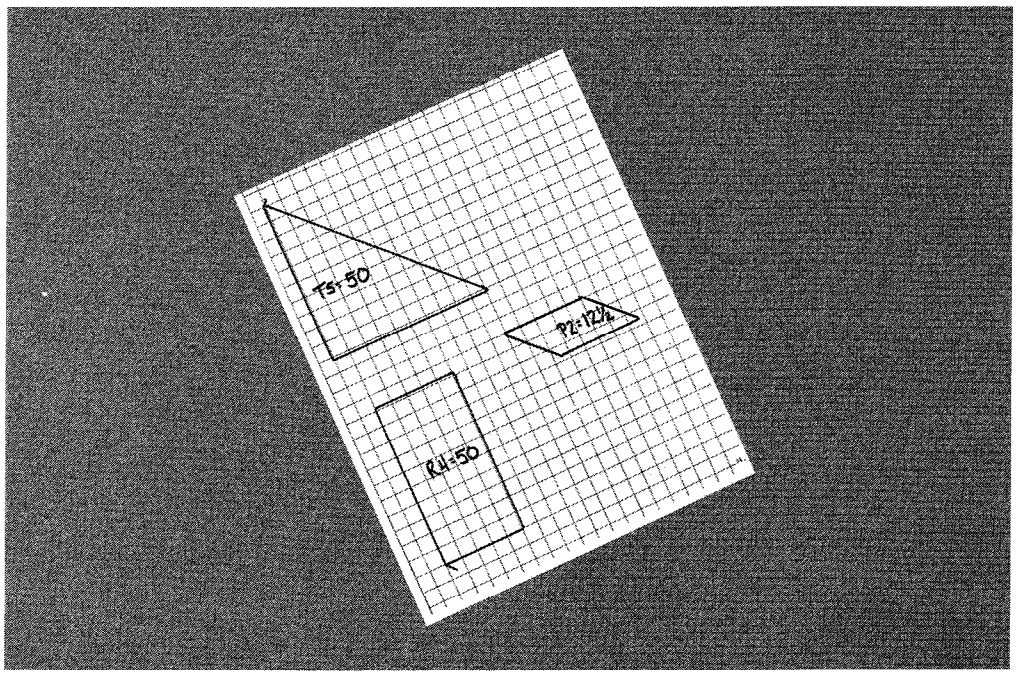
Purpose: To develop a sense of the magnitude of a metric unit by measuring the area of the blocks using centimeter grid paper

Activity: **Teacher:** Take out T1. Estimate the number of square centimeters it would take to cover the T1. Write it down.
Student: (Assorted numbers are volunteered.)
Teacher: Now place T1 on your grid paper and trace around it. How many square centimeters does it cover?
Student: Three and a little bit more.
Teacher: Estimate how much the little bit is and record your result on your drawing. Compare your estimate with your measurement. Measure the area of the other blocks using centimeter grid paper.

To do the following, the class must have been taught how to find an average. When the class has measured the area of each block, the teacher collects the data and consolidates it. She/he shows the class a list of all the areas for a given shape.

Teacher: When the class measured the area of P3, it got a range of different areas. Here is a list of your data. Find the average area of P3 and record it in a Table of Relative Areas.

When the students understand what they are to do, the teacher lists the class results for the other blocks. They determine the average area for each of the blocks and record it in a Table of Relative Areas. The teacher saves the averages so they can be compared to the data in the next lesson.



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AREA: STANDARD UNITS

Materials: Power Blocks
Table of Relative Areas (blackline master)
Calculators
Metric rulers graduated in millimeters

Purpose: To develop a sense of the magnitude of a metric unit by measuring the length of the sides of the blocks using a ruler

Activity: To do the activity that follows, students need to be able to measure with a metric ruler and be familiar with the formulas for the areas of squares, rectangles, parallelograms, and triangles.

Teacher: I want you to work in groups of two or three. Take out T3. Measure the base to the nearest millimeter.
Student: Which side is the base?
Teacher: Stand your triangle up so a side with the right angle is resting on the desktop. We will call that side the base. Record your measurement.
Student: Okay.
Teacher: Now measure the height to the nearest millimeter. Let's call the height the other side of the right angle.
Student: Okay.
Teacher: Remember the formula for the area of a triangle A=1/2 bh. Calculate the area of T3 based on your measurements. Write the area of T3 in a Table of Relative Areas.
Student: Okay.
Teacher: Calculate the area of each of the other triangles using the formula. Write the results in a Table of Relative Areas.

