PART II

A Look Ahead
An Overview of the Activities

Activity-centered learning is a valuable approach because it utilizes the child’s natural style of learning, allowing the child to develop concepts from the actual manipulation of the environment. The child can be helped to move gradually from a “hands on” experience to ever-increasing levels of abstraction and symbolism.

In using Workjobs, ideally, each child uses the activities to explore number cycling from lesser to greater abstraction. Beginning with concrete experiences, the child is gradually led to more symbolic ones.

After a few weeks in a typical kindergarten, first or second grade classroom of thirty or more children, students should be working at their appropriate level (determined by the assessment on page 95) side-by-side with children working at other levels— as opposed to working with children at the same level. (Second grade children move toward greater abstraction at a quicker pace than do kindergarten or first grade students, which is quite natural at this age level.)

The first level (the Concept Level) allows a child to confront number and all its various relationships as represented by different concrete materials. This stage can be thought of as intuitive.

The second level (the Connecting Level) attempts to link the concept of number as represented concretely in the familiar materials with the traditional mathematical symbols.

The third level (the Symbolic Level) gives the children an opportunity to express their mathematical thinking in their own handwritten work, and thereby grow to feel at ease with abstraction.

Each of the three levels represents different types of thinking almost more than different levels or sequential steps. Potentially, they could be experienced in any order and still produce the same end results, but the three levels are a helpful sequence and structure which guarantees success for all learners.

It is important to remember that children learn in a geometric fashion, not a linear one. A three-year-old will sometimes use remarkably advanced vocabulary years before it would be expected to appear as a natural part of the child’s language. A lock-step approach to language development with “mastery tests” for advancement never produces such wonderfully rich, out-of-order progress!

You should get a feel for the three levels by reading the rest of this chapter and looking carefully at the sequence of pictures; but make a conscious effort to prevent your thinking from getting locked into the steps. Children who skip around actually learn more than children forced through each level step-by-step with “proof of mastery” demanded of them before being allowed to progress to the next level. Such a rigid approach would be completely inappropriate here, but it is helpful for a teacher to have a clear understanding of the different types of thinking and uniquely different focus represented by each level. With this understanding, the teacher will be more sensitive to and aware of the experiences which are appropriate for individual children at particular stages of development and thereby guide the children more skillfully.
A Photographic Collage of the Developmental Levels

A child could explore the numbers from 0–9 with the 20 activities at each of the three separate levels (Concept, Connecting and Symbolic), and then explore the operations of addition and subtraction, again, at these same three levels. This is a six-fold recycling of each Workjob activity.

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The Connecting Level bridges the familiar concrete experience of the child’s world to the adult world of abstract symbols.

The pictures on the following pages show children at work at each of these different levels, moving from lesser to greater abstraction. This overview is intended to help you visualize each of the different levels as they would occur in your classroom.
Exploring the numbers from 0–9 at the Concept Level: The children count out the same number of objects onto each counting area.

"One, two, three, four. One, two, three, four. One, two, three, four. One...."

"...five, six, seven, eight. One, two, three, four, five, six, seven, eight. One...."

"...five, six, seven. One, two, three, four, five, six, seven. One, two...."
"... six. One, two, three, four, five, six. One, two..."

"... four, five. One, two, three, four, five. One, two, three, four, five. One, two..."

"... four, five. One, two, three, four, five. One, two, three, four, five. One, two..."

"One, two, three. One, two, three. One, two, three. One, two, three. One, two, three..."
Exploring the numbers from 0-9 at the Connecting Level: The children count out the appropriate numbers of counters to match each numeral.
Exploring the numbers from 0–9 at the Symbolic Level: The children record the total number of objects on each counting area on a tiny piece of paper.
Exploring the operation of addition at the Concept Level: The children take turns in pairs verbalizing the combinations created on a counting area—no totals are given at this stage.

"We went to the pet store and bought one fish for our new tank."

"Then we bought five more."

"That's one and five."

"It's your turn now. I went fishing with my Uncle Bill. We could see six fish in the pool."

"Four more swam in... six and four."
Exploring the operation of subtraction at the Concept Level: The children take turns playing a "take away" game verbalizing the process of subtraction to their partner—no remainders are given at this stage.

"Mama bird laid four baby bird eggs."

"One egg fell out of the nest by accident ... four minus one."

