

Index

A

Advanced Addition	286
Assessment	298, 323
Base means	288
Calculators for checking	308
Changing bases	298
Checking students' work	304
Chip trading	311
Creating addition problems	303, 315
Creating problems in base 10	307
Multibase blocks	310
Names for the numbers	316
Number patterns	292
Opportunities to use addition skills	317
Palindromes	325
Pattern searches	328
Patterns in recording strips	324
Patterns in the bases	312
Place value game	300
Plus one	288
Plus one in base 10	295
Plus one with recording	291
Racing up	296
Why teach different bases	286, 305
Advanced Division	
Assessment	354
Creating division problems with dice	351
Dividing large numbers	349
Not just multiplication in reverse	348
Problems for students to solve	352
Remainders	350
Advanced Multiplication	
Finding the answer	339
Knowing when to multiply	346
Lattice multiplication	342
Matrices in different bases	334
Matrix box for finding answers	341
Patterns for multiplying	343
Patterns in the matrices	337
Problems for students to solve	346
Reasonableness	345
Using knowledge of place value	334
Advanced Subtraction	286
Assessment	323
Creating problems in base 10	308
Creating subtraction problems	305, 315
Minus one	289
Minus one with recording	292
Number patterns	292
Opportunities to use subtraction skills	317
Racing back	298
Algebra	
Algebra is	359, 363
Area	363
Battleships	366
Cartesian coordinates	366
Connecting equations to the graphs	374
Coordinate graphs	385
Coordinate tic-tac-toe	366
Difference between arithmetic & algebra	365
Equivalencies	383
Exponents and roots	375
Function machine	370
Functions and variables	370
Grade-school algebra	361

Graphing calculator	378
Graphing numbers	372
Negative numbers - adding & subtracting	367
Negative numbers - graphing	367
Negative numbers – multiplying	368
Parenthesis	369
Pi (π)	373
Pick's Theorem	365
Proportionality	383
Radicals	376
Rates	382
Ratios	382
Science opportunities	387
Solving equations	379
Volume	364
Writing formulas	371
$y = mx + b$	384
Assessment	4, 64
Advanced Addition and Subtraction	431
Advanced Multiplication and Division	432
Algebra	432
Assessing ourselves	44, 65
Beginning Addition and Subtraction	430
Beginning Multiplication and Division	431
Beginning Number	430
Fractions, Ratios, Money, Decimals	431
Geometry, Shapes, Relationships	430
Graphing, Probability and Statistics	431
Initial assessment	33, 42, 44
Measurement, Estimation and Time	431
Patterns and Connections	430
Portfolios	432
Problems out the door	127
Sorting, Classifying	430
A Thousand Days	451
B	
Balances	
Board and fulcrum	202
Bowl	202
Double-arm	203
Student-made	204
Beginning Addition	
Chalkboards	125
Checking student work	127
Consecutive numbers	133
Creating problems	125, 126, 142
Families of facts	56
Flash cards	55
Number facts	138
Number games	139
Patterns	129
People problems	139
Problems in a stream	128
Problems out the door	127
Questions we might ask	153
Start with, go by	129
Start with, go by, both	131
The class answer	140
True beginning	124
Ways to make eight	133
Writing Stories	144
Beginning Division	
Assessment	239
Creating problems	230
Creating stories	233