The process is repeated for the other shapes. Each shape is measured, the measurement plugged into the appropriate formula, and the area calculated.

Teacher: When the class calculated the area of T3, it came up with different answers. Here they are. What is the average of these areas?
Student: Calculate the average.
Teacher: Take out the Table of Relative Areas from yesterday's lesson. Write the average area of T3 beside yesterday's answer.

When the students understand what they are to do, they determine the average area of each of the blocks. They record it in the table with the average measurements from the previous lesson.

Questions to explore with students:

- Were any of the results of your measurements with graph paper the same as those with the ruler? Which one(s)?
- Which method of determining the area of the blocks was more accurate, the graph paper or the ruler? Why?
- Which method of determining area do you think would be the most useful outside a classroom?

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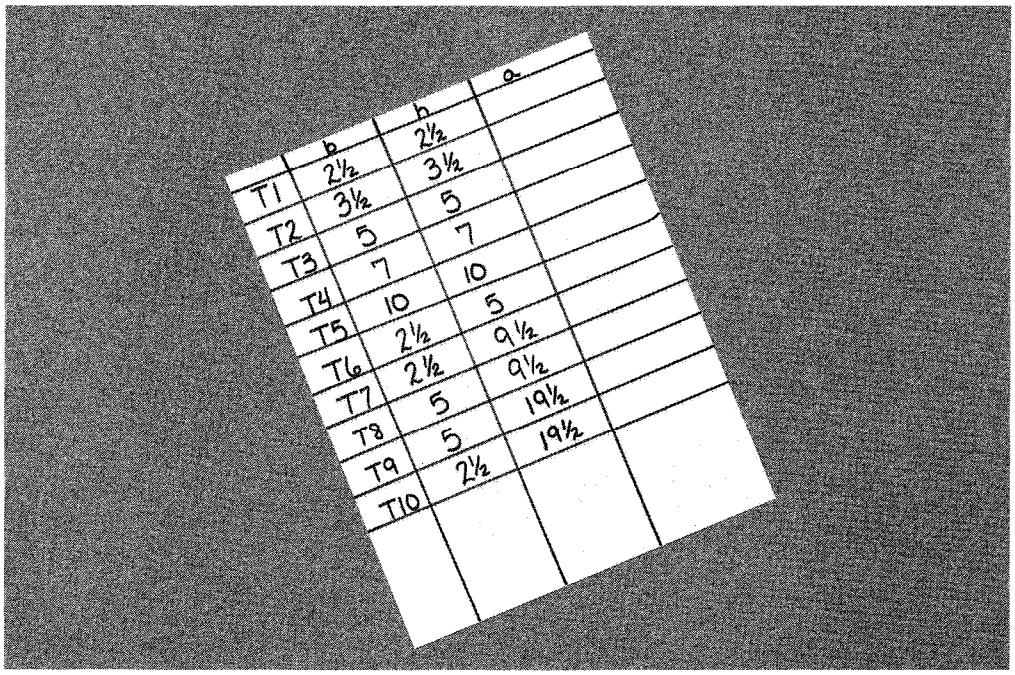

	b	h	a
T1	2½	2½	
T2	3½	3½	
T3	5	5	
T4	7	7	
T5	10	10	
T6	2½	5	
T7	2½	9½	
T8	5	9½	
T9	5	19½	
T10	2½	19½	

AREA: STANDARD UNITS

Materials: Power Blocks
Calculators
Unlined paper

Purpose: To determine the area of Power Blocks by logical thinking

Activity: **Teacher:** Today we are going to see if we can determine the area of the blocks by doing as little measuring as possible. When we used the centimeter graph paper, you said the area of S5 was 100 square centimeters. When we measured S5 with a ruler, you said that it was 10 x 10 centimeters and had an area of 100 square centimeters. Let's see if we can use that information to figure out the other areas. How many R4s does it take to make S5?
Student: Two.
Teacher: What fraction of S5 is R4?
Student: One-half.
Teacher: So R4 is half of S5. If S5 is 100 square centimeters, what is the area of R4?
Student: Fifty square centimeters.

Student determine the area of each piece based on S5 being 100 square centimeters. They record the information with the data from the previous two methods.

