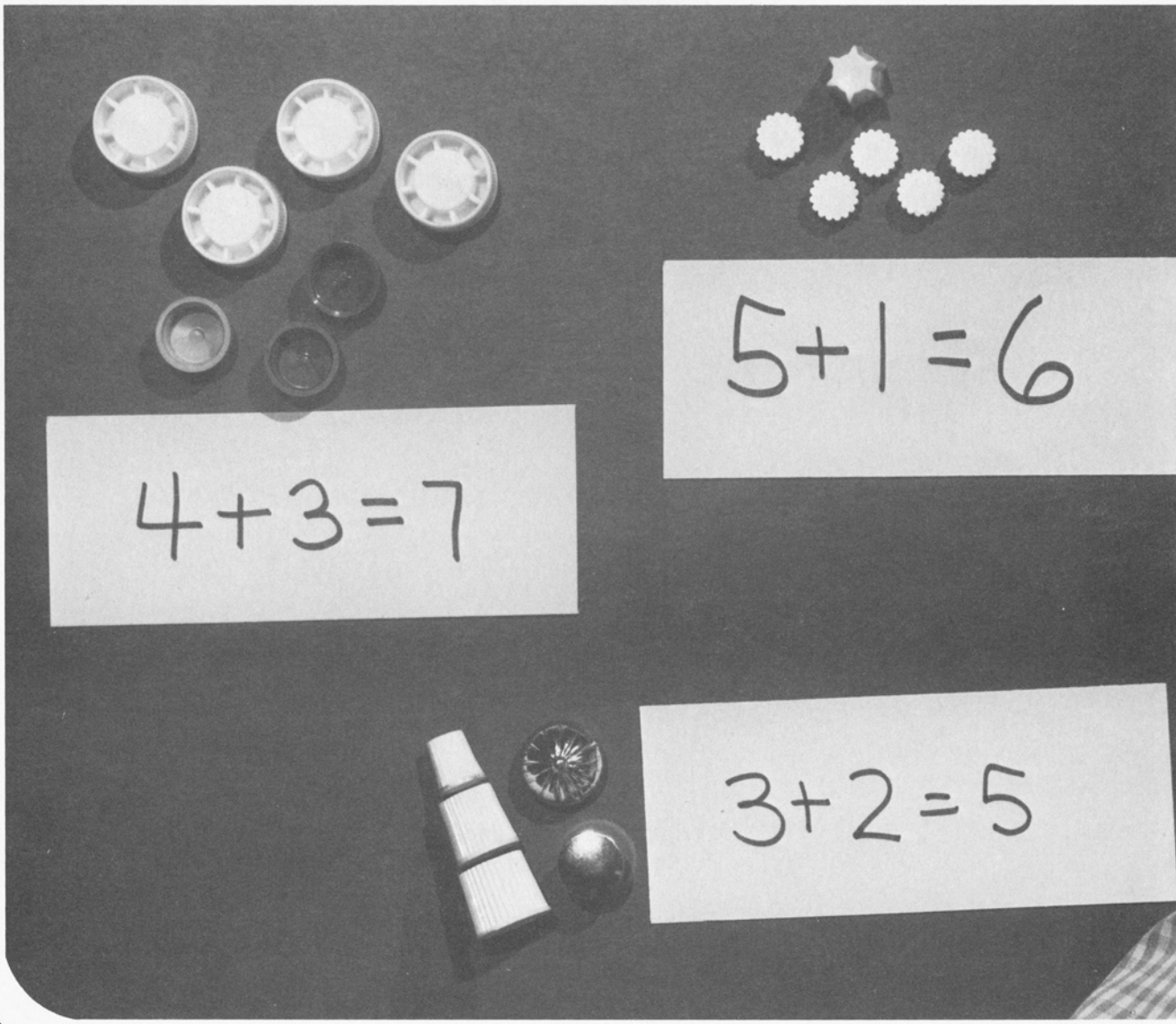
9

NUMBER AT THE SYMBOLIC LEVEL



9

SKILLS AND CONCEPTS

Extending the concept of number
Measuring time, water displacement, volume, and quantity with non-standard units
Adding and subtracting
Comparing
Making and testing predictions
Using symbols to record events

SELF CONCEPT AND SOCIAL INTERACTION

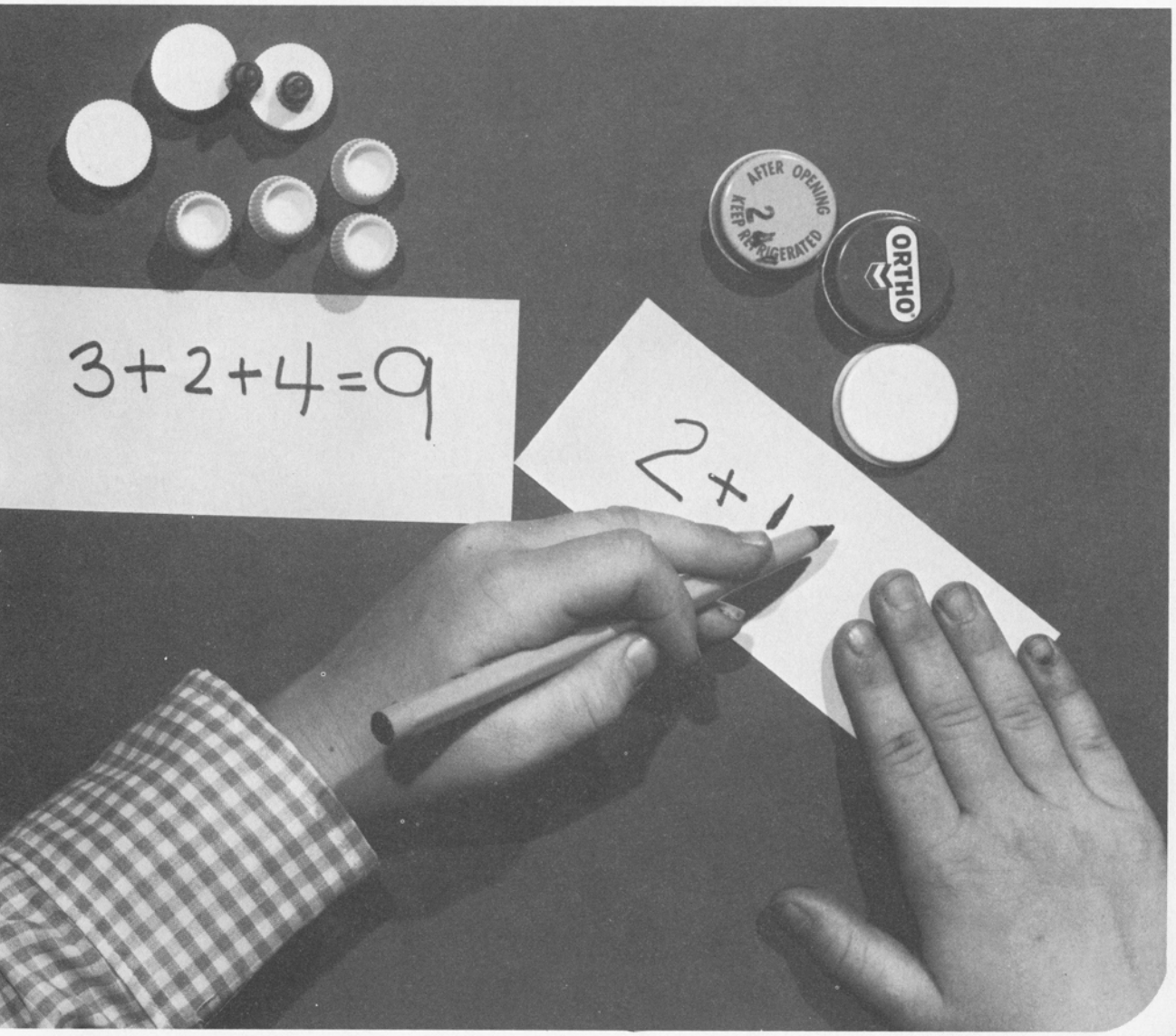
Accepting the responsibility of one's own actions by learning to operate in an independent, self-directed manner
Heightening awareness and visual imagery as a result of using all five senses in learning