Index

Remainders	224	Introducing materials	7
Sideways L	221		G
Three questions.....	223		
Beginning Multiplication		Geoboards	
Assessment	222, 239	Area	58
Beginning means	215	Recording geoboard designs	62
Creating problems	216, 230	Ways to make three	63
Creating stories	233	Ways to make two	60
Families of facts	229		
Flash cards	229	Geometry	
In our lives	235, 237	Angles	107
Matrix	217, 219, 221, 228, 231, 232	360°	113
Multiplication is	215	Compass	115
Multiplication means.....	216	Degrees.....	110
Sideways L	221	Protractor.....	111
Three questions.....	223	What is an angle	108
Beginning Subtraction		Art.....	103
Checking student work.....	136	Assessment.....	120
Creating problems	135, 143	Beginning geometry	93
Problems in a stream	138	Building.....	91
Start with 100, go by	136	With What	92
Writing stories	143	Definition	91
Building behind the shield	85	Geometric awareness	116
	C	Geometric constructions	115
Calculators	128, 285	Geometry is	90
Chalkboards	70	Spatial sense.....	90
Class answers.....	70, 151	Symmetry	96
Classroom management		Tessellation	98
Responsibility	423	Patterns for	99
She did it first!.....	425	Procedures.....	101
The point system	423	Why tessellate	102
Concept, connecting, symbolic	51, 146	Visual imagery	104
	D	Graphing	161
Decimals	268	Asking questions	168
Decimals are	268, 269	Assessment.....	186
Equivalencies.....	270	Build behind the shield	168
Multiplying	271	Connections.....	186
One is what we say it is.....	269, 272	Graphing Canvas	166
Relation between decimals and fractions	262	How to graph.....	161
Teaching decimals.....	269	Kinds of Questions	166
	F	Real graphs.....	160
Fractals	102	Reasons why	161
Fractions.....	240	Spontaneous	165
Adding fractions.....	247	Stream of ideas.....	162
Assessment	248, 252, 281	Writing questions	170
Cube sticks.....	256		H
Equivalencies.....	253, 256	Homework	
Factors.....	259	Advanced Addition and Subtraction	439
Fifty-four marbles	249	Advanced Multiplication and Division.....	439
First lesson.....	241	Algebra	439
Fractions are	240, 249	Beginning Addition and Subtraction	438
Fractions everywhere	251	Beginning Multiplication and Division	438
Geoboards.....	244	Beginning Number	437
Measurement	262	Fractions, Ratios, Money, Decimals	439
One is what we say it is.....	253, 262	Free Exploration and Creative Learning.....	437
People fractions	247	Geometry, Shapes	438
Relation between fractions and decimals	262	Graphing, Probability and Statistics	438
Remainders	250	Measurement, Estimation and Time	438
Start with, go by, both	258	Patterns and Connections	437
Stories for the numbers	261	Sorting, Classifying	437
Subtracting fractions	248		M
Triangle areas	245		
Writing fractions	243	Math in our lives	146
Free Exploration	6, 8, 11, 12	Measurement	
Eight questions	9	Assessment.....	212
		Balances	201
		Creating the environment	192
		Estimation	211

Index

Experiences with	189	Rate tables	267
Map of the class	198	Rates	266
Mapping	195	Rates are	268
Measurement is	189	Ratios are	264
Measuring tools	201	Shadow ratios	264
Questions that we ask	191, 193	Reasonableness	147, 148, 331
Rice and sand	191	S	
Mental Arithmetic	328, 330	Science	390
Money	277	A way of thinking	392
Assessment	280	Boats	400
Money questions	278	Building	408
Teach money by using money	280	Candles	412
N		Changes	402
Ninety-nine lessons	455	Friction, force and motion	418
Kindergarten Sequence	456	Ice cubes	414
First—Second Grade Sequence	459	Liquids	393
Third—Fourth Grade Sequence	465	Magnifying	396
Fifth—Sixth Grade Sequence	473	Moonshine	406
Numbers	34	Objects in water	404
Area - geoboard	58	Paper planes and kites	409
Assessment for Counting	36	Pendulums	410
Cardinal	39	Safety first	391
Counting	34, 36	Seeds and plants	397
Creating the Environment	34	Shadows	416
Doing numbers	40	Teacher questions	392
In columns	37	Worms and other animals	399
Names for numbers	38	Sorting	68
Ordinal	39	Assessment	87
Recording	45	Attribute blocks	76
Saying means knowing what is said	35	Attributes are	72
Surrounding with the concept	44	Expanding thinking and vocabulary	73
P		People sorting	81
Parental involvement		Sharing ways	72
Power Block Lesson	446	Sorting by attributes	69
Three elements	434	Sorting in life	74
Patterns	15	Sorting tree	82
0-99 Matrix	26	Students describe themselves	79
A different way	22, 23	Teaching definitions	75
A-B patterns	17	To teach the meaning of selected words	83
A-A-B patterns	57	Why do we sort and classify	68
Art	28	Speaking math	171
Assessment	19, 26	Spelling notebooks	49, 73
Knowing we can know	15	Statistics	160
Patterns and connections	16	Students checking students work	21, 41
Patterns everywhere	20	Surrounding with the concept	22
Patterns recorded	24	Symmetry	
Searching for patterns	17	Definition	96
Ways to make three	57	Four types	100
Percent	274	Hinged mirrors	97
Are decimals equivalent to percents?	274	Mirrors	96
As a part of life	275	T	
Percent means	275	Time	205
Probability	160, 174	Minutes, miles and meaning	208
Making sense out of patterns	185	Questions that we ask	207
Predicting	174, 184	Teaching time	207
Rolling dice	178	U	
The study of	185	Unifix break-aparts	226
Tossing cardboard squares	174		
R			
Ratios	264		
Bouncing balls	265		
Diagonals	265		