Questions to explore with students:

- Were any of the results of your measurements with graph paper and the ruler the same as those we determined by logical thinking? Which one(s)?
- Which method of determining the areas of the blocks was more accurate? Why?
- Is the third method useful outside the classroom?

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SURFACE AREA OF SOLIDS: STANDARD UNITS

Materials: Power Blocks
Centimeter grid paper
Calculators
Cellophane tape or masking tape
Small gummed labels (removable)
Scissors

Purpose: To measure the surface area of a solid

Activity: Prior to the lesson, students choose three identical blocks and stack the blocks on top of one another. When they have made enough stacks, they tape each stack of three blocks together to make a solid. They stick a gummed label on each surface of the solid and number the surfaces by writing on each label. The purpose is to make sets of solid shapes for use in the next lesson.

Teacher: Here is a stack of three S5s that have been taped together. Number each surface. How many surfaces does the shape have?
Student: Six.
Teacher: Place S5 on top of a piece of centimeter grid paper. Trace this surface of the solid. Flip it ninety degrees, and trace the new surface. Continue this for each surface. As you trace each surface, write the surface's number on the tracing. When you have finished tracing each surface of the shape, you should be able to be cut it out as a single piece of paper.
Student: I can't make it come out that way.
Teacher: It is difficult. After you trace a surface, flip it ninety degrees. Keep doing that until all the surfaces have been traced.
Student: Okay.
Teacher: Cut it out.
Student: Okay.
Teacher: Can you make a "jacket" for the solid with the piece of paper you have just cut out? It should fit the solid as snugly as possible. Use tape to hold it in place.
Student: It's not very snug.
Teacher: It looks close enough. How could you measure the surface area of this solid?
Student: Count the squares.
Teacher: Which squares?
Student: All the squares on the "jacket".
Teacher: Okay. Let's do that.
Students: Provide different answers.
Teacher: When the class measured the surface area of S5, these are the results. Calculate the average area for S5.

When students understand what they are to measure, they work together to measure the surface areas of other solids. The measurements are shared and an average calculated for each solid.

Questions to explore with students:

- Is it possible to use formulas to calculate the surface areas of the solids?
- What formulas would you use?
- Compare the results of the measurements made with "jackets" to those calculated using formulas.

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MASS: NONSTANDARD UNITS

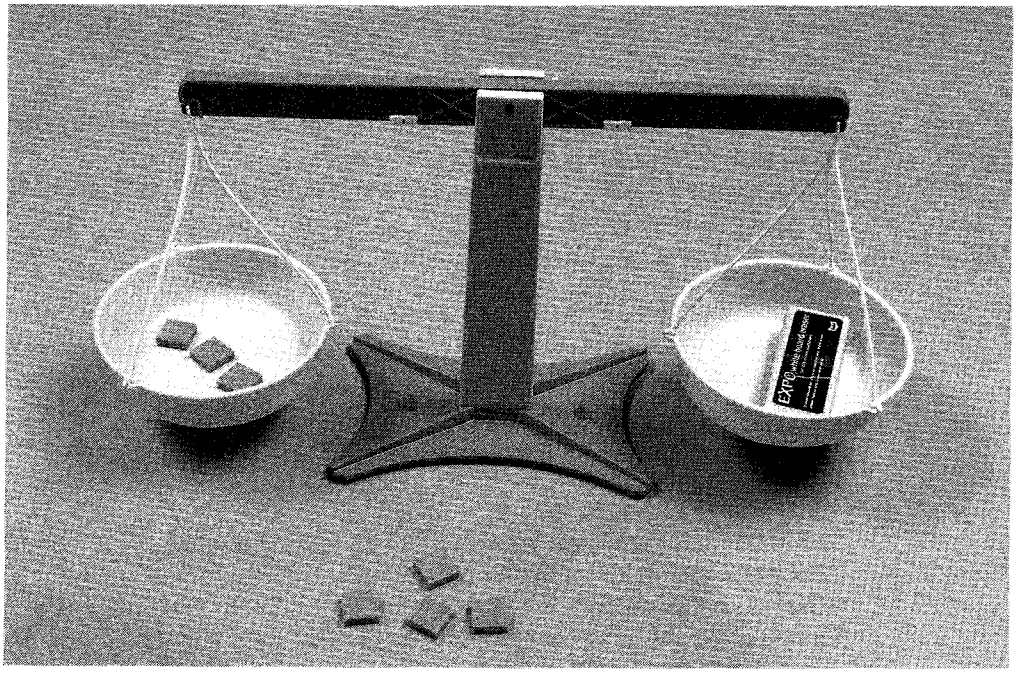
Materials: Power Blocks
Balance
Common objects