FUTURE APPLICATIONS

Understanding functions
Measuring with standard units

PREREQUISITE CHAPTERS

Free Exploration
Counting
Comparing
Number at the Concept Level



9

NUMBER AT THE SYMBOLIC LEVEL

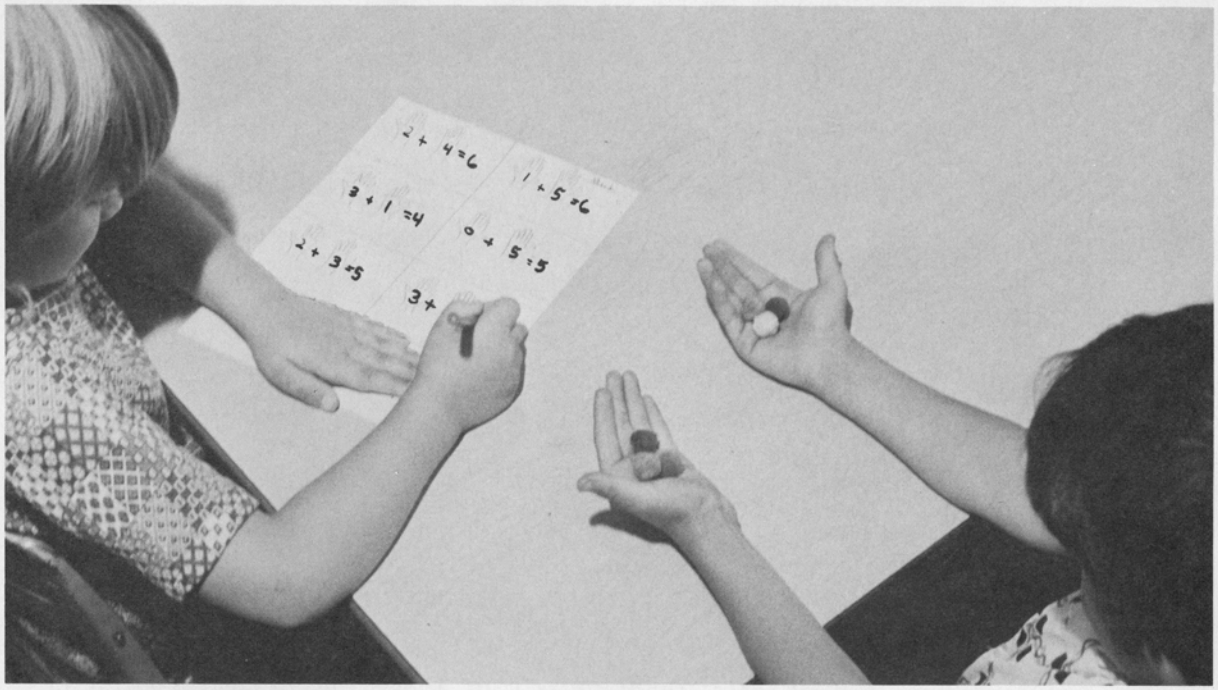
INTRODUCTION

At the symbolic level of numbers, the child learns to record a known concept abstractly. The following activities give children many opportunities to write the mathematical symbols which they have related to various number concepts at the connecting level.

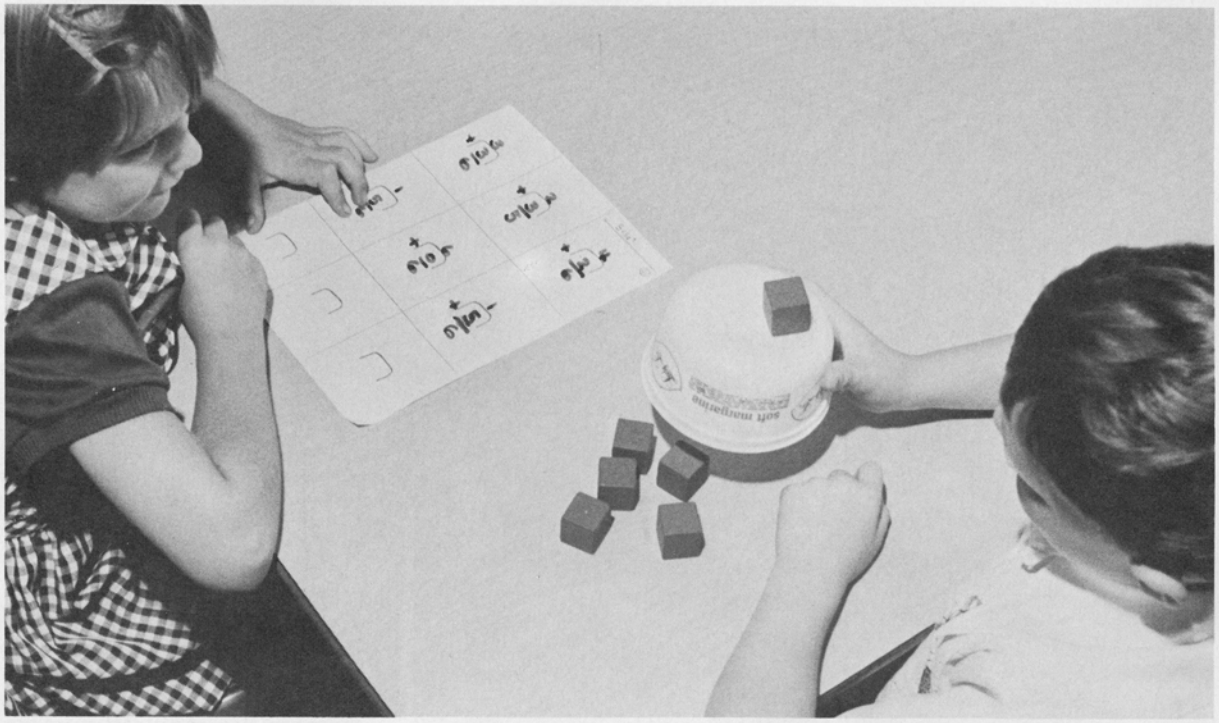
See page 43 for a description of the development of the necessary motor coordination for writing each numeral. Those activities are prerequisites to the following activities.