Index

Stories and Philosophical Asides

1,000 words (Are we really as terrible at math as we may think we are?)	123
(23•32) — (13•32) (The x in algebra is more than just a letter.).....	384
A	
A father and his son (Letting students find answers for themselves.).....	141
A hundred years from now (Preparing our students for the future or preparing them for the past?)	283
A single sandwich (There is much more math in a lunch bag than on any workbook page.).....	352
A soccer story (As teachers, we are the makers of the rules.).....	74
A way of thinking (Science is mathematical thinking applied.)	392
As real as language (Why reteach math concepts in each new grade?).....	124
Awareness means (Learning to look and learning to ask why.).....	116
B	
Basing our curriculum on mediocrity (Basing our curriculum on the end-of-year test.)	348
Black-eyed peas (A lesson is a flexible guide and not a frozen script.).....	106
Boys and girls (We can get what we accept or we can get what we expect.).....	422
Brenda's story (Using students' writing as an assessment tool.).....	145
Building (What our students learn from building.).....	91
C	
Calculators (What calculators can and cannot do.).....	128
Checking your neighbor (Students helping each other makes it possible for everyone to learn.).....	21
Concept, connecting, symbolic (Linking concepts to symbols.).....	51
Confusion (Knowing how multiply or divide does not teach us how to use these skills.).....	214
Connections (How do we get learning into memory?)	453
Conversation between two parents (Making word problems real.).....	151
Credibility (We insure the materials we use have credibility with our students.).....	126
D	
Da da wee (Tolerance for answers not yet right.)	33
Driving down the street (The math involved in simply driving down the street.).....	188
E	
Eight questions (Eight questions as a starting point for explorations of all kinds.)	9
Everybody Counts (A Report to the Nation on the Future of Mathematics Education,)	442
Everybody wants to learn (Even repeated failure does not discourage our natural desire to learn.)	421
Excuses (Rationales for not letting students use calculators in class.).....	285
F	
Fast finishers (Right answers do not always measure understanding.).....	277
Fifty-four marbles (Thinking about what numbers actually represent.)	249
First up, last up (Giving everyone the chance to feel that he or she can win.)	297
Fraction (Children are familiar with fractions from their earliest years.).....	240
G	
Geoboards and Mr. Pick (Pick's theorem for areas on the geoboard.)	364
Geometry is (Geometry is more than we learned it was in high school.)	90
Grade-school algebra (Why teach algebra in the elementary grades?).....	361
Guides and nothing more (The lessons in this book are guides and nothing more.)	240
H	
Hayley (Understanding or misunderstanding the concept of area.)	59
How long do we linger? (When to move on and still leave no one behind.)	44
How many times? (What does it mean to multiply?)	333
I	
I believed (When we provide the answers, we eliminate our students' need to think.)	327
I wasn't really reading it (Focusing on what is right and not on what is wrong.)	65
J	
Jill (Experiencing shapes.)	94
John's story (Why do we learn algebra in school?).....	359
K	
Kevin and the teacher, of course (Kevin uses his graphing skills to prove a point.).....	161
Knowing we can know (Children who know they can know are children who will learn.)	15
L	
Learning from the lesson (What we can learn from the lessons that we teach.)	309
Let the answer be (We must learn to let our students be wrong sometimes.).....	140
M	
Mathematics is in (An overview of the math in our lives.)	1
Meaning (What makes sense is kept in long term memory.)	454
Michelle (Calm teacher, out-of-control class.).....	422