Purpose: To measure the mass of objects in the classroom using S1 as the unit of measure

Activity: **Teacher:** Today we are going to measure the mass of this eraser. Estimate its mass. (Teacher demonstrates adding S1s to one side of a balance until it is balanced.) What is the mass of the eraser?
Student: Students count S1s.
Teacher: I have collected common objects for you to measure. Measure each object and record the results of your measurements.

Questions to explore with students:

- How could you measure the mass of an object if it was heavier than all your S1s put together?
- What is the mass of each Power Block if S1 has a mass of one?



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MASS: 1, 2, 4, 8, 16...

Materials: Power Blocks
Balance(s)
Lined paper
Common objects
Gummed labels (removable)

Purpose: To measure the mass of objects using T1, T2, T3, T4, and T5

Activity: Prior to the lesson, the teacher needs to make sets of blocks consisting of T1, T2, T3, T4, and T5. One set is required for each balance. A gummed label needs to be placed on each side of each blocks.

Teacher: If T1 is equal to one unit of mass, how could you find the mass of T2?
Student: Weigh it with T1s.
Teacher: When you do that, what is the mass of T2?
Student: Two T1s.
Teacher: Okay. Write 2 on the gummed labels on T2, and 1 on the labels on T1,
Student: Okay.
Teacher: What is the mass of each of the other blocks?
Student: T3 is four. T4 is eight and T5 is sixteen.
Teacher: Record the block's mass on the label. Take a piece of paper and draw vertical lines on it to make seven columns. The second column from the left should be a little wider than the others.
Student: Okay.
Teacher: At the top of the third column from the left, write T5. In the fourth column write T4, in the next write T3, then T2, and finally T1. Write the numbers from one to thirty-one in the left hand column. Write one number on each line starting with the second line.
Student: Got it.
Teacher: I want you to measure the mass of objects found in the room. Use only the five triangles with the gummed labels.
Student: Can we mix sets to get more pieces?
Teacher: No. What is the mass of this ruler? If I put T1 on the other side of the balance, nothing happens. If I add T2, nothing happens. If I add T3, the balance is now out of balance in the other direction. What can I do to make it balance?
Student: Take out T1.
Teacher: Okay. Now it is balanced. T2 has a mass of 2. T3 has a mass of 4. What is the mass of the ruler?
Student: Six.
Teacher: Next to the number six on your record sheet, write the word "ruler". How many T5s did I use?
Student: None.
Teacher: Write 0 in the T5 column next to the word ruler. How many T4s did I use?
Student: None.
Teacher: Write zero in the T4 column next to the other zero. How many T3s did I use?
Student: One.
Teacher: Write one in the T3 column. How many T2s did I use?
Student: One.
Teacher: Write one in the T2 column. How many T1s did I use ?
Student: Zero.
Teacher: Write zero in the T1 column. Measure the mass of other objects found in the room. When you record your results, what is the largest number you will ever have to write in a column?
Student: One.