In the previous section, Introducing Number at the Connecting Level, the children labeled groups of objects with numerals but were not required to *write* any of those numerals. They were merely linking up the concept of various numbers with the appropriate mathematical symbols. The activities at this level, Number at the Symbolic Level, assumes the connection was made and enables children to make written records in the same format, recording various mathematical experiences.

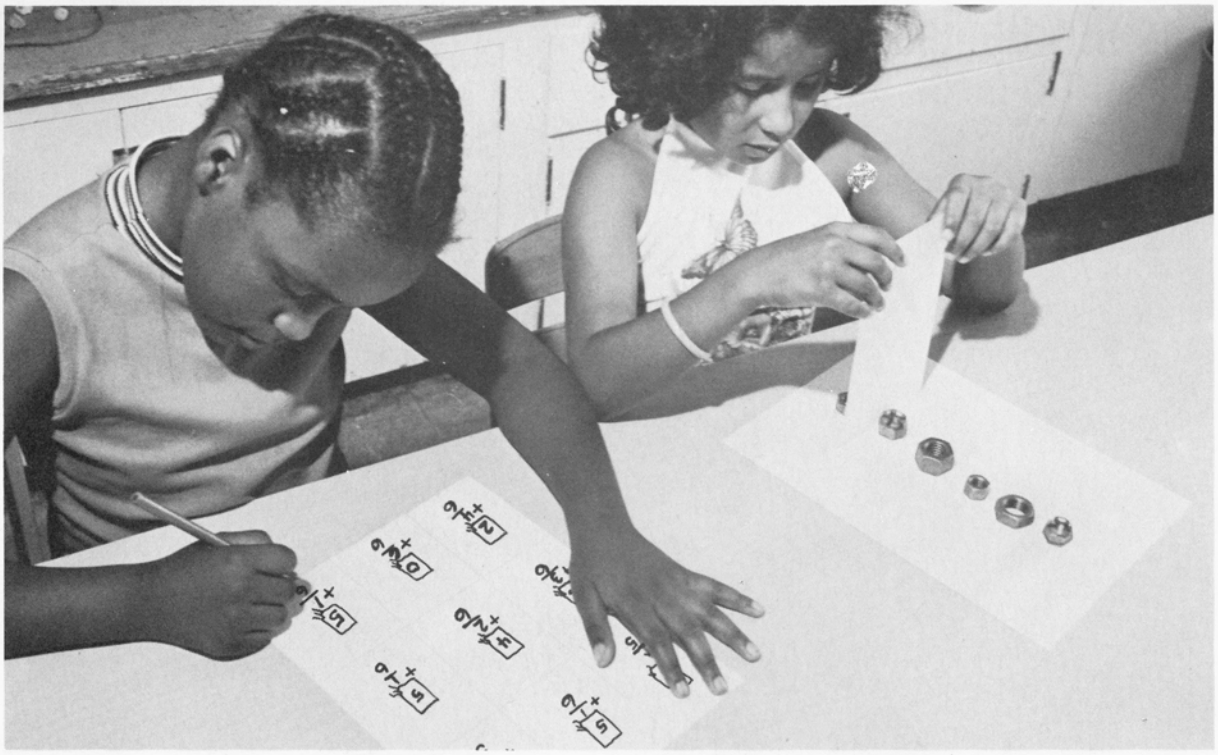
The Old Games



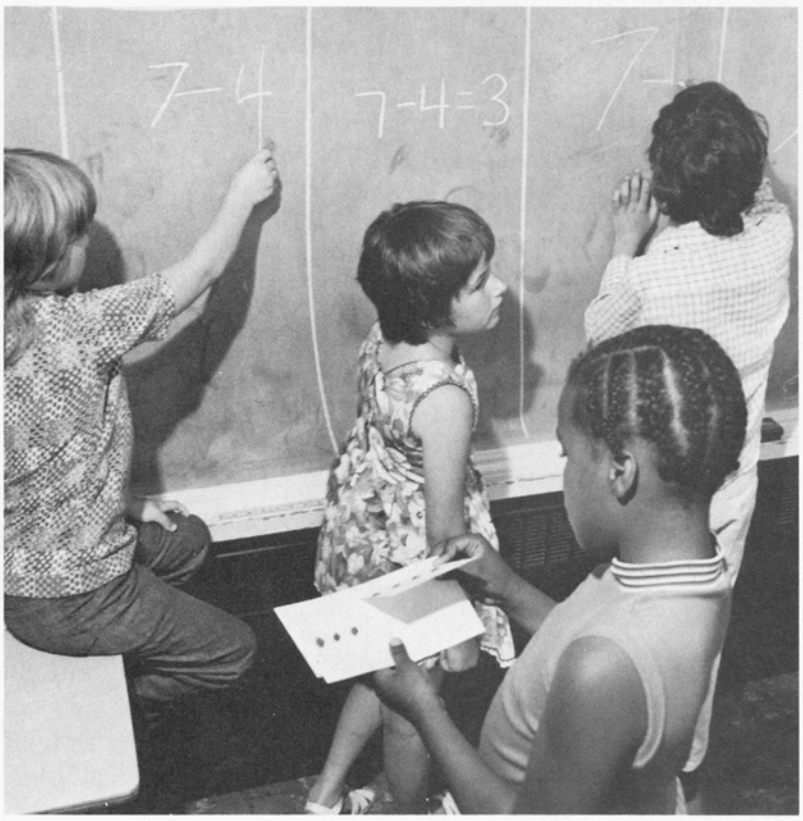
The Hand Game: (p. 180, 221) One child creates combinations in hisorher hands with a given number of objects. A second child or group of children writes down the combinations formed (see Worksheet 43).



