Curriculum and Evaluation Standards for School Mathematics Grades K - 4

Note: The information that follows is a summary of the thirteen standards recommended for K-4 by the National Council of Teachers of Mathematics, *Curriculum and Evaluation Standards for School Mathematics,* pp. 15 - 62. It is highly recommended for every school to purchase copies of the *Curriculum and Evaluation Standards for School Mathematics* and *Professional Standards for Teaching Mathematics* for every teacher (or, at the very least, the schools could purchase several copies of each book and place them in the school's professional library for teachers to sign out). Ordering information: NCTM, 1906 Association Drive, Reston, VA 22091; (800) 253-7566.

The Need for Change

The need for curricular reform in K-4 mathematics is clear. Such reform must address both the content and emphasis of the curriculum as well as approaches to instruction. A long-standing preoccupation with computation and other traditional skills has dominated both *what* mathematics is taught and *the way* mathematics is taught at this level. As a result, the present K-4 curriculum is narrow in scope; fails to foster mathematical insight, reasoning, and problem solving; and emphasizes rote activities. Even more significant is that children begin to lose their belief that learning mathematics is a sense-making experience. They become passive receivers of rules and procedures rather than active participants in creating knowledge.

The Direction of Change

The introduction describes a vision for school mathematics built around five overall curricular goals for students to achieve: learning to value mathematics, becoming confident in one's own ability, becoming a mathematical problem solver, learning to communicate mathematically, and learning to reason mathematically. This vision addresses what mathematics is, what it means to know and do mathematics, what teachers should do when they teach mathematics, and what children should do when they learn mathematics. The K-4 standards reflect the implications of this vision for the curriculum in the early grades and present a coherent viewpoint about mathematics, about children, and about the learning of mathematics by children.

Children and Mathematics: Implications for the K-4 Curriculum

An appropriate curriculum for young children that reflects the *Standards'* overall goals must do the following:

1. *Address the relationship between young children and mathematics.*

   Children enter kindergarten with considerable mathematical experience, a partial understanding of many concepts, and some important skills, including counting. Nonetheless, it takes careful planning to create a curriculum that capitalizes on children's intuitive insights and language in selecting and teaching mathematical ideas and skills. It is clear that children's intellectual, social, and emotional development should guide the kind of mathematical experiences they should have in light of the overall goals for learning mathematics. The notion of a *developmentally appropriate* curriculum is an important one.

   A developmentally appropriate curriculum encourages the exploration of a wide variety of mathematical ideas in such a way that children retain their enjoyment of, and curiosity about, mathematics. It incorporates real-world contexts, children's experiences, and children's language in developing ideas. It recognizes that children need considerable time to construct sound understandings and develop the ability to reason and communicate mathematically. It looks beyond what children appear to know to determine how they think about ideas. It provides repeated contact with important ideas in varying contexts throughout the year and from year to year.

   Programs that provide limited developmental work, that emphasize symbol manipulation and computational rules, and that rely heavily on paper and-pencil worksheets do not fit the natural learning patterns of children and do not contribute to important aspects of children's mathematical development.

2. *Recognize the importance of the qualitative dimensions of children's learning.*

   The mathematical ideas that children acquire in grades K-4 form the basis for all further study of mathematics. Although quantitative considerations have frequently dominated discussions in recent years, qualitative considerations have greater significance. Thus, how well children come to understand mathematical ideas is far more important than how many skills they acquire. The success with which programs at later grade levels achieve their goals depends largely on the quality of the foundation that is established during the first five years of school .

3. *Build beliefs about what mathematics is, about what it means to know and do mathematics, and about children's view of themselves as mathematics learners.*

   The beliefs that young children form influence not only their thinking and performance during this time but also their attitude and decisions about studying mathematics in later years. Beliefs also become more resistant to change as children grow older. Thus, affective dimensions of learning play a significant role in, and must influence, curriculum and instruction.

ASSUMPTIONS

Several basic assumptions governed the selection and shaping of the K-4 standards.

1. *The K-4 curriculum should be conceptually oriented.*

   The view that the K-4 curriculum should emphasize the development of mathematical understandings and relationships is reflected in the discussions about the content and emphasis of the curriculum. A conceptual approach enables children to acquire clear and stable concepts by constructing meanings in the context of physical situations and allows mathematical abstractions to emerge from empirical experience. A strong conceptual framework also provides anchoring for skill acquisition. Skills can be acquired in ways that make sense to children and in ways that result in more effective learning. A strong emphasis on mathematical concepts and understandings also supports the development of problem solving.