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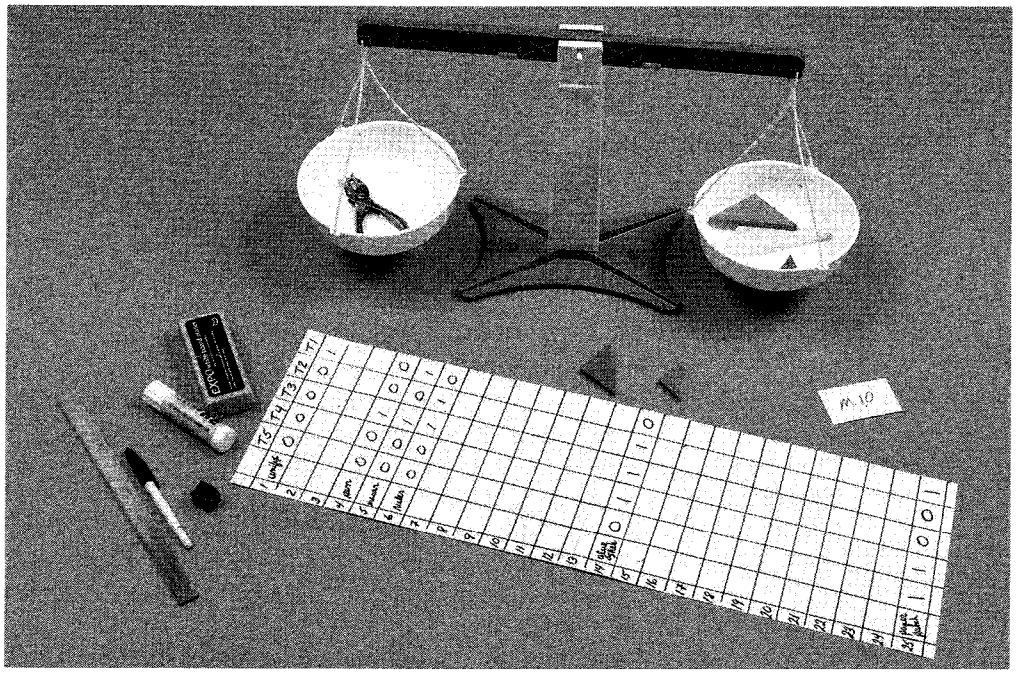
Teacher: What is the smallest?
Student: Zero.

Questions to explore with students:

- Were there any whole numbers between one and thirty-one that you could not make with the five blocks? Which ones?
- Look at the matrix we have made. Do you see a pattern going down any of the columns?
- What base uses only ones and zeros to record quantities?
- We used blocks with masses of 1, 2, 4 ,8, and 16. If we wanted to add to our collection, what would the mass of the next block have to be to keep the pattern going?



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MEASURING ANGLES

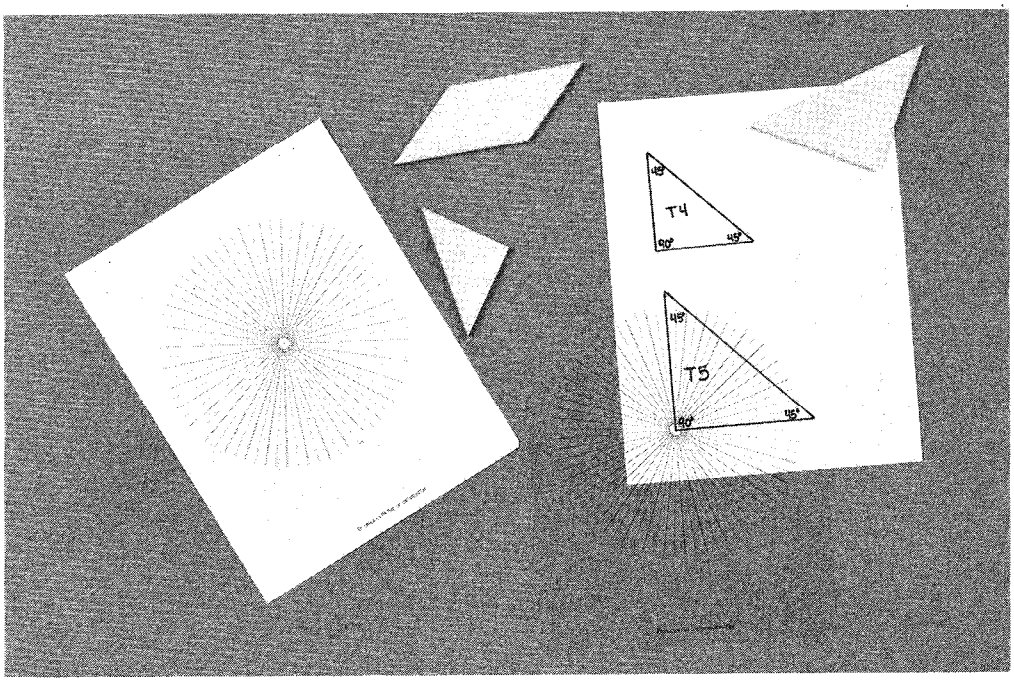
Materials: Power Blocks
Transparent protractors (blackline master)