Lift the Bowl: (p. 181, 222) One child arranges the blocks and a second child or group of children writes the combinations (see Worksheet 42).

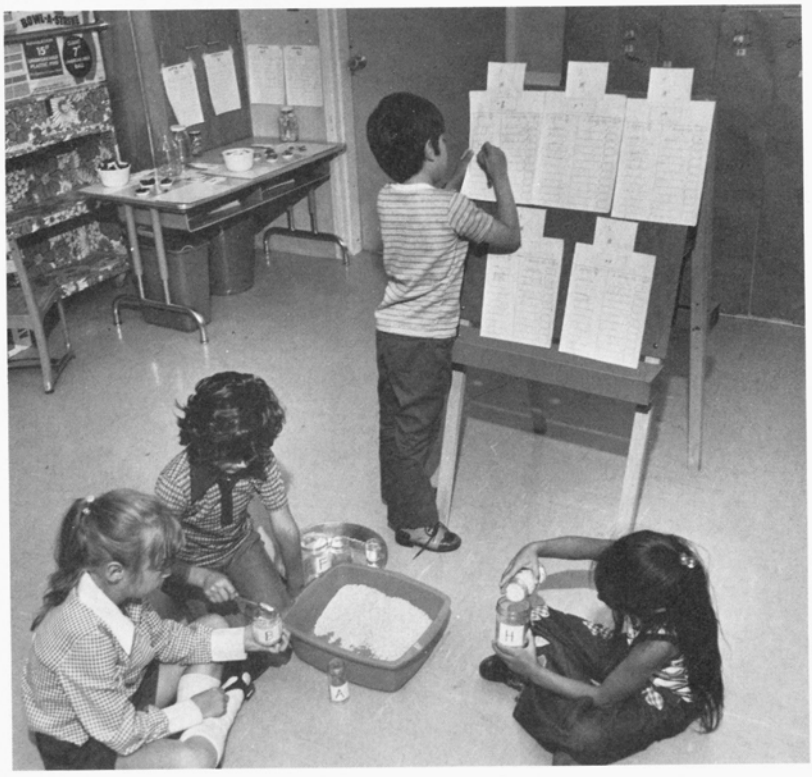


Peek Through the Wall: (p. 183, 222) One child walls off a particular number with a piece of tagboard. The solid card encourages the child to recall the images recorded on earlier occasions using the clear acetate. A second child or group of children writes down the combinations formed (see Worksheet 44).



Subtraction Cards: (p. 193) One child reads the subtraction problem. A second child or group of children subtracts the required number of objects from a work space and then records the equation.

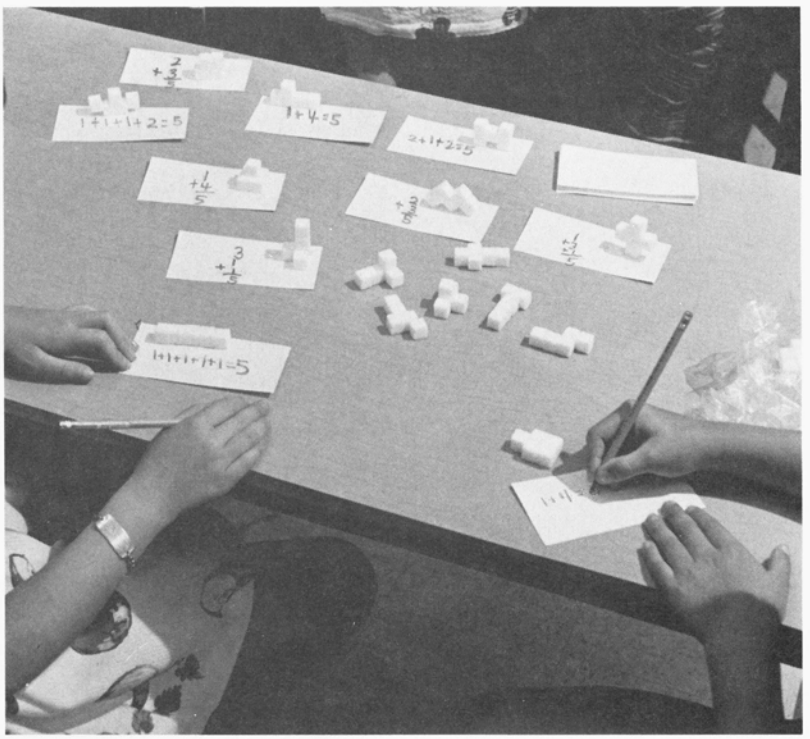
The bean, Unifix, jewel, and Say it Fast books: (p. 178) The children make little books from records made at the number stations. The combinations are recorded on each page.



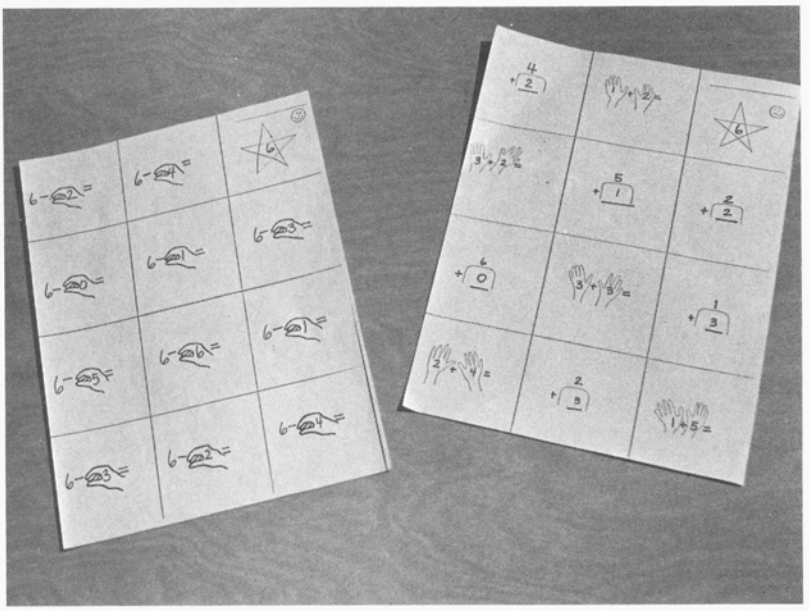
Measuring with the Jars: (p. 131) Pin four or five records from the "too little" section on a nearby bulletin board. Each child selects a record to work with. The earlier activity is repeated and then the child experiments to find the number of spoons of rice that must be added to fill the second jar. This number is recorded in the square at the top of the recording sheet (Worksheet 23).