"It's my turn now. My nest has five eggs in it. If I take out three of them, that's five minus three."
Exploring the operation of addition at the Connecting Level: The children use counters representing each equation concretely.
Exploring the operation of subtraction at the Connecting Level: The children use counters representing each equation concretely.
Exploring the operation of addition at the Symbolic Level: The children build an addition problem from an equation card, recording the combination and total on a separate piece of paper.
Exploring the operation of addition at the Symbolic Level: The children use a numeral card to indicate the total number of objects to be used on each counting area. After building this number with their objects the children record the combination and total on a separate piece of paper.
Exploring the operation of subtraction at the Symbolic Level: The children build a subtraction problem from an equation card, recording the process and remainder on a separate piece of paper.
Exploring the operation of subtraction at the Symbolic Level: The children use a numeral card to indicate the total they will build on each counting area. They subtract whatever amount they wish and then record both the subtraction process and the remainder on a separate piece of paper.
Prerequisite Skills

In order for children to benefit fully from WORKJOBS II they need to have sufficient skill with 1:1 correspondence to comfortably count out four objects. Children will eventually need to write the numbers and, consequently, need to begin activities to develop this skill months before they need to have acquired it.

COUNTING TO FOUR

A child who cannot count four objects confidently and consistently needs to do the following series of lessons before using the Workjobs.

Give the child thirty squares of tagboard and some wooden cubes. Ask the child to put two cubes on each paper. If this is easily accomplished, the next day ask the child to put three cubes on each.

If this is difficult (e.g., the child puts one cube or two cubes on some squares rather than consistently putting three), the teacher should put two dots on each square of tagboard after school, using a black crayon (a felt tip pen bleeds through to the other side). The next day the child matches the cubes to the dots, guaranteeing success.

If matching cubes to the two dots is easy, the following day the child should be asked to put out the squares with the dotted sides face down, and again to try to put two cubes on each square. If this is easy, the teacher can add one more dot to each tagboard square after school; the next day the child will match three cubes to the three dots on each piece of tagboard.
When this seems easy, ask the child to put three cubes on the plain side of the tagboard. If this is too hard, the child can turn the cards over, one at a time, match the cubes to the dots, then remove the blocks, turn the card back over and place three blocks on the plain side.

When the child comfortably and successfully counts out three blocks onto the plain side of the squares three consecutive times, the teacher can add another dot to the cards and ask the child to put down groups of four cubes. Once this is accomplished, the child is ready to begin using the WORKJOBS II activities as described in Part IV.

NUMERAL FORM

During the first two weeks of school the teacher will want to assess every child in class (kindergarten, first and second grade) for correct numeral form. It is helpful and interesting to have this early information with which to make later comparisons.

The teacher should cut ten 3” × 5” pieces of tagboard and write one number (0,1,2,3,4,5,6,7,8 or 9) on each card with a black marking pen.

Three children at a time are asked to go to the chalkboard while the rest of the class works on a variety of independent activities which require no adult supervision (drawing, looking at books, etc.). The teacher says to the three children at the chalkboard, “Show me how you write a number five.” The teacher holds up a card for them to see with the number five written on it. (If the teacher fails to show the number, visual memory as well as numeral form is being tested and you can’t isolate accurately one skill from the other.)
The teacher should pay close attention to the strokes each child makes to form each number. The numbers from zero to nine are given one at a time out of order. The teacher may ask the children to make a second number to double check a child she or he is unsure of or who was missed. This is a good check on an individual child’s consistency with the correct form.

A number made skillfully will be flowing and effortless. Keep a record of any numbers which were made facing the wrong direction (reversed), or from incorrect position (starting from the bottom rather than the top), or with great hesitation and effort.

Once the teacher knows which children need practice and help with which numbers, this needed practice can begin.

THE THEORY BEHIND THE PURPLE-GREEN SYSTEM

The goal is for each child to internalize the starting position, order and direction of the strokes or parts that combine to form the numerals. By practicing these strokes, the child will also strengthen the small muscles which are used in writing.

If the order of the strokes of each number is analyzed and the teacher writes the first stroke in one color and the second stroke in a second color, the child quickly learns the pattern and can work independently learning to make all the numbers correctly and smoothly (letters and shapes, too, if desired).

Having a consistent color pattern or sequence to rely on, the child no longer must reinvent or memorize the process, thinking “How do I make that?” or
“Where do I start?” or “Which direction do I go?”. The child’s practice is more effective since every time the child makes the strokes they are in the same order. This reinforces the correct pattern and helps the child internalize the most efficient way of writing each numeral.