   Emphasizing mathematical concepts and relationships means devoting substantial time to the development of understandings. It also means relating this knowledge to the learning of skills by establishing relationships between the conceptual and procedural aspects of tasks. The time required to build an adequate conceptual base should cause educators to rethink when children are expected to demonstrate a mastery of complex skills. A conceptually oriented curriculum is consistent with the overall curricular goals in this report and can result in programs that are better balanced, more dynamic, and more appropriate to the intellectual needs and abilities of children.

2. *The K-4 curriculum should actively involve children in doing mathematics.*

   Young children are active individuals who construct, modify, and integrate ideas by interacting with the physical world, materials, and other children. Given these facts, it is clear that the learning of mathematics must be an active process. Throughout the Standards, such verbs as *explore, justify, represent, solve, construct, discuss, use, investigate, describe, develop,* and *predict* are used to convey this active physical and mental involvement of children in learning the content of the curriculum.

   The importance of active learning by children has many implications for mathematics education. Teachers need to create an environment that encourages children to explore, develop, test, discuss, and apply ideas. They need to listen carefully to children and to guide the development of their ideas. They need to make extensive and thoughtful use of physical materials to foster the learning of abstract ideas.

   K-4 classrooms need to be equipped with a wide variety of physical materials and supplies.… …Simple household objects, such as buttons, dried beans, shells, egg cartons, and milk cartons, also can be used.

3. *The K-4 curriculum should emphasize the development of children's mathematical thinking and reasoning abilities.*

   An individual's future uses and needs for mathematics make the ability to think, reason, and solve problems a primary goal for the study of mathematics. Thus, the curriculum must take seriously the goal of instilling in students a sense of confidence in their ability to think and communicate mathematically, to solve problems, to demonstrate flexibility in working with mathematical ideas and problems, to make appropriate decisions in selecting strategies and techniques, to recognize familiar mathematical structures in unfamiliar settings, to detect patterns, and to analyze data. The K-*4* standards reflect the view that mathematics instruction should promote these abilities so that students understand that knowledge is empowering and that individual pieces of content are all related to this broader perspective.

   Developing these characteristics in children requires that schools build appropriate reasoning and problem-solving experiences into the curriculum from the outset. Further, this goal needs to influence the way mathematics is taught and the way students encounter and apply mathematics throughout their education.

4. *The K-4 curriculum should emphasize the application of mathematics.*

   If children are to view mathematics as a practical, useful subject, they must understand that it can be applied to a wide variety of real world problems and phenomena. Even though most mathematical ideas in the K-4 curriculum arise *from* the everyday world, they must be regularly applied *to* real-world situations. Children also need to understand that mathematics is an integral part of real-world situations and activities in other curricular areas. The mathematical aspects of that work should be highlighted.

   Learning mathematics has a purpose. At the K-4 level, one major purpose is helping children understand and interpret their world and solve problems that occur in it. Children learn computation to solve problems; they learn to measure because measurement helps them answer questions about how much, how big, how long, and so on; and they learn to collect and organize data because doing so permits them to answer other questions. By applying mathematics, they learn to appreciate the power of mathematics.

5. *The K-4 curriculum should include a broad range of content.*

   To become mathematically literate, students must know more than arithmetic. They must possess a knowledge of such important branches of mathematics as measurement, geometry, statistics, probability, and algebra. These increasingly important and useful branches of mathematics have significant and growing applications in many disciplines and occupations.

   The curriculum at all levels needs to place substantial emphasis on these branches of mathematics. Mathematical ideas grow and expand as children work with them throughout the curriculum. The informal approach at this level establishes the foundation for further study and permits children to acquire additional knowledge they will need. These topics are highly appropriate for young learners because they make important contributions to children's mathematical development and help them see the usefulness of mathematics. They also provide productive, intriguing activities and applications.

The inclusion of a broad range of content in the curriculum also allows children to see the interrelated nature of mathematical knowledge. When teachers take advantage of the opportunity to relate one mathematical idea to others and to other areas of the curriculum, as will be described in Standard 4, children acquire broader notions about the interconnectedness of mathematics and its relationships to other fields. The curriculum should enable all children to do a substantial amount of work in each of these topics at each grade level.