Other children duplicate the experiments and list their names under agree or disagree.

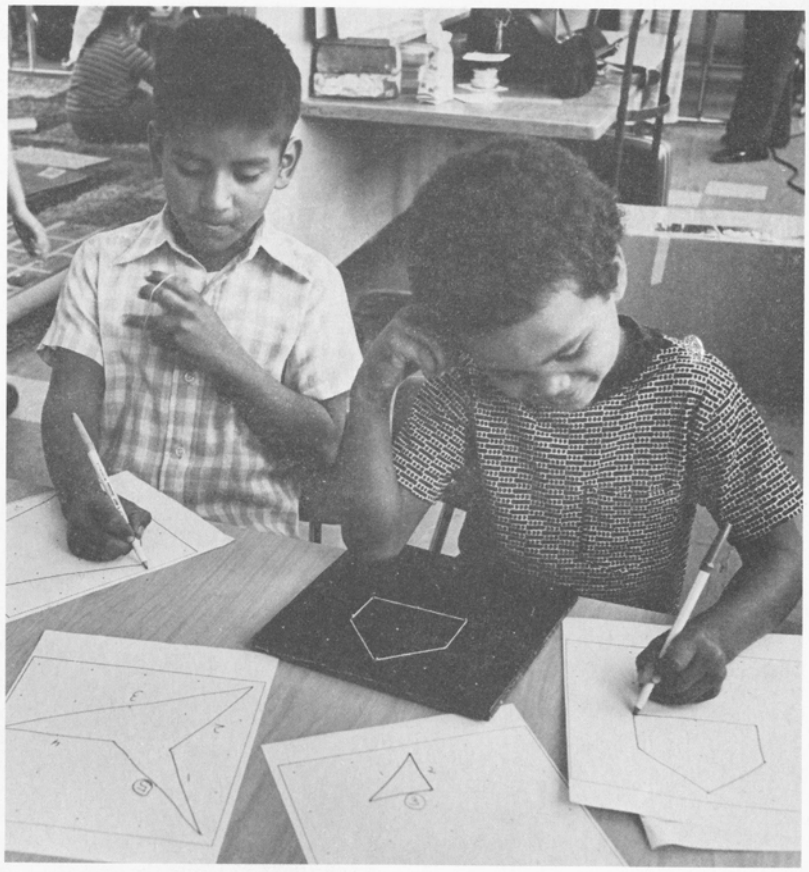
This activity is repeated many times, changing the records used each time and alternating recordings from the "too little" and "too much" sections to allow the children to experiment with both addition and subtraction.

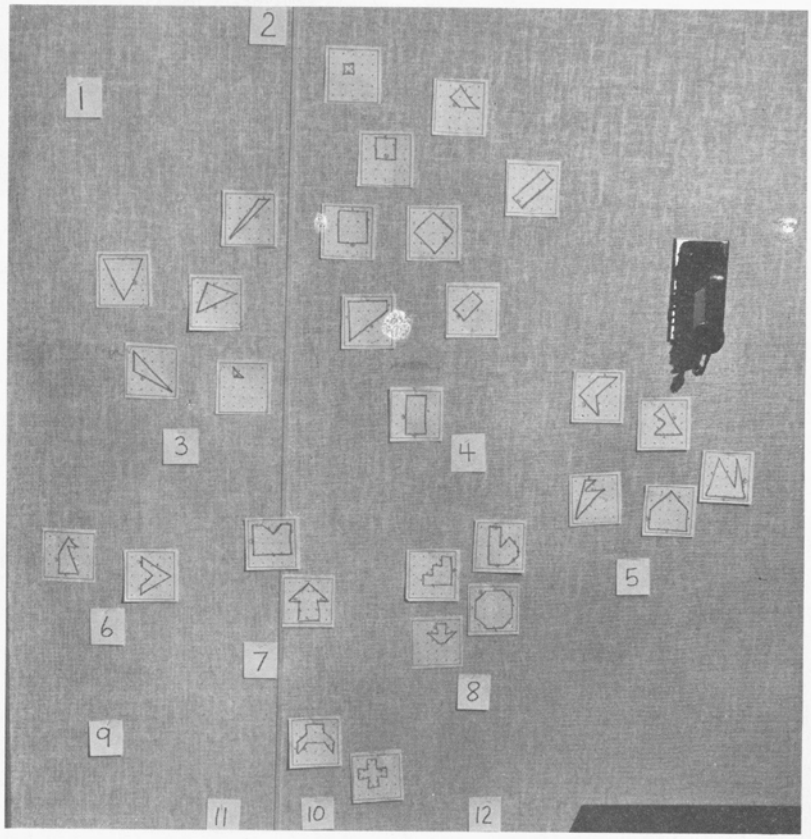


Wooden Blocks: (p. 178) The children write the combinations formed with the sugar cube records.



Symbolic Level "Test" Dittos: (See Worksheets 45–53.) The children compute the answers to these worksheets without the aid of manipulative materials.