Index

Monica and Andrew	147
Mr. Smith (Making word problems real.)	150
N	
No limit to the learning (Is the time spent on free exploration worth it?)	6
No nonsense (We use our knowledge of how the brain works in our lessons.)	454
No one knows anything (Our initial assessment.)	33
Not equivalent (Why decimals are not equivalent to percents.)	274
Number-one responsibility (Our students are our number-one responsibility.)	451
Numbers are different than letters (Numbers are learned in patterns to be seen.)	37
O	
One busy teacher says (Do we have the time to teach this way?)	3
One question (An example of the questions that can follow once one question is asked.)	388
P	
Painting the house (Thinking about the numbers or knowing the algebra to use?)	253
Playing with the blocks (The value of free exploration.)	241
Problems out the door (A quick assessment tool.)	127
R	
Rats (Mind's growth comes from a rich and varied environment with which to interact.)	453
Ryan (Understanding or misunderstanding the concept of area.)	58
S	
Science (Science is a way of thinking, too.)	390
Seventeen and seventy (Why we assume no one knows anything.)	38
Something we see (Why is seeing-patterns important.)	14
Sort and classify (Sorting and classifying as a part of life.)	68
Speak math (Looking for the math that surrounds us all the time.)	171
Speaking math again (We can teach the parents of our students to speak math, as well.)	435
Stories (Stories of people using math with and without understanding.)	122
Surrounding the child with the concept (Creating the learning environment.)	22
T	
Terry (Our pattern-seeking minds are lightning-quick.)	67
The awareness we create (Is the purpose of the question the answer or stimulating thought?)	252
The basics (What is a 'basic' education, anyway?)	434
The eighteen year old (People do not always know what we think they know.)	206
The house across the street (Opportunities that arise or that we create show math in use.)	118
The letter carrier (Teaching negative numbers.)	367
The math that is already there (Encouraging our students to see the math in their own lives.)	146
The SCANS basics (What Work Requires of School - A SCANS Report)	439
The story told again (Why do we learn algebra in school?)	360
They do not know what we think they know (Children's thinking is not like ours.)	32
Thirty children, thirty teachers (None of us is as smart as all of us.)	19
TIMSS (Third International Mathematics and Science Study)	357
To think mathematically (What does it mean to think mathematically?)	16
Too late for the beginning (Billy is three, and so is Billy's dad.)	32
True stories (Examples of adults who have not learned to see patterns in math.)	342
Twenty problems or just one (Encouraging our students to see the math in their own lives.)	152
V	
Vending machine (Students learn about money by using it.)	278
W	
We can be thankful (We use sorting and classifying nearly everywhere in life.)	87
We sort (What do we sort in life?)	74
We take away the math from girls (The math in toys matters.)	92
What does it mean to begin? (Learning starts before students start school.)	124
What might we learn (Learning is a natural part of everybody's life.)	208
What we have to understand (Do we have the background for teaching math?)	2
Why tessellate? (Patterns in math are visual as well as numerical.)	102
Windows (Mathematics is all around us all the time.)	68
Wondering means (Learning to ask why.)	117

Index

Questions from Teachers

Free Exploration and Creative Learning

How much time should we allow for free exploration?.....	11
What materials do we allow our students to explore?	12
Is there any particular assessment we should be making as our students are free exploring? If so, what should we be assessing?.....	12

Patterns and Connections

Most of my students can see the A-A-B and A-A-A-B patterns, but I'm not so sure about two or three children in particular. Do I wait until I am sure these children see, or do I go on to the next lessons? I hate to hold everyone else back for these few.....	28
Some of my students still want to play with the materials when I am trying to get them to create patterns. What do I do if some of my students are still at the free exploration level when I want to begin work with patterns?.....	28
The dialogs in this chapter show the teacher using words like diagonal, parallel, symmetry, create, predict, extend, matrix and so on. At what grade level would you recommend beginning to use words like this with students.	29
Is there a particular sequence of A-B patterns that is recommended?	29
What if I don't see the patterns my students see?.....	30