6. *The K-4 curriculum should make appropriate and ongoing use of calculators and computers.*

Calculators must be accepted at the K-4 level as valuable tools for learning mathematics. Calculators enable children to explore number ideas and patterns, to have valuable concept-development experiences, to focus on problem-solving processes, and to investigate realistic applications. The thoughtful use of calculators can increase the quality of the curriculum as well as the quality of children's learning.

Calculators do not replace the need to learn basic facts, to compute mentally, or to do reasonable paper-and-pencil computation. Classroom experience indicates that young children take a common sense view about calculators and recognize the importance of not relying on them when it is more appropriate to compute in other ways. The availability of calculators means, however, that educators must develop a broader view of the various ways computation can be carried out and must place less emphasis on complex paper-and-pencil computation. Calculators also highlight the importance of teaching children to recognize whether computed results are reasonable.

The power of computers also needs to be used in contemporary mathematics programs. Computer languages that are geometric in nature help young children become familiar with important geometric ideas. Computer simulations of mathematical ideas, such as modeling the renaming of numbers, are an important aid in helping children identify the key features of the mathematics. Many software programs provide interesting problem-solving situations and applications.

The thoughtful and creative use of technology can greatly improve both the quality of the curriculum and the quality of children's learning. Integrating calculators and computers into school mathematics programs is critical in meeting the goals of a redefined curriculum.

STANDARD 1
MATHEMATICS AS PROBLEM SOLVING

In grades K-4, the study of mathematics should emphasize problem solving so that students can—

- ***use problem-solving approaches to investigate and understand mathematical content;***
- ***formulate problems from everyday and mathematical situations;***
- ***develop and apply strategies to solve a wide variety of problems;***
- ***verify and interpret results with respect to the original problem;***
- ***acquire confidence in using mathematics meaningfully.***

STANDARD 2
MATHEMATICS AS COMMUNICATION

In grades K-4, the study of mathematics should include numerous opportunities for communication so that students can—

- ***relate physical materials, pictures, and diagrams to mathematical ideas;***
- ***reflect on and clarify their thinking about mathematical ideas and situations;***
- ***relate their everyday language to mathematical language and symbols;***
- ***realize that representing, discussing, reading, writing, and listening to mathematics are a vital part of learning and using mathematics.***

STANDARD 3
MATHEMATICS AS REASONING

In grades K-4, the study of mathematics should emphasize reasoning so that students can—

- ***draw logical conclusions about mathematics;***
- ***use models, known facts, properties, and relationships to explain their thinking;***
- ***justify their answers and solution processes;***
- ***use patterns and relationships to analyze mathematical situations;***
- ***believe that mathematics makes sense.***

STANDARD 4
MATHEMATICAL CONNECTIONS

In grades K-4, the study of mathematics should include opportunities to make connections so that students can—

- ***link conceptual and procedural knowledge;***
- ***relate various representations of concepts or procedures to one another;***
- ***recognize relationships among different topics in mathematics;***
- ***use mathematics in other curriculum areas;***
- ***use mathematics in their daily lives.***

STANDARD 5
ESTIMATION

In grades K-4, the curriculum should include estimation so students

- ***explore estimation strategies;***
- ***recognize when an estimate is appropriate;***
- ***determine the reasonableness of results;***
- ***apply estimation in working with quantities, measurement, computation, and problem solving.***

STANDARD 6
NUMBER SENSE AND NUMERATION

In grades K-4, the mathematics curriculum should include whole number concepts and skills so that students can—

- ***construct number meanings through real-world experiences and the use of physical materials;***
- ***understand our numeration system by relating counting, grouping, and place value concepts;***
- ***develop number sense;***
- ***interpret the multiple uses of numbers encountered in the real world.***

STANDARD 7
CONCEPTS OF WHOLE NUMBER OPERATIONS

In grades K-4, the mathematics curriculum should include concepts of addition, subtraction, multiplication, and division of whole numbers so that students can—

- ***develop meaning for the operations by modeling and discussing a rich variety of problem situations;***
- ***relate the mathematical language and symbolism of operations to problem situations and informal language;***
- ***recognize that a wide variety of problem structures can be represented by a single operation;***
- ***develop operation sense.***

STANDARD 8
WHOLE NUMBER COMPUTATION

In grades K-4, the mathematics curriculum should develop whole number computation so that students can—