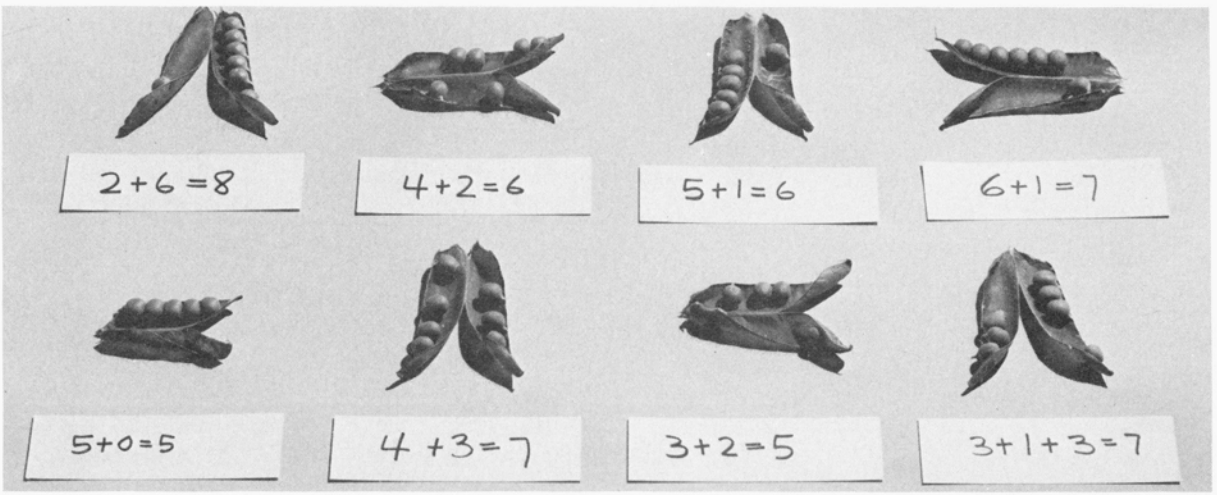




Geoboard Patterns: The children make shapes on their geoboard and sort them by the number of sides.

APPLICATION AND EXTENSION OF NUMBER AT THE SYMBOLIC LEVEL

Introduce the following activities when the children know the symbols up to ten. The goal of these activities is to develop the children's ability to use symbols in a natural way for recording a variety of experiences.



Numbered Squares

SKILLS

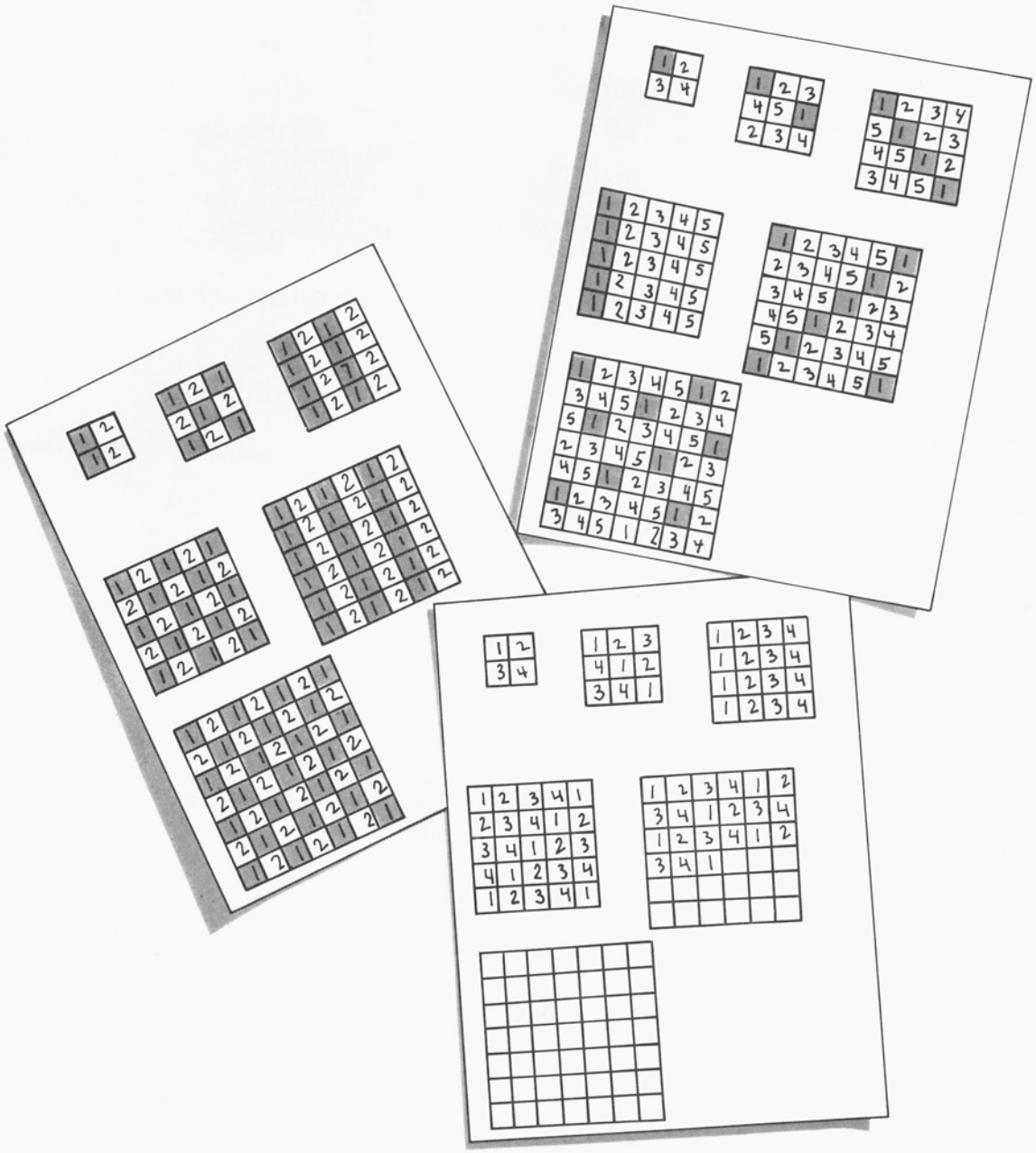
Pattern
Writing mathematical symbols
Ordering

MATERIALS

Worksheet 54, pencils, crayons

ACTIVITY

The children write numbers on the worksheet beginning each matrix with "1." If a child is practicing the numerals up to six, the series, 0,1,2,3,4,5,6, is repeated in each matrix as many times as possible. When each matrix is complete, the children color in all the "1's" and look for patterns.



Growing Plants

SKILLS

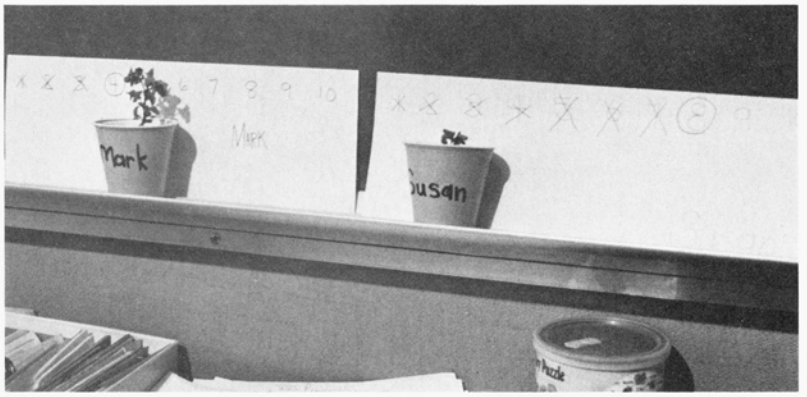
Labeling concepts with mathematical symbols
Measurement
Time

MATERIALS

Seeds, growing containers, paper, number line template*

ACTIVITY

The children plant some seeds and observe the growth each day. Cross out the days when there is no visible growth above ground. Circle the day when growth is first seen.