The teacher makes a large numeral in front of the children every few days using a purple crayon for the first stroke and a green crayon for the second. (These particular colors were chosen because they are easily distinguishable and are both available as ditto masters for readily preparing related classroom materials.)

The children practice all the numbers by tracing them four times: in the air, on their palms, on one another’s backs and on their individual chalkboards.*

While tracing the number in the air and in the palms of their hands the children say the name of the color with which each stroke is made, not the name of the number. This helps the children internalize the pattern; it is not the point of the lesson to learn the names of the numbers.

*Individual chalkboards can be easily made by painting pieces of heavy chipboard (with a brush, not spray), using slate paint or chalkboard paint, available in all large hardware stores. Ask the children to bring an old sock from home which becomes the eraser and holder for their chalk. Buy chalk at the dime store to use; school chalk has a hardener added to make it dustless and will scratch the boards.
The children should trace each number three times in the air and three times in the palms of their hands, each time repeating the colors:

Teacher: "In the air."
Children: "Purple, green, purple, green,
purple, green."

Teacher: "On your hand."
Children: "Purple, green, purple, green,
purple, green."

Now the children turn and trace the number on one another’s backs three times, saying the name of the number rather than the colors.
The teacher should check carefully to be sure all the children have the pattern internalized before directing them to say the number names instead of the colors. Saying the colors reinforces the pattern. Saying the number names assumes the child consistently uses the internalized pattern and is ready for another step.

The final step is for the children to write the number several times on their individual chalkboards.

**INDIVIDUALIZED NUMBER PRACTICE**

The teacher groups together the children who need help on a certain number for a lesson.
The children who need help writing numbers other than the one presently being worked on should work on individualized number practice activities.*

*See Appendix, pages 125–132. Run on tagboard when using macaroni and on paper when using yarn.

†Available at cost from The Center for Innovation in Education, 19225 Vineyard Lane, Saratoga, CA 95070.

Look for ideas under “Learning to Write Numerals” on pages 43–51 in MATHMATICS THEIR WAY.
It is imperative that the teacher clearly understand that the purpose of the purple and green pattern is to be a teaching tool or guide. It is made available to the child only as a sequencing reference. It is never to be copied with purple and green colors when actually writing. The child writes in black with a crayon, pen or pencil, *never* with the purple and green colors. This mistaken procedure would actually distract the child’s attention from the sequential pattern.

**ADVANCED NUMBER PRACTICE**

Children who know how to write all the numbers benefit nonetheless from a five-to six-minute review lesson with all their classmates once a week. The rest of the time they should be asked to do one of the following activities independently while the rest of the class works on the numbers they need to practice, either independently or in the teacher-directed group.

1. **Dice Graph**
   
   **Materials:** Dotted dice or wooden cubes with 0, 1, 2, 3, 4, 5 dots (made with a permanent marking pen)
   
   Dittoed graph paper

   The child shakes one die and records the number rolled above the appropriate dot pattern. When one column reaches the top of the graph the child stops.

2. **Number Sequences**
   
   **Materials:** Each group of four or five children needs ten wooden cubes with numbers written on the faces (five dice with numbers zero to five and five dice with numbers four to nine)
   
   Paper and pencils

   When interest in making individual records wanes have several children work together adding to a huge piece of graph paper.
One child in a small group of children shakes the ten dice.

Everyone works to arrange the dice in as many pairs and sequences as possible.

When finished, the children record the sequences made on their paper.

The children in the group take turns rolling the ten dice. They record each resulting number sequence on their paper. Previously rolled sequences are tallied and new sequences are added to the lists.

3. Repeating Number Patterns

Materials: A three foot sheet of butcher paper, rolled up and cut to two inches wide
Pencils and crayons

The teacher begins a repeating number pattern on the chalkboard such as 133813381338. The children predict the next five or six numbers verbally, and then begin working on their own.
Each day a new pattern can be started.

The teacher should be careful to balance over time the inclusion of each number from zero to nine in the various patterns. Very soon the children can be encouraged to make up patterns for one another.

Another variation of writing patterns is to have two or three children work together to make one four-inch strip. The paper is laid out on the floor and each child takes a turn writing one number. It is actually more difficult to add a part of a part to a pattern rather than repeating the entire part, which makes this in effect a completely different activity.