- ***model, explain, and develop reasonable proficiency with basic facts and algorithms;***
- ***use a variety of mental computation and estimation techniques;***
- ***use calculators in appropriate computational situations;***
- ***select and use computation techniques appropriate to specific problems and determine whether the results are reasonable.***

STANDARD 9
GEOMETRY AND SPATIAL SENSE

In grades K-4, the mathematics curriculum should include two- and three-dimensional geometry so that students can—

- ***describe, model, draw, and classify shapes;***
- ***investigate and predict the results of combining, subdividing, and changing shapes;***
- ***develop spatial sense;***
- ***relate geometric ideas to number and measurement ideas;***
- ***recognize and appreciate geometry in their world.***

STANDARD 10
MEASUREMENT

In grades K-4, the mathematics curriculum should include measurement so that students can—

- ***understand the attributes of length, capacity, weight, area, volume, time, temperature, and angle;***
- ***develop the process of measuring and concepts related to units of measurement;***
- ***make and use estimates of measurement;***
- ***make and use measurements in problem and everyday situations.***

STANDARD 11
STATISTICS AND PROBABILITY

In grades K-4, the mathematics curriculum should include experiences with data analysis and probability so that students can—

- ***collect, organize, and describe data;***
- ***construct, read, and interpret displays of data;***
- ***formulate and solve problems that involve collecting and analyzing data;***
- ***explore concepts of chance.***

STANDARD 12
FRACTIONS AND DECIMALS

In grades K-4, the mathematics curriculum should include fractions and decimals so that students can—

- ***develop concepts of fractions, mixed numbers, and decimals;***
- ***develop number sense for fractions and decimals;***
- ***use models to relate fractions to decimals and to find equivalent fractions;***
- ***use models to explore operations on fractions and decimals;***
- ***apply fractions and decimals to problem situations.***

STANDARD 13
PATTERNS AND RELATIONSHIPS

In grades K-4, the mathematics curriculum should include the study of patterns and relationships so that student can—

- ***recognize, describe, extend, and create a wide variety of patterns;***
- ***represent and describe mathematical relationships;***
- ***explore the use of variables and open sentences to express relationships.***

The New Vision Of Assessment

ASSUMPTIONS

Six basic assumptions made by the working groups underlie the development of this document.

Assumption 1

Every student is capable of achieving mathematical power.

Assumption 2

Evidence about student mathematical performance is needed for a variety of purposes. However, the type and quality of evidence varies with each purpose and with the consequences for the students related to each purpose.

Assumption 3

For each of these various purposes, information needs to be collected from multiple sources using a variety of methods and formats. There are only three basic sources of such information: observations, student responses to questions, and examinations of student products. However, for each source there are a variety of methods for gathering evidence. At present, there is often too much reliance on a single source or method of assessment. Balance is the use of multiple sources and methods is essential for most purposes.

Assumption 4

All evidence about student performance must be considered as a sample of the possible evidence that could have been gathered. As such, there is considerable potential for error when inferences are drawn from the evidence. The degree of error that can be tolerated depends on the consequences of the inferences. Multiple source of information can be used to provide breadth to the sample of evidence so that more reliable inferences can be made.

Assumption 5

Teachers should be the primary assessors of student performance. No one else is in a better position to judge the development of students' mathematical power than a professional teacher who frequently observes, challenges, and listens to students as they investigate problems. Other assessors (e.g., administrators, state departments of education, test publishers) should also draw heavily on the expertise of professional teachers in their development of methods for evaluating students' mathematical performance.

Assumption 6

During their schooling, students should grow in confidence and in their ability to evaluate their own progress and performance. We challenge all persons interested in the quality of school mathematics to work collaboratively to use these Assessment Standards as a basis for the design of new assessment systems so that the teaching of mathematics in our schools will produce growth in mathematical power for all students.

> In order to develop mathematical power in all students, assessment needs to support the continued mathematics learning of each student. This is the central goal of assessment in school mathematics. When done equitably, assessment of a student's progress will further learning.
>
> Assessment occurs at the intersection of the important mathematics that is taught with how it is taught, what is learned, and how it is learned. It is a dynamic process that continuously yields information about student progress toward the achievement of mathematical power. When the information gathered is consistent with learning goals and is used appropriately to inform teaching, it can enhance learning as well as document it. The process of gathering evidence in order to make inferences about student learning communicates to students and all of those concerned with their learning what is valued in mathematics and how students are progressing to ward specific goals. Assessment also enhances mathematics learning when there is a shared understanding of the learning goals and of the methods used for demonstrating progress toward those goals.