On which day did your plant begin growing above ground?

Dice Toss

SKILLS

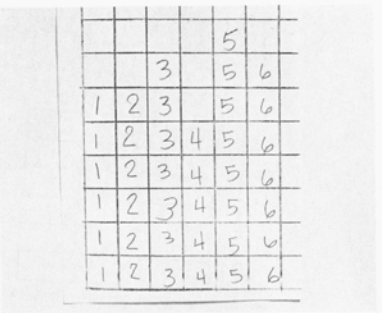
Probability
Graphing
Writing mathematical symbols

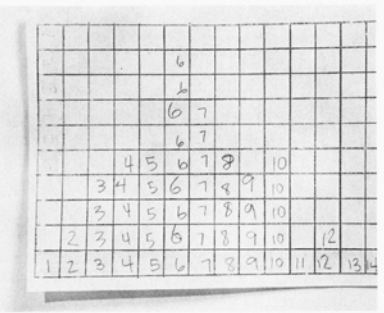
MATERIALS

Dice, paper

ACTIVITY

The children toss a die and record the results in the form of a graph. Some children will enjoy tossing two dice and recording the total number on a graph.





A Rock Bath

SKILLS
Water displacement
Measurement
Ordering
Comparing

MATERIALS
Rocks, a wide mouthed plastic container, paper

ACTIVITY
A child places rocks, one at a time, into a wide mouthed container and records the distance the water rises for each rock.

The Grab Bag

SKILLS

Developing the sense of touch
Comparing
Money

MATERIALS

A penny, nickel, and dime in a paper bag for each child participating; more-and-less spinner*

ACTIVITY

The teacher turns the more-and-less spinner over and divides the circle into three equal sections, in which sheorhe writes 1¢, 5¢ and 10¢ and then reattaches the circle to the spinner.

One child spins the spinner. If it lands on 5¢, the children reach into their bags and attempt to remove the nickel using only their sense of touch. The child with the spinner enters the value of each coin on a graph.

Addition with Unifix Cubes

SKILLS

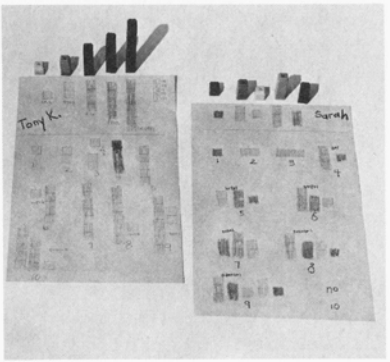
Addition
Problem solving
Writing mathematical symbols

MATERIALS

Unifix cubes,* paper, number line templates*

ACTIVITY

The children build five stacks of Unifix cubes using a different color for each stack. The stacks do not necessarily have to be different heights. The child records the stacks at the top of hisorher paper and then combines the stacks attempting to make each number on the number line. Each time the child is able to build a number, sheorhe records it.



This activity should be repeated many times using different stacks of cubes each time.

Time Trials

SKILLS

Counting
Comparing
Writing mathematical symbols
Time

MATERIALS

Clothespins, an index card, a timer such as a metronome, a tape recording of a bell ringing in one second intervals or a pendulum

ACTIVITY

The children sit in a circle with one child designated as the starter. This child wears a hat or a yarn necklace to clearly delineate himorher from the rest of the children. Each child picks up a clothespin and the starter passes a card to the next child. This child takes the card with hisorher clothespin and passes it on. The card is passed around the circle from clothespin to clothespin until it gets back to the starter. When the children understand the activity, ask another child to be the timer to count the bells, the swings of the pendulum, or the clicks of the metronome. After each trial the timer enters the time.

This activity can be repeated frequently, changing the activity that is timed.

Number Sequences

SKILLS
Ordering
Sorting

MATERIALS
Numbered dice, Unifix cubes*, more-less spinners*

ACTIVITY
The children take turns rolling six dice and arranging them in sequence. Each child takes the same number of Unifix cubes as the number of dice put in sequence. When the children want to end the game, they spin the spinner to determine the overall winner.

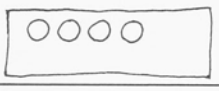
The Magic Box

SKILLS
Pattern
Addition
Subtraction
Making and checking predictions
Counting on

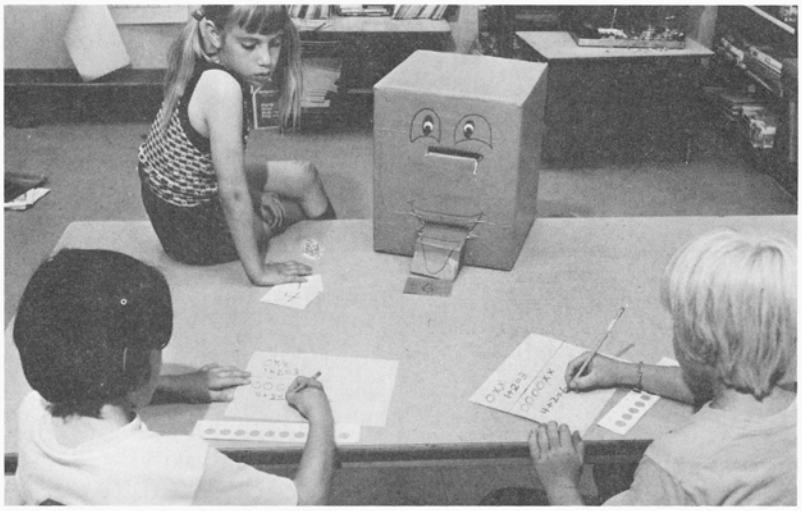
MATERIALS
The magic box,* magic box cards,* number line templates,* pencils and lined paper