Beginning Number

Don't we have an obligation to move every child along as fast as that child is ready to move? Are not we doing a disservice to all children by having the slowest children be the determiners of the progress of the whole class?.....	64
Do we really move everyone at once?.....	65
Not all children are ready to explore the same number at the same time. Shouldn't we either assess each child to determine that child's appropriate starting number, or at least determine a range of numbers for each child to explore?	65
How do we know when to move on? How do we know when we have left no one behind?	65
You indicate children should write numbers to record their designs. Do you mean kindergarten children as well?	66
How do we keep track of where our students are, for ourselves, for the parents, for our administrators and for next year's teacher?	66

Sorting, Classifying, Expanding Language

How do we assess that a child is learning what he or she is supposed to be learning as we are teaching about sorting and classifying? ..	87
You refer to the sortings or the rules for sortings as "patterns." Isn't this confusing? When my students and I use the word <i>patterns</i> , we mean things like A, A, B, A, A, B and not two-hole, four-hole sorting.....	88
It seems that all of the sortings in this chapter use only two groups. When do students sort into groups of threes and fours and fives?	88
When would you use Venn diagrams?	88

Geometry, Shapes, Relationships and Constructions

How do we assess geometry?.....	119
---------------------------------	-----

Beginning Addition and Subtraction

How do we assess?	156
In Lesson One, all students worked for the same amount of time and not for the number of problems to be solved. But they were making up their own problems with squares. When we make up the problems, what is a good number of problems to give? How long should our students work?	157
Within each lesson there appears to be no grade-level separation. The advanced problems are mixed in with the easier ones. In Lesson One, for example, taking handfuls of squares to make up addition problems is included right along with finding series of consecutive whole numbers. How do we know which problems to do for which grade levels?	157
I use something I call "Mad Minute" to time test my students on the basic facts. I find it very useful for drilling my students on quick recall. The students are given a minute to do the problems on each of several increasingly difficult pages. One page per minute, one minute per day. Once they can do one page at a mastery level, they pass on to the next sheet of problems for the next Mad Minute drill. My students love it and they learn so much from this test of speed. What harm is there in this?	158

Graphing, Probability and Statistics

The probability in this chapter involves tossing cardboard squares and rolling dice and comparing the charts and graphs our students make.	
The only conclusion was that some things are more likely to happen than others. What about odds and ratios and all the rest of probability?	186
As we teach our students to graph, what is our assessment to be?.....	186

Measurement, Estimation and Time

How do we assess measurement activities?	212
How do we know what to do in measurement for each grade level when the lessons do not say what grade level they are for?	213
I am not sure what it is we are supposed to teach our students about measurement. Are they just supposed to problem solve and make maps, or are there particular measuring skills to teach, like knowing how to use a ruler?.....	213

Beginning Multiplication and Division

Some of the multiplication and division problems the children encounter as they look for problems in their own environment may be very, very large. Should we delay trying to find answers to the larger problems until we are teaching advanced multiplication and division?.....	237
Shouldn't we put more emphasis on memorizing the facts? Knowing all the facts will help our students pass the tests they face in school.	
Knowing all the facts will help our students find the answers faster when they leave our room.	237
Is there a particular assessment we should use?	238

Fractions, Ratios, Money, Decimals and Percent

How do we assess for fractions? What kind of assessments might we use?	281
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Index

In Lesson Ten, for the multiplication problems on the geoboard, the fractions in the answers were not reduced. Why not? Isn't the teaching of the reducing of fractions an extension of equivalencies?	281
Advanced Addition and Subtraction	
In the books <i>Mathematics Their Way</i> and <i>Mathematics a Way of Thinking</i> , children made up words like "yuck" and "zurkle" to describe the groupings for the different bases. Why are made-up words not used in this book?	331
Doesn't it confuse students to see 10 and not have it mean ten? Wouldn't it be easier to have our students say "ten" whenever they see 10 written, or "eleven" whenever they see 11, regardless of the base? Saying "one cup, one" seems so artificial.	331
Is it really fair to let our students use calculators in school any time they want?	331
What do we do when there are report cards we must fill out?	332
Advanced Multiplication and Division	
What is our assessment of division to be?	354
It seems that the more advanced the chapter, the fewer the problems we give the students to practice what they have learned. The <i>one-sandwich</i> problem could occupy our students for a week. Why do we seem to do fewer problems as we advance and not more?	355
Algebra	
At what grade level should we begin teaching algebra?	387