Purpose: To measure the interior angles of Power Blocks

Activity: Each team of students needs a transparent protractor. These can be made by using the blackline at the back of the book to make overhead projector transparencies.
Teacher: Today we are going to measure angles. How many sections are there in your protractor?
Student: Seventy-two.
Teacher: How many degrees are there in a circle?
Student: Three-hundred-sixty.
Teacher: How many degrees are set off by each section of your protractor?
Student: Five.
Teacher: Take out T5. To measure an interior angle of T5, place the dot at the center of the protractor on the vertex of one of the angles. Line up one side of T5 with any line on the protractor. Count each section of the protractor as five degrees. Count until you reach the other side of the triangle. How many degrees are there in each of the angles of T5?
Student: Forty-five, forty-five, and ninety.
Teacher: Make a record by tracing T5 and recording your results on your drawing. Measure the interior angles of each of the other blocks.

Questions to explore with students:

- Measure the exterior angles of the shapes. Do you see a pattern that would enable you to predict the exterior angle of a shape if you knew the interior angle?
- Do you see a pattern that would enable you to predict the sum of the angles of a block?



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MEASURING ANGLES

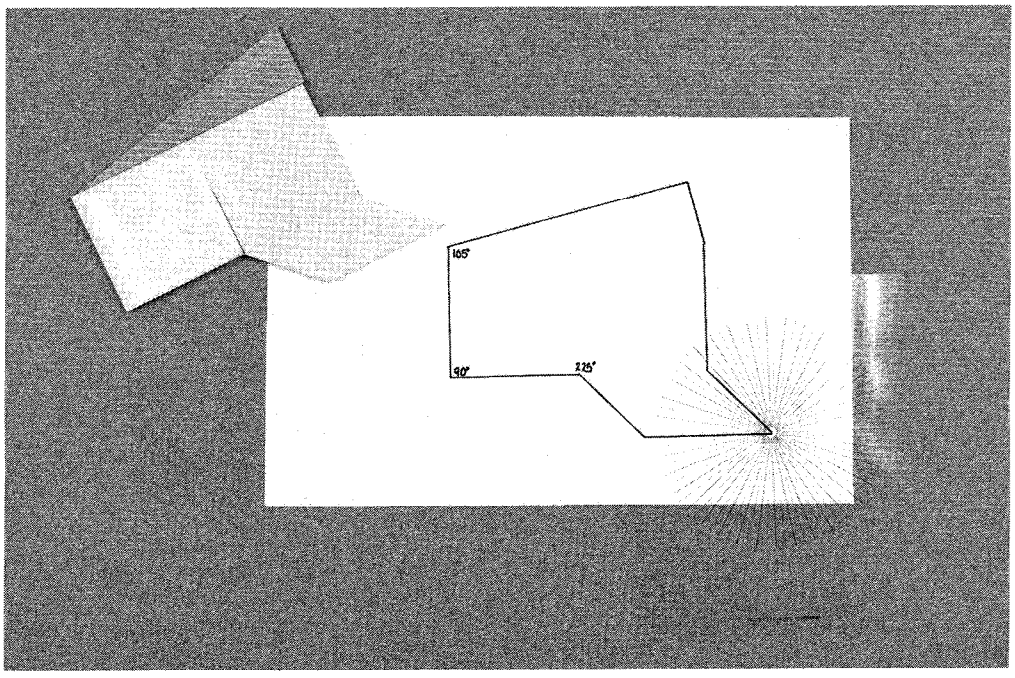
Materials: Power Blocks
Transparent protractors
Calculators

Purpose: To measure the interior angles of student generated polygons

Activity: **Teacher:** Today I want you to make polygons using several Power Blocks. I have made one using two S5s, one P4, and one T9. Make the same shape. Trace it and measure its interior angles.
Student: Okay.
Teacher: How many degrees in each of the angles?
Student: 105, 90, 90, 225, 135, 45, 225, and 165.
Teacher: What is the sum of the interior angles?
Student: 1080.
Teacher: How many sides did it have?
Student: Eight.
Teacher: Make a two column table. Label one column "number of sides". Label the other column "sum of angles". Make as many polygons as you can. Measure the interior angles of each polygon, and calculate their sum. Count its sides and record your information in your table.

Questions to explore with students:

- Do all shapes with the same number of sides have the same sum for the interior angles?
- Do you see a pattern that would help you predict the sum of the interior angles if you knew the number of sides?



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