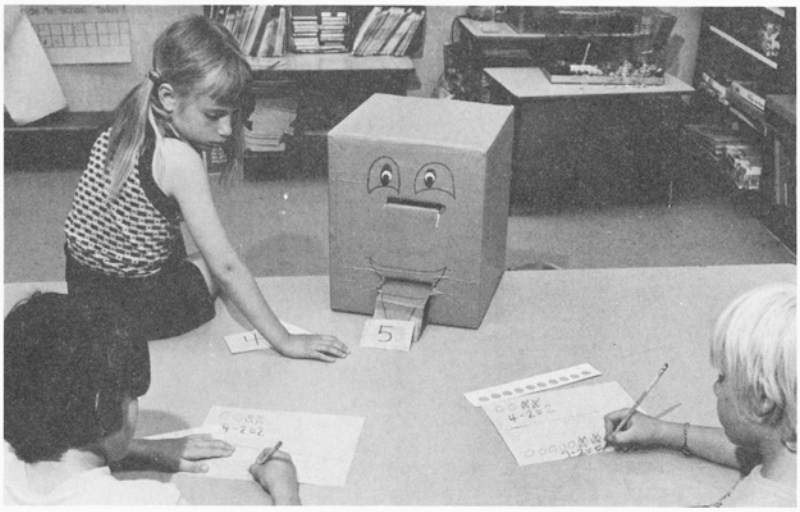
ACTIVITY
The children work in a group. The teacher puts the cards into the magic box and directs the activity.

SAMPLE TEACHING STRATEGY

TEACHER	CHILDREN
"What number is on the white side of the magic box card?"	"Four."
"Make four circles with your number line template."	
"Are you ready? Watch me put the card in the magic box." (The teacher puts the card into the top of the box, and the children watch it slide out of the bottom showing the number six.	"Six."
"Make enough "x's" to get to number six."	
"Slide your template down and read the problem."	"Four and two is six."
"Now write the problem."	The children write, 4 + 2 = 6.

Repeat this with all the cards in one set, but ask a child to take over your role as teacher. When the children have gone through all the cards in this set, put the cards in the box the opposite way, red side first, and have the children write the subtraction problems.





SAMPLE TEACHING STRATEGY

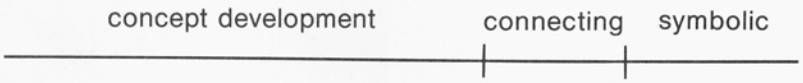
TEACHER	CHILDREN
"In goes number . . ."	"Seven."
"Make that many circles with your number line template."	
"Watch what number comes out this time." (The card slides out the bottom.)	"Five!"
"Cross off enough circles to get down to five."	
"Slide your template down and read the problem."	"Seven take away two is five."
"Write the problem."	The children write, $7 - 2 = 5$.

QUESTIONS FROM TEACHERS

ABOUT HOW LONG SHOULD CHILDREN WORK AT THE CONNECTING STAGE BEFORE GOING ON TO THE SYMBOLIC STAGE?

The purpose of the connecting stage is to link the concept level and the symbolic level. It provides the children an opportunity to see the symbols which visually represent the concept being explored. In the Three Games, for example, the concept level gives children an opportunity to hear and create various addition combinations for a particular number. The connecting stage allows children to see the mathematical symbols linked up with those now familiar combinations. This level can be vividly recalled at the symbolic level when the child makes a written record of hisorher combinations or experiments to find the answer to various written problems.

I think of the proportion of the three stages in this way:



In practice, we play the Three Games at the concept level for two or three days, at the connecting level for one day, and at the symbolic level for one or two days. Then I assess the child's mastery of the combinations for that number.

IS IT NECESSARY FOR THE CHILDREN TO CONNECT *EVERY* NUMBER WHEN PLAYING THE THREE GAMES?

Yes, it seems to be crucial. If you feel your children need to move more quickly, perhaps they could connect for part of the period and work symbolically for the rest of the time. Connecting doesn't need to be long to be effective, but it is a critically important step.

IS THE TEXTBOOK EVER AN APPROPRIATE ACTIVITY OR SHOULD THESE ACTIVITIES COMPLETELY REPLACE TEXTBOOK ACTIVITIES?

If you have access to a textbook for your children, *now* is an appropriate time to introduce it. The children have had an opportunity to discover the number combinations in a variety of situations and have recorded those combinations at both the connecting and symbolic levels. Because they have already discovered and internalized these concepts, they will have no difficulty with a textbook. The pages will merely be a reinforcement of concepts they already understand. The children will merely have to decode the *form* of the pages and not the mathematical process required.

As long as this kind of work is freely selected by children from a variety of other options and not imposed upon the children as their sole experience in mathematics, it does no harm. The real danger is when *any* symbolic-level material is the child's initial introduction to a concept and when it completely substitutes for real-life experiences and concrete mathematical materials. Keep in mind that no matter how attractive and compelling pages of drill may be to a child, they cannot provide the exploration which leads to the discovery of the relationships of volume, weight, height, length, duration, logical thinking, or pattern. Computation is but a small part of the real world of mathematics. Nonetheless, it *is* a part and should not be slighted.

AT WHAT POINT SHOULD CHILDREN DEMONSTRATE "MASTERY" OF NUMBER FACTS OR IS THAT IDEA OUTDATED?

No, mastery will never be outdated, although the focus has changed. Speed of calculation is no longer the primary focus. The emphasis now is on understanding in a variety of circumstances, not just being able to "fill in the empty box." Accuracy, being able to figure out the correct answer consistently, is also more important than speed.

You should expect mastery at the symbolic level of the Three Games. If you do not get mastery, do not move to a new number. Give the child more *variety,* not more of the same. Then reassess for mastery. (See p 187)

The only thing "outdated" is the idea of relentless repetitious "drill" in the form of either workbook pages or speed-oriented flash cards (unless of course it is freely *chosen* as an activity by individual children). The availability and widespread use of inexpensive electronic calculators make the skill of speed no longer necessary as a human mathematical skill. Our minds are needed for understanding the processes involved in calculations and in estimating so we can recognize a reasonable answer when we see one.

Once children have explored a given number thoroughly at both the concept and connecting levels, they should achieve mastery of the combinations for this number before exploring the next higher number. The Symbolic Level Test Dittos (see Worksheets 45–53) give children an opportunity to show mastery of their facts at any given level. These worksheets are to be filled in without the use of manipulative materials as the final test of the child's having progressed from the concrete to the abstract level. These worksheets may be repeated as many times as is necessary. A child should solve these problems quickly, confidently, and correctly after experiencing the concept and connecting level activities for any given number.