This statement succinctly summarizes the assessment vision of the National Council of Teachers of Mathematics (NCTM) in its efforts to guide reform in the teaching and learning of mathematics. NCTM has produced this document on assessment for two reasons: the first is to present teachers of mathematics with a vision of assessment that is consistent with the earlier Standards documents produced by NCTM-Curriculum and Evaluation Standards for School Mathematics (1989) and Professional Standards for Teaching Mathematics (1991). The second reason is to align this vision of assessment with different educational purposes.

Central to NCTM's efforts is the development of mathematical power. For all students to achieve mathematical power, they need to become mathematical problem solvers, to value mathematics, to reason and communicate mathematically, and to be confident in using mathematics to make sense of real-world problem situations. The mathematics curriculum, the instructional methods, and the strategies used to assess student performance must be congruent with this notion of mathematical power.

To reach these goals for students, the Curriculum Standards present a vision of the mathematical content that the mathematical sciences education community believes students should have the opportunity to learn. The Evaluation Standards describe the general features of an evaluation system that complement the content vision. And the Teaching Standards portray a vision of an instructional environment and pedagogy related to that content.

Assessment is the process of gathering evidence about a student's knowledge of, ability to use, and disposition towards mathematics and of making inferences based on that evidence for a variety of purposes. Mathematics assessors, primarily teachers, should be proficient in developing assessment plans for specific educational purposes, in gathering such evidence, and in using that evidence to facilitate the growth of mathematical power for all students. Additionally, assessors should assist students to grow in confidence and in their ability to evaluate their own progress and performance.

The six Assessment Standards presented in the second section of this document are, as the term "standard" implies, offered as criteria to judge the appropriateness of assessment activities. Criteria for evaluating appropriate assessment activities are needed for at least three reasons. First, we are convinced that current commonly used assessment instruments (norm-references standardized tests, textbook tests, state and national profile examinations) and inferences based on their use would fail miserably when judged against these standards. Second, teachers and others need guidance and models of assessment to inform their practice. Third, the standards are tools that can, and should, be used to develop new effective assessment systems.

Standard 1
Important Mathematics

Assessment should reflect the mathematics that is most important for students to learn.

Most items on current tests are both uninteresting and superficial. What is needed are tasks that capture both what students know and what they are able to do with respect to "big" mathematical ideas.

Standard 2
Enhanced Learning

Assessment should enhance mathematics learning.

Current tests are purely asssessment tools, not teaching tools. What is needed are assessment practices that connect assessment with instruction.

Standard 3
Equity

Assessment should promote equity by giving each student optimal opportunities to demonstrate mathematical power and by helping each student meet the profession's high expectations.

Current procedures ignore differences in students' experiences and backgrounds and are used to limit students' access to important mathematics. What is needed are practices that support each student's learning.

Standard 4
Openness

All aspects of the mathematics assessment process should be open to review and scrutiny.

Currently, students, parents, teachers, and other stakeholders are often not informed about expectations, the way in which performance will be judged, or the consequences of assessment. What is needed are shared understandings of goals and assessment practices among students, teachers, and the public.

Standard 5
Valid Inferences

Evidence from assessment activities should yield valid inferences about students' mathematical learning.

Currently, too many inferences are based on superficial evidence about a student's mathematical power. What is needed is a balance of evidence from multiple sources to capture a broad picture of a student's in-depth knowledge and ability to do mathematics.

Standard 6
Consistency

Every aspect of an assessment process should be consistent with the purposes of assessment.

Currently, instruments developed for one purpose are used inappropriately and create invalid inferences with respect to other purposes. It is essential that assessors be vigilant about the role of evidence for specific purposes. Also, because this final standard focuses on the importance of the interrelationships among purpose, evidence, inferences, and consequences, it serves as a transition to the third section of the document (Use of Assessment Standards for Different Purposes).

These standards are a "vision for the future." They reflect the values and goals associated with an assessment system that must be achieved if the reforms envisioned in the teaching and learning of mathematics are to become reality. We challenge teachers, school district staff, and state or province education leaders to read and reflect on this vision; critically examine their current assessment systems; and then work to develop new assessment systems that meet these six Assessment Standards. The teaching and learning of mathematics, as expressed in the Curriculum and Teaching Standards, depend upon the development of assessment systems based on the values and goals reflected in these Assessment